Cooperative Connectivity Models and Bounds for Wireless Relay Networks

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Abstract

Cooperative communication is a recent paradigm for wireless relay networks wherein multiple wireless terminals share their resources to jointly transmit information of which they are not the source in order to achieve an improvement in overall performance. This is realized via the application of cooperative diversity techniques that take advantage of the spatial diversity offered by cooperation between wireless terminals. This dissertation addresses wireless relay networks with an arbitrary number of cooperating wireless terminals, arbitrary sets of communication links between pairs of cooperating terminals, and arbitrary numbers of antennas at each cooperating terminal.

The starting point for any analysis of wireless relay networks is the development of a system model that allows comparison of key performance characteristics such as signal to noise ratio, probability of error, and probability of outage. This dissertation develops a general system model for arbitrarily connected wireless relay networks. Since increased spatial diversity is one of the key benefits expected of cooperative connectivity it is especially important to understand the bounds on the maximum achievable diversity order of wireless relay networks with arbitrary cooperative connectivity. This dissertation develops upper bounds on the maximum diversity order for arbitrarily connected wireless relay networks. Each of the cooperative diversity techniques presented in the literature places different requirements on the system resources that must be available, and therefore system resource constraints that limit the connectivity of individual terminals also constrain the cooperative diversity techniques that can be applied. This dissertation develops a framework for modeling cooperative connectivity that exposes the relationship between available system resources and achievable cooperative connectivity.

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List of Acronyms

General Terminology

AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
CDMA	Code-Division Multiple Access
DSP	Digital Signal Processing
FDMA	Frequency-Division Multiple Access
ISI	Inter-Symbol Interference
LOS	Line-of-Sight
MRC	Maximal Ratio Combining
SER	Symbol Error Rate
SIR	Signal-to-Interference Ratio
SINR	Signal-to-Interference-plus-Noise Ratio
SNR	Signal-to-Noise Ratio
TDMA	Time-Division Multiple Access

System Resource Constraint Terminology

NCA	N Channels Available
KCA	K Channels Available
2CA	2 Channels Available
RCC	Relay Common Channel Combination
NRCC	No Relay Common Channel Combination
DCC	Destination Common Channel Combination

NDCC	No Destination Common Channel Combination
ROC	Relay Orthogonal Channel Combination
NROC	No Relay Orthogonal Channel Combination
DOC	Destination Orthogonal Channel Combination
NDOC	No Destination Orthogonal Channel Combination
MCT	Multiple Channel Transmission
NMCT	No Multiple Channel Transmission
IIC	Interhop Interference Cancellation
NIIC	No Interhop Interference Cancellation

Cooperative Connectivity Model Terminology

1R	Single Relay Connectivity
KR	*K* Channel Relay Connectivity
CR	Common Channel Relay Connectivity
NR	Non-Identical Relay Connectivity
FR	Full Relay Connectivity
1D	Single Destination Connectivity
KD	*K* Channel Destination Connectivity
CD	Common Channel Destination Connectivity
ND	Non-Identical Destination Connectivity
FD	Full Destination Connectivity
KH	*K* Hop Network Connectivity
FH	Full Hop Network Connectivity

List of Symbols

$a_{k,i}$ — the composite signal amplitude attenuation factor associated with the link between terminal T_k and terminal T_i

$a_{kl,ij}$ — the composite signal amplitude attenuation factor associated with the link between the l^{th} antenna of terminal T_k and the j^{th} antenna of terminal T_i

A_i — the set of antennas of terminal T_i

A_{ij} — the j^{th} antenna of terminal T_i

d — the diversity order of a network

$d_{\max}$ — the maximum diversity order of a network

$d_{k,i}$ — the distance between terminal T_k and terminal T_i normalized with respect to the reference distance

$E[x]$ — the expectation of x

F_S — the minimum number of degrees of freedom in the channel across all cut sets

F_T — the minimum number of degrees of freedom in the channel across all terminals

G_i — the amplification gain factor of terminal T_i

I_i — the mutual information at terminal T_i or across cut set S_i

K — the number of orthogonal channels required to operate a given transmission scheme

L_i the set of inter-terminal antenna links associated with terminal T_i or cut set S_i

$L_{k,i}$ the inter-terminal link between terminal T_k and terminal T_i (when there is only one antenna per terminal)

$L_{kl,ij}$ the inter-terminal antenna link between the l^{th} antenna of terminal T_k and the j^{th} antenna of terminal T_i

L_R the complete set of inter-terminal antenna links associated with the directed network graph

M_S the minimum number of incident inter-terminal antenna links across all cut sets

M_T the minimum number of incident inter-terminal antenna links across all terminals

$N_{k,i}$ the noise power spectral density associated with the link between terminal T_k and terminal T_i (when there is only one antenna per terminal)

$N_{kl,ij}$ the noise power spectral density associated with the link between the l^{th} antenna of terminal T_k and the j^{th} antenna of terminal T_i

N_0 the reference noise power spectral density

$O(g(n))$ the asymptotic complexity of a function

p the signal propagation exponent

P_e the total probability of error

$P_{e,i}$ the probability of error of terminal T_i

$P_e(\gamma)$	the probability of error given an SNR of γ
P_o	the total probability of outage
$P_{o,i}$	the probability of outage of terminal T_i
$P_o(\gamma, R)$	the probability of outage given an SNR of γ and a target rate of R
$Q(x)$	the Q function of *x*, related to the error function erfc(x)
r	the multiplexing gain of a network
$r_{\max}$	the maximum multiplexing gain of a network
$r_{k,i}$	the received complex baseband signal amplitude associated with the link between terminal T_i and terminal T_k (when there is only one antenna per terminal)
$r_{kl,ij}$	the received complex baseband signal amplitude associated with the link between the l^{th} antenna of terminal T_k and the j^{th} antenna of terminal T_i
$R_{k,i}$	the Rayleigh random variable associated with the link between terminal T_i and terminal T_k (when there is only one antenna per terminal)
$R_{kl,ij}$	the Rayleigh random variable associated with the link between the l^{th} antenna of terminal T_k and the j^{th} antenna of terminal T_i
$\underline{R}_{\gamma}$	the covariance matrix of the signal to noise ratio $\gamma_{P(i),i}$
s_i	the transmitted complex baseband signal amplitude of terminal T_i (when there is only one antenna per terminal)

s_{ij} the transmitted complex baseband signal amplitude of the j^{th} antenna of terminal T_i

S_i the identifier of the ith cut set

$S_{O(m)}$ the set of cut sets that are associated with m incident inter-terminal antenna links

S_R the complete set of distinct cut sets associated with the directed network graph

T_i the identifier of the ith terminal

T_A the set of all terminals

T_C the set of terminals that decode correctly

$T_{C(i)}$ the subset of terminals of $T_{P(i)}$ that decode correctly

T_D the set of destination terminals

T_E the set of terminals that decode incorrectly

$T_{E(i)}$ the subset of terminals of $T_{P(i)}$ that decode incorrectly

T_I the set of intermediate (relay) terminals

$T_{IN(i)}$ the set of terminals on the input side of cut set S_i that are associated with at least one of the inter-terminal links in the corresponding set L_i

$T_{OUT(i)}$ the set of terminals on the output side of cut set S_i that are associated with at least one of the inter-terminal links in the corresponding set L_i

$T_{O(m)}$ the set of receiving terminals that are associated with m incident inter-terminal antenna links

$T_{P(i)}$	the set of terminals that transmit a signal directly received by terminal T_i
T_R	the set of receiving terminals
T_S	the set of source terminals
T_T	the set of transmitting terminals
$T_{U(i)}$	the set of transmitting terminals that are not a member of $T_{P(i)}$
W	the bandwidth of the communication channel
$\bar{x}$	the expected value of x
$\tilde{x}$	the instantaneous value of x
$\lvert x \rvert$	the absolute value of scalar x
$\lvert X \rvert$	the cardinality of set X
$\underline{X}^{\mathrm{H}}$	the Hermitian transpose of matrix $\underline{X}$
$z_{k,i}$	the additive white Gaussian noise random variable associated with the link between terminal T_k and terminal T_i (when there is only one antenna per terminal)
$z_{kl,ij}$	the additive white Gaussian noise random variable associated with the link between the l^{th} antenna of terminal T_k and the j^{th} antenna of terminal T_i
α_i	the complex amplitude of the information symbol transmitted by terminal T_i over a given signaling interval
β_i	the complex amplitude of the propagated noise transmitted by terminal T_i over a given signaling interval

δ^2	the free space signal power attenuation factor between a transmitting terminal and an arbitrary reference distance
ε_0	the reference total power constraint
ε_i	the transmitted power of terminal T_i
γ	the threshold signal to noise ratio
$\gamma_{k,i}$	the aggregate signal to noise ratio received at terminal T_i as a result of the signal from terminal T_k (when there is only one antenna per terminal)
$\gamma_{kl,ij}$	the aggregate signal to noise ratio received at the j^{th} antenna of terminal T_i as a results of the signal from the l^{th} antenna of terminal T_k
$\gamma_{P(i),i}$	the aggregate signal to noise ratio received at terminal T_i as a result of the signals from all the terminals in set $T_{P(i)}$
κ	the rank of a MIMO channel matrix
κ_i	the rank of the channel matrix associated with cut set S_i
$\{\lambda_k\}$	the eigenvalues of $\underline{\Gamma}_\gamma$
$\mu_{k,i}$	the scaling factor for the SNR of the link between terminal T_k and terminal T_i with respect to the reference SNR ε_0/N_0 (when there is only one antenna per terminal)
$\mu_{kl,ij}$	the scaling factor for the SNR of the link between the l^{th} antenna of terminal T_k and the j^{th} antenna of terminal T_i with respect to the reference SNR ε_0/N_0

σ_x^2	the variance of random variable x
$\underline{\Gamma}_\gamma$	the positive definite matrix related to the covariance matrix $\underline{R}_\gamma$
$\psi_{k,i}$	the link signal to noise ratio received at terminal T_i as a result of the signal from terminal T_k (when $\beta_k = 0$ and there is only one antenna per terminal)
$\psi_{kl,ij}$	the link signal to noise ratio received at the j^{th} antenna of terminal T_i as a results of the signal from the l^{th} antenna of terminal T_k (when $\beta_k = 0$)
$\psi_{P(i),i}$	the link signal to noise ratio received at terminal T_i as a result of the signals from all the terminals in set $T_{P(i)}$ (when $\beta_k = 0, \forall T_k \in T_{P(i)}$)

Chapter 1 - Introduction

Cooperative communication is a recent paradigm for wireless relay networks wherein multiple wireless terminals share their resources to jointly transmit information of which they are not the source in order to achieve an improvement in overall performance. This is realized via the application of cooperative diversity techniques that take advantage of the spatial diversity offered by cooperation between wireless terminals. This manuscript is a doctoral dissertation in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Electrical Engineering at Carleton University, and develops cooperative connectivity models and bounds for wireless relay networks with an arbitrary number of cooperating wireless terminals and arbitrary sets of communication links between pairs of cooperating terminals.

In the context of a given cooperative communication exchange, a wireless relay network is generally composed of a source terminal, a destination terminal, and a variable number of intermediate relay terminals. It is the participation of these relay terminals in the communication exchange, which forward information of which they are neither the source nor destination, that classifies it as being cooperative. A pair of terminals is defined to be connected to each other whenever there is a direct communication link between them that is used during the transmission of an information signal from a source terminal to a destination terminal. The cooperative connectivity of a wireless relay network is defined as the set of such communication links between all pairs of connected terminals. Relay terminals are defined to cooperate with a source terminal whenever they transmit a signal that helps the corresponding destination terminal to successfully decode the original information signal.

1.1 Motivation

Recently, cooperation between wireless terminals has been proposed in cellular and ad-hoc networks as a method of achieving the benefits of spatial diversity without requiring the use of physical antenna arrays. These cooperative diversity techniques involve the sharing of information on inter-terminal channels such that each user sends information using multiple terminals. In general, it has been shown that the capacity and outage performance of wireless relay networks can be increased through the application of cooperative techniques that leverage cooperation between wireless terminals, and that these techniques can therefore be used to achieve higher rates and lower probabilities of error.

However, the results presented up to this point have for the most part been focused on very simple networks of terminals, for example the traditional three-terminal relay channel [73], or straightforward extensions such as multiple relays in parallel with diversity combination only at the destination [5], or multiple relays in serial with full (complete) connectivity [15]. Even works that have attempted to address more general cooperation strategies have relied on simplifying assumptions for how terminals are connected to each other, for example many multihop paths in parallel [91], or multiple relay tiers with full intra-tier connectivity [40]. Furthermore, there has been minimal consideration of wireless relay networks where some or all of the cooperating terminals employ multiple antennas, and a wide and inconsistent range of assumptions with respect to the wireless terminal hardware and channel resources required to operate the different cooperative diversity techniques.

There is a need for more general results that address arbitrarily connected wireless relay networks – that is, wireless relay networks with an arbitrary number of cooperating terminals, an arbitrary set of communication links between pairs of cooperating terminals, and an arbitrary number of antennas at each terminal. This dissertation addresses arbitrary cooperative connectivity between cooperating terminals in a wireless relay network. In fact, the analysis presented in this dissertation considers the most general case and can therefore be used to contrast and compare all of the simpler models considered thus far.

The starting point for any analysis of arbitrarily connected wireless relay networks is the development of a system model that allows comparison of key performance characteristics such as signal to noise ratio, probability of error, and probability of outage. This dissertation develops a general system model for arbitrarily connected wireless relay networks that incorporates important wireless channel environmental factors such as multipath propagation, distance dependent path loss, shadowing, fading, receiver noise, and other forms of interference. Furthermore, performance characterizations such as the aggregate signal to noise ratio, probability of error, and probability of outage are developed for networks employing various relaying methods, codebook generation schemes, and numbers of antennas per terminal.

Since increased spatial diversity is one of the key benefits expected of cooperative connectivity it is especially important to understand the bounds on the maximum achievable diversity order of wireless relay networks with arbitrary, but generally less than full, cooperative connectivity. This dissertation determines upper bounds on the maximum diversity order for arbitrarily connected wireless relay networks with two classes of relaying method (comprehensive or destination decoding), and two codebook generation schemes (common or independent codebook generation). Although previous work has derived the diversity order for specific combinations of cooperative connectivity and relaying method, this dissertation develops a more general formulation applicable to arbitrarily connected cooperative networks with any number of relays. Furthermore, this dissertation presents some related bounds for the more general diversity-multiplexing tradeoff formulation.

Each of the cooperative diversity techniques presented in the literature places different requirements on the wireless terminal hardware capabilities, channel availability, and multiple access schemes used to implement mesh connectivity between terminals, and thus places different requirements on the system resources that must be available. Therefore, system resource constraints that limit the connectivity of individual terminals constrain the cooperative diversity techniques that can be applied, and the cooperative connectivity needed to support each technique can often be achieved with different combinations of system resources. This dissertation develops a framework for modeling cooperative connectivity that exposes the relationship between available system resources and achievable cooperative connectivity, considers the cost and performance benefit of different possibilities for cooperative connectivity, and therefore supports qualitative analysis of which possibilities for cooperative connectivity are promising targets for further investigation and which are not. Additionally, the developed framework allows comparison of the system resource requirements and assumptions of the different cooperative diversity techniques proposed in the literature, and analysis of the efficiency, with respect to the minimum cost constraint sets, of the assumed available system resources.

1.2 Motivating Example

In order to further highlight the motivation for this dissertation it is helpful to consider the example wireless relay network shown in Fig. 1. This example network represents a particular instant in time within some larger and more enduring system, and is composed of source terminal T_S, intermediate relay terminals T_{R1}, T_{R2}, T_{R3}, T_{R4}, T_{R5}, and destination terminal T_D, with arrows indicating the directed transmission links between the terminals. This example network has irregular connectivity between terminals, perhaps due to significant macroscopic shadowing between terminals, wireless terminal hardware capabilities, channel availability, or other environmental or system factors. The existing literature does not provide results for this kind of arbitrarily connected wireless relay network, which in fact may occur quite often in practice for ad-hoc or other multihop wireless relay networks exposed to common wireless channel impairments.

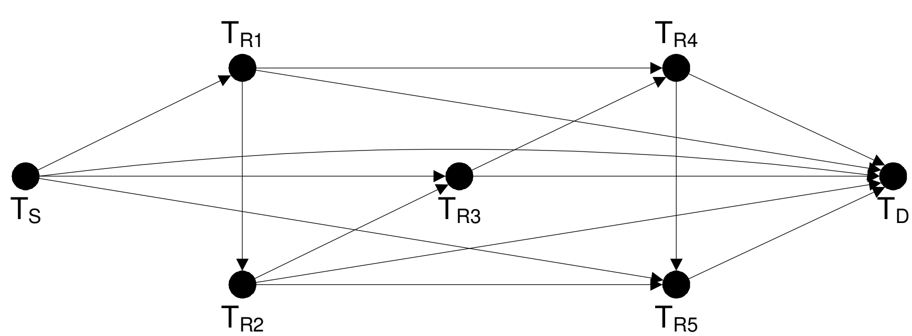

Fig. 1. Example Wireless Relay Network

There are various performance characteristics of this example wireless relay network that might be of interest, including signal to noise ratio, probability of error, probability of outage, and the maximum achievable diversity order. For example, an initial intuitive response might be that the diversity order of this example network is six, because that is how many inter-terminal links are received by the destination terminal. However, we will show that this response is incorrect, because it does not take into account the correlation between the inter-terminal links received by the destination terminal. Furthermore, these results depend on the relaying method and coding scheme employed, and will in general be different from the optimal performance achievable over this composite communication channel. The situation becomes even more complex when some terminals may have more than one physical antenna.

Also of significant interest are the wireless terminal hardware capabilities, channel availability, and multiple access schemes required to achieve the connectivity between terminals shown in this example network. There may be different combinations of system resources that could be used to achieve this example connectivity, but it is not immediately obvious what the combinations are, nor which combination would be considered optimal from the perspective of system cost. Perhaps it would be possible to use a transmission scheme with slightly less connectivity between terminals with a significantly reduced system cost, but it is not clear what the performance impact of that choice would be. The results developed in this dissertation allow us to address these issues in a consistent fashion.

1.3 Overview

This dissertation is composed of three primary areas of research: development of a general system model for arbitrarily connected wireless relay networks, analysis of upper bounds on the maximum diversity order and diversity-multiplexing tradeoff for arbitrarily connected wireless relay networks, and development of a framework for modeling cooperative connectivity that exposes the relationship between constraints on available system resources and the achievable combinations of communication links between cooperating terminals.

The developed system model for arbitrarily connected wireless relay networks incorporates multipath propagation, distance dependent path loss, shadowing, fading, Gaussian noise, and other forms of interference. The probability of error and information theoretic probability of outage are developed for amplified relaying, also known as non-regenerative or amplify-and-forward relaying, decoded relaying with error propagation, also known as regenerative or fixed decode-and-forward relaying, and decoded relaying without error propagation, also known as adaptive or selective decode-and-forward relaying. Additionally, the aggregate signal to noise ratio of relay networks employing amplified relaying is developed as an intermediate result.

The maximum diversity order analysis derives limits on the maximum achievable diversity order and diversity-multiplexing tradeoff of wireless relay networks with arbitrary link connectivity between cooperating terminals. Two classes of relaying method are considered, those requiring all cooperating terminals to correctly decode the transmitted information signal in order for the destination terminal to correctly decode (comprehensive decoding) and those requiring only a subset of cooperating terminals to correctly decode the transmitted information signal in order for the destination to correctly decode (destination decoding). The first class includes decoded relaying with error propagation, and the second class includes decoded relaying without error propagation and amplified relaying. Two general schemes for how codebooks are generated by and partitioned among terminals in the relay network are considered and compared: common codebook generation and independent codebook generation.

The system resource constraints considered in the cooperative connectivity modeling framework are the available number of orthogonal relaying channels, the ability of terminals to diversity combine signals on a single common channel, the ability of terminals to diversity combine signals on orthogonal channels, the ability of terminals to transmit signals on multiple orthogonal channels, and the ability of terminals to cancel the effects of interhop interference. Cooperative connectivity models defined by the achievable combinations of links are derived, associated with their minimum cost constraint sets, and mapped to diversity techniques presented in the literature. Simulations are provided that illustrate the value of the framework as a modeling tool by comparing the performance impact of the constraints for some example network topologies and relaying methods.

1.4 Contributions

This dissertation addresses arbitrary connectivity between cooperating terminals in wireless relay networks. The key high-level contributions of this dissertation are the following:

- Development of a general system model for arbitrarily connected wireless relay networks that derives important performance characterizations such as the aggregate signal to noise ratio, probability of error, and probability of outage and incorporates important wireless channel environmental factors such as multipath propagation, distance dependent path loss, shadowing, fading, Gaussian noise, and other forms of interference.

- Development of upper bounds on the maximum achievable diversity order, high SNR probability of outage, and diversity-multiplexing tradeoff for arbitrarily connected wireless relay networks with comprehensive or destination decoding, and common or independent codebook generation.
- Development of a framework for modeling cooperative connectivity that exposes the complex relationships between available system resources and achievable cooperative connectivity and allows comparison of the system resource requirements and assumptions of the different cooperative diversity techniques proposed in the literature.

1.5 Published, Submitted, and Proposed Manuscripts

A number of published and submitted manuscripts have been completed thus far from the research contained in this dissertation, and a number of additional proposed manuscripts are in progress for future submission. Following is a summary of these manuscripts with their publication status at the time of writing, including the reference numbers in this dissertation for published and submitted manuscripts:

- J. Boyer, D. D. Falconer, and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. on Communications*, vol. 52, no. 10, pp. 1820-1830, October 2004. This manuscript is [15] in the list of references, and contains results that were partially produced in fulfillment of a previous Master's thesis [13] and partially produced as part of the current dissertation. The novel portion of the work relevant to the current dissertation is an enhanced and refined system model that appears in Chapter 3 of this dissertation.

- J. Boyer, D. D. Falconer, and H. Yanikomeroglu, "On the aggregate SNR of amplified relaying channels," Proc. of *IEEE Global Telecommunications Conference*, vol. 5, pp. 3394-3398, November 2004. This manuscript is [16] in the list of references, and contains results that correspond to the aggregate signal to noise ratio derivation that appears in Appendix A and underlies part of the system model that appears in Chapter 3 of this dissertation.
- J. Boyer, D. D. Falconer, and H. Yanikomeroglu, "On the impact of system resource constraints on wireless relaying channels," Proc. of *IEEE International Conference on Communications*, vol. 5, pp. 3266-3270, May 2005. This manuscript is [17] in the list of references, and contains preliminary results, later refined, that correspond to the cooperative connectivity modeling framework that appears in Chapter 5 of this dissertation.
- J. Boyer, D. Falconer, and H. Yanikomeroglu, "Cooperative connectivity models for wireless relay networks," *IEEE Trans. on Wireless Communications*, vol. 6, no. 6, pp. 1992-2000, June 2007. This manuscript is [18] in the list of references, and contains results that correspond to the cooperative connectivity modeling framework that appears in Chapter 5 of this dissertation.
- J. Boyer, D. Falconer, and H. Yanikomeroglu, "Diversity order bounds for wireless relay networks," Proc. of *IEEE Wireless Communications and Networking Conference*, pp. 1800-1804, March 2007. This manuscript is [19] in the list of references, and contains preliminary results, later refined, that correspond to the maximum diversity order analysis that appears in Chapter 4 of this dissertation.

- J. Boyer, D. Falconer, and H. Yanikomeroglu, "Diversity order bounds for arbitrarily connected wireless relay networks," submitted to *IEEE Trans. on Wireless Communications*, 2007. This manuscript is [20] in the list of references, and contains results that correspond to the maximum diversity order analysis that appears in Chapter 4 of this dissertation.
- J. Boyer, D. Falconer, and H. Yanikomeroglu, "Bounds on the diversity-multiplexing tradeoff of arbitrarily connected wireless relay networks," in progress. This manuscript will contain results that correspond to the diversity-multiplexing analysis that appears in Chapter 4 of this dissertation, along with some more recent refinements.
- J. Boyer, D. Falconer, and H.Yanikomeroglu, "A comparison of cooperative connectivity models for wireless relay networks," in progress. This manuscript will contain results that correspond to the simulations, comparative results, and qualitative discussion that appears in Chapter 6 of this dissertation.

1.6 Organization

The remainder of this dissertation is organized as follows. Chapter 2 summarizes the contributions and important literature relevant to the problems studied in this dissertation and provides background on many of the related issues and considerations. Chapter 3 presents the developed general system model for arbitrarily connected wireless relay networks and derives the probability of error and information theoretic probability of outage, along with intermediate results, for decoded relaying with error propagation, decoded relaying without error propagation, and amplified relay. Chapter 4 presents the developed maximum diversity order analysis for arbitrarily connected wireless relay networks with comprehensive or destination decoding, and common or independent codebook generation. Chapter 5 presents the developed cooperative connectivity modeling framework that exposes the complex relationships between available system resources and achievable cooperative connectivity. Chapter 6 applies the cooperative connectivity modeling framework in a series of simulations that provide a comparison with respect to probability of error and information theoretic probability of outage for various network topologies and allow the performance impact of the individual system resource constraints to be isolated. Chapter 7 provides some concluding remarks, wherein the dissertation is summarized, the contributions and main qualitative results are described, and a number of areas for future research are identified.

Chapter 2 - Background and Literature Review

This chapter summarizes the contributions of important literature relevant to the problems studied in this dissertation and provides background on many of the related issues and considerations. Section 2.1 summarizes the significant characteristics of the wireless environment, describing a fairly rich wireless channel model and discussing the relevant constraints on wireless hardware in both infrastructure and ad hoc networks. Section 2.2 summarizes the information theoretic work on relay channels, introducing the concept of channel capacity in fading environments and describing capacity related results for classical channel configurations including the classical relay channel and multiple terminal networks. Section 2.3 summarizes some relevant work on diversity and multiple antenna systems, introducing the fundamental diversity techniques, describing capacity and performance related results for multiple antenna systems, and discussing the more recent concept of the diversity-multiplexing tradeoff. Section 2.4 summarizes the current state of work on cooperative diversity, introducing the classical two-user cooperative diversity model and discussing various extensions in the areas of cooperative coding, non-orthogonal signaling, multiple hops, and relay networks.

2.1 Wireless Environment

This section summarizes the significant characteristics of the wireless environment, describing a fairly rich wireless channel model and discussing the relevant constraints on wireless hardware in both infrastructure and ad hoc networks. It is these environmental factors and implementation constraints that result in the relatively high error and outage rates of wireless systems and make diversity techniques in general, and cooperative diversity techniques in particular, so appealing.

2.1.1 Wireless Channel Model

The wireless channel model used throughout this dissertation is the traditional one described in [86], [90], and [96] that experiences channel effects such as distance dependent path loss, fading due to multipath propagation, and additive noise. Using standard conceptual techniques, the continuous-time bandwidth-limited information transmissions and channel effects are modeled as discrete-time baseband signals transmitted over discrete-time baseband channels. The effects of distance dependent path loss and fading are modeled using the convolution of the discrete-time signals with time-varying discrete-time linear filters. The fading coefficients are modeled as zero-mean complex Gaussian (Rayleigh) random variables and the additive noise coefficients are modeled as zero-mean Guassian random variables that capture the combined effects of local thermal noise and other interference.

Although this Rayleigh fading model is chosen for its simplicity and wide acceptance, the techniques and results of this dissertation are equally applicable to other fading models. In general, it is assumed that the links between different sets of mobile terminals experience mutually independent attenuation, fading, and noise, although some results are presented later in the dissertation for correlated fading environments. The focus is on frequency non-selective (within the context of the transmitted signal of a given terminal) time non-selective (within the context of the transmissions of a given information block) wireless channels since they are relatively common in wireless environments and clearly highlight the benefits and tradeoffs of the described cooperative diversity techniques. For an extensive summary of performance and information theoretic results for general wireless fading channels we refer the reader to [9] and [101].

2.1.2 Wireless Hardware Constraints

Wireless terminal hardware in general suffers from a number of practical constraints such as half-duplex operation, limited channel state information, limited antenna physical geography, average and peak power constraints, and bandwidth limitations. These constraints result from a combination of physical implementation complexity and cost and regulatory compliance requirements. Half-duplex operation results from practical limitations in current signal processing capabilities that are unable to mitigate for the fact that since transmitted power is generally many orders of magnitude greater than received power, full-duplex operation would result in so much feedback that the received signal may be completely obscured. The non-symmetric and dynamically changing nature of the wireless environment means that it is generally impractical to assume channel state information at transmitters, although channel state information at receivers can be leveraged when the time coherence of the channel is relatively large (time non-selective channels).

The small size of mobile wireless terminals generally limits the use of multiple antennas due to channel correlation, although fixed wireless terminals may not experience these same constraints. Average and peak power constraints are imposed by the limited battery capacity of mobile wireless terminals and regulatory limitations for both mobile and fixed wireless terminals. Limited bandwidth availability may result from regulatory limitations and physical hardware implementation cost. The combination of these hardware constraints, a challenging wireless environment, and increasing demand for ubiquitous high-rate communication motivates the introduction of the novel wireless system architectures considered in this dissertation.

2.2 Relay Channels

This section summarizes the information theoretic work on relay channels, introducing the concept of channel capacity in fading environments and describing capacity related results for the classical relay channel, various extensions to the classical relay channel, and multiple terminal networks.

2.2.1 Channel Capacity and Information Theoretic Outage Probability

The concept of Shannon capacity [100] is a measure of the maximum achievable rate of reliable communication over a channel, wherein the probability of decoding error approaches zero asymptotically with codeword length. This is an important metric as it allows comparison of different channels without dependency on any specific encoding scheme, as well as comparison of practical encoding schemes with the fundamental performance bounds of the corresponding channel. However, Shannon capacity is less relevant in non-ergodic or delay limited channels where asymptotically long codewords are either not beneficial or not possible. To address this issue the concept of capacity-versus-outage probability was introduced in [83]. In this formulation, the outage probability is the probability that the instantaneous rate (mutual information) of the channel falls below a given fixed rate threshold.

The probability that the mutual information falls below a given rate threshold is defined as:

$$P_o(\gamma, R) = \Pr[I(\gamma) < R],$$

where γ is the signal to noise ratio resulting in the mutual information $I(\gamma)$, and R is a fixed rate threshold. Without loss of generality, if we assume a randomly generated iid circularly symmetric, complex Gaussian codebook, then the mutual information is of the form $I(\gamma) = \log(1+\gamma)$ and the probability of outage is given by

$$P_o(\gamma, R) = \Pr\left[\gamma < 2^R - 1\right].$$

This probability of outage can be solved by integrated over all fading realizations of the signal to noise ratio γ that result in a rate less than the target rate R. A closed form solution for this probability of outage formulation can be found for many probability distributions of interest, including the exponential distributions that result from the Rayleigh fading coefficients of the wireless channel model used in this dissertation. This formulation for outage probability was first applied to cooperative diversity channels in [64] and has been reused in much subsequent work. This dissertation applies this information theoretic probability of outage formulation repeatedly during the development of the general system model, maximum diversity order analysis, and cooperative connectivity model simulations for arbitrarily connected wireless relay networks.

2.2.2 Classical Channel Configurations

The classical three-terminal relay channel was first introduced in [73], and is shown in Fig. 2. Lower and upper bounds on the capacity of this relay channel with additive white Gaussian noise were developed in [25] using random coding arguments, and three random coding schemes for the relay channel were introduced, generally known as facilitation, cooperation, and compression (or observation). In the facilitation scheme the relay generates as little interference as possible. In the cooperation scheme the relay fully decodes the source message and retransmits some information to the destination. In the compression scheme the relay retransmits a compressed or quantized version on the source message to the destination.

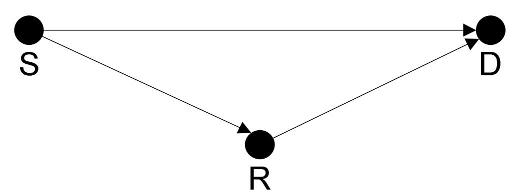

Fig. 2. Relay Channel

A slightly more symmetric version of the classical relay channel, the parallel relay channel shown in Fig. 3, was introduced in [95] along with a number of corresponding coding techniques.

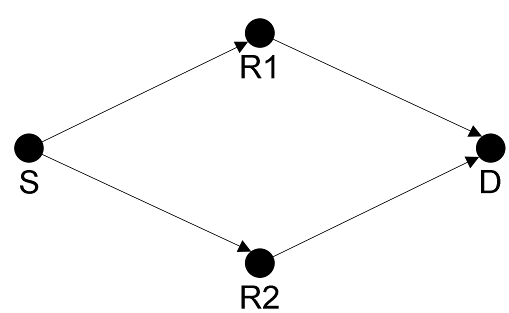

Fig. 3. Parallel Relay Channel

A number of related results have been developed in the literature for well-know classical channel configurations. The broadcast channel, shown Fig. 4 for two destinations, involves a single source transmitting concurrently to multiple destinations.

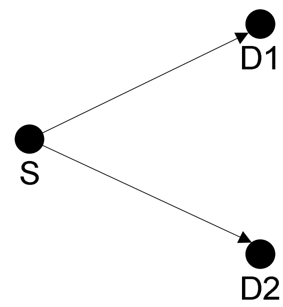

Fig. 4. Broadcast Channel

The multiple access channel, shown in Fig. 5 for two sources, involves multiple sources transmitting concurrently to a single destination.

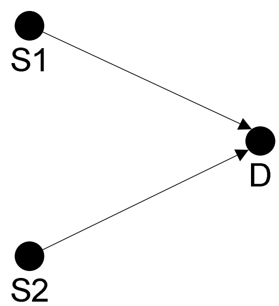

Fig. 5. Multiple Access Channel

The interference channel, shown in Fig. 6 for two source-destination pairs, involves multiple sources transmitting concurrently to multiple destinations (in pairs), but mutually interfering with each other.

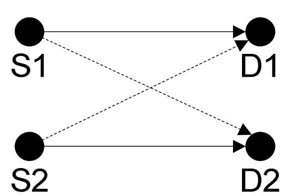

Fig. 6. Interference Channel

For example, information theoretic results have been provided for the broadcast channel in [24] and the multiple-access channel with generalized feedback in [113], [114], and

[115]. Additionally, the relay channel capacity results of [25] were extended in [49] to the case where terminals are not able to transmit and receive on the same channel but operate in half-duplex mode, a constraint which is also assumed throughout this dissertation.

As noted in [64] and other publications, in the relay channel and other classical channel configurations we see the basis for the more recent work on cooperative diversity initiated in [97], as well as the more complex wireless relay networks considered in this dissertation. For example, the cooperation scheme of [25] motivates the decode-and-forward cooperative diversity protocol introduced in [64] for the case where the retransmission is the entire source message, and the compression scheme of [25] motivates the amplify-and-forward cooperative diversity protocol introduced in [64] for the case where the retransmission is the entire source message. The basic cooperative diversity channel shown in Fig. 7 is effectively the combination of two classical relay channels, or alternatively the classical interference channel with cooperation between sources. It is interesting to note that the exact channel capacity is unknown even for the classical relay channel without fading, although various upper and lower bounds such as those provided in the references above have been developed.

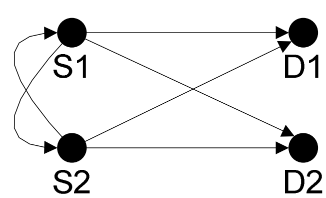

Fig. 7. Cooperative Diversity Channel

2.2.3 Multiple Terminal Networks

The classical channel configurations shown in the previous section have since been extended to more complex networks with additional participating terminals. Packet radio networks, a precursor to ad hoc networks, were analyzed in various scenarios in [55], [68], [72], [87], and [108], although this work has only recently been considered in the context of information and communication theory. In [39], it was initially indicated that without cooperation between terminals, the throughput per terminal decreases to zero asymptotically with the number of terminals and therefore the total capacity of the large networks does not scale with the number of member terminals. However, it was subsequently shown in [35] that when the point-to-point transmission protocol of [39] is extended to allow for arbitrarily complex cooperation between the nodes then the asymptotic capacity can be significantly increased, and in [38] that when the terminals in the network have high mobility the throughput per terminal can be kept constant asymptotically with the number of terminals as long as arbitrarily long delays are acceptable, simply by waiting until the chosen relay is close to the destination before transmitting.

The same general result based on the user with the best instantaneous channel transmitting was indicated in [36], [44], and [69] in the context of the capacity of multiple user systems. The capacity results for the classical relay channel were first extended to networks with multiple relays in [40], followed by incrementally improved coding schemes developed in [59], [121], [122], and [123]. All of these results indicated that the general encoding schemes introduced in [25] can also be extended to more complex channel configurations, and that in order for the total capacity of large networks to scale with the number of member terminals it is necessary to apply more complex network protocols involving cooperation between terminals. This further motivates our analysis of the application of cooperative diversity techniques to arbitrarily connected wireless relay networks.

2.3 Diversity Techniques and Multiple Antenna Systems

This section summarizes some relevant work on diversity and multiple antenna systems, introducing the fundamental diversity techniques, describing capacity and performance related results for multiple antenna systems, and discussing the more recent concept of the diversity-multiplexing tradeoff.

2.3.1 Diversity Techniques

The channel fading in wireless environments means that significant gains can be achieved using diversity techniques that take advantage of the probabilistic independence of the fading on different inter-terminal links. Diversity increases capacity and provides robustness against fading by averaging over multiple fading realizations (traditionally in time, frequency, or space) using some diversity combination method. The application of diversity can result in the probability of symbol error decreasing with the dth power of the SNR, where d is defined as the diversity order. The probability of error then has the behavior

$$P_e(\gamma) \propto \gamma^{-d}.$$

The most common diversity methods are spatial diversity using multiple antennas, temporal diversity using automatic repeat request schemes or block interleaving with error correction, and frequency diversity using frequency spreading, frequency hopping, or orthogonal frequency division multiplexing techniques. These techniques are described in detail in [86], [90], and [96]. In general, the choice of diversity combination method depends on the desired tradeoff between complexity and performance and the amount of available channel state information. Selection combining (SC) just chooses the diversity branch with the highest instantaneous single amplitude, and is the simplest but has the worst performance. Maximal ratio combining (MRC) provides optimal performance in the presence of AWGN noise but is the most complex and requires perfect channel state information. Equal gain combining (EGC) also combines weighted versions of the different diversity branches as in MRC, but has reduced complexity since it provides an equal weight to all branches so that it is not dependent on knowledge of the instantaneous signal envelope amplitudes. Generalized selection combining (GSC) involves MRC amongst a selected subset of the overall diversity branches. The performance of optimal maximal ratio combining when diversity branches are correlated was analyzed in [30], [129], and [130], extending the classical non-correlated results summarized in [86]. The results from [30], [86], [129], and [130] on the performance of diversity combination with MRC in the presence of various level of correlation are applied in the dissertation.

2.3.2 Multiple-Antenna Systems

One form of diversity, spatial diversity, can be provided via physical antenna arrays at either transmitters or receivers. When there are multiple antennas at the receiver, the diversity combination techniques and results of the previous section are directly applicable. However, when there are multiple antennas at the transmitter, it generally requires more hardware complexity because the resulting arrays of signals are superimposed at the receive antennas. There has been significant work on the design of practical coding methods for multiple-antenna systems that can take advantage of the offered spatial diversity of multiple transmit antennas to achieve similar diversity gains to receive antenna arrays, including the space-time codes described in [3], [34], [79], [105], and [106].

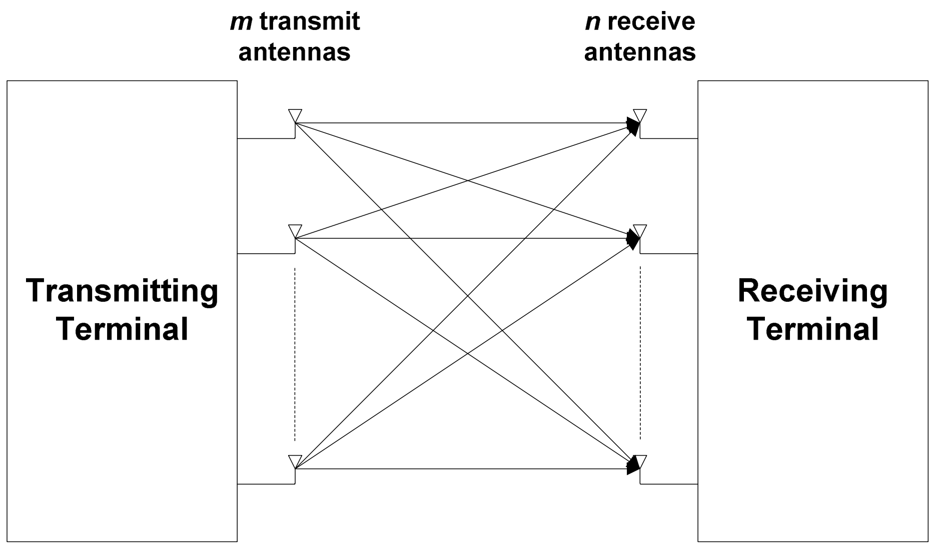

Fig. 8. MIMO Channel

Fig. 8 shows an example multiple-antenna system, also known as a multiple-input multiple-output (MIMO) channel, with m transmit antennas and n receive antennas. There has been much recent work on determining the performance limits of MIMO channels, for example [10], [34], [79], [107], and [117] which generally indicate that significant improvements in capacity are possible. It has been shown that in the high SNR regime, the capacity of a MIMO channel with m transmit antennas, n receive antennas, and independent identically distributed Rayleigh fading between each pair of antennas is given by

$$C(\gamma) = \min\{m, n\} \log(\gamma) + O(1) ,$$

where the number of degrees of freedom in the channel is thus the minimum of m and n, and $O(1)$ is a set of terms that go to zero asymptotically as the signal to noise ratio increases. It has also been shown that the maximum diversity order is given by

$$d_{\max} = mn,$$

where this is also the number of independent fading signals between the transmitting and receiving terminals. Wireless relay networks that employ cooperative diversity have sometimes been referred to as virtual MIMO systems, so it is natural to consider whether the arbitrarily connected wireless relay networks considered in this dissertation can achieve this same performance improvement.

2.3.3 Diversity-Multiplexing Tradeoff

The redundancy introduced by additional antennas can be used either to increase the diversity gain for a particular data rate or to increase the multiplexing gain (data rate) for a particular diversity gain. To increase the multiplexing gain, instead of using additional antennas for diversity, the transmit antennas are used to send independent data streams simultaneously to all receive antennas, which are then able to jointly decode the data streams. The concept of diversity-multiplexing tradeoff that was introduced in [133] describes this fundamental tradeoff between diversity and multiplexing gains. It was shown that the diversity-multiplexing tradeoff curve of a MIMO channel with m transmit antennas, n receive antennas, and independent identically distributed Rayleigh fading between each pair of antennas is given by the piecewise linear function connecting the points $(r, d(r)), r = 0,1,..., \min\{m,n\}$, where

$$d(r) = (m-r)(n-r),$$

$d(r)$ is the diversity order, and r is the multiplexing gain

In this formulation, the rate of the system scales with SNR according to $R(\gamma) = r\log(\gamma)$ such that

$$r = R(\gamma)/\log(\gamma)$$

and the probability of error or outage of the system scales with SNR according to $P_e(\gamma) \propto \gamma^{-d(r)}$ such that

$$d(r) = \lim_{\gamma\to\infty}(-\log P_e(\gamma)/\log(\gamma)).$$

We can again see that the maximum diversity order of the MIMO channel is $d_{\max} = mn$ and the maximum multiplexing gain is $r_{\max} = \min(m,n)$. This characterization indicates that in multiple-antennas systems when each additional antenna is introduced into a system it can be used to increase the diversity gain or multiplexing gain (under some conditions), but not both at the same time. It is also shown in [133] that asymptotically at high SNR the probability of error is of the same order as the probability of outage, indicating that for many problems of interest it is sufficient to determine the high SNR probability of outage instead of the probability of error.

This diversity-multiplexing formulation was extended to multiple-access channels in [110] and to relay (simple cooperative diversity) channels in [6], [63], [85]; however all of these works considered fully connected networks. For wireless relay networks that employ cooperation between terminals it is shown in [127] that although cooperative networks can achieve the diversity order performance of MIMO channels, this is not possible for all multiplexing gains. Specifically, it is indicated that for cooperative networks where each terminal has a single antenna, the maximum achievable multiplexing gain is 1, regardless of the channel model or coding scheme. This is due to the finite capacity link between the single antenna source and all receivers (or correspondingly, the finite capacity link between the single antenna destination and all transmitters). We consider the maximum diversity order and diversity-multiplexing tradeoff of arbitrarily connected wireless relay networks in this dissertation, including the possibility of multiple antennas per terminal.

2.4 Cooperative Diversity

This section summarizes the current state of work on cooperative diversity, introducing the classical two-user cooperative diversity model and discussing various extensions in the areas of cooperative coding, non-orthogonal signaling, multiple hops, and relay networks.

2.4.1 Classical Cooperative Diversity

As previously noted in the discussion on wireless terminal hardware constraints, the physical size of most mobile hardware devices limits the number of antennas that can be co-deployed due to probabilistic correlation between antennas. This is due to the fact that the separation between antennas must be at least on the order of half the wavelength of the carrier frequency to avoid correlated fading. It is for systems where physical antenna arrays at individual terminals are severely limited that virtual antenna arrays enabled by cooperation between distributed wireless terminals have the most promise. Cooperative diversity refers to the class of techniques where diversity benefits are gained via the sharing of information between multiple cooperating terminals in a wireless network. In general, cooperative diversity can be applied to either fixed or mobile relays in infrastructure or ad hoc networks respectively. Additionally, one area where cooperative diversity systems are in fact better than multiple-antenna systems is for combating macroscopic fading effects such as shadowing. Although multiple collocated antennas will be ineffective against macroscopic shadowing since they may all suffer from the same shadowing, the widely separated virtual antennas of cooperative systems do not have the same limitation. This dissertation incorporates shadowing in the system model as a factor that limits the connectivity between terminals.

The concept of cooperative diversity was introduced in [97] by extending the results of [115] on the multiple access channel with feedback to the case of relaying and fading in a multiple access channel. The basic cooperative diversity channel is shown in Fig. 7. This is effectively the cooperation case of the classical relay channel in [25] extended to two concurrent users in a fading environment, and is shown to enlarge the achievable rate region. Various protocols to implement cooperative diversity with two cooperating terminals were developed in [61], [62], and [64], generally indicating that full diversity order of two can be achieved. As noted previously, the decode-and-forward relaying protocol corresponds to the cooperation scheme for the classical relay channel and the amplify-and-forward relaying protocol corresponds to a degraded version of the compression scheme for the classical relay channel. Additionally, a number of minor related results have since been provided, for example the simple extension to the decode-and-forward scheme of [64] that is proposed in [47] that achieves full diversity order with slightly lower implementation complexity, and the analysis in [66] of the network coding gain (as opposed to the diversity order) of the basic cooperative diversity scheme.

2.4.2 Cooperative Coding Extensions

Although simple in implementation, the repetition-based protocols proposed in [64] are not efficient with respect to bandwidth utilization. As a result, a number of coded cooperative diversity schemes have been proposed with the intention of improving the spectral efficiency. These have included a coded cooperative diversity scheme that partitions the transmitted codewords via puncturing techniques and then has each partnering terminal generate the punctured symbols for retransmission to the destination in [51], the application of space-time codes to cooperative diversity channels in [7], [27], [28], [29], [63], and [104], and the application of space-time transmission with iterative (turbo) decoding techniques in [53] and [132]. It has been clearly shown that coded cooperative diversity can achieve full diversity order for an arbitrary number of cooperating terminals at higher data rates than repetition-based schemes [52], [74], for various relay terminal placement regions [71], and that space-time signaling techniques can be used to combine the different copies of a cooperative transmission from different transmitting terminals without the spectral efficiency loss caused by repetition coding over orthogonal channels [63], [78]. Although this dissertation does not explicitly consider coded cooperative diversity schemes, the developed cooperative connectivity modeling framework is equally applicable to systems that employ traditional source and channel coding techniques.

2.4.3 Non-Orthogonal Signaling Extensions

Another set of techniques that can be used to improve the spectral efficiency of cooperative diversity is non-orthogonal transmission. Some techniques such as delay diversity [37], [98], [118], were original proposed for MIMO channels but have since been considered for use in wireless relay networks, for example in [81], [88], [89], [94], [102], [119], and [120], while other techniques such as random relay phase rotation [42], non-orthogonal space-time communication [50], [77], and superposition modulation [67] were introduced specifically for wireless relay networks. In general, these techniques are based on the artificial generation of a rich multi-path fading environment and allow a common channel to be reused by multiple transmitters, with delay or phase separation introduced by propagation between terminals and retransmission processing at terminals.

As noted in [93], the delay between the signals received from different relay terminals is an additional source of diversity that can be exploited. Utilizing these techniques generally requires ensuring that the transmitted signal has low autocorrelation properties and that the different multi-path components are separated in time by at least the duration of one symbol. This can be accomplished by intelligent signal design and relay terminals introducing fixed or random delays before retransmitting. These techniques can achieve spectral efficiency upper bounded by the scheme proposed in [111] where transmitters have full channel state information and can adjust their transmissions so that the signals add coherently at the intended receiver. These techniques can be applied to cooperative wireless relay networks to achieve diversity while improving the spectral efficiency of cooperative diversity, and are considered for the common channel combination system resources constraints incorporated in the cooperative connectivity modeling framework of this dissertation. However, it is important to note that depending on the specific channel conditions, there may be some degradation of performance in comparison to that of maximal ratio combining, for example in the scheme of [50] the diversity achieved is only $d/2$.

2.4.4 Multihop and Network Extensions

The basic cooperative diversity channel has been extended to multihop scenarios and more complexly connected relay networks. Fundamental results relying on derivations of the harmonic and geometric means of composite fading distributions were used in [45], [46], and [57] to examine the performance of multihop, non-diversity channels using amplify-and-forward relaying and indicated a significant performance increase over non-relay channels, although without an increase in diversity order. Multi-user diversity, introduced in [4] and [31], extends the results of [64] to multiple relays in parallel. It was shown in [125] that the diversity order of a fully connected relay network with two relays is three for amplified relaying. It was shown in [15] and [126] that the maximum diversity order of a fully connected relay network with $N-1$ relays is N, and that higher diversity order can only be achieved when clustering results in a change in the underlying channel model assumptions. A class of multihop cooperative diversity protocols wherein only a subset of the previous relays is connected to each receiver was introduced in [92], and again was able to achieve a maximum diversity order of N when there are $N-1$ relays due to the fact that all relays are connected to the source and destination. The symbol error probability for an arbitrary number of parallel multihop paths was analyzed in [91] and indicated similar diversity order results. The connectivity between terminals assumed in each of these works is generalized in the analysis of arbitrarily connected wireless networks considered in this dissertation.

2.4.5 Selected Other Cooperative Diversity Literature

A number of other results have been provided in the broad area of cooperative diversity; we briefly note a few of these publications here to provide a flavor for the ongoing research and summary papers not referenced elsewhere in this dissertation. A different strategy wherein cooperative diversity improvements can be obtained by dynamic relay selection at the network level is proposed in [8] and shown to also achieve full diversity order. A modeling framework for diversity in wireless relay networks is presented in [41] that leverages the fact that parallel and serial relay combinations are analogous to serial and parallel resistor combinations respectively, an interesting relationship previously noted in [15], [16], and [64]. A technique is proposed in [2] and [70] wherein a linear code is mapped to a network graph for multicast transmission. The use of collaboration between terminals is shown in [103] to significantly improve the asymptotic connectivity of both sparse and dense wireless ad hoc networks. A MAC-level technique wherein diversity is achieved by MAC level collision, retransmission, and recombination is proposed in [109] and [131]; however this technique is only appropriate for channels with coherence time less that the packet transmission period. Practical processing and detection schemes for MIMO relay channels, as well as dynamic routing techniques wherein only the best instantaneous relays retransmit, were considered in [32]. Finally, [48], [80], and [84] provide a good high-level introduction and overview to the current cooperative diversity research.

Chapter 3 - System Model

This chapter develops a general system model for arbitrarily connected wireless relay networks and derives probability of error and information theoretic probability of outage results for decoded relaying with error propagation, decoded relaying without error propagation, and amplified relaying. Section 3.1 introduces the system model. Section 3.2 develops terminology and intermediate results for the system model. Section 3.3 presents the fundamental probability of error results. Section 3.4 presents the fundamental probability of outage results. Section 3.5 derives the probability of error and probability of outage for wireless relay networks that employ amplified relaying. Section 3.6 derives the probability of error and probability of outage for wireless relay networks that employ decoding relaying with error propagation. Section 3.7 derives the probability of error and probability of outage for wireless relay networks that employ decoding relaying without error propagation. Section 3.8 extends the system model to include terminals with multiple antennas. Section 3.9 summarizes the contributions of the chapter.

3.1 Introduction

The foundation for any advanced research in the area of cooperative connectivity for wireless relay networks is the development of an appropriate underlying system model. Although a number of system models have been developed in the cooperative diversity literature, for example [4], [15], [41], [45], [46], [52], [64], [91], and [97], existing work has so far not addressed arbitrary, and especially irregular, connectivity between cooperating terminals in a wireless relay network. This section develops a general system model for arbitrarily connected wireless relay networks with various relaying methods.

Probability of error and information theoretic probability of outage results are presented for arbitrarily connected wireless relay networks that employ amplified relaying, decoded relaying with error propagation, and decoded relaying without error propagation.

The scope of the system model is limited to a single user communication in isolation and the potential multiple access interference created by the signals of other concurrent users is not considered. Even so, the nature of multihop relaying channels creates some unique channel allocation requirements. Due to the assumed half-duplex operation of the wireless terminals, at least one separate channel needs to be allocated for relaying, and depending on the specific multiple-access schemes and cooperative connectivity models chosen, additional channels may be required. However, for the purposes of this chapter it is sufficient to assume that orthogonal channels are achieved through some combination of time, frequency, and code division multiple access as part of an overall channel and resource allocation scheme. The orthogonal channel requirements associated with cooperative connectivity will be addressed in the following work on cooperative connectivity models.

3.2 Terminology and Intermediate Results

The system model is composed of a source terminal, a destination terminal, and a variable number of intermediate relay terminals. Let T_S, T_I, and T_D respectively denote the sets of source, intermediate (relay), and destination terminals, let $T_T = T_S \cup T_I$ denote the set of all transmitting terminals, let $T_R = T_I \cup T_D$ denote the set of all receiving terminals, and let T_C denote the subset of intermediate terminals that have decoded correctly (retrieved without error the original information transmitted by the source).

Furthermore, let $T_{P(i)}$ denote the set of previous terminals that transmit a signal received by terminal T_i, let $T_{C(i)} = T_{P(i)} \bigcap (T_S \bigcup T_C)$ denote the subset of terminals of $T_{P(i)}$ (including the source) that have decoded correctly (not generated an error or outage event), and let $T_{R(i)}$ denote the set of terminals that includes terminal T_i and the receiving terminals that precede terminal T_i in the channel. Fig. 9 shows an example network, annotated with this terminal set terminology.

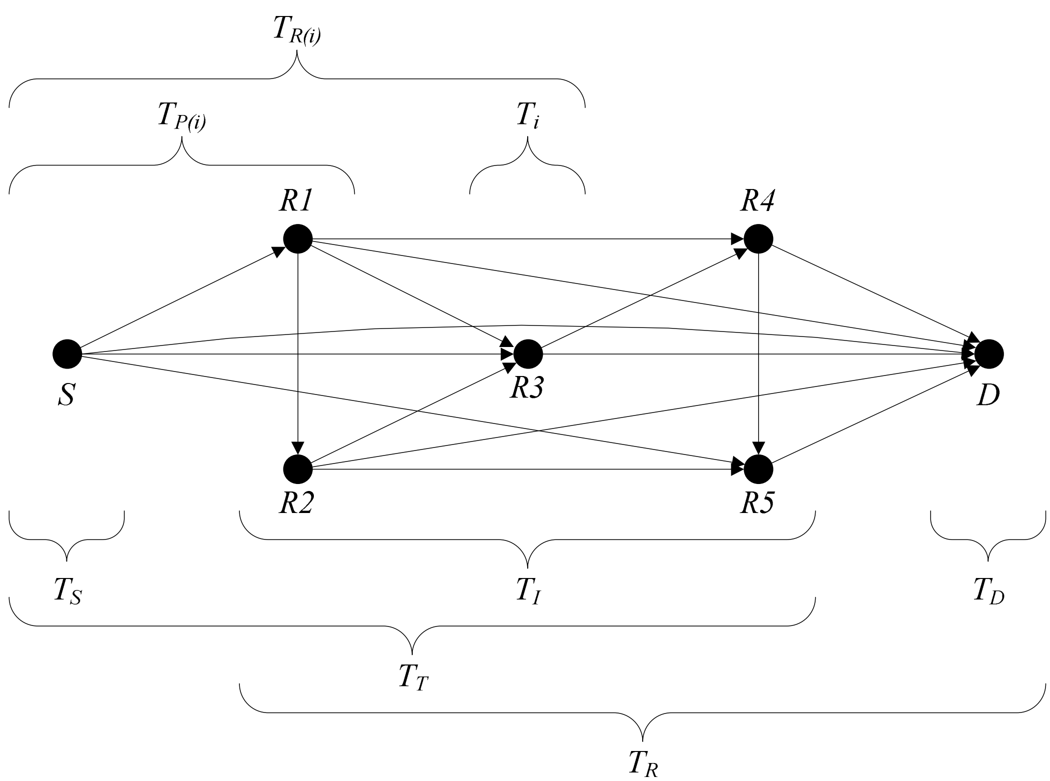

Fig. 9. Wireless Relay Network Terminal Set Terminology

Also, let S_R denote the complete set of distinct cut sets associated with the directed network graph, let L_R denote the complete set of inter-terminal links associated with the directed network graph, let L_i denote the set of inter-terminal links associated with a particular cut set S_i, let each $L_{k,l} \in L_i$ denote the inter-terminal link that joins terminals

T_k and T_l across cut set S_i, and let $T_{IN(i)}$ and $T_{OUT(i)}$ respectively denote the sets of terminals on the input and output sides of cut set S_i that are associated with at least one of the inter-terminal links in the corresponding set L_i. Finally, let $T_{O(m)}$ denote the set of receiving terminals that have m immediately preceding terminals (i.e., $\left|T_{P(i)}\right| = m, \forall T_i \in T_{O(m)}$ where $\left|T_{P(i)}\right|$ is the cardinality of $T_{P(i)}$), and let $S_{O(m)}$ denote the set of cut sets that are associated with m inter-terminal links (i.e., $\left|L_i\right| = m, \forall S_i \in S_{O(m)}$ where $\left|L_i\right|$ is the cardinality of L_i). Fig. 10 shows an example network, annotated with this cut set terminology.

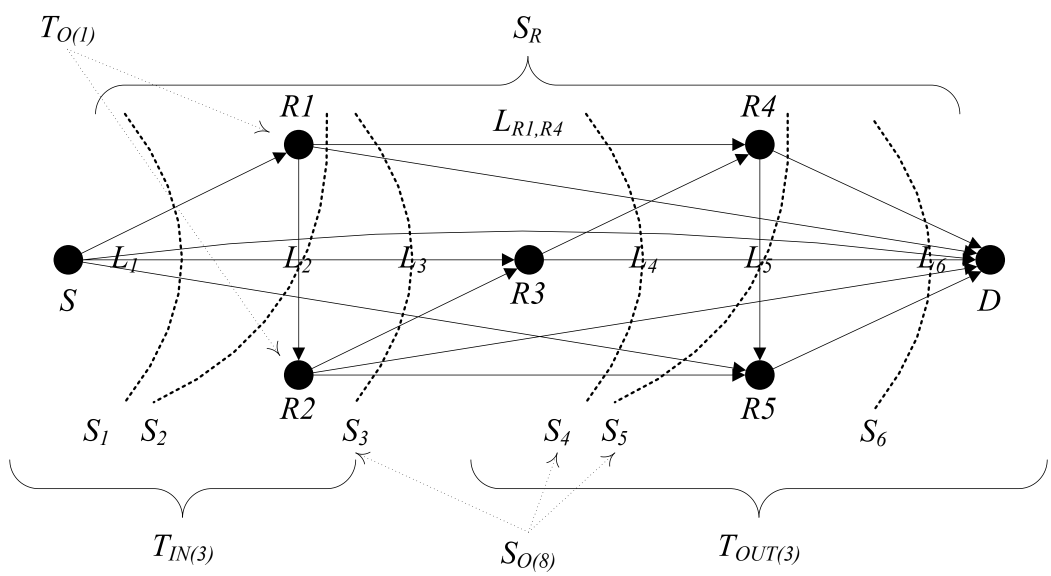

Fig. 10. Wireless Relay Network Cut Set Terminology

This defined set notation is used in variable subscripts of the form x_{T_i} to denote specific terminals, groups of terminals, or cut sets. Notation of the form x_{T_i} is abbreviated to x_i for simplicity of exposition. Furthermore, it is assumed that all relays operate in half-duplex mode and that a network with N transmitters in general requires

$2 \le K \le N$ orthogonal channels to be allocated, resulting in a rate factor of $1/K$. The system model does not imply any particular method by which the set of relay terminals, or set of active links between the pairs of terminals, are chosen. It is equally applicable to possible methods where the links are set up in advance via some static allocation scheme, where the links are dynamically chosen for each signal transmission based on current channel conditions, and where all available links (those not experiencing significant shadowing or obstruction) are always used, among other possible methods.

Each terminal T_i transmits a discrete-time signal with complex baseband amplitude given by

$$s_i = \sqrt{\varepsilon_i}(\alpha_i + \beta_i), \tag{1}$$

where ε_i is the transmitted power, α_i is the complex amplitude of the information symbol over a given signaling interval, and β_i is propagated noise from previous terminals. Propagated noise only occurs in wireless relay networks where relays amplify in an analog fashion the received signals from preceding terminals before retransmission, therefore retransmitting both information and noise portions of this composite received signal. The noise introduced at each received terminal may in general be composed of local thermal noise and other interference resulting from other users operating in the network. This system model normalizes the transmitted signal such that $|\alpha_i|^2 + E[|\beta_i|^2] = 1$, in order to ensure that ε_i is the transmitted power irrespective of what portion of the transmitted signal is information and what portion is propagated noise. For terminals which do not propagate noise, $\beta_i = 0$ and $|\alpha_i|^2 = 1$. For terminals that do propagate noise, the propagated noise is the total received noise of the terminal multiplied by the amplification gain factor of the terminal.

Each inter-terminal link experiences distance-dependent attenuation, shadowing, and fading. A simple wireless channel model with flat (frequency non-selective), slow (time non-selective) fading is chosen in order to clearly highlight the benefits of cooperative diversity. Each inter-terminal channel experiences mutually independent attenuation, fading, and noise. Each terminal T_i then receives from each immediately preceding terminal $T_k \in T_{P(i)}$ a discrete-time signal with complex baseband amplitude given by

$$r_{k,i} = a_{k,i}\sqrt{\varepsilon_k}\,(\alpha_k + \beta_k) + z_{k,i}\,, \tag{2}$$

where $a_{k,i}$ captures the effects of distance-dependent attenuation, shadowing, and fading between T_k and T_i, and $z_{k,i}$ is a zero-mean Gaussian random variable with variance $N_{k,i}$ that captures the combined effects of local thermal noise and other interference resulting from other users operating in the network. For the case of mutually independent flat slow Rayleigh fading and distance-dependent attenuation each $a_{k,i}$ can be modeled as

$$a_{k,i} = (\delta\sqrt{d_{k,i}^{-p}}\,R_{k,i}) \tag{3}$$

where δ^2 is the free space signal power attenuation factor between the transmitting terminal and an arbitrary reference distance [90], $d_{k,i} \geq 1$ is the inter-terminal distance normalized with respect to this reference distance, p is the signal propagation exponent, and $R_{k,i}$ is a complex Gaussian (Rayleigh envelope) random variable with variance $\sigma_{k,i}^2$. This simple distance-dependent Rayleigh fading model is used throughout the remainder of this dissertation.

Shadowing is included in this simple fading channel model in two ways. First, it is assumed that the considered inter-terminal links are those with a reasonable path loss, such that when there is significant shadowing between a pair of terminals it is modeled that the corresponding inter-terminal link does not exist and the pair of terminals is not directly connected. The path loss threshold above which a pair of terminals is modeled as not being directly connected can be set arbitrarily. Second, shadowing below this path loss threshold can be incorporated in the model as a factor of the inter-terminal distance metrics, resulting in an equivalent network geometry that incorporates both physical inter-terminal distances and inter-terminal shadowing. It is assumed that the shadowing is fixed for a given placement of terminals so that it remains constant across multiple information blocks. Fig. 11 shows an example of this signal terminology.

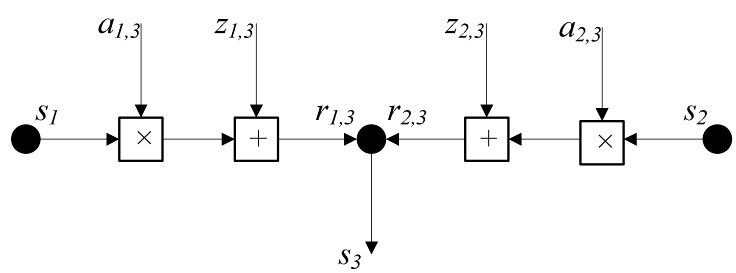

Fig. 11. Wireless Relay Network Signal Terminology

The link signal to noise ratio at T_i for the link from each immediately preceding terminal $T_k \in T_{P(i)}$ is defined as

$$\psi_{k,i} = \frac{\varepsilon_k}{N_{k,i} / \left| a_{k,i} \right|^2}, \tag{4}$$

or alternatively represented by

$$\psi_{k,i} = SNR \mu_{k,i} \left| a_{k,i} \right|^2, \tag{5}$$

where $SNR = \varepsilon_0 / N_0$ is a reference SNR and $\mu_{k,i} = SNR^{-1} \varepsilon_k / N_{k,i}$ is a scaling factor for each link SNR with respect to the reference SNR. The propagated noise β_k is not present because the link SNR by definition does not include propagated noise (i.e., $\beta_k = 0$ and $|\alpha_k|^2 = 1$) and is an upper bound on the SNR for relaying methods that involve propagated noise.

The aggregate signal to noise ratio at T_i for the signal from each immediately preceding terminal $T_k \in T_{P(i)}$ is defined as

$$\gamma_{k,i} = \frac{\varepsilon_k |\alpha_k|^2}{\varepsilon_k E[|\beta_k|^2] + N_{k,i} / |a_{k,i}|^2}, \tag{6}$$

where the term aggregate refers to the inclusion of propagated noise terms generated as amplified relaying terminals amplify both the information and noise portions of received signals indiscriminately. Note that the link SNR is identical to the aggregate SNR when there is no propagated noise, $\beta_k = 0$. The aggregate signal to noise ratio at T_i from all immediately preceding terminals assuming maximal ratio combining is therefore given by

$$\gamma_{P(i),i} = \sum_{T_k \in T_{P(i)}} [\frac{\varepsilon_k |\alpha_k|^2}{\varepsilon_k E[|\beta_k|^2] + N_{k,i} / |a_{k,i}|^2}]. \tag{7}$$

3.3 Probability of Error

Let $P_{e,i}(\gamma_{P(i),i})$ generally denote the probability of decoding error at terminal T_i for a given modulation scheme for incident SNR $\gamma_{P(i),i}$ and assuming maximal ratio combining. For example, for independent Rayleigh fading of the signals on each input branch of the diversity combiner and BPSK modulation, when the different signal to noise ratio values are distinct the probability of decoding error according to the method in [86] is given by

$$P_{e,i}(\gamma_{P(i),i}) = \frac{1}{2}\Big(\sum_{T_k \in T_{P(i)}} \Big[\prod_{\substack{T_j \in T_{P(i)} \\ T_j \neq T_k}} \frac{\overline{\gamma_{k,i}}}{\overline{\gamma_{k,i}} - \overline{\gamma_{j,i}}}\Big(1 - \sqrt{\frac{\overline{\gamma_{k,i}}}{2 + \overline{\gamma_{k,i}}}}\Big)\Big]\Big), \qquad (8)$$

where $\overline{\gamma_{k,i}}$ is the expected received signal to noise ratio at T_i for the branch of the diversity combiner associated with the signal incident from terminal T_k. We note that the 2 in the denominator inside the summation is different from the 1 in the standard BPSK results in [86]. The reason for this difference is that the additive white Gaussian noise in our model has a variance of $N_{k,i}$, while in [86] the additive white Gaussian noise has a variance of $N_{k,i}/2$.

The probability of error performance of maximal ratio combining is generally a lower bound on the achievable probability of error when using common channel combination techniques. Depending on the specific channel conditions, there may be some degradation of performance in comparison to that of maximal ratio combining when applying these techniques. When terminals have the capability to cancel interhop interference (a form of inter-symbol interference created when channels are reused at different hops within a single multihop path [15]), it is assumed that the interference cancellation is perfect. This assumption provides a tractable lower bound on the achievable probability of error or outage. It would be possible to extend the system model to include an additional factor that incorporates in a more granular fashion the fraction of interhop interference that can be cancelled by each receiver. This would allow further analysis of the performance impact of partial interhop interference cancellation, but is outside the scope of this dissertation.

3.4 Probability of Outage

Information theoretic probability of outage formulations have a number of advantages over probability of symbol error formulations, including independence from any particular coding or modulation scheme, applicability for arbitrary transmission rates, and straightforward derivation of asymptotic diversity order results. The method used in this dissertation to calculate the diversity order based on the $SNR \to \infty$ behavior of the probability of the mutual information falling below a target rate R is similar to that used in [63]. Different from [6], for practicality the model is constrained to non-overlapping symbol periods. Two general schemes for how codebooks are generated by and partitioned among terminals in the relay network are considered and compared: common codebook generation and independent codebook generation.

Common codebook generation involves the source and all relays sharing a common randomly generated iid circularly symmetric, complex Gaussian codebook, and encompasses many practical encoding schemes, including both repetition coding and space-time coding. In the common codebook generation scheme, for a given information block, all transmitters use a codeword generated from the same jointly designed codebook, where the codeword may optionally be the same for each transmitter. Receivers perform maximum-ratio combining. Independent codebook generation involves the source and all relays employing independent randomly generated iid circularly symmetric, complex Gaussian codebooks, and provides an information theoretic lower bound on the probability of outage of all achievable codebook generation schemes. In the independent codebook generation scheme, for a given information block, all transmitters use a codeword generated from a different and independent codebook. Receivers combine mutual information from the different independent codewords. As noted in [63], the generation of independent random codebooks corresponds to utilizing parallel spatial channels, and can achieve better spectral efficiency at the cost of increased complexity. This distinction between common codebook generation and independent codebook generation is analogous to the distinction between repetition coding and parallel channel coding in [65].

3.4.1 Common Codebook Generation

When the source and all relays share a common randomly generated iid circularly symmetric, complex Gaussian codebook, the mutual information between terminal T_i and all immediately preceding terminals $T_{P(i)}$ for incident SNR $\gamma_{P(i),i}$ is given by

$$I_i(\gamma_{P(i),i}) = \frac{1}{K}\log\left(1 + \sum_{T_k \in T_{P(i)}} \gamma_{k,i}\right), \tag{9}$$

and the information theoretic probability of outage at terminal T_i is the probability of the mutual information falling below a target rate R, and is given by

$$P_{o,i}(\gamma_{P(i),i}, R) = \Pr\left[\sum_{T_k \in T_{P(i)}} \gamma_{k,i} < 2^{KR} - 1\right]. \tag{10}$$

This can be lower bounded by

$$P_{o,i}(\gamma_{P(i),i}, R) \geq \Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR}-1}{SNR}\right], \tag{11}$$

where the lower bound results from the potential presence of propagated noise.

Using standard results from [86] for the probability distribution function of the sum of the distinct and independent exponential random variables $\psi_{k,i} = SNR\mu_{k,i}\left|a_{k,i}\right|^2$, the probability of outage can be solved by integrated over all fading realizations that result in a rate less than the target rate R. When the different signal to noise ratio values are distinct, the probability of outage is given by

$$
\begin{aligned}
P_{o,i}(\gamma_{P(i),i}, R) &\geq \int_0^{2^{KR}-1} \sum_{T_k \in T_{P(i)}} \left(\frac{\pi_k}{\overline{\psi_{k,i}}} \exp(-\frac{x}{\overline{\psi_{k,i}}}) \right) \partial x \\
&\geq \sum_{T_k \in T_{P(i)}} \left(\int_0^{2^{KR}-1} \frac{\pi_k}{\overline{\psi_{k,i}}} \exp(-\frac{x}{\overline{\psi_{k,i}}}) \partial x \right), \\
&\geq \sum_{T_k \in T_{P(i)}} \pi_k \left(1 - \exp(-\frac{2^{KR}-1}{\overline{\psi_{k,i}}}) \right)
\end{aligned} \tag{12}
$$

where $\pi_k = \prod_{\substack{T_j \in T_{P(i)} \\ T_j \neq T_k}} \left(\overline{\psi_{k,i}} \big/ \left(\overline{\psi_{k,i}} - \overline{\psi_{j,i}} \right) \right)$. A simpler approximate result that more clearly exposes the diversity order behavior can be generated by taking the limit of the probability of outage as $SNR \to \infty$ to result in

$$
\begin{aligned}
\frac{P_{o,i}(\gamma_{P(i),i}, R)}{\left(\frac{2^{KR}-1}{SNR} \right)^{\left|T_{P(i)}\right|}} &\geq \underbrace{\left(\frac{2^{KR}-1}{SNR} \right)^{-\left|T_{P(i)}\right|} \Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i} \left|a_{k,i}\right|^2 < \frac{2^{KR}-1}{SNR} \right]}_{\to \left(\left|T_{P(i)}\right|!\right)^{-1} \prod_{T_k \in T_{P(i)}} \left(\mu_{k,i}\sigma_{k,i}^2\right)^{-1}}, \\
&\to \frac{1}{\left|T_{P(i)}\right|!} \prod_{T_k \in T_{P(i)}} \frac{1}{\mu_{k,i}\sigma_{k,i}^2}
\end{aligned} \tag{13}
$$

where $\left|T_{P(i)}\right|$ is the number of previous terminals that transmit a signal received by terminal T_i, and the asymptotic approximation uses cumulative distribution function results for the sum of independent exponential random variables [63]. At high SNR the lower bound becomes tight and the probability of outage can therefore be approximated by

$$P_{o,i}(\gamma_{P(i),i}, R) \approx \left(\frac{1}{\left|T_{P(i)}\right|!} \prod_{T_k \in T_{P(i)}} \frac{1}{\mu_{k,i}\sigma_{k,i}^2} \right) \left(\frac{2^{KR}-1}{SNR} \right)^{\left|T_{P(i)}\right|}, \tag{14}$$

and the diversity order [133] is given by

$$d_i = -\lim_{SNR \to \infty} \frac{\log P_{o,i}(\gamma_{P(i),i}, R)}{\log SNR} = \left|T_{P(i)}\right|. \tag{15}$$

3.4.2 Independent Codebook Generation

When the source and all relays employ independent randomly generated iid circularly symmetric, complex Gaussian codebooks, the mutual information between terminal T_i and all immediately preceding terminals $T_{P(i)}$ for incident SNR $\gamma_{P(i),i}$ is upper bounded by

$$I_i(\gamma_{P(i),i}) \leq \frac{1}{K} \sum_{T_k \in T_{P(i)}} \log\left(1+\gamma_{k,i}\right), \tag{16}$$

and the information theoretic probability of outage at terminal T_i is the probability of the mutual information falling below a target rate R, and is lower bounded by

$$P_{o,i}(\gamma_{P(i),i}, R) \geq \Pr\left[\prod_{T_k \in T_{P(i)}} \left(1+\gamma_{k,i}\right) < 2^{KR} \right]. \tag{17}$$

This can be further lower bounded by

$$P_{o,i}(\gamma_{P(i),i}, R) \geq \Pr\left[\prod_{T_k \in T_{P(i)}} \left(1+SNR\mu_{k,i}\left|a_{k,i}\right|^2\right) < 2^{KR} \right]. \tag{18}$$

where the lower bound results from the potential presence of propagated noise.

Using results from [65] for the high SNR approximation of the probability of outage for parallel channel coding, the probability of outage can be approximated by

$$P_{o,i}(\gamma_{P(i),i}, R) \approx g_{|T_{P(i)}|}(KR)\left(\prod_{T_k \in T_{P(i)}} \frac{1}{\mu_{k,i}\sigma_{k,i}^2}\right)\left(\frac{1}{SNR}\right)^{|T_{P(i)}|}, \tag{19}$$

where $g_{|T_{P(i)}|}(KR) = \sum_{j=2}^{|T_{P(i)}|}\left((-1)^{|T_{P(i)}|-j}(\frac{2^{KR}}{j-1})(KR\ln 2)^{j-1}\right) + (-1)^{|T_{P(i)}|-1}(2^{KR}) + (-1)^{|T_{P(i)}|}$.

Appendix A presents a detailed derivation of this high SNR approximation of the probability of outage using the results from [65]. The diversity order [133] is given by

$$d_i = -\lim_{SNR \to \infty} \frac{\log P_{o,i}(\gamma_{P(i),i}, R)}{\log SNR} = |T_{P(i)}|. \tag{20}$$

Although this result for the probability of outage is tight at high SNR, we can find a looser but simpler approximation for the probability of outage by taking the lower bound

$$\begin{aligned} P_{o,i}(\gamma_{P(i),i}, R) &\geq \prod_{T_k \in T_{P(i)}} \Pr\left[(1+\gamma_{k,i}) < 2^{KR/|T_{P(i)}|}\right] \\ &\geq \prod_{T_k \in T_{P(i)}} \Pr\left[\left(1 + SNR\mu_{k,i}|a_{k,i}|^2\right) < 2^{KR/|T_{P(i)}|}\right], \\ &\geq \prod_{T_k \in T_{P(i)}} \Pr\left[\mu_{k,i}|a_{k,i}|^2 < \frac{2^{KR/|T_{P(i)}|}-1}{SNR}\right] \end{aligned} \tag{21}$$

where the lower bound results from the potential presence of propagated noise and the relation $\Pr\left[\prod_k x_k < \prod_k y_k\right] \geq \prod_k \Pr[x_k < y_k]$ for independent x_k and y_k.

Again using standard results from [86] for the probability distribution function of the sum of the independent exponential random variables $\psi_{k,i} = SNR\mu_{k,i}|a_{k,i}|^2$, the probability of outage can be solved by integrated over all fading realizations that result in a rate less than the target rate R. When the different signal to noise ratio values are distinct, the probability of outage is given by

$$\begin{aligned} P_{o,i}(\gamma_{P(i),i}, R) &\geq \prod_{T_k \in T_{P(i)}} \left(\int_0^{2^{KR/|T_{P(i)}|}-1} \frac{\pi_k}{\overline{\overline{\psi_{k,i}}}} \exp(-\frac{x}{\overline{\overline{\psi_{k,i}}}}) \partial x \right) \\ &\geq \prod_{T_k \in T_{P(i)}} \left(1 - \exp(-\frac{2^{KR/|T_{P(i)}|}-1}{\overline{\overline{\psi_{k,i}}}}) \right) \end{aligned} . \tag{22}$$

Again, a simpler approximate result that more clearly exposes the diversity order behavior can be generated by taking the limit of the probability of outage as $SNR \to \infty$ to result in

$$\begin{aligned} \frac{P_{o,i}(\gamma_{P(i),i}, R)}{\left(\frac{2^{KR/|T_{P(i)}|}-1}{SNR} \right)^{-|T_{P(i)}|}} &\geq \underbrace{\left(\frac{2^{KR/|T_{P(i)}|}-1}{SNR} \right)^{-|T_{P(i)}|} \prod_{T_k \in T_{P(i)}} \Pr\left[\mu_{k,i} |a_{k,i}|^2 < \frac{2^{KR/|T_{P(i)}|}-1}{SNR} \right]}_{\to \prod_{T_k \in T_{P(i)}} \left(\mu_{k,i}\sigma_{k,i}^2\right)^{-1}} , \\ &\to \prod_{T_k \in T_{P(i)}} \frac{1}{\mu_{k,i}\sigma_{k,i}^2} \end{aligned} \tag{23}$$

where the asymptotic approximation again uses cumulative distribution function results for the sum of independent exponential random variables [63]. At high SNR this probability of outage can therefore be approximated by

$$P_{o,i}(\gamma_{P(i),i}, R) \approx \left(\prod_{T_k \in T_{P(i)}} \frac{1}{\mu_{k,i}\sigma_{k,i}^2} \right) \left(\frac{2^{KR/|T_{P(i)}|}-1}{SNR} \right)^{|T_{P(i)}|}, \tag{24}$$

and the diversity order [133] is given by

$$d_i = -\lim_{SNR \to \infty} \frac{\log P_{o,i}(\gamma_{P(i),i}, R)}{\log SNR} = |T_{P(i)}|. \tag{25}$$

We note that this approximation for the probability of outage when the source and all relays employ independent randomly generated codebooks is in general not tight. However, the corresponding diversity order result derived from this approximation is tight. This is clear, since even though (17) is strictly less than (10) and (21) is strictly less than (18), the resulting diversity orders (15), (20), and (25) are identical. Although the approximate probability of outage (24) is used throughout the remainder of the dissertation for its simplicity, it can be readily replaced with the tighter probability of outage result (19) when required.

3.5 Amplified Relaying

This section derives the probability of error and outage for wireless relay networks that employ amplified relaying. For amplified relaying, also known as non-regenerative or amplify-and-forward relaying, each intermediate terminal simply combines and amplifies the received signals from preceding terminals before retransmission, relaying both information and noise portions towards the destination terminal. The amplification gain factor at each intermediate terminal T_i is simply the transmitted power over the received power and is given by

$$G_i = \frac{\varepsilon_i}{\sum_{T_k \in T_{P(i)}} (|a_{k,i}|^2 \varepsilon_k + N_{k,i})}, \tag{26}$$

the aggregate SNR at T_i for the set of preceding terminals $T_k \in T_{P(i)}$ is given recursively by

$$\gamma_{P(i),i} \approx \sum_{T_k \in T_{P(i)}} (\psi_{k,i}^{-1} + \gamma_{P(k),k}^{-1} + \psi_{k,i}^{-1} \gamma_{P(k),k}^{-1})^{-1}, \tag{27}$$

where a detailed derivation of this recursive formula for the aggregate signal to noise ratio is provided in Appendix B. The total probability of decoding error is given by

$$P_e = P_{e,D}(\gamma_{P(D),D}), \tag{28}$$

where $P_{e,D}$ is the probability of decoding error at the destination terminal, and the total information theoretic probability of outage is given by

$$P_o = P_{o,D}(\gamma_{P(D),D}, R), \tag{29}$$

where $P_{o,D}$ is the probability of outage at the destination terminal.

It is important to note that with amplified relaying the signals on each input branch of the diversity combiner may be correlated. This occurs when the signal transmitted over a given fading link is further transmitted and amplified on multiple parallel transmission paths that are later recombined. For correlated Rayleigh fading of the signals on each input branch of the diversity combiner and BPSK modulation, the probability of decoding error according to the method in [30] and [129] is given by

$$P_{e,i}(\gamma_{P(i),i}) = \frac{1}{2}\Big(\sum_{\lambda_k \in \underline{\Gamma}_\gamma} \Big[\prod_{\substack{\lambda_j \in \underline{\Gamma}_\gamma \\ \lambda_j \neq \lambda_k}} \frac{\lambda_k}{\lambda_k - \lambda_j}\Big(1 - \sqrt{\frac{\lambda_k}{2+\lambda_k}}\Big)\Big]\Big), \tag{30}$$

where the λ_k are the eigenvalues of $\underline{\Gamma}_\gamma$, a positive definite matrix related to the input branch SNR covariance matrix $\underline{R}_\gamma$ by $\Gamma_\gamma(i,j) = \sqrt{R_\gamma(i,j)}$.

3.6 Decoded Relaying with Error Propagation

This section derives the probability of error for wireless relay networks that employ decoding relaying with error propagation. For decoded relaying with error propagation, also known as regenerative or fixed decode-and-forward relaying, each intermediate terminal combines, digitally decodes, and re-encodes the received signals from preceding terminals before retransmission and error or outages events at relays propagate as error or outage events at the destination. The aggregate SNR at T_i when all preceding terminals have decoded correctly is given by

$$\gamma_{P(i),i} = \sum_{T_k \in T_{P(i)}} \psi_{k,i} \, , \tag{31}$$

the total probability of decoding error across all terminals is given by

$$P_e = 1 - \prod_{T_i \in T_R} (1 - P_{e,i}(\gamma_{P(i),i})) \, , \tag{32}$$

where $P_{e,i}$ is the probability of decoding error at T_i, and the total information theoretic probability of outage across all terminals is given by

$$P_o = 1 - \prod_{T_i \in T_R} (1 - P_{o,i}(\gamma_{P(i),i}, R)) \, , \tag{33}$$

where $P_{o,i}$ is the probability of outage at T_i.

3.7 Decoded Relaying without Error Propagation

This section derives the probability of error for wireless relay networks that employ decoding relaying without error propagation. For decoded relaying without error propagation, also known as adaptive or selective decode-and-forward relaying, each intermediate terminal combines, digitally decodes, and re-encodes the received signals from preceding terminals before retransmission and error or outage events at relays do not propagate as error or outage events at the destination. This can be achieved in practice through the use of various techniques that stop relays from retransmitting under conditions that would result (with high probability) in errors, for example error detection with CRC checks or stronger error detection codes, or alternatively via the application of SNR thresholds [82]. We make the simplifying assumption that error detection is perfect in order to make the presented system model more tractable. Although perfect error detection is not feasible, the comparison of the idealized decoded relaying with error propagation and decoded relaying without error propagation relaying methods clearly isolates the key impact of error propagation on the performance of wireless relay networks. The aggregate SNR at T_i for the set of preceding terminals that have decoded correctly is given by

$$\gamma_{C(i),i} = \sum_{T_k \in T_{C(i)}} \psi_{k,i} \, , \tag{34}$$

and the total probability of decoding error is lower bounded by

$$P_e \geq \sum_{T_C \in T_I} \left[P_{e,D}(\gamma_{C(D),D}) \prod_{\substack{T_j \in T_I \\ T_j \notin T_C}} P_{e,j}(\gamma_{C(j),j}) \prod_{\substack{T_k \in T_I \\ T_k \in T_C}} (1 - P_{e,k}(\gamma_{C(k),k})) \right], \tag{35}$$

where $P_{e,D}$ is the probability of decoding error at the destination terminal, the summation of terms is over all possible sets of intermediate terminals $T_C \in T_I$ that decode correctly, and the lower bound is due to the simplifying assumption that error detection is perfect. The lower bound can be made arbitrarily tight with successively stronger error detection codes. The total probability of decoding error can be further expanded and lower bounded by

$$
\begin{aligned}
P_e \geq\ & P_{e,D}(\gamma_{P(D),D}) \prod_{T_k \in T_I} (1 - P_{e,k}(\gamma_{P(k),k})) \\
& + \sum_{\substack{T_m \in T_I \\ T_C = T_I - T_m}} \left[P_{e,D}(\gamma_{C(D),D}) P_{e,m}(\gamma_{C(m),m}) \prod_{\substack{T_k \in T_I \\ T_k \in T_C}} (1 - P_{e,k}(\gamma_{C(k),k})) \right] \\
& + \sum_{\substack{T_m, T_n \in T_I, T_m \neq T_n \\ T_C = T_I - T_m - T_n}} \left[P_{e,D}(\gamma_{C(D),D}) P_{e,m}(\gamma_{C(m),m}) P_{e,n}(\gamma_{C(n),n}) \prod_{\substack{T_k \in T_I \\ T_k \in T_C}} (1 - P_{e,k}(\gamma_{C(k),k})) \right] \\
& + \ldots
\end{aligned}
, \quad (36)
$$

where the first term is the set where all preceding terminals decode correctly, the first summation of terms includes all possible sets where exactly one preceding terminal decodes incorrectly, the second summation of terms includes all possible sets where exactly two preceding terminals decode incorrectly, and so on. The total information theoretic probability of outage is lower bounded by

$$
P_o = \geq \sum_{T_C \in T_I} \left[P_{o,D}(\gamma_{C(D),D}, R) \prod_{\substack{T_j \in T_I \\ T_j \notin T_C}} P_{o,j}(\gamma_{C(j),j}, R) \prod_{\substack{T_k \in T_I \\ T_k \in T_C}} (1 - P_{o,k}(\gamma_{C(k),k}, R)) \right], \quad (37)
$$

which can be expanded in similar fashion to the probability of error.

3.8 Extension to Terminals with Multiple Antennas

Recent wireless base-stations, whether they are the source terminal in downlink transmissions or the destination terminal in uplink transmissions, are already starting to incorporate multiple antennas. Additionally, mobile wireless relay terminals are starting to incorporate dual antennas, and the introduction of cooperative diversity techniques in fixed relay networks raises the possibility that some fixed wireless relay terminals may support even more antennas. These trends are expected to become even more prevalent in the future. Therefore, wireless relay networks where some of the cooperating terminals employ multiple antennas will be of increasing interest. A discussion of the rationale for using fixed relays with multiple antennas in wireless relay networks, as well as an analysis of the performance of these systems, is presented in [1].

We now consider the extension of the system model to the case where there may be more than one physical antenna at each terminal. An expansion of terminology is required. Let A_i denote the set of antennas at terminal T_i, let A_{ij} denote the j^{th} antenna of terminal T_i, let L_i denote the set of inter-terminal antenna links associated with a particular terminal T_i or cut set S_i, let $T_{O(m)}$ denote the set of receiving terminals that have m incident inter-terminal antenna links (i.e., $|L_i| = \sum_{T_k \in T_{P(i)}} |A_k||A_i| = m, \forall T_i \in T_{O(m)}$), and let $S_{O(m)}$ denote the set of cut sets that are associated with m inter-terminal antenna links (i.e., $|L_i| = m, \forall S_i \in S_{O(m)}$). Furthermore, $\mu_{kl,ij} = SNR^{-1} \varepsilon_k / N_{kl,ij}$ is the SNR scaling factor with respect to the reference SNR, and $a_{kl,ij}$ and $z_{kl,ij}$ are respectively the composite signal amplitude attenuation factor and the additive white Gaussian noise random variable with variance $N_{kl,ij}$ associated with the link $L_{kl,ij} \in L_i$ between the l^{th} antenna of terminal T_k and the j^{th} antenna of terminal T_i. It is assumed that the total transmit power at each terminal is kept constant with respect to the single antenna case such that the set of transmit antennas equally partition the transmit power. With this revised terminology, the results when there may be more than one physical antenna at each terminal are straightforward generalizations of the results for one physical antenna at each terminal.

When there may be more than one physical antenna at each terminal, the transmitted complex baseband signal amplitude associated with the j^{th} antenna of terminal T_i is given by

$$s_{ij} = \sqrt{\varepsilon_i / |A_i|}(\alpha_i + \beta_i), \tag{38}$$

and the received complex baseband signal amplitude associated with the link between the l^{th} antenna of terminal T_k and the j^{th} antenna of terminal T_i is given by

$$r_{kl,ij} = a_{kl,ij}\sqrt{\varepsilon_k / |A_k|}(\alpha_k + \beta_k) + z_{kl,ij}. \tag{39}$$

The link signal to noise ratio received at the j^{th} antenna of terminal T_i as a result of the signal from the l^{th} antenna of terminal T_k is given by

$$\begin{aligned}\psi_{kl,ij} &= \frac{\varepsilon_k / |A_k|}{N_{kl,ij} / |a_{kl,ij}|^2} \\ &= SNR\mu_{kl,ij}|a_{kl,ij}|^2 / |A_k|\end{aligned}, \tag{40}$$

the aggregate signal to noise ratio received at the j^{th} antenna of terminal T_i as a result of the signal from the l^{th} antenna of terminal T_k is given by

$$\gamma_{kl,ij} = \frac{\varepsilon_k |\alpha_k|^2 / |A_k|}{\varepsilon_k E[|\beta_k|^2] / |A_k| + N_{kl,ij} / |a_{kl,ij}|^2}, \tag{41}$$

and the aggregate signal to noise ratio at T_i from all immediately preceding terminals is therefore given by

$$\gamma_{P(i),i} = \sum_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} \left[\frac{\varepsilon_k |\alpha_k|^2 / |A_k|}{\varepsilon_k E[|\beta_k|^2] / |A_k| + N_{kl,ij} / |a_{kl,ij}|^2}\right]. \tag{42}$$

Furthermore, let $P_e(\gamma_{P(i),i})$ generally denote the probability of decoding error at T_i for a given modulation scheme for incident SNR $\gamma_{P(i),i}$ and assuming maximal ratio combining. For example, for independent Rayleigh fading of the signals on each input branch (each separate inter-terminal antenna link) of the diversity combiner and BPSK modulation, when the different signal to noise ratio values are distinct the probability of decoding error is given by

$$P_{e,i}(\gamma_{P(i),i}) = \frac{1}{2}\left(\sum_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} \left[\prod_{\substack{T_m \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{mn} \in A_m \\ A_{mn} \neq A_{kl}}} \frac{\overline{\gamma_{kl,ij}}}{\overline{\gamma_{kl,ij}} - \overline{\gamma_{mn,ij}}}\left(1 - \sqrt{\frac{\overline{\gamma_{kl,ij}}}{2 + \overline{\gamma_{kl,ij}}}}\right)\right]\right), \tag{43}$$

where $\overline{\gamma_{kl,ij}}$ is the expected received signal to noise ratio at the j^{th} antenna of terminal T_i as a result of the signal from the l^{th} antenna of terminal T_k.

When the source and all relays share a common randomly generated iid circularly symmetric, complex Gaussian codebook, the mutual information between terminal T_i and all immediately preceding terminals $T_{P(i)}$ for incident SNR $\gamma_{P(i),i}$ is given by

$$I_i(\gamma_{P(i),i}) = \frac{1}{K}\log\left(1 + \sum_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} \gamma_{kl,ij}\right), \tag{44}$$

and the information theoretic probability of outage at terminal T_i is given by

$$P_{o,i}(\gamma_{P(i),i}, R) = \Pr\left[\sum_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} \gamma_{kl,ij} < 2^{KR} - 1\right], \tag{45}$$

which at high SNR can be approximated by

$$P_{o,i}(\gamma_{P(i),i}, R) \approx \left(\frac{1}{|L_i|!} \prod_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} \frac{|A_k|}{\mu_{kl,ij}\sigma^2_{kl,ij}}\right)\left(\frac{2^{KR}-1}{SNR}\right)^{|L_i|}. \tag{46}$$

When the source and all relays employ independent randomly generated iid circularly symmetric, complex Gaussian codebooks, the mutual information between terminal T_i and all immediately preceding terminals $T_{P(i)}$ for incident SNR $\gamma_{P(i),i}$ according to the method in [34] is given by

$$\begin{aligned} I_i(\gamma_{P(i),i}) &= \frac{1}{K}\log\left(\det\left[\underline{I}_{|T_{P(i)}|} + \underline{\gamma}_{P(i),i}\underline{\gamma}^{\mathrm{H}}_{P(i),i}\right]\right) \\ &\le \frac{1}{K}\sum_{\substack{T_k \in T_{P(i)} \\ A_{kl} \in A_k}} \log\left(1 + \sum_{A_{ij} \in A_i} \gamma_{kl,ij}\right) \\ &\le \frac{1}{K}\sum_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} \log\left(1 + \gamma_{kl,ij}\right) \end{aligned}, \tag{47}$$

where $\underline{I}_{|T_{P(i)}|}$ is the $|T_{P(i)}| \times |T_{P(i)}|$ identity matrix, $\underline{\gamma}_{P(i),i}$ is the matrix form of the incident SNR between terminal T_i and all immediately preceding terminals $T_{P(i)}$, and $\underline{\gamma}^{\mathrm{H}}_{P(i),i}$ is the Hermitian transpose of $\underline{\gamma}_{P(i),i}$. The information theoretic probability of outage at terminal T_i is therefore lower bounded by

$$P_{o,i}(\gamma_{P(i),i}, R) \geq \Pr\left[\prod_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A \\ A_{kl} \in A_k}} \left(1 + \gamma_{kl,ij}\right) < 2^{KR}\right], \tag{48}$$

which at high SNR can be approximated by

$$P_{o,i}(\gamma_{P(i),i}, R) \approx \left(\prod_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} \frac{|A_k|}{\mu_{kl,ij}\sigma^2_{kl,ij}}\right)\left(\frac{2^{KR/|L_i|} - 1}{SNR}\right)^{|L_i|}. \tag{49}$$

As noted in [34], the upper bound on mutual information in the second line of (47) corresponds to an artificial scenario where the signal components transmitted by each transmit antenna are received by the receive antennas with no interference from the other transmit antennas, in effect utilizing separate parallel channels for each transmit antenna. The third line of (47) is a further upper bound that similarly corresponds to the effective utilization of separate parallel channels for each combination of transmit and receive antennas. Although these scenarios are not realistic, they again result in tractable lower bounds on the probability of outage of more practical scenarios, and as noted previously do not affect the corresponding diversity order results that are our primary focus.

For amplified relaying, the amplification gain factor at each intermediate terminal T_i is still the transmitted power over the received power and is given by

$$G_i = \frac{\varepsilon_i}{\sum_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} \left(\left|a_{kl,ij}\right|^2 \varepsilon_k \big/ \left|A_k\right| + N_{kl,ij}\right)}, \tag{50}$$

the aggregate SNR at T_i for the set of preceding terminals $T_k \in T_{P(i)}$ is given recursively by

$$\gamma_{P(i),i} \approx \sum_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} (\psi_{kl,ij}^{-1} + \gamma_{P(k),k}^{-1} + \psi_{kl,ij}^{-1}\gamma_{P(k),k}^{-1})^{-1}, \tag{51}$$

the total probability of decoding error is given by

$$P_e = P_{e,D}(\gamma_{P(D),D}), \tag{52}$$

and the total information theoretic probability of outage is given by

$$P_o = P_{o,D}(\gamma_{P(D),D}, R). \tag{53}$$

For decoded relaying with error propagation, the aggregate SNR at T_i when all preceding terminals have decoded correctly is given by

$$\gamma_{P(i),i} = \sum_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} (\psi_{kl,ij}), \tag{54}$$

the total probability of decoding error remains given by

$$P_e = 1 - \prod_{T_i \in T_R} (1 - P_{e,i}(\gamma_{P(i),i})), \tag{55}$$

and the total information theoretic probability of outage is given by

$$P_o = 1 - \prod_{T_i \in T_R} (1 - P_{o,i}(\gamma_{P(i),i}, R)). \tag{56}$$

For decoded relaying without error propagation, the aggregate SNR at T_i for the set of preceding terminals that have decoded correctly is given by

$$\gamma_{C(i),i} = \sum_{\substack{T_k \in T_{C(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} (\psi_{kl,ij}), \tag{57}$$

the total probability of decoding error remains lower bounded by

$$P_e \geq \sum_{T_C \in T_I} \left[P_{e,D}(\gamma_{C(D),D}) \prod_{\substack{T_j \in T_I \\ T_j \notin T_C}} P_{e,j}(\gamma_{C(j),j}) \prod_{\substack{T_k \in T_I \\ T_k \in T_C}} (1 - P_{e,k}(\gamma_{C(k),k})) \right], \qquad (58)$$

and the total information theoretic probability of outage remains lower bounded by

$$P_o = \geq \sum_{T_C \in T_I} \left[P_{o,D}(\gamma_{C(D),D}, R) \prod_{\substack{T_j \in T_I \\ T_j \notin T_C}} P_{o,j}(\gamma_{C(j),j}, R) \prod_{\substack{T_k \in T_I \\ T_k \in T_C}} (1 - P_{o,k}(\gamma_{C(k),k}, R)) \right]. \qquad (59)$$

3.9 Summary

This chapter has developed a general system model for arbitrarily connected wireless relay networks employing various relaying methods. These expressions provide a method for analyzing the impact of varying the link connectivity or power allocation for a given set of terminals, and support the comparison of different relaying methods. Probability of error and information theoretic probability of outage results are presented for arbitrarily connected wireless relay networks that employ amplified relaying, decoded relaying with error propagation, and decoded relaying without error propagation. The developed expressions are applicable for a given set of source, destination, and relaying terminals with any number of antennas, link connectivity, link attenuation, transmit power, and receiver noise, and can be used to extend many of the traditional two-hop cooperative diversity results to more than two hops, arbitrary connectivity, and the possibility of more than one antenna per terminal. These expressions are applied in the maximum diversity order analysis of Chapter 4 and the cooperative connectivity model simulations of Chapter 6.

Chapter 4 - Maximum Diversity Order Analysis

This chapter presents the developed maximum diversity order analysis for networks with comprehensive or destination decoding. Section 4.1 introduces the maximum diversity order analysis. Sections 4.2 and 4.3 respectively derive bounds on the maximum achievable diversity order of wireless relay networks employing common codebook generation, and independent codebook generation, with comprehensive or destination decoding. Section 4.4 extends the maximum diversity order analysis to the case where there may be multiple antennas per terminal. Section 4.5 derives bounds on the diversity-multiplexing tradeoff of wireless relay networks with comprehensive and destination decoding. Section 4.6 comments on the complexity of algorithms for minimization of the diversity order or diversity-multiplexing tradeoff across all terminals or cut sets in the network. Section 4.7 summarizes the contributions of the chapter.

4.1 Introduction

Full link connectivity between all cooperating terminals in wireless relay networks employing multiple half-duplex relays will generally not be implemented in practice due to excessive channel allocation requirements. Therefore, an important area of analysis is the maximum achievable diversity order of wireless relay networks with arbitrary, but generally less than full, cooperative connectivity. This section derives bounds on the maximum achievable diversity order and high signal to noise ratio (SNR) probability of outage of wireless relay networks with arbitrary link connectivity between cooperating terminals. Two classes of relaying method are considered, those requiring all cooperating terminals to correctly decode the transmitted information signal in order for the destination to correctly decode (comprehensive decoding), and those requiring only a subset of cooperating terminals to correctly decode the transmitted information signal in order for the destination terminal to correctly decode (destination decoding). For the destination decoding class, individual relays may attempt to decode the information signal before retransmission, but the key point is that only a subset are required to do so correctly in order for the destination to correctly decode.

The comprehensive decoding class includes decoded relaying with error propagation, also known as fixed decode-and-forward (DF) relaying. The destination decoding class includes decoded relaying without error propagation, also known as adaptive or selective decode-and-forward (DF) relaying, and amplified relaying, also known as amplify-and-forward (AF) relaying. From the perspective of diversity order, this classification of comprehensive decoding versus destination decoding is more fundamental than that of decode-and-forward versus amplify-and-forward relaying. As noted in many existing publications, the maximum achievable diversity order is not dependent on the physical layer relaying method (amplified or decoded), but instead on the criteria for retransmission at each relay and the presence of error propagation. In general, this dissertation focuses on physical layer cooperative diversity techniques, and does not explicitly consider recently proposed network level cooperative diversity techniques, for example the opportunistic relaying protocol proposed in [11]. However, it is expected that the analysis of this protocol would be similar to the analysis of networks with destination decoding, as at a high level the behavior is the same as the selective decode-and-forward protocol when only a single relay correctly decodes the source transmission.

Previous work has derived the diversity order for specific combinations of cooperative connectivity and relaying method. It is shown in [6] and [63] that the diversity order of a relay network with *N-1* relays connected to the source and destination in parallel with the source and destination also connected is *N* for both decoded relaying without error propagation and amplified relaying. It is shown in [33] that the diversity order of a relay network with *K* relays connected to the source and destination in parallel, *N* antennas distributed across the relays, and leveraging transmit beam-forming techniques, is *N*. It is shown in [64] that the diversity order of a fully connected relay network with one relay is one for decoded relaying with error propagation, and two for decoded relaying without error propagation and amplified relaying. It is shown in [125] that the diversity order of a fully connected relay network with two relays is three for amplified relaying. It is shown in [126] that the maximum diversity order of a fully connected relay network with *N-1* relays is *N*. This dissertation presents a more general formulation applicable to cooperative networks with any number of relay terminals and any possible combination of links between cooperating terminals.

Although the results for networks with comprehensive decoding may appear quite intuitive, there is value in a more formal derivation that provides definitive results for any number of relay terminals and any possible combination of links between cooperating terminals. The high SNR outage probability is a more general intermediate result that is less intuitive. Additionally, the results are applicable not only to the decode-and-forward protocol that is generally understood to offer inferior diversity performance to other cooperative diversity protocols, but also to cooperative broadcasting protocols that by definition require all cooperating terminals to correctly decode. Finally, it is quite interesting to note the parallels of the formulations based on mutual information across all terminals and mutual information across all cut sets for comprehensive decoding and destination decoding respectively.

4.2 Common Codebook Generation

This section derives bounds on the maximum achievable diversity order and high SNR probability of outage of wireless relay networks employing common codebook generation with comprehensive or destination decoding. This form of codebook generation involves the source and all relays sharing a common randomly generated iid circularly symmetric, complex Gaussian codebook, and encompasses practical combination schemes that do not require a separate orthogonal channel for each inter-terminal link, including repetition coding and space-time coding.

4.2.1 Networks with Comprehensive Decoding

The probability of outage of relay networks employing relaying methods with comprehensive decoding is the probability of outage at any cooperating terminal in the network, since an outage event at any cooperating terminal will result in an outage event at the destination. Since outage events at different cooperating terminals are independent, the total probability of outage at any cooperating terminal in the network is given by

$$P_o(SNR, R) = 1 - \prod_{T_i \in T_R} \left(1 - \Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i} \left| a_{k,i} \right|^2 < \frac{2^{KR} - 1}{SNR} \right] \right), \tag{60}$$

which when expanded to show all possible terminal outage event combinations is expressed as

$$\begin{aligned}P_o(SNR,R) = &\sum_{T_i \in T_R}\left(\Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR}-1}{SNR}\right]\right)\\ &-\sum_{\substack{T_i,T_j \in T_R\\ T_i \neq T_j}}\left(\Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR}-1}{SNR}\right]\Pr\left[\sum_{T_k \in T_{P(j)}} \mu_{k,j}\left|a_{k,j}\right|^2 < \frac{2^{KR}-1}{SNR}\right]\right) \quad .(61)\\ &+\sum \textit{other terms with an odd \# of outage events} - \sum \textit{other terms with an even \# of outage events}\end{aligned}$$

This can be further expanded to separate out terms involving terminals with the minimum number of immediately preceding terminals M_T, and taken to the limit as $SNR \to \infty$ to result in

$$\begin{aligned}\frac{P_o(SNR,R)}{\left(\frac{2^{KR}-1}{SNR}\right)^{M_T}} = &\left(\begin{aligned}&\sum_{\substack{T_i \in T_R\\ T_i \in T_{O(M_T)}}}\left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_T}\Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to (M_T!)^{-1}\prod_{T_k \in T_{P(i)}}\left(\mu_{k,i}\sigma_{k,i}^2\right)^{-1}}\right)\\ &+\sum_{\substack{T_i \in T_R\\ T_i \notin T_{O(M_T)}}}\left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_T}\Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0}\right)\end{aligned}\right)\\ &-\left(\begin{aligned}&\sum_{\substack{T_i,T_j \in T_R\\ T_i \neq T_j\\ T_i \in T_{O(M_T)}}}\left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_T}\Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to (M_T!)^{-1}\prod_{T_k \in T_{P(i)}}\left(\mu_{k,i}\sigma_{k,i}^2\right)^{-1}}\underbrace{\Pr\left[\sum_{T_k \in T_{P(j)}} \mu_{k,j}\left|a_{k,j}\right|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0}\right)\\ &+\sum_{\substack{T_i,T_j \in T_R\\ T_i \neq T_j\\ T_i \notin T_{O(M_T)}}}\left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_T}\Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0}\underbrace{\Pr\left[\sum_{T_k \in T_{P(j)}} \mu_{k,j}\left|a_{k,j}\right|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0}\right)\end{aligned}\right), (62)\\ &+\sum \underbrace{\textit{other terms with an odd \# of outage events}}_{\to 0} - \sum \underbrace{\textit{other terms with an even \# of outage events}}_{\to 0}\\ &\to \sum_{\substack{T_i \in T_R\\ T_i \in T_{O(M_T)}}}\left(\frac{1}{M_T!}\prod_{T_k \in T_{P(i)}}\frac{1}{\mu_{k,i}\sigma_{k,i}^2}\right)\end{aligned}$$

where the asymptotic approximation uses CDF results for the sum of independent exponential random variables [63].

The probability of outage at high SNR can therefore be approximated by

$$P_o(SNR,R) \approx \sum_{\substack{T_i \in T_R \\ T_i \in T_{O(M_T)}}} \left(\frac{1}{M_T!} \prod_{T_k \in T_{P(i)}} \frac{1}{\mu_{k,i}\sigma_{k,i}^2} \right) \left(\frac{2^{KR}-1}{SNR} \right)^{M_T}, \tag{63}$$

and the maximum diversity order [133] of the network is given by

$$d_{\max} = -\lim_{SNR \to \infty} \frac{\log P_o(SNR,R)}{\log SNR} = M_T = \min_{T_i \in T_R} \left\{ \left| T_{P(i)} \right| \right\}, \tag{64}$$

the minimum number of immediately preceding terminals across all terminals in the network. Since at least one relay receives only from the source, the maximum diversity order of the network is $M_T = 1$. Furthermore, inter-terminal links that are not associated with terminals with the minimum number of immediately preceding terminals do not asymptotically (at high SNR) affect the probability of outage. Fig. 12 shows an example network, annotated with the achievable diversity order of each terminal.

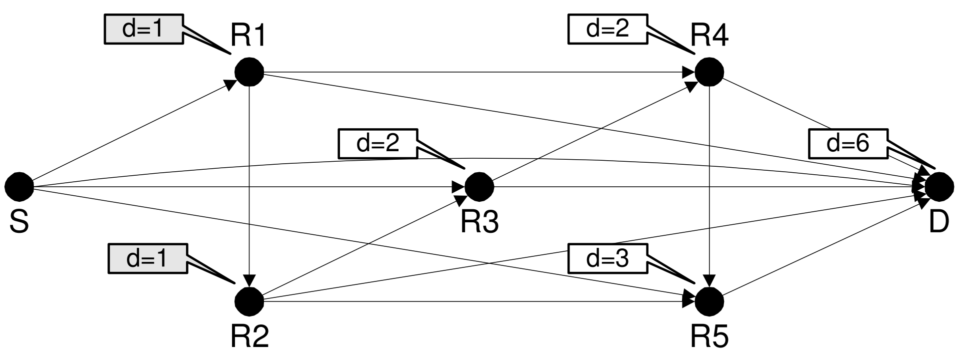

Fig. 12. Diversity Order of Example Network with Comprehensive Decoding

4.2.2 Networks with Destination Decoding

The probability of outage of relay networks employing relaying methods with destination decoding is the probability of outage at any cut set in the network, since an outage event at any cut set will result in an outage event at the destination. The mutual information at cut set S_i (across all the inter-terminal links associated with cut set S_i) is given by

$$I_i = \frac{1}{K}\log\left(1 + SNR \sum_{L_{k,l}\in L_i} \mu_{k,l}\left|a_{k,l}\right|^2\right), \tag{65}$$

and the probability of outage at S_i is the probability of the mutual information falling below a target rate R, and is given by

$$P_{o,i}(SNR,R) = \Pr\left[\sum_{L_{k,l}\in L_i} \mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right]. \tag{66}$$

Since outage events at different cut sets are not necessarily independent due to the possibility of shared inter-terminal links, the total probability of outage at any cut set in the network is given by

$$P_o(SNR,R) = \Pr\left[\bigcup_{S_i\in S_R}\left(\sum_{L_{k,l}\in L_i} \mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right)\right], \tag{67}$$

which when expanded to show all possible cut set outage event combinations is expressed as

$$\begin{aligned}
P_o(SNR,R) = &\sum_{S_i\in S_R}\left(\Pr\left[\sum_{L_{k,l}\in L_i} \mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right]\right) \\
&- \sum_{\substack{S_i,S_j\in S_R\\ S_i\neq S_j}}\left(\begin{array}{l}\Pr\left[\sum_{L_{k,l}\in L_i} \mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right] \\ \times\Pr\left[\left(\sum_{L_{k,l}\in L_j} \mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right)\Big|\left(\sum_{L_{k,l}\in L_i} \mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right)\right]\end{array}\right) \\
&+ \sum \textit{other terms with an odd \# of outage events} - \sum \textit{other terms with an even \# of outage events}
\end{aligned} \tag{68}$$

This can be further expanded to separate out terms involving cut sets with the minimum number of inter-terminal links M_S, and taken to the limit as $SNR \to \infty$ to result in

$$\frac{P_o(SNR,R)}{\left(\frac{2^{KR}-1}{SNR}\right)^{M_S}} = \left(\begin{array}{l} \sum\limits_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_S} \Pr\left[\sum\limits_{L_{k,l}\in L_i} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to (M_S!)^{-1} \prod_{L_{k,l}\in L_i} (\mu_{k,l}\sigma_{k,l}^2)^{-1}} \right) \\ + \sum\limits_{\substack{S_i \in S_R \\ S_i \notin S_{O(M_S)}}} \left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_S} \Pr\left[\sum\limits_{L_{k,l}\in L_i} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0} \right) \end{array} \right)$$
$$- \left(\begin{array}{l} \sum\limits_{\substack{S_i,S_j \in S_R \\ S_i \neq S_j \\ S_i \in S_{O(M_S)}}} \left(\begin{array}{l} \underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_S} \Pr\left[\sum\limits_{L_{k,l}\in L_i} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to (M_S!)^{-1} \prod_{L_{k,l}\in L_i} (\mu_{k,l}\sigma_{k,l}^2)^{-1}} \\ \times \underbrace{\Pr\left[\left(\sum\limits_{L_{k,l}\in L_j} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right) | \left(\sum\limits_{L_{k,l}\in L_i} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right)\right]}_{\to 0} \end{array} \right) \\ + \sum\limits_{\substack{S_i,S_j \in S_R \\ S_i \neq S_j \\ S_i \notin S_{O(M_S)}}} \left(\begin{array}{l} \underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_S} \Pr\left[\sum\limits_{L_{k,l}\in L_i} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0} \\ \times \underbrace{\Pr\left[\left(\sum\limits_{L_{k,l}\in L_j} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right) | \left(\sum\limits_{L_{k,l}\in L_i} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right)\right]}_{\to 0} \end{array} \right) \end{array} \right), \quad (69)$$
$$+ \sum \underbrace{other\ terms\ with\ an\ odd\ \#\ of\ outage\ events}_{\to 0} - \sum \underbrace{other\ terms\ with\ an\ even\ \#\ of\ outage\ events}_{\to 0}$$
$$\to \sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\frac{1}{M_S!} \prod_{L_{k,l}\in L_i} \frac{1}{\mu_{k,l}\sigma_{k,l}^2} \right)$$

where the asymptotic approximation again uses CDF results for the sum of independent exponential random variables [63].

The probability of outage at high SNR can therefore be approximated by

$$P_o(SNR,R) \approx \sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\frac{1}{M_S!} \prod_{L_{k,l}\in L_i} \frac{1}{\mu_{k,l}\sigma_{k,l}^2} \right) \left(\frac{2^{KR}-1}{SNR} \right)^{M_S}, \quad (70)$$

and the maximum diversity order of the network is given by

$$d_{\max} = -\lim_{SNR \to \infty} \frac{\log P_o(SNR, R)}{\log SNR} = M_S = \min_{S_i \in S_R} \{ |L_i| \}, \quad (71)$$

the minimum number of inter-terminal links across all cut sets in the network. This is equivalent to the number of disjoint paths through the network joining the source and destination. Furthermore, inter-terminal links that are not associated with cut sets with the minimum number of inter-terminal links do not asymptotically (at high SNR) affect the probability of outage. Fig. 13 shows an example network, annotated with the achievable diversity order of each cut set that is relevant given the directed connectivity. We note that [126] and [128] independently use a similar cut set bound argument, although the approach taken is different and is limited to fully connected relay networks.

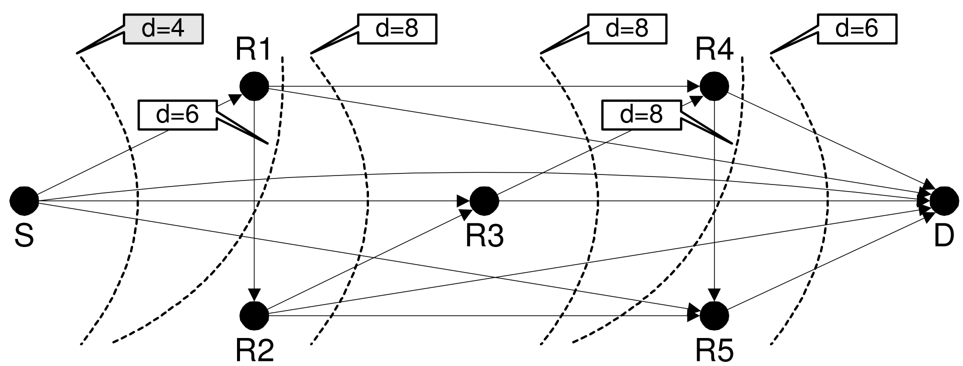

Fig. 13. Diversity Order of Example Network with Destination Decoding

4.3 Independent Codebook Generation

This section derives bounds on the maximum achievable diversity order and high SNR probability of outage of wireless relay networks employing independent codebook generation with comprehensive or destination decoding. This form of codebook generation involves the source and all relays employing independent randomly generated iid circularly symmetric, complex Gaussian codebooks, and provides an information theoretic lower bound on the probability of outage of all achievable codebook generation schemes.

4.3.1 Networks with Comprehensive Decoding

The probability of outage of relay networks employing relaying methods with comprehensive decoding is the probability of outage at any cooperating terminal in the network. Since outage events at different cooperating terminals are independent, the total probability of outage at any cooperating terminal in the network is lower bounded by

$$P_o(SNR,R) \geq 1 - \prod_{T_i \in T_R}\left(1 - \Pr\left[\prod_{T_k \in T_{P(i)}}\left(1 + SNR\mu_{k,i}\left|a_{k,i}\right|^2\right) < 2^{KR}\right]\right), \tag{72}$$

which when expanded to show all possible terminal outage event combinations and using the lower bound of (21) is expressed as

$$\begin{aligned} P_o(SNR,R) \geq & \sum_{T_i \in T_R}\left(\prod_{T_k \in T_{P(i)}} \Pr\left[\mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR/\left|T_{P(i)}\right|}-1}{SNR}\right]\right) \\ & - \sum_{\substack{T_i,T_j \in T_R \\ T_i \neq T_j}}\left(\begin{array}{l} \displaystyle\prod_{T_k \in T_{P(i)}} \Pr\left[\mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR/\left|T_{P(i)}\right|}-1}{SNR}\right] \\ \displaystyle\times \prod_{T_k \in T_{P(j)}} \Pr\left[\mu_{k,j}\left|a_{k,j}\right|^2 < \frac{2^{KR/\left|T_{P(j)}\right|}-1}{SNR}\right] \end{array}\right) \\ & + \sum \textit{other terms with an odd \# of outage events} - \sum \textit{other terms with an even \# of outage events} \end{aligned} \tag{73}$$

This can be further expanded to separate out terms involving terminals with the minimum number of immediately preceding terminals M_T, and taken to the limit as $SNR \to \infty$ to result in

$$\frac{P_o(SNR,R)}{\left(\frac{2^{KR/M_T}-1}{SNR}\right)^{M_T}} \ge \left(\begin{array}{l} \displaystyle\sum_{\substack{T_i\in T_R \\ T_i\in T_{O(M_T)}}} \left(\underbrace{\left(\frac{2^{KR/M_T}-1}{SNR}\right)^{-M_T} \prod_{T_k\in T_{P(i)}} \Pr\left[\mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR/\left|T_{P(i)}\right|}-1}{SNR}\right]}_{\to \prod_{T_k\in T_{P(i)}}\left(\mu_{k,i}\sigma_{k,i}^2\right)^{-1}} \right) \\ \displaystyle + \sum_{\substack{T_i\in T_R \\ T_i\notin T_{O(M_T)}}} \left(\underbrace{\left(\frac{2^{KR/M_T}-1}{SNR}\right)^{-M_T} \prod_{T_k\in T_{P(i)}} \Pr\left[\mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR/\left|T_{P(i)}\right|}-1}{SNR}\right]}_{\to 0} \right) \end{array} \right)$$

$$- \left(\begin{array}{l} \displaystyle\sum_{\substack{T_i,T_j\in T_R \\ T_i\neq T_j \\ T_i\in T_{O(M_T)}}} \left(\begin{array}{l} \underbrace{\left(\frac{2^{KR/M_T}-1}{SNR}\right)^{-M_T} \prod_{T_k\in T_{P(i)}} \Pr\left[\mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR/\left|T_{P(i)}\right|}-1}{SNR}\right]}_{\to \prod_{T_k\in T_{P(i)}}\left(\mu_{k,i}\sigma_{k,i}^2\right)^{-1}} \\ \times \underbrace{\prod_{T_k\in T_{P(j)}} \Pr\left[\mu_{k,j}\left|a_{k,j}\right|^2 < \frac{2^{KR/\left|T_{P(j)}\right|}-1}{SNR}\right]}_{\to 0} \end{array} \right) \\ \displaystyle + \sum_{\substack{T_i,T_j\in T_R \\ T_i\neq T_j \\ T_i\notin T_{O(M_T)}}} \left(\begin{array}{l} \underbrace{\left(\frac{2^{KR/M_T}-1}{SNR}\right)^{-M_T} \prod_{T_k\in T_{P(i)}} \Pr\left[\mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR/\left|T_{P(i)}\right|}-1}{SNR}\right]}_{\to 0} \\ \times \underbrace{\prod_{T_k\in T_{P(j)}} \Pr\left[\mu_{k,j}\left|a_{k,j}\right|^2 < \frac{2^{KR/\left|T_{P(j)}\right|}-1}{SNR}\right]}_{\to 0} \end{array} \right) \end{array} \right)$$

$$+ \sum \underbrace{other\ terms\ with\ an\ odd\ \#\ of\ outage\ events}_{\to 0} - \sum \underbrace{other\ terms\ with\ an\ even\ \#\ of\ outage\ events}_{\to 0}$$

$$\to \sum_{\substack{T_i\in T_R \\ T_i\in T_{O(M_T)}}} \left(\prod_{T_k\in T_{P(i)}} \frac{1}{\mu_{k,i}\sigma_{k,i}^2} \right) \quad , \quad (74)$$

where the asymptotic approximation uses CDF results for exponential random variables [63].

The probability of outage at high SNR can therefore be approximated by

$$P_o(SNR,R) \approx \sum_{\substack{T_i\in T_R \\ T_i\in T_{O(M_T)}}} \left(\prod_{T_k\in T_{P(i)}} \frac{1}{\mu_{k,i}\sigma_{k,i}^2} \right) \left(\frac{2^{KR/M_T}-1}{SNR} \right)^{M_T}, \quad (75)$$

and the maximum diversity order [133] of the network is given by

$$d_{\max} = -\lim_{SNR\to\infty} \frac{\log P_o(SNR,R)}{\log SNR} = M_T = \min_{T_i \in T_R} \left\{ \left| T_{P(i)} \right| \right\}, \tag{76}$$

the minimum number of immediately preceding terminals across all terminals in the network. We see that the maximum diversity order of independent codebook generation is identical to that of common codebook generation. This result indicates that for networks with comprehensive decoding, independent codebook generation does not offer any diversity gain over common codebook generation.

The high SNR probability of outage gain factor of independent codebook generation over common codebook generation is upper bounded by

$$\begin{aligned} \frac{P_{o,C}(SNR,R)}{P_{o,I}(SNR,R)} &\leq \frac{\sum\limits_{\substack{T_i \in T_R \\ T_i \in T_{O(M_T)}}} \left(\frac{1}{M_T!} \prod\limits_{T_k \in T_{P(i)}} \frac{1}{\mu_{k,i}\sigma_{k,i}^2} \right) \left(\frac{2^{K_C R} - 1}{SNR} \right)^{M_T}}{\sum\limits_{\substack{T_i \in T_R \\ T_i \in T_{O(M_T)}}} \left(\prod\limits_{T_k \in T_{P(i)}} \frac{1}{\mu_{k,i}\sigma_{k,i}^2} \right) \left(\frac{2^{K_I R/M_T} - 1}{SNR} \right)^{M_T}} \\ &\leq \frac{1}{M_T!} \left(\frac{2^{K_C R} - 1}{2^{K_I R/M_T} - 1} \right)^{M_T} \\ &\leq \frac{2^{K_C R} - 1}{2^{K_I R} - 1} \end{aligned}, \tag{77}$$

where K_C is the number of channels required for common codebook generation and K_I is the number of channels required for independent codebook generation. The last simplification results from the fact that for networks with comprehensive decoding $M_T = 1$. It is interesting to note that if independent codebook generation requires more channels than common codebook generation it will result in an increase in the probability of outage. If the same number of channels is required for common and independent codebook generation then the probability of outage gain factor is equal to unity.

4.3.2 Networks with Destination Decoding

The probability of outage of relay networks employing relaying methods with destination decoding is the probability of outage at any cut set in the network. The mutual information at cut set S_i (across all the inter-terminal links associated with cut set S_i) is upper bounded by

$$\begin{aligned} I_i &\leq \frac{1}{K} \sum_{L_{k,l} \in L_i} \log\left(1 + SNR\mu_{k,l} |a_{k,l}|^2\right) \\ &\leq \frac{1}{K} \log \prod_{L_{k,l} \in L_i} \left(1 + SNR\mu_{k,l} |a_{k,l}|^2\right) \end{aligned}, \tag{78}$$

and the probability of outage at S_i is the probability of the mutual information falling below a target rate R, and is lower bounded by

$$P_{o,i}(SNR, R) \geq \Pr\left[\prod_{L_{k,l} \in L_i} \left(1 + SNR\mu_{k,l} |a_{k,l}|^2\right) < 2^{KR}\right]. \tag{79}$$

This can be further lower bounded by

$$\begin{aligned} P_{o,i}(SNR, R) &\geq \prod_{L_{k,l} \in L_i} \Pr\left[\left(1 + SNR\mu_{k,l} |a_{k,l}|^2\right) < 2^{KR/|L_i|}\right] \\ &\geq \prod_{L_{k,l} \in L_i} \Pr\left[\mu_{k,l} |a_{k,l}|^2 < \frac{2^{KR/|L_i|} - 1}{SNR}\right] \end{aligned}, \tag{80}$$

where the lower bound again results from applying the relation $\Pr\left[\prod_k x_k < \prod_k y_k\right] \geq \prod_k \Pr[x_k < y_k]$ for independent x_k and y_k.

Since outage events at different cut sets are not necessarily independent due to the possibility of shared inter-terminal links, the total probability of outage at any cut set in the network is given by

$$P_o(SNR,R) = \Pr\left[\bigcup_{S_i \in S_R}\left(\prod_{L_{k,l} \in L_i}\left(1 + SNR\mu_{k,l}\left|a_{k,l}\right|^2\right) < 2^{KR}\right)\right], \tag{81}$$

which when expanded to show all possible cut set outage event combinations and using the lower bound of (80) is expressed as

$$\begin{aligned} P_o(SNR,R) \geq & \sum_{S_i \in S_R}\left(\prod_{L_{k,l} \in L_i} \Pr\left[\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right]\right) \\ & - \sum_{\substack{S_i, S_j \in S_R \\ S_i \neq S_j}}\left(\begin{array}{l} \prod_{L_{k,l} \in L_i} \Pr\left[\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right] \\ \times \Pr\left[\bigcap_{L_{k,l} \in L_j}\left(\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR/|L_j|}-1}{SNR}\right) \mid \bigcap_{L_{k,l} \in L_i}\left(\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right)\right] \end{array}\right) \\ & + \sum \textit{other terms with an odd \# of outage events} - \sum \textit{other terms with an even \# of outage events} \end{aligned} \tag{82}$$

This can be further expanded to separate out terms involving cut sets with the minimum number of inter-terminal links M_S, and taken to the limit as $SNR \rightarrow \infty$ to result in

$$\frac{P_o(SNR,R)}{\left(\frac{2^{KR/M_S}-1}{SNR}\right)^{-M_S}} \geq \left(\begin{array}{l} \sum\limits_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\underbrace{\left(\frac{2^{KR/M_S}-1}{SNR}\right)^{-M_S} \prod\limits_{L_{k,l}\in L_i} \Pr\left[\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right]}_{\to \prod_{L_{k,l}\in L_i}\left(\mu_{k,l}\sigma_{k,l}^2\right)^{-1}} \right) \\ + \sum\limits_{\substack{S_i \in S_R \\ S_i \notin S_{O(M_S)}}} \left(\underbrace{\left(\frac{2^{KR/M_S}-1}{SNR}\right)^{-M_S} \prod\limits_{L_{k,l}\in L_i} \Pr\left[\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right]}_{\to 0} \right) \end{array} \right)$$

$$- \left(\begin{array}{l} \sum\limits_{\substack{S_i,S_j \in S_R \\ S_i \neq S_j \\ S_i \in S_{O(M_S)}}} \left(\begin{array}{l} \underbrace{\left(\frac{2^{KR/M_S}-1}{SNR}\right)^{-M_S} \prod\limits_{L_{k,l}\in L_i} \Pr\left[\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right]}_{\to \prod_{L_{k,l}\in L_i}\left(\mu_{k,l}\sigma_{k,l}^2\right)^{-1}} \\ \underbrace{\times \Pr\left[\bigcap\limits_{L_{k,l}\in L_j}\left(\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR/|L_j|}-1}{SNR}\right) \,\Big|\, \bigcap\limits_{L_{k,l}\in L_i}\left(\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right)\right]}_{\to 0} \end{array} \right) \\ + \sum\limits_{\substack{S_i,S_j \in S_R \\ S_i \neq S_j \\ S_i \notin S_{O(M_S)}}} \left(\begin{array}{l} \underbrace{\left(\frac{2^{KR/M_S}-1}{SNR}\right)^{-M_S} \prod\limits_{L_{k,l}\in L_i} \Pr\left[\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right]}_{\to 0} \\ \underbrace{\times \Pr\left[\bigcap\limits_{L_{k,l}\in L_j}\left(\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR/|L_j|}-1}{SNR}\right) \,\Big|\, \bigcap\limits_{L_{k,l}\in L_i}\left(\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR/|L_i|}-1}{SNR}\right)\right]}_{\to 0} \end{array} \right) \end{array} \right), \quad (83)$$

$$+ \sum \underbrace{\text{other terms with an odd \# of outage events}}_{\to 0} - \sum \underbrace{\text{other terms with an even \# of outage events}}_{\to 0}$$

$$\to \sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\prod_{L_{k,l}\in L_i} \frac{1}{\mu_{k,l}\sigma_{k,l}^2} \right)$$

where the asymptotic approximation again uses CDF results for the sum of independent exponential random variables [63].

The probability of outage at high SNR can therefore be approximated by

$$P_o(SNR,R) \approx \sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\prod_{L_{k,l}\in L_i} \frac{1}{\mu_{k,l}\sigma_{k,l}^2} \right) \left(\frac{2^{KR/M_S}-1}{SNR}\right)^{M_S}, \quad (84)$$

and the maximum diversity order of the network is given by

$$d_{\max} = -\lim_{SNR\to\infty} \frac{\log P_o(SNR,R)}{\log SNR} = M_S = \min_{S_i \in S_R}\{|L_i|\}, \quad (85)$$

the minimum number of inter-terminal links across all cut sets in the network. Again, we see that the maximum diversity order of independent codebook generation is identical to that of common codebook generation. This result indicates that for networks with destination decoding, independent codebook generation does not offer any diversity gain over common codebook generation. This is a generalization of the similar result in [65] to cooperative networks with any number of relay terminals and any possible combination of links between cooperating terminals.

However, there is a clear difference in the probability of outage at high SNR of independent codebook generation in comparison to that of common codebook generation. The high SNR probability of outage gain factor of independent codebook generation over common codebook generation is upper bounded by

$$\begin{aligned}\frac{P_{o,C}(SNR,R)}{P_{o,I}(SNR,R)} &\leq \frac{\sum\limits_{\substack{S_i\in S_R \\ S_i\in S_{O(M_S)}}} \left(\frac{1}{M_S!}\prod\limits_{L_{k,l}\in L_i}\frac{1}{\mu_{k,l}\sigma_{k,l}^2}\right)\left(\frac{2^{K_C R}-1}{SNR}\right)^{M_S}}{\sum\limits_{\substack{S_i\in S_R \\ S_i\in S_{O(M_S)}}} \left(\prod\limits_{L_{k,l}\in L_i}\frac{1}{\mu_{k,l}\sigma_{k,l}^2}\right)\left(\frac{2^{K_I R/M_S}-1}{SNR}\right)^{M_S}}, \\ &\leq \frac{1}{M_S!}\left(\frac{2^{K_C R}-1}{2^{K_I R/M_S}-1}\right)^{M_S}\end{aligned} \tag{86}$$

where again K_C is the number of channels required for common codebook generation and K_I is the number of channels required for independent codebook generation. The high SNR probability of outage gain factor indicates that independent codebook generation can offer a significant probability of outage improvement over common codebook generation, but only under the conditions that the minimum number of inter-terminal links across all cut sets in the network is high and the number of channels required for independent codebook generation is not significantly larger than the number of channels required for common codebook generation. If independent codebook generation requires more channels than common codebook generation and the diversity order is low, then the high SNR probability of outage gain factor may be less than unity (increased outage). If the same number of channels is required for both independent codebook generation and common codebook generation then the high SNR probability of outage gain factor will always be greater than unity (decreased outage).

4.4 Extension to Terminals with Multiple Antennas

As noted previously, wireless relay networks where some or all of the cooperating terminals employ multiple antennas are expected to be of increasing interest. We now consider the extension of the maximum diversity order analysis to the case where there may be more than one physical antenna at each terminal. When there are an arbitrary number of physical antennas per terminal, the process for calculating the maximum achievable diversity order is identical to that with a single antenna per terminal with the exception that the calculation is over the links between all pairs of antennas.

4.4.1 Networks with Comprehensive Decoding

For networks with common codebook generation and comprehensive decoding the probability of outage at high SNR can be approximated by

$$P_o(SNR,R) \approx \sum_{\substack{T_i \in T_R \\ T_i \in T_{O(M_T)}}} \left(\frac{1}{M_T!} \prod_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} \frac{|A_k|}{\mu_{kl,ij}\sigma^2_{kl,ij}} \right) \left(\frac{2^{KR}-1}{SNR} \right)^{M_T}, \tag{87}$$

and the maximum diversity order of the network is given by

$$d_{\max} = -\lim_{SNR\to\infty} \frac{\log P_o(SNR,R)}{\log SNR} = M_T = \min_{T_i \in T_R} \left\{ \sum_{T_k \in T_{P(i)}} |A_k| \|A_i| \right\}, \tag{88}$$

where M_T is the minimum number of incident inter-terminal antenna links across all terminals.

For networks with independent codebook generation and comprehensive decoding the probability of outage at high SNR can be approximated by

$$P_o(SNR,R) \approx \sum_{\substack{T_i \in T_R \\ T_i \in T_{O(M_T)}}} \left(\prod_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} \frac{|A_k|}{\mu_{kl,ij} \sigma_{kl,ij}^2} \right) \left(\frac{2^{KR/M_T} - 1}{SNR} \right)^{M_T}, \tag{89}$$

and the maximum diversity order of the network is given by

$$d_{\max} = -\lim_{SNR\to\infty} \frac{\log P_o(SNR,R)}{\log SNR} = M_T = \min_{T_i \in T_R} \left\{ \sum_{T_k \in T_{P(i)}} |A_k| \|A_i| \right\}, \tag{90}$$

where M_T is the minimum number of incident inter-terminal antenna links across all terminals. These results indicate that independent codebook generation does not offer any diversity gain over common codebook generation for networks with comprehensive decoding even when there are an arbitrary number of physical antennas per terminal.

The high SNR probability of outage gain factor of independent codebook generation over common codebook generation for networks with comprehensive decoding is upper bounded by

$$\frac{P_{o,C}(SNR,R)}{P_{o,I}(SNR,R)} \le \frac{\displaystyle\sum_{\substack{T_i \in T_R \\ T_i \in T_{O(M_T)}}} \left(\frac{1}{M_T!} \prod_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} \frac{|A_k|}{\mu_{kl,ij}\sigma^2_{kl,ij}} \right) \left(\frac{2^{K_C R}-1}{SNR} \right)^{M_T}}{\displaystyle\sum_{\substack{T_i \in T_R \\ T_i \in T_{O(M_T)}}} \left(\prod_{\substack{T_k \in T_{P(i)} \\ A_{ij} \in A_i \\ A_{kl} \in A_k}} \frac{|A_k|}{\mu_{kl,ij}\sigma^2_{kl,ij}} \right) \left(\frac{2^{K_I R/M_T}-1}{SNR} \right)^{M_T}} \le \frac{1}{M_T!}\left(\frac{2^{K_C R}-1}{2^{K_I R/M_T}-1} \right)^{M_T} . \tag{91}$$

4.4.2 Networks with Destination Decoding

For networks with common codebook generation and destination decoding the probability of outage at high SNR can be approximated by

$$P_o(SNR,R) \approx \sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\frac{1}{M_S!} \prod_{L_{kl,ij} \in L_i} \frac{|A_k|}{\mu_{kl,ij}\sigma^2_{kl,ij}} \right) \left(\frac{2^{KR}-1}{SNR} \right)^{M_S}, \tag{92}$$

and the maximum diversity order of the network is given by

$$d_{\max} = -\lim_{SNR \to \infty} \frac{\log P_o(SNR,R)}{\log SNR} = M_S = \min_{S_i \in S_R} \{ |L_i| \}, \tag{93}$$

where M_S is the minimum number of inter-terminal antenna links across all cut sets.

For networks with independent codebook generation and destination decoding the probability of outage at high SNR can be approximated by

$$P_o(SNR,R) \approx \sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\prod_{L_{kl,ij} \in L_i} \frac{|A_k|}{\mu_{kl,ij}\sigma^2_{kl,ij}} \right) \left(\frac{2^{KR/M_S}-1}{SNR} \right)^{M_S}, \tag{94}$$

and the maximum diversity order of the network is given by

$$d_{\max} = -\lim_{SNR \to \infty} \frac{\log P_o(SNR, R)}{\log SNR} = M_S = \min_{S_i \in S_R} \{|L_i|\}, \tag{95}$$

where M_S is the minimum number of inter-terminal antenna links across all cut sets. These results indicate that independent codebook generation does not offer any diversity gain over common codebook generation for networks with destination decoding even when there are an arbitrary number of physical antennas per terminal.

The high SNR probability of outage gain factor of independent codebook generation over common codebook generation for networks with destination decoding is upper bounded by

$$\frac{P_{o,C}(SNR,R)}{P_{o,I}(SNR,R)} \le \frac{\sum\limits_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\frac{1}{M_S!} \prod\limits_{L_{kl,ij} \in L_i} \frac{|A_k|}{\mu_{kl,ij} \sigma_{kl,ij}^2} \right) \left(\frac{2^{K_C R} - 1}{SNR} \right)^{M_S}}{\sum\limits_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\prod\limits_{L_{kl,ij} \in L_i} \frac{|A_k|}{\mu_{kl,ij} \sigma_{kl,ij}^2} \right) \left(\frac{2^{K_I R / M_S} - 1}{SNR} \right)^{M_S}} \le \frac{1}{M_S!} \left(\frac{2^{K_C R} - 1}{2^{K_I R / M_S} - 1} \right)^{M_S}. \tag{96}$$

Now that M_T may be greater than 1, the high SNR probability of outage gain factor results indicate similar behavior for both networks with comprehensive decoding and networks with destination decoding. In both cases, independent codebook generation can offer a significant probability of outage improvement over common codebook generation under certain conditions. This has a significantly higher likelihood of being the case when there are multiple antennas per terminal as the value of M_T or M_S respectively will in general be larger.

4.5 Diversity-Multiplexing Tradeoff

The diversity-multiplexing tradeoff [133] describes the maximum achievable diversity order for a range of achievable rates, and is a more general result than the maximum diversity order. In general, is not possible to directly determine the diversity-multiplexing tradeoff of wireless networks with arbitrary link connectivity between cooperating terminals from the high SNR probability of outage results. This is due to the fact that the terminals (for comprehensive decoding) or cut sets (for destination decoding) that limit the maximum achievable diversity order may not necessarily be the same terminals or cut sets that limit the maximum achievable multiplexing gain. The key difference is that although the maximum achievable diversity order depends on the number of independent fading realizations that can be combined, the maximum achievable multiplexing gain instead depends on the number of degrees of freedom in the channel. The terminal, or cut set, with the minimum number of independent fading realizations is not necessarily the terminal, or cut set, with the minimum number of degrees of freedom across all terminals, or cut sets, in the network.

The method used to bound the diversity-multiplexing tradeoff of wireless networks with arbitrary link connectivity between cooperating terminals applies the main result of [133] where it is shown that the diversity-multiplexing tradeoff curve of a MIMO channel with m transmit antennas, n receive antennas, and independent identically distributed Rayleigh fading between each pair of antennas is given by the piecewise linear function connecting the points $(r, d(r)), r = 0,1,..., \min\{m, n\}$, where $d(r) = (m-r)(n-r)$. We note that although this result was generated for MIMO channels, it is equivalent to the

MISO/SIMO diversity-multiplexing tradeoff curve of $d(r) = mn(1-r)$ for the relevant range $r = 0,1$.

For analysis of the diversity-multiplexing tradeoff of wireless networks with arbitrary link connectivity between cooperating terminals we can dispense with the distinction between common codebook generation and independent code generation, since increasing the multiplexing gain requires leveraging the available inter-terminal antenna links for the transmission of parallel spatial channels. The method of bounding and minimization across cut sets is similar to that applied independently in [128], although more general in that it is not limited to the classical three-terminal relay channel. We note again that the presented results are different from those in [6] due to the previously mentioned non-overlapping symbol period constraint.

4.5.1 Networks with Comprehensive Decoding

For networks with comprehensive decoding the diversity-multiplexing tradeoff curve can be upper bounded by performing a minimization across the diversity-multiplexing tradeoff curves of all terminals in the network. Applying the main result of [133], the diversity-multiplexing tradeoff curve of the channel at terminal T_i (between terminal T_i and all immediately preceding terminals $T_{P(i)}$) is given by the piecewise linear function connecting the points $(r, d(r)), r = 0, 1/K, ..., \min\left\{\sum_{T_k \in T_{P(i)}} |A_k|, |A_i|\right\}/K$, where

$$d(r) = (\sum_{T_k \in T_{P(i)}} |A_k| - Kr)(|A_i| - Kr). \tag{97}$$

For networks with comprehensive decoding the diversity-multiplexing tradeoff curve is therefore upper bounded by the piecewise linear function connecting the points $(r, d(r)), r = 0, 1/K, \ldots, \min_{T_i \in T_R}\left\{\min\left\{\sum_{T_k \in T_{P(i)}} |A_k|, |A_i|\right\}\right\}/K$, where

$$d(r) \leq \min_{T_i \in T_R}\left\{(\sum_{T_k \in T_{P(i)}} |A_k| - Kr)(|A_i| - Kr)\right\}. \tag{98}$$

From the diversity-multiplexing tradeoff curve it can be seen that the maximum achievable diversity order is again given by

$$d_{\max} = \min_{T_i \in T_R}\left\{\sum_{T_k \in T_{P(i)}} |A_k||A_i|\right\}, \tag{99}$$

the minimum number of incident inter-terminal antenna links across all terminals, and the maximum achievable multiplexing gain is given by

$$r_{\max} = \min_{T_i \in T_R}\left\{\sum_{T_k \in T_{P(i)}} |A_k|, |A_i|\right\}/K, \tag{100}$$

the minimum number of transmit or receive antennas for the incident multiple antenna channels across all terminals, divided by the number of orthogonal channels required to operate the given transmission scheme. Fig. 14 shows an example network, indicating those terminals with more than one antenna and annotated with the achievable multiplexing gain (not including the rate factor $1/K$) of each terminal.

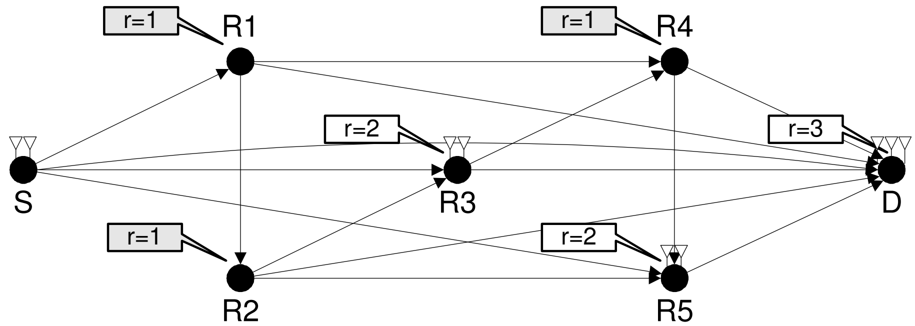

Fig. 14. Multiplexing Gain of Example Network with Comprehensive Decoding

4.5.2 Networks with Destination Decoding

For networks with destination decoding the diversity-multiplexing tradeoff curve can be upper bounded by performing a minimization across the diversity-multiplexing tradeoff curves of all cut sets in the network. Applying the main result of [133], the diversity-multiplexing tradeoff curve of the channel at cut set S_i (across all the inter-terminal links associated with cut set S_i) is upper bounded by the piecewise linear function connecting the points $(r, d(r)), r = 0, 1/K, \ldots, \min\left\{ \sum_{T_k \in T_{IN(i)}} |A_k|, \sum_{T_l \in T_{OUT(i)}} |A_l| \right\} / K$, where

$$d(r) \le \Big(\sum_{T_k \in T_{IN(i)}} |A_k| - Kr \Big)\Big(\sum_{T_l \in T_{OUT(i)}} |A_l| - Kr \Big), \tag{101}$$

where the upper bound results from the fact that the sets of terminals on the input and output sides of cut set S_i, $T_{IN(i)}$ and $T_{OUT(i)}$ respectively, are not necessarily fully connected. For example, in the network shown in Fig. 15, for the cut set S_4 where terminals T_S, T_{R1}, T_{R2}, and T_{R3} belong to $T_{IN(4)}$ and terminals T_{R4}, T_{R5}, and T_D belong to $T_{OUT(4)}$, although terminal T_{R3} is directly connected to terminals T_{R4} and T_D, it is not directly connected to terminal T_{R5}. This reduction in connectivity in comparison to a fully connected MIMO channel between the sets of terminals on the input and output sides of cut set S_4 means that the diversity-multiplexing tradeoff of that corresponding fully connected MIMO channel is an upper bound on the actual diversity-multiplexing tradeoff of the channel at cut set S_4. In general, the diversity-multiplexing tradeoff of the channel at cut set S_i is upper bounded by the diversity-multiplexing tradeoff of the corresponding fully connected MIMO channel involving the same sets of terminals on the input and output sides of cut set S_i. This upper bound is tight when the sets of terminals on the input and output sides of cut set S_i are fully connected.

For networks with destination decoding the diversity-multiplexing tradeoff is therefore upper bounded by the piecewise linear function connecting the points $(r, d(r)), r = 0, 1/K, \ldots, \min_{S_i \in S_R}\left\{\min\left\{\sum_{T_k \in T_{IN(i)}} |A_k|, \sum_{T_l \in T_{OUT(i)}} |A_l|\right\}\right\}/K$, where

$$d(r) \le \min_{S_i \in S_R}\left\{\left(\sum_{T_k \in T_{IN(i)}} |A_k| - Kr\right)\left(\sum_{T_l \in T_{OUT(i)}} |A_l| - Kr\right)\right\}. \tag{102}$$

From the diversity-multiplexing tradeoff curve it can be seen that the maximum achievable diversity order is again given by

$$d_{\max} = \min_{S_i \in S_R}\left\{\sum_{T_k \in T_{IN(i)}} |A_k| \sum_{T_l \in T_{OUT(i)}} |A_l|\right\} = \min_{S_i \in S_R}\{|L_i|\}, \tag{103}$$

the minimum number of inter-terminal antenna links across all cut sets, and the maximum achievable multiplexing gain is given by

$$r_{\max} = \min_{S_i \in S_R}\left\{\sum_{T_k \in T_{IN(i)}} |A_k|, \sum_{T_l \in T_{OUT(i)}} |A_l|\right\}/K, \tag{104}$$

the minimum number of transmit or receive antennas for the incident multiple antenna channels across all cut sets, divided by the number of orthogonal channels required to operate the given transmission scheme. Fig. 15 shows an example network, indicating those terminals with more than one antenna and annotated with the achievable multiplexing gain (not including the rate factor $1/K$) of each cut set that is relevant given the directed connectivity.

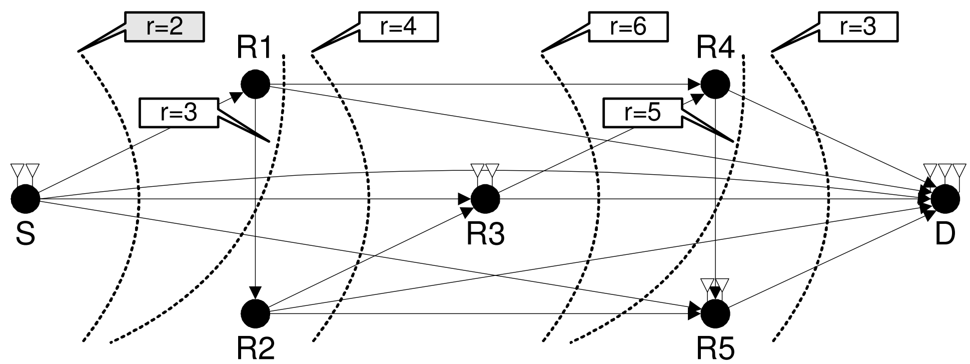

Fig. 15. Multiplexing Gain of Example Network with Destination Decoding

We now consider a refinement that provides a more precise result than the upper bound of (101). As noted in [22] and [133], the diversity-multiplexing tradeoff of a MIMO channel degrades when the channel matrix is rank deficient. It is shown in [22] that the diversity-multiplexing tradeoff curve of a rank deficient MIMO channel with m transmit antennas, n receive antennas, and independent identically distributed Rayleigh fading between each pair of antennas is given by the piecewise linear function connecting the points $(r, d(r)), r = 0,1,..., \kappa$, where $\kappa \leq \min\{m, n\}$ is the rank of the MIMO channel matrix and $d(r) = (m+n-\kappa-r)(\kappa-r)$.

Since the fact that the sets of terminals on the input and output sides of cut set S_i are not fully connected may result in the corresponding MIMO channel being rank deficient, the diversity-multiplexing tradeoff curve of the channel at cut set S_i is given by the piecewise linear function connecting the points $(r, d(r)), r = 0, 1/K, ..., \kappa_i / K$, where

$$d(r) = (\sum_{T_k \in T_{IN(i)}} |A_k| + \sum_{T_l \in T_{OUT(i)}} |A_l| - \kappa_i - Kr)(\kappa_i - Kr), \tag{105}$$

and $\kappa_i \le \min\{\sum_{T_k \in T_{IN(i)}} |A_k|, \sum_{T_l \in T_{OUT(i)}} |A_l|\}$ is the rank of the corresponding MIMO channel matrix at cut set S_i. From this diversity-multiplexing tradeoff curve it can be seen that the maximum achievable diversity order at cut set S_i is given by

$$d_{\max} = (\sum_{T_k \in T_{IN(i)}} |A_k| + \sum_{T_l \in T_{OUT(i)}} |A_l| - \kappa_i)\kappa_i . \tag{106}$$

However, it was shown earlier that the maximum achievable diversity order at cut set S_i is $d_{\max} = |L_i|$, so it is clear that $(\sum_{T_k \in T_{IN(i)}} |A_k| + \sum_{T_l \in T_{OUT(i)}} |A_l| - \kappa_i)\kappa_i = |L_i|$. Therefore, we can rewrite the diversity-multiplexing tradeoff curve at cut set S_i with

$$\begin{aligned} d(r) &= (\sum_{T_k \in T_{IN(i)}} |A_k| + \sum_{T_l \in T_{OUT(i)}} |A_l| - \kappa_i - Kr)(\kappa_i - Kr) \\ &= (\sum_{T_k \in T_{IN(i)}} |A_k| + \sum_{T_l \in T_{OUT(i)}} |A_l| - \kappa_i)\kappa_i + (Kr)^2 - (\sum_{T_k \in T_{IN(i)}} |A_k| + \sum_{T_l \in T_{OUT(i)}} |A_l|)Kr \\ &= |L_i| + (Kr)^2 - (\sum_{T_k \in T_{IN(i)}} |A_k| + \sum_{T_l \in T_{OUT(i)}} |A_l|)Kr \\ &= |L_i| - (\sum_{T_k \in T_{IN(i)}} |A_k| + \sum_{T_l \in T_{OUT(i)}} |A_l| - Kr)Kr \end{aligned} , \tag{107}$$

for $r = 0, 1/K, ..., \kappa_i / K$. This result can be applied across all cut sets in the network, such that for networks with destination decoding the diversity-multiplexing tradeoff is therefore given by the piecewise linear function connecting the points $(r, d(r)), r = 0, 1/K, ..., \min_{S_i \in S_R} \{\kappa_i\} / K$, where

$$d(r) = \min_{S_i \in S_R} \left\{ |L_i| - (\sum_{T_k \in T_{IN(i)}} |A_k| + \sum_{T_l \in T_{OUT(i)}} |A_l| - Kr)Kr) \right\}. \tag{108}$$

From the diversity-multiplexing tradeoff curve it can be seen that the maximum achievable diversity order is again given by

$$d_{\max} = \min_{S_i \in S_R} \{|L_i|\}, \tag{109}$$

the minimum number of inter-terminal antenna links across all cut sets, and the maximum achievable multiplexing gain is given by

$$r_{\max} = \min_{S_i \in S_R} \{\kappa_i\} / K , \tag{110}$$

the minimum rank of the channel matrix across all cut sets, divided by the number of orthogonal channels required to operate the given transmission scheme.

4.6 Complexity of Minimization Algorithms

In the analysis in this chapter we have repeatedly used results that include a minimization of diversity order or diversity-multiplexing tradeoff across all terminals or cut sets in a wireless relay network. It is therefore very relevant to consider the algorithmic complexity of this minimization process for the parameters and relaying method classes of interest. We now consider the complexity of the relevant algorithms for minimization across all terminals or cut sets in the network. We use the O-notation and definition from [23] to describe the asymptotic complexity of a function $f(n)$ with respect to n. Let $f(n)$ and $g(n)$ be two functions defined on some subset of the real numbers. Then $f(n) \in O(g(n))$ as $n \to \infty$ if an only if $\exists n_0, \exists M > 0$, $|f(n)| \leq M|g(n)|$ for $n > n_0$. This can be read that $f(n)$ is of the order of $g(n)$.

The number of possible cut sets in a wireless relay network with N terminals is given by

$$\begin{aligned} \left|S_R\right| &= \sum_{i=0}^{N-2} \frac{(N-2)!}{(N-2-i)!i!}, \\ &= 2^{N-2} \end{aligned} \tag{111}$$

where the reason that this is a function of $N-2$ instead of N is that the source and destination terminals are always on the source and sink sides of all cut sets respectively.

When performing a minimization of the diversity-multiplexing tradeoff across all terminals, respectively cut sets, in the network there is a first comparison cycle to calculate the value of the minimum number of degrees of freedom in the channel across all terminals F_T, respectively the minimum number of degrees of freedom in the channel across all cut sets F_S, and then a subsequent F_T, respectively F_S, comparison cycles to perform the minimization for each integer value of the multiplexing gain $r \le F_T$, respectively $r \le F_S$. The minimum number of degrees of freedom in the channel across all terminals in the network is given by

$$F_T = \min_{T_i \in T_R} \left\{ \min\left\{ \sum_{T_k \in T_{P(i)}} \left|A_k\right|, \left|A_i\right| \right\} \right\}, \tag{112}$$

and the minimum number of degrees of freedom in the channel across all cut sets in the network is given by

$$F_S = \min_{S_i \in S_R} \left\{ \min\left\{ \sum_{T_k \in T_{IN(i)}} \left|A_k\right|, \sum_{T_k \in T_{OUT(i)}} \left|A_k\right| \right\} \right\}, \tag{113}$$

assuming the corresponding channel matrices are full rank.

Calculating the complexity of minimization across all terminals for networks with comprehensive decoded is straightforward. For the maximum diversity order analysis of networks with comprehensive decoding, the minimization algorithm includes $N-1$ comparisons between terminals. The complexity of algorithms for minimization of the diversity order across all terminals in a network with N terminals is therefore $(N-1)\in O(N)$. For the diversity-multiplexing tradeoff analysis of networks with comprehensive decoding, the algorithm includes $(N-1)(F_T+1)$ comparisons between terminals. The complexity of algorithms for minimization of the diversity-multiplexing tradeoff across all terminals in a network with N terminals is therefore $(N-1)(F_T+1)\in O(N)$. In general, the complexity of algorithms to minimize across all terminals is linear.

The baseline complexity of minimization across all cut sets for networks with destination decoding is an algorithm that brute-force searches across all cut sets. For the maximum diversity order analysis of networks with destination decoding, the minimization algorithm includes $2^{N-2}-1$ comparisons between cut sets. The complexity of algorithms for minimization of the diversity order across all cut sets in network with N terminals is therefore $(2^{N-2}-1)\in O(2^N)$. For the diversity-multiplexing tradeoff analysis of networks with destination decoding, the algorithm includes $(2^{N-2}-1)(F_S+1)$ comparisons between cut sets. The complexity of algorithms for minimization of the diversity-multiplexing tradeoff across all cut sets in a network with N terminals is therefore $(2^{N-2}-1)(F_S+1)\in O(2^N)$.

The brute-force algorithm of minimization across all cut sets in a network is exponential in complexity and therefore not very practical for large networks. However, a number of more efficient algorithms for determining the maximum flow of a directed network graph have been developed and are reported in [23]. These can be applied in a more efficient manner for the minimization of the diversity order or diversity-multiplexing tradeoff across all cut sets with an intelligent choice of edge weights applied to the inter-terminal antenna links. The Ford-Fulkerson algorithm [23] has a complexity of $O(|L_R|d_{\max})$. The Edmonds-Karp algorithm [23] has a complexity of $O(N|L_R|^2)$. The Relabel-to-Front algorithm [23] has a complexity of $O(N^3)$. The theoretically minimum complexity algorithm for determining the maximum flow of a directed network graph has a complexity of $O(N|L_R|)$, but so far no practical algorithms has been developed that achieve that complexity [23]. In large networks where the number of terminals and inter-terminal links is large, the Relabel-to-Front algorithm will generally have the lowest complexity of existing practical algorithms, so we assume a baseline complexity of $O(N^3)$ for minimization across all cut sets in a network with N terminals. In general, the complexity of algorithms to minimize across all cut sets is polynomial.

4.7 Summary

This chapter has derived bounds on the maximum achievable diversity order of wireless relay networks with arbitrary link connectivity between cooperating terminals. When all cooperating terminals must correctly decode it is shown that the maximum achievable diversity order is constrained by the minimum number of immediately preceding terminals across all receiving terminals in the network, which is one.

Furthermore, inter-terminal links that are not associated with terminals with the minimum number of immediately preceding terminals do not asymptotically (at high SNR) affect the probability of outage. When only the destination terminal must correctly decode the maximum achievable diversity order is constrained by the minimum number of inter-terminal links across all cut sets in the network, which is the number of disjoint paths through the network. Furthermore, inter-terminal links that are not associated with cut sets with the minimum number of inter-terminal links do not asymptotically (at high SNR) affect the probability of outage.

When there are an arbitrary number of antennas per terminal and all cooperating terminals must correctly decode it is shown that the maximum achievable diversity order is constrained by the minimum number of incident inter-terminal antenna links across all receiving terminals in the network, and the achievable multiplexing gain is constrained by the minimum number of transmit or receive antennas for the incident multiple antenna channels across all terminals, divided by the number of orthogonal channels required to operate the given transmission scheme. When there are an arbitrary number of antennas per terminal and only the destination terminal must correctly decode it is shown that the maximum achievable diversity order is constrained by the minimum number of associated inter-terminal antenna links across all cut sets in the network, and the achievable multiplexing gain is constrained by the minimum number of transmit or receive antennas for the incident multiple antenna channels across all cut sets, divided by the number of orthogonal channels required to operate the given transmission scheme.

These diversity results do not depend on the codebook generation scheme, and are intuitively satisfying as it is natural to think of the diversity order of a network as being equivalent to the minimum number of inter-terminal links that have to “fail” for the network to “fail”. The results also indicate that although independent codebook generation does not offer any diversity gain over common codebook generation, it can offer a significant probability of outage improvement under the conditions that the minimum number of inter-terminal antenna links across all cut sets in the network is high and the number of channels required for independent codebook generation is not significantly larger than the number of channels required for common codebook generation.

Finally, it is important to note that diversity order is only one aspect of the total probability of outage or error of relay networks. The presented results clearly indicate that although the diversity order of various configurations of cooperative connectivity between terminals in multihop relay networks may be the same, the total probability of outage or error will in general be different. This must be kept in mind when comparing the performance of relay networks with different cooperative connectivity operating at specific target rates and signal to noise ratios. It is not possible to increase the diversity order beyond that achievable with all relay terminals connected in parallel between the source and destination terminals, but it is still possible to decrease the probability of error or outage, and sometimes significantly as indicated by the cooperative connectivity model simulations of Chapter 6.

Chapter 5 - Cooperative Connectivity Modeling Framework

This chapter presents the developed cooperative connectivity modeling framework, describing the various system resource constraints that limit cooperative connectivity and the resultant cooperative connectivity models. Section 5.1 introduces the cooperative connectivity modeling framework. Section 5.2 describes the considered system resource constraints in detail. Section 5.3 analyzes the possible combinations of system resource constraints and derives cooperative connectivity models defined by the achievable combinations of communication links between cooperating wireless terminals. Section 5.4 derives the sets of constraints that result in the different cooperative connectivity models while minimizing the system cost (the minimum cost constraint sets). Section 5.5 presents cooperative connectivity model transition diagrams that show the transitions between cooperative connectivity models with respect to the lifting of system resource constraints. Section 5.6 describes the mapping of the cooperative connectivity models to various distributed spatial diversity techniques presented in the literature in order to highlight the general richness of the problem domain. Section 5.7 summarizes the contributions of the chapter.

5.1 Introduction

Recent findings in the literature have shown that the performance of wireless relay networks can be improved through the application of distributed spatial diversity techniques that leverage cooperation between wireless terminals. Multi-user diversity [5] and virtual antenna arrays [27], [28] achieve spatial diversity by relaying the signal along multiple routes in parallel. Multihop diversity [14], [15] achieves spatial diversity from the reception of signals that have been transmitted by multiple relays in serial along a single multihop route. Cooperative diversity [47], [61], [62], [64], [97], [99] achieves spatial diversity by sharing information between the source terminal and cooperating relay terminals such that each user of the cooperation group sends information to the destination using all of the cooperating terminals. Coded cooperative diversity [51], [52], [53], [63], [71], [104], [132] uses various coding techniques to improve the performance over that of basic repetition coding. Non-orthogonal signaling techniques based on the artificial generation of a rich multi-path fading environment, for example [42], [50], [67], [81], [88], [94], can be leveraged to reclaim some of the spectral efficiency loss caused by repetition coding over orthogonal relaying channels. Information theoretic results for the classical relay channel [25] have been extended to include more general multi-hop relay channels [40], [60]. Results characterizing the diversity-multiplexing tradeoff of multiple-antenna [133] and multiple-access [110] channels have been extended to half-duplex cooperative channels [6]. Basic strategies for the connectivity between cooperating wireless terminals have been extended to include cascaded [41], [48], [92], and multihop multi-branch [91] cooperation strategies. These distributed spatial diversity techniques contrast with non-distributed techniques such as classical MIMO transmission that require individual terminals to host multiple physical antennas.

Each of the distributed spatial diversity techniques proposed in the literature places different requirements on the wireless terminal hardware capabilities, channel availability, and multiple access schemes used to implement connectivity between terminals, and thus places different requirements on the system resources that must be available. Therefore, system resource constraints that limit the ways that cooperating terminals can be connected to each other also constrain the distributed spatial diversity techniques that can be applied and the increased capacity expected from these techniques. Additionally, the cooperation between terminals needed to support each technique can often be achieved with different combinations of system resources. Instead of generating results for one specific assumed set of available system resources, this dissertation explicitly specifies the system resources that must be available for cooperating terminals to be connected to each other in different ways, and therefore to utilize different distributed spatial diversity techniques.

This chapter develops a framework for modeling the ways that cooperating terminals can be connected to each other in wireless relay networks and the relationship between constraints on the available system resources and achievable cooperative connectivity. The system resource constraints considered are the available number of orthogonal relaying channels, the ability of relays and destinations to diversity combine incident signals on a single common channel, the ability of relays and destinations to diversity combine incident signals on different orthogonal channels, the ability of transmitters to transmit signals on multiple orthogonal channels, and the ability of receivers to cancel the effects of interhop interference [15]. Generally, it is of significant relevance to understand the different ways that cooperating terminals can be connected to each other in wireless relay networks, which of those options for cooperative connectivity show the most promise for further investigation and eventual adoption, and which system resources can enable those options for cooperative connectivity in the most efficient manner. The developed framework is a valuable first step in the analysis of these issues, and enables the formulation of interesting and relevant, but previous unexpressed, questions with respect to cooperative connectivity.

For example, the framework allows us to formulate and solve the following classes of problems: 1) given an available set of system resources, determine the achievable cooperative connectivity, 2) given a desired level of cooperative connectivity, determine the possible (and lowest cost) sets of system resources that can be used to achieve it, 3) given a baseline set of system resources and level of cooperative connectivity, determine the impact on the achievable cooperative connectivity of incrementally adding or removing different system resources, and 4) given a baseline set of system resources and level of cooperative connectivity, determine the possible (and lowest cost) sets of system resources that can be incrementally added or removed to achieve different levels of cooperative connectivity. The first problem class and its inverse, the second problem class, consider the core relationship between system resources and achievable cooperative connectivity. The third problem class and its inverse, the fourth problem class, consider the value of reusing existing capabilities of the network infrastructure as incremental modifications are made. Each of the different system resources has an associated cost, so it is important that existing investment is leveraged when modifications are made to network infrastructure. We note that these problem classes do not inherently express the philosophy that more connectivity is better, but are simply concerned with the relationship between achievable cooperative connectivity and available system resources. Even so, in the context of a given source-destination terminal pair when the system resources used are constant, it is clear that more cooperative diversity will result in better performance. However, this is not necessarily the case when considered in the context of a system with many source-destination terminal pairs, where many other factors such as spatial reuse and inter-user interference must be considered.

The developed framework is equally applicable to systems employing either fixed or mobile relays, but does assume that all relays in a given network have the same capabilities. Therefore, hybrid systems employing a combination of both fixed and mobile relays within a single multihop path are not explicitly considered. Although the focus of this dissertation is on uncoded systems, the developed framework is equally applicable to systems that employ traditional source and channel coding techniques. These coding techniques can be employed regardless of the ways that cooperating terminals can be connected to each other. However, the choice of coding technique should depend on the composite characteristics of the multihop relay channel formed by a given set of cooperating terminals and communication links between them. For cooperative coding techniques, where different relays generate different re-encoded versions of the original transmitted information signal, there are strong requirements on the ways that cooperating terminals can be connected to each other. For example, the distributed space-time code protocol of [63] requires that all relays be directly connected to both the source and destination in parallel. General analysis of the cooperative connectivity requirements for different cooperative coding techniques is outside the scope of this dissertation. However, the developed framework can be used to determine the possible sets of system resources that can be used to achieve the required cooperative connectivity of a given cooperative coding technique.

The developed framework does not address the relationship between cooperative connectivity and spatial reuse of channels, a mechanism that can be applied in wireless relay networks to improve the overall spectral efficiency. Spatial reuse of channels occurs when different signals can be transmitted concurrently on the same channels in spatially separated portions of multihop paths (or networks). However, spatial reuse is less feasible in systems where terminals are connected by long direct communication links that do not allow spatial partitioning of the network, especially when using a multiple access scheme that does not allow spatial overlap of concurrent transmissions on the same channel. Systems with higher levels of cooperative connectivity, which generally have more long direct communication links and enjoy higher levels of spatial diversity, are generally less able to leverage spatial reuse due to a corresponding larger region of interference. This tradeoff between improved spatial diversity and improved spatial reuse is an important topic for further analysis.

5.2 System Resource Constraints

The considered system resource constraints are described in detail in this section. The motivation for each constraint is discussed in terms of system complexity and cost. Options for each constraint are introduced, along with their corresponding relative cost and connectivity impact. In all cases, constraint options with lower cost have higher connectivity impact. Connectivity impact is defined as the reduction in achievable cooperative connectivity caused by absence of a system resource, and is measured in comparison to a fully connected (complete) relay network with links between all terminals. The underlying assumption when comparing the system cost is that the modulation scheme, total power, and rate are kept constant for fair comparison. The term ‘preceding terminal’ denotes any terminal that is earlier along the multihop transmission path than the candidate terminal. The term ‘following terminal’ denotes any terminal that is later along the multihop transmission path than the candidate terminal.

5.2.1 Number of Channels Available

This constraint defines the number of orthogonal relaying channels available for the transmission of a signal between a single source-destination pair. The half-duplex nature of wireless terminal hardware requires that each relay transmit and receive with different channels, implying a minimum of two orthogonal channels. When the modulation scheme, total power, and rate are kept constant, use of more than two orthogonal channels for relaying increases the system cost since more bandwidth is necessary to achieve a given rate of transmission for each source-destination pair. However, when the symbol rate loss due to subdivision of the original bandwidth can be compensated for by an improved end-to-end SNR, the requirement for additional orthogonal channels may not necessarily result in an increase in overall required bandwidth. A special case is when the number of channels available equals the number of relay levels (equivalent to the number of hops in the longest multihop path) in the network.

- *N Channels Available (NCA):* The source and relays transmit using N orthogonal channels, where $N+1$ is the number of terminals. There is no connectivity impact.
- *K Channels Available (KCA):* The source and all relays transmit using K orthogonal channels, where $2 < K < N$. The connectivity impact is that each terminal may only be connected to preceding terminals that transmit on a different subset of channels ($K-1$ possible channels for relays and K possible channels for destinations).
- *2 Channels Available (2CA):* The source and all relays transmit using 2 orthogonal channels. The connectivity impact is that each terminal may only be connected to

preceding terminals that transmit on a different subset of channels (1 possible channel for relays and 2 possible channels for destinations).

5.2.2 Common Channel Combination

This constraint defines the ability of terminals to diversity combine incident signals from multiple preceding terminals on a single common channel. Common channel combination can be achieved using various orthogonal and non-orthogonal signaling techniques including space-time coding, random relay phase rotation, and artificial multi-path generation with adaptive equalization, spatial processing, or RAKE reception [42], [50], [67], [81], [88], [94]. One interesting result of many of these references is that diversity combination can be achieved even without the use of orthogonal channels, although in some cases, depending on the specific channel conditions, there may be some degradation of performance in comparison to that of maximal ratio combining. Use of common channel combination increases the system cost since advanced common channel processing and combination hardware is required at cooperating terminals.

- *Relay Common Channel Combination (RCC):* Relays are able to perform common channel combination. Common channel combination hardware is required at relays. There is no connectivity impact.
- *No Relay Common Channel Combination (NRCC):* Relays are not able to perform common channel combination. The connectivity impact is that each relay may only be connected to one preceding terminal on each channel.
- *Destination Common Channel Combination (DCC):* Destinations are able to perform common channel combination. Common channel combination hardware is required at destinations. There is no connectivity impact.

- *No Destination Common Channel Combination (NDCC):* Destinations are not able to perform common channel combination. The connectivity impact is that each destination may only be connected to one preceding terminal on each channel.

5.2.3 Orthogonal Channel Combination

This constraint defines the ability of terminals to diversity combine incident signals from multiple preceding terminals on different orthogonal channels. Orthogonal channel combination can be achieved using classical combination techniques and possibly buffering of multiple orthogonal channels. Use of orthogonal channel combination increases the system cost since classical combination hardware is required at cooperating terminals.

- *Relay Orthogonal Channel Combination (ROC): Relays* are able to perform orthogonal channel combination. Orthogonal channel combination hardware is required at relays. There is no connectivity impact.
- *No Relay Orthogonal Channel Combination (NROC):* Relays are not able to perform orthogonal channel combination. The connectivity impact is that each relay may only be connected to the subset of preceding terminals that transmit on one common channel.
- *Destination Orthogonal Channel Combination (DOC):* Destinations are able to perform orthogonal channel combination. Orthogonal channel combination hardware is required at destinations. There is no connectivity impact.
- *No Destination Orthogonal Channel Combination (NDOC):* Destinations are not able to perform orthogonal channel combination. The connectivity impact is that each

destination may only be connected to the subset of preceding terminals that transmit on one common channel.

5.2.4 Multiple Channel Transmission

This constraint defines the ability of transmitters to transmit a given signal on multiple orthogonal channels. Use of multiple channel transmission increases the system cost since more complex channel transmission hardware may be required at cooperating terminals (for example, transmitting concurrently at multiple frequencies) and each terminal that transmits on multiple channels generates additional energy and interference (when the modulation scheme and rate are kept constant).

- *Multiple Channel Transmission (MCT):* Terminals are able to transmit on multiple orthogonal channels. Multiple channel transmission hardware is required. There is no connectivity impact.
- *No Multiple Channel Transmission (NMCT):* Terminals are not able to transmit on multiple orthogonal channels. The connectivity impact is that each terminal may only be connected to the subset of following terminals that receive on one common channel.

5.2.5 Interhop Interference Cancellation

This constraint defines the ability of receivers to cancel the effects of interhop interference created by the retransmission of signals on the same channel at different hops along a multihop transmission path [15]. Interhop interference is a special case of intersymbol interference (ISI) that affects wireless relay networks where channels are reused, and therefore in theory it is expected that practical mitigation techniques will be based on those used for ISI, including traditional equalization techniques. Use of interhop interference cancellation increases the system cost since more complex equalization hardware is required at cooperating terminals. Note that the analysis of this constraint is somewhat simplistic. First, it does not consider that in practice channel reuse may be possible without interhop interference cancellation in some circumstances due to sufficient spatial separation or attenuation between terminals. Second, it is assumed that interference cancellation is perfect when channels are reused at different hops within a single multihop path. Although this is not feasible in practice, even partial interference cancellation allows the same level of cooperative connectivity to be achieved, therefore resulting in the same cooperative connectivity models.

- *Interhop Interference Cancellation (IIC):* Terminals are able to cancel interhop interference. Interhop interference mitigation hardware is required. There is no connectivity impact.
- *No Interhop Interference Cancellation (NIIC):* Terminals are not able to cancel interhop interference. The connectivity impact is that networks with K channels available have a maximum of K hops in the longest multihop path of the network, since it is not possible to reuse channels within a single multihop path.

5.3 Cooperative Connectivity Models

The possible system resource constraint combinations are analyzed in this section and a set of resultant cooperative connectivity models is derived from the combinations. Although a combinatorial approach to the system resource constraints indicates 192 possible combinations, intersection between the achievable cooperative connectivity of different combinations results in only 34 distinct cooperative connectivity models. The cooperative connectivity models are fully characterized according to three parameters: the achievable cooperative connectivity of the relays, the achievable cooperative connectivity of the destination, and the maximum achievable length of the longest multihop path of the network. The cooperative connectivity models can therefore be identified using the form '*x*R*y*D*z*H', where '*x*R' indicates the achievable cooperative connectivity of the relays, '*y*D' indicates the achievable cooperative connectivity of the destination, and '*z*H' indicates the maximum achievable length of the longest multihop path of the network. The relationship between this parameterization and the maximum achievable diversity order of the network is not simple, although higher levels of achievable relay connectivity and especially achievable destination connectivity definitely do correspond to an increase in maximum achievable diversity order. As shown in the previous chapter, the maximum achievable diversity order of wireless relay networks is constrained by either the terminal or cut set with the minimum number of incident inter-terminal links, depending on the class of relaying method.

Fig. 16 describes the possible values of x, y, and z used when characterizing the achievable cooperative connectivity of the models and shows graphical examples of the cooperative connectivity terminology, with transmitting channel allocations indicated in brackets. In general, models where destinations have more capabilities than relays are more likely to be deployed for systems with mobile relays, and models where relays have more capabilities than destinations are more likely to be deployed for systems with fixed relays. Appendix C provides tables summarizing all possible constraint combinations and the resultant cooperative connectivity models. Appendices D, E, and F respectively present graphical examples illustrating the connectivity of each cooperative connectivity model with *K*, 2, and *N* channels available, with the chosen transmitting channel allocations indicated in brackets. Other channel allocations are possible, in some cases with better performance. However, the indicated channel allocations have been chosen since they can be consistently applied across all cooperative connectivity models to illustrate the connectivity differences between the models.

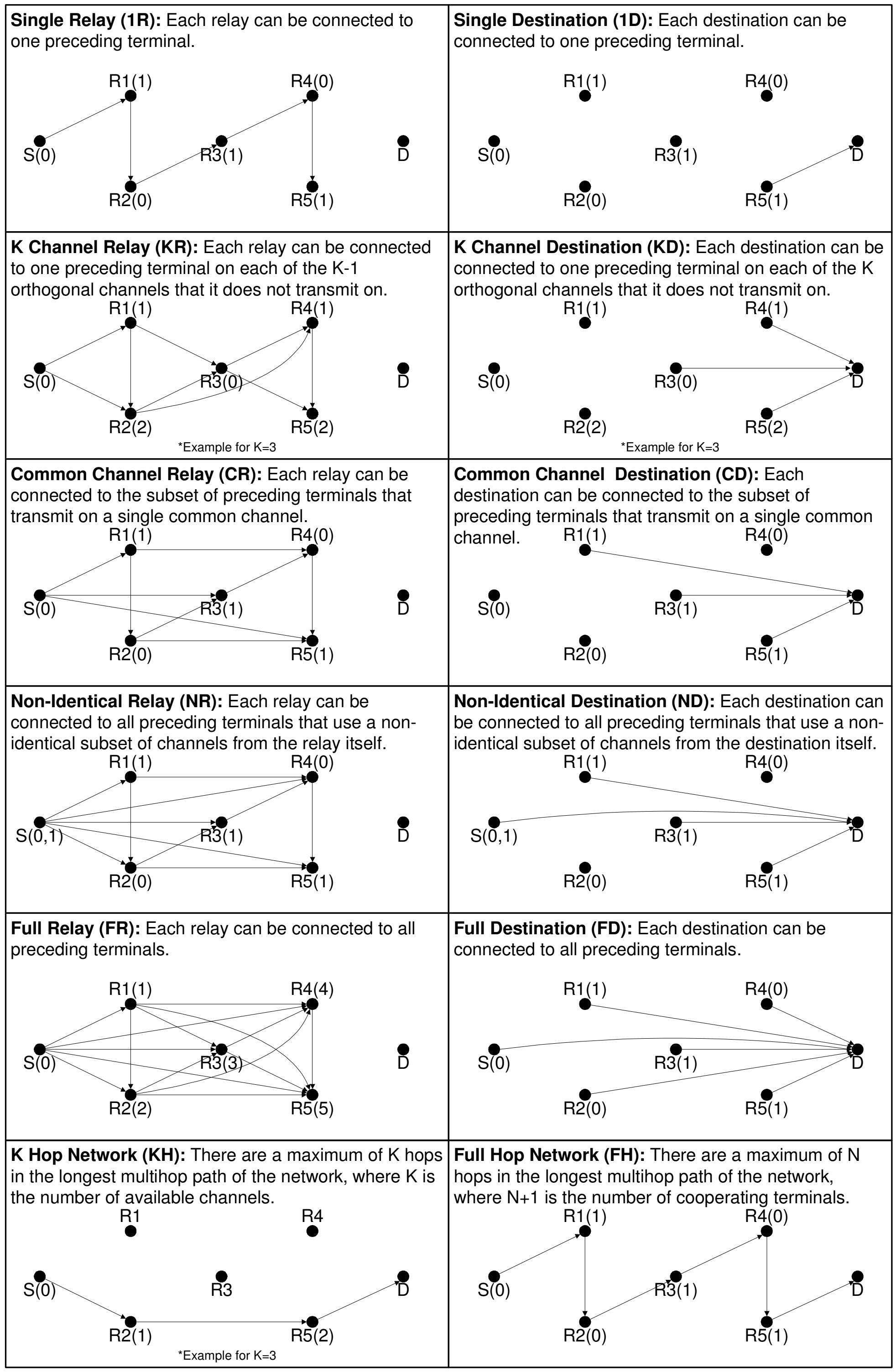

Fig. 16. Cooperative Connectivity Model Terminology Examples

The cooperative connectivity equations presented below specify the relationship between the system resource constraints and the characterizing parameters of the cooperative connectivity models. These equations address the first problem class listed in the introduction: given an available set of system resources, determine the achievable cooperative connectivity. The number of available channels (NCA, KCA, and 2CA) has a structuring influence on the connectivity impact of the other constraints. The results are therefore presented classified by the number of available channels in order to avoid complex and monolithic cooperative connectivity and minimum cost constraint set equations.

Connectivity Equations for Models with KCA: The connectivity equations for cooperative connectivity models with *K* channels available are:

```
xR:  if (NRCC & NROC) then x = 1
     else if (NRCC & ROC) then x = K
     else if (RCC & NROC & NMCT) then x = C
     else if (RCC & (ROC | MCT)) then x = N
yD:  if (NDCC & NDOC) then y = 1
     else if (NDCC & DOC) then y = K
     else if (DCC & NDOC & NMCT) then y = C
     else if (DCC & (DOC | MCT)) then y = F
zH:  if (IIC) then z = F
     else if (NIIC) then z = K
```

Connectivity Equations for Models with 2CA: The connectivity equations for cooperative connectivity models with 2 channels available are:

```
xR:  if (NRCC | NIIC) then x = 1
     else if (RCC & NMCT & IIC) then x = C
     else if (RCC & MCT & IIC) then x = N
yD:  if (NDCC & NDOC) then y = 1
     else if (NDCC & DOC) then y = 2
     else if (DCC & NDOC & NMCT) then y = C
     else if (DCC & NDOC & MCT) then y = N
     else if (DCC & DOC) then y = F
zH:  if (IIC) then z = F
     else if (NIIC) then z = 2
```

Connectivity Equations for Models with NCA: The connectivity equations for cooperative connectivity models with *N* channels available are:

```
xR:  if (ROC | (RCC & MCT)) then x = F
     else x = 1
yD:  if (DOC | (DCC & MCT)) then y = F
     else y = 1
zH:  z = F
```

The cooperative connectivity models derived when there are *K* channels available (Fig. 17) are the most general set. The cooperative connectivity models derived when there are 2 channels available (Fig. 18) are a subset of the models derived when there are *K* channels available, with the reduction resulting from additional system resource constraints and intersection between models. The majority of cooperative connectivity models result from constraint combinations with less than *N* channels available (Fig. 19). Of the 34 possible cooperative connectivity models, 18 models are achievable when there are 2 channels available, a further 14 models (for a total of 32) are achievable when there are 3 or more channels available, and the final 2 models are achievable only when there are *N* channels available. Only the models with full relay connectivity, FR1DFH and FRFDFH, are exclusive to constraint combinations with *N* channels available. Of particular interest is that full cooperative connectivity is only achievable when N orthogonal channels are available, implying that the FRFDFH model will be very expensive to implement in practice for even a moderate number of relay terminals. Also of interest is that the 1RNDFH and NRNDFH models are exclusive to constraint combinations with 2 channels available since the additional availability of at least a third channel results in full destination (FD) connectivity when the remainder of the constraints are kept constant.

5.4 Minimum Cost Constraint Sets

The sets of constraints that result in the different cooperative connectivity models while minimizing the system cost (the minimum cost constraint sets) are derived in this section. These minimum cost constraint sets are important in that they indicate the optimal ways to implement the different cooperative connectivity models. Derivation of the minimum cost constraint sets inherently requires the assignment of a system cost weight to each of the system resource constraints for the purposes of comparison. However, since the considered system resource constraints involve a variety of terminal hardware and channel resources that are difficult to compare directly, it is necessary that the system cost weights incorporate at least some amount imprecise qualitative analysis. This implies that the system cost weights may be variable in different contexts, depending on the fundamental assumptions on which this qualitative analysis is based.

As a baseline for this dissertation, the minimum cost constraint sets for each cooperative connectivity model are derived using the system resource constraint ordering listed in Table 1, with the indicated increasing system cost weights. The chosen system cost weights are normalized such that maintaining all system resource constraints results in a total cost of 0, but lifting all system resource constraints results in a total cost of 1. Although the absolute cost weights are somewhat arbitrary, justification for the order of system cost weights is provided in Table 1, and is based on the following fundamental assumptions:

- Constraints involving additional power, interference, or channels within the network itself have a greater system cost than constraints involving more complex terminals. This is based on the underlying assumption that the cost of hardware complexity generally decreases with time while the cost of bandwidth is fixed [75].
- Constraints involving more complex relay terminals have a greater system cost than constraints involving more complex destination terminals because in general they affect more terminals in a multihop transmission path.

Order	System Resource Constraint	Cost
1	Destination Orthogonal Channel Combination (DOC): Orthogonal channel combination hardware is required on the destination.	0.02
2	Destination Common Channel Combination (DCC): Common channel combination hardware is required on the destination. The incremental system cost is considered to be greater than destination orthogonal channel combination because it involves more complex non-classical combination hardware.	0.04
3	Relay Orthogonal Channel Combination (ROC): Orthogonal channel combination hardware is required on every relay. The incremental system cost is considered to be greater than destination common channel combination because it involves combination hardware on every relay instead of combination hardware only on the destination.	0.06
4	Interhop Interference Cancellation (IIC): Inter-symbol interference equalization hardware is required on every relay. The incremental system cost is considered to be greater than relay orthogonal channel combination because it involves more complex equalization hardware.	0.08
5	Relay Common Channel Combination (RCC): Common channel combination hardware is required on every relay. The incremental system cost is considered to be greater than interhop interference cancellation because it involves leveraging the feed-forward part of the interhop interference for diversity combination.	0.10
6	Multiple Channel Transmission (MCT): Multiple channel transmission hardware is required on transmitters. The incremental system cost is considered to be greater than relay common channel combination because it involves each transmitter generating comparatively more power and interference within the network.	0.15
7	K Channels Available (KCA): K orthogonal channels are available. The incremental system cost is considered to be greater than multiple channel transmission because it involves K-2 more channels being provided within the network for every active source-destination pair.	0.20
8	N Channels Available (NCA): N orthogonal channels are available. The incremental system cost is considered to be greater than K channel available because it involves N-K more channels being provided within the network for every active source-destination pair.	0.35

Table 1. System Cost Weight of System Resource Constraints

The minimum cost constraint sets are therefore the sets of constraints that result in the different cooperative connectivity models while minimizing the sum of the corresponding system cost weights. The minimum cost constraint set equations presented below specify the relationship between the characterizing parameters of the cooperative connectivity models and the presence of each system resource in the minimum cost constraint set. Together with the previously presented cooperative connectivity equations, these equations address the second problem class listed in the introduction: given a desired level of cooperative connectivity, determine the possible (and lowest cost) sets of system resources that can be used to achieve it.

Minimum Cost Constraint Set Equations for Models with KCA: The minimum cost constraint set equations for cooperative connectivity models with K channels available are:

```
RCC: if (CR | NR) then RCC = TRUE
     else if (1R | KR) then RCC = FALSE
DCC: if (CD | FD) then DCC = TRUE
     else if (1D | KD) then DCC = FALSE
ROC: if (KR | NR) then ROC = TRUE
     else if (1R | CR) then ROC = FALSE.
DOC: if (KD | FD) then DOC = TRUE
     else if (1D | CD) then DOC = FALSE
MCT: MCT = FALSE
IIC: if (FH) then IIC = TRUE
     else if (KH) then IIC = FALSE
```

Minimum Cost Constraint Set Equations for Models with 2CA: The minimum cost constraint set equations for cooperative connectivity models with 2 channels available are:

RCC: `if (CR | NR) then` RCC `= TRUE`
`else if (1R) then` RCC `= FALSE`
DCC: `if (CD | ND | FD) then` DCC `= TRUE`
`else if (1D | 2D) then` DCC `= FALSE`
ROC: ROC `= FALSE`
DOC: `if (2D | FD) then` DOC `= TRUE`
`else if (1D | CD | ND) then` DOC `= FALSE`
MCT: `if (NR | ND) then` MCT `= TRUE`
`else` MCT `= FALSE`
IIC: `if (FH) then` IIC `= TRUE`
`else if (2H) then` IIC `= FALSE`

Minimum Cost Constraint Set Equations for Models with NCA: The minimum cost constraint set equations for cooperative connectivity models with *N* channels available are:

RCC: RCC `= FALSE`
DCC: DCC `= FALSE`
ROC: `if (FR) then` ROC `= TRUE`
`else if (1R) then` ROC `= FALSE`
DOC: `if (FD) then` DOC `= TRUE`
`else if (1D) then` DOC `= FALSE`
MCT: MCT `= FALSE`
IIC: IIC `= FALSE`

It is interesting to note that not all of the system resource constraints are included ("TRUE") in the minimum cost constraint sets for a given number of channels available. Multiple channel transmission is not part of any of the minimum cost constraint sets when there are *K* channels available. Relay orthogonal channel combination is not part of any of the minimum cost constraint sets when there are 2 channels available. Relay and destination common channel combination, multiple channel transmission, and interhop interference cancellation are not part of any of the minimum cost constraint sets when there are N channels available. This does not imply that the corresponding system resource does not have any impact, only that the same cooperative connectivity can be achieved using an alternative set of system resources with lower system cost.

Table 2 summarizes the minimum cost constraint sets for the cooperative connectivity models with K channels available, indicating the system cost weight sums.

Model	Minimum Cost Constraint Set	Cost
1R1DKH	{KCA, NRCC, NDCC, NROC, NDOC, NMCT, NIIC}	0.20
1R1DFH	{KCA, NRCC, NDCC, NROC, NDOC, NMCT, IIC}	0.28
1RKDKH	{KCA, NRCC, NDCC, NROC, DOC, NMCT, NIIC}	0.22
1RKDFH	{KCA, NRCC, NDOC, NROC, DOC, NMCT, IIC}	0.30
1RCDKH	{KCA, NRCC, DCC, NROC, NDOC, NMCT, NIIC}	0.24
1RCDFH	{KCA, NRCC, DCC, NROC, NDOC, NMCT, IIC}	0.32
1RFDKH	{KCA, NRCC, DCC, NROC, DOC, NMCT, NIIC}	0.26
1RFDFH	{KCA, NRCC, DCC, NROC, DOC, NMCT, IIC}	0.34
KR1DKH	{KCA, NRCC, NDCC, ROC, NDOC, NMCT, NIIC}	0.26
KR1DFH	{KCA, NRCC, NDCC, ROC, NDOC, NMCT, IIC}	0.34
KRKDKH	{KCA, NRCC, NDCC, ROC, DOC, NMCT, NIIC}	0.28
KRKDFH	{KCA, NRCC, NDCC, ROC, DOC, NMCT, IIC}	0.36
KRCDKH	{KCA, NRCC, DCC, ROC, NDOC, NMCT, NIIC}	0.30
KRCDFH	{KCA, NRCC, DCC, ROC, NDOC, NMCT, IIC}	0.38
KRFDKH	{KCA, NRCC, DCC, ROC, DOC, NMCT, NIIC}	0.32
KRFDFH	{KCA, NRCC, DCC, ROC, DOC, NMCT, IIC}	0.40
CR1DKH	{KCA, RCC, NDCC, NROC, NDOC, NMCT, NIIC}	0.30
CR1DFH	{KCA, RCC, NDCC, NROC, NDOC, NMCT, IIC}	0.38
CRKDKH	{KCA, RCC, NDCC, NROC, DOC, NMCT, NIIC}	0.32
CRKDFH	{KCA, RCC, NDCC, NROC, DOC, NMCT, IIC}	0.40
CRCDKH	{KCA, RCC, DCC, NROC, NDOC, NMCT, NIIC}	0.34
CRCDFH	{KCA, RCC, DCC, NROC, NDOC, NMCT, IIC}	0.42
CRFDKH	{KCA, RCC, DCC, NROC, DOC, NMCT, NIIC}	0.36
CRFDFH	{KCA, RCC, DCC, NROC, DOC, NMCT, IIC}	0.44
NR1DKH	{KCA, RCC, NDCC, ROC, NDOC, NMCT, NIIC}	0.36
NR1DFH	{KCA, RCC, NDCC, ROC, NDOC, NMCT, IIC}	0.44
NRKDKH	{KCA, RCC, NDCC, ROC, DOC, NMCT, NIIC}	0.38
NRKDFH	{KCA, RCC, NDCC, ROC, DOC, NMCT, IIC}	0.46
NRCDKH	{KCA, RCC, DCC, ROC, NDOC, NMCT, NIIC}	0.40
NRCDFH	{KCA, RCC, DCC, ROC, NDOC, NMCT, IIC}	0.48
NRFDKH	{KCA, RCC, DCC, ROC, DOC, NMCT, NIIC}	0.42
NRFDFH	{KCA, RCC, DCC, ROC, DOC, NMCT, IIC}	0.50

Table 2. Minimum Cost Constraint Sets for KCA

Table 3 summarizes the minimum cost constraint sets for the cooperative connectivity models with 2 channels available, indicating the system cost weight sums.

Model	Minimum Cost Constraint Set	Cost
1R1D2H	{2CA, NRCC, NDCC, NROC, NDOC, NMCT, NIIC}	0.00
1R1DFH	{2CA, NRCC, NDCC, NROC, NDOC, NMCT, IIC}	0.08
1R2D2H	{2CA, NRCC, NDCC, NROC, DOC, NMCT, NIIC}	0.02
1R2DFH	{2CA, NRCC, NDOC, NROC, DOC, NMCT, IIC}	0.10
1RCD2H	{2CA, NRCC, DCC, NROC, NDOC, NMCT, NIIC}	0.04
1RCDFH	{2CA, NRCC, DCC, NROC, NDOC, NMCT, IIC}	0.12
1RND2H	{2CA, NRCC, DCC, NROC, NDOC, MCT, NIIC}	0.19
1RNDFH	{2CA, NRCC, DCC, NROC, NDOC, MCT, IIC}	0.27
1RFD2H	{2CA, NRCC, DCC, NROC, DOC, NMCT, NIIC}	0.06
1RFDFH	{2CA, NRCC, DCC, NROC, DOC, NMCT, IIC}	0.14
CR1DFH	{2CA, RCC, NDCC, NROC, NDOC, NMCT, IIC}	0.18
CR2DFH	{2CA, RCC, NDCC, NROC, DOC, NMCT, IIC}	0.20
CRCDFH	{2CA, RCC, DCC, NROC, NDOC, NMCT, IIC}	0.22
CRFDFH	{2CA, RCC, DCC, NROC, DOC, NMCT, IIC}	0.24
NR1DFH	{2CA, RCC, NDCC, NROC, NDOC, MCT, IIC}	0.33
NR2DFH	{2CA, RCC, NDCC, NROC, DOC, MCT, IIC}	0.35
NRNDFH	{2CA, RCC, DCC, NROC, NDOC, MCT, IIC}	0.37
NRFDFH	{2CA, RCC, DCC, NROC, DOC, MCT, IIC}	0.39

Table 3. Minimum Cost Constraint Sets for 2CA

Table 4 summarizes the minimum cost constraint sets for the cooperative connectivity models with *N* channels available, indicating the system cost weight sums.

Model	Minimum Cost Constraint Set	Cost
1R1DFH	{NCA, NRCC, NDCC, NROC, NDOC, NMCT, NIIC}	0.35
1RFDFH	{NCA, NRCC, NDCC, NROC, DOC, NMCT, NIIC}	0.37
FR1DFH	{NCA, NRCC, NDCC, ROC, NDOC, NMCT, NIIC}	0.41
FRFDFH	{NCA, NRCC, NDCC, ROC, DOC, NMCT, NIIC}	0.43

Table 4. Minimum Cost Constraint Sets for NCA

The system cost weights allow further comparison of the minimum cost constraint sets for cooperative connectivity models with different numbers of channels available. Table 5 summarizes the composite minimum cost constraint sets for cooperative connectivity models with different numbers of channels available, indicating and sorted by the baseline system cost weight sums.

Model	Minimum Cost Constraint Set	Cost
1R1D2H	{2CA, NRCC, NDCC, NROC, NDOC, NMCT, NIIC}	0.00
1R2D2H	{2CA, NRCC, NDCC, NROC, DOC, NMCT, NIIC}	0.02
1RCD2H	{2CA, NRCC, DCC, NROC, NDOC, NMCT, NIIC}	0.04
1RFD2H	{2CA, NRCC, DCC, NROC, DOC, NMCT, NIIC}	0.06
1R1DFH	{2CA, NRCC, NDCC, NROC, NDOC, NMCT, IIC}	0.08
1R2DFH	{2CA, NRCC, NDOC, NROC, DOC, NMCT, IIC}	0.10
1RCDFH	{2CA, NRCC, DCC, NROC, NDOC, NMCT, IIC}	0.12
1RFDFH	{2CA, NRCC, DCC, NROC, DOC, NMCT, IIC}	0.14
CR1DFH	{2CA, RCC, NDCC, NROC, NDOC, NMCT, IIC}	0.18
1RND2H	{2CA, NRCC, DCC, NROC, NDOC, MCT, NIIC}	0.19
CR2DFH	{2CA, RCC, NDCC, NROC, DOC, NMCT, IIC}	0.20
1R1DKH	{KCA, NRCC, NDCC, NROC, NDOC, NMCT, NIIC}	0.20
CRCDFH	{2CA, RCC, DCC, NROC, NDOC, NMCT, IIC}	0.22
1RKDKH	{KCA, NRCC, NDCC, NROC, DOC, NMCT, NIIC}	0.22
CRFDFH	{2CA, RCC, DCC, NROC, DOC, NMCT, IIC}	0.24
1RCDKH	{KCA, NRCC, DCC, NROC, NDOC, NMCT, NIIC}	0.24
1RFDKH	{KCA, NRCC, DCC, NROC, DOC, NMCT, NIIC}	0.26
KR1DKH	{KCA, NRCC, NDCC, ROC, NDOC, NMCT, NIIC}	0.26
1RNDFH	{2CA, NRCC, DCC, NROC, NDOC, MCT, IIC}	0.27
KRKDKH	{KCA, NRCC, NDCC, ROC, DOC, NMCT, NIIC}	0.28
1RKDFH	{KCA, NRCC, NDOC, NROC, DOC, NMCT, IIC}	0.30
KRCDKH	{KCA, NRCC, DCC, ROC, NDOC, NMCT, NIIC}	0.30
CR1DKH	{KCA, RCC, NDCC, NROC, NDOC, NMCT, NIIC}	0.30
KRFDKH	{KCA, NRCC, DCC, ROC, DOC, NMCT, NIIC}	0.32
CRKDKH	{KCA, RCC, NDCC, NROC, DOC, NMCT, NIIC}	0.32
NR1DFH	{2CA, RCC, NDCC, NROC, NDOC, MCT, IIC}	0.33
KR1DFH	{KCA, NRCC, NDCC, ROC, NDOC, NMCT, IIC}	0.34
CRCDKH	{KCA, RCC, DCC, NROC, NDOC, NMCT, NIIC}	0.34
NR2DFH	{2CA, RCC, NDCC, NROC, DOC, MCT, IIC}	0.35
KRKDFH	{KCA, NRCC, NDCC, ROC, DOC, NMCT, IIC}	0.36
CRFDKH	{KCA, RCC, DCC, NROC, DOC, NMCT, NIIC}	0.36
NR1DKH	{KCA, RCC, NDCC, ROC, NDOC, NMCT, NIIC}	0.36
NRNDFH	{2CA, RCC, DCC, NROC, NDOC, MCT, IIC}	0.37
KRCDFH	{KCA, NRCC, DCC, ROC, NDOC, NMCT, IIC}	0.38
NRKDKH	{KCA, RCC, NDCC, ROC, DOC, NMCT, NIIC}	0.38
NRFDFH	{2CA, RCC, DCC, NROC, DOC, MCT, IIC}	0.39
KRFDFH	{KCA, NRCC, DCC, ROC, DOC, NMCT, IIC}	0.40
CRKDFH	{KCA, RCC, NDCC, NROC, DOC, NMCT, IIC}	0.40
NRCDKH	{KCA, RCC, DCC, ROC, NDOC, NMCT, NIIC}	0.40
FR1DFH	{NCA, NRCC, NDCC, ROC, NDOC, NMCT, NIIC}	0.41
NRFDKH	{KCA, RCC, DCC, ROC, DOC, NMCT, NIIC}	0.42
FRFDFH	{NCA, NRCC, NDCC, ROC, DOC, NMCT, NIIC}	0.43
NRKDFH	{KCA, RCC, NDCC, ROC, DOC, NMCT, IIC}	0.46
NRCDFH	{KCA, RCC, DCC, ROC, NDOC, NMCT, IIC}	0.48

Table 5. Composite Minimum Cost Constraint Sets

It is important to note that a different ordering of system resource constraints with respect to increasing system cost would result in different minimum cost constraint sets for some of the cooperative connectivity models, specifically those involving non-identical relay (NR), full relay (FR), or full destination (FD) connectivity. For example, consider that in some context bandwidth is very cheap such that instead of a system cost weight of 0.35, the NCA system resource constraint is assigned a system cost weight of 0.10. In the baseline context, the minimum cost constraint set for the 1RFDFH cooperative connectivity model would be {2CA, NRCC, DCC, NROC, DOC, NMCT, IIC}. In this alternate context, the minimum cost constraint set for the 1RFDFH cooperative connectivity model would instead be {NCA, NRCC, NDCC, NROC, DOC, NMCT, NIIC}. Since bandwidth is very cheap, the desired connectivity would be less costly to achieve by allocating more bandwidth instead of requiring more complex wireless terminal hardware.

For any assumed ordering/weighting of system resource constraints, the minimum cost constraint set associated with a particular cooperative connectivity model can be determined by minimizing the system cost weight sum across all corresponding system resource constraint combinations in Appendix C. In general, the minimum cost constraint sets are not very sensitive to small changes in the system cost weights of the different system resource constraints, but are sensitive to significant reordering of the system resource constraints with respect to system cost. The only minimum cost constraint sets that are not at all sensitive to reordering of the system resource constraints with respect to system cost are those where only a single combination of system resource constraints results in the corresponding cooperative connectivity model, or those where a single combination of system resource constraints is a subset of all other combinations that result in the corresponding cooperative connectivity model. An example of the first case is the minimum cost constraint set {KCA, RCC, DCC, NROC, DOC, NMCT, NIIC}, that is the only combination that results in the CRFDKH model. An example of the second case is the minimum cost constraint set {2CA, NRCC, NDCC, NROC, DOC, NMCT, NIIC}, that is a subset of all other combinations that result in the 1R2D2H model.

5.5 Cooperative Connectivity Model Transitions

The transitions between cooperative connectivity models with respect to the lifting of system resource constraints are shown in this section. Figs. 17-19 respectively show the transitions between the different cooperative connectivity models when there are K, 2, and N channels available. These transition diagrams show the paths of increasing cooperative connectivity that can be followed without wasting any system resources, which would occur if a particular allocated system resource does not contribute to improving the cooperative connectivity. Increasing cooperative connectivity refers to the fact that as more system resources are available it is possible to add more active links between pairs of terminals. Each transition in the transition diagrams corresponds to the inclusion of an additional system resource and the possibility of adding more active links that rely on that system resource, therefore increasing the achievable cooperative connectivity. The boxes with ‘KH/FH’ represent two cooperative connectivity models with a different maximum length of the longest multihop path of the network. Transitions between ‘KH’ and ‘FH’ cooperative connectivity models correspond to the IIC system resource constraint being lifted. The boxes with only ‘FH’ indicate that the corresponding

‘KH’ cooperative connectivity model does not exist for the given number of available channels. Transitions are in the direction of decreased system resource constraints.

Transitions that decrease system resource constraints without improving the cooperative connectivity are not shown. For example, adding MCT when the cooperative connectivity model is 1R1DFH brings no benefit since the terminals are not able to diversity combine the new channels. Transitions that do not follow the minimum cost constraint sets derived in the previous section are also not shown. For example, adding DOC when the cooperative connectivity model is 1RNDFH is not optimal since the resultant 1RFDFH cooperative connectivity model does not require MCT. Consider that the minimum cost constraint set of the originating 1RNDFH model includes the DCC, MCT, and IIC system resources. When the DOC system resource is added it results in the 1RFDFH model implemented with the DCC, MCT, IIC, and DOC system resources. However, the minimum cost constraint set of the 1RFDFH model includes the DCC, DOC, and IIC system resources but not the MCT system resource, meaning that the MCT system resource does not provide any increase in achievable cooperative connectivity.

Together with the previously presented cooperative connectivity and minimum cost constraint set equations, these transition diagrams address the third and fourth problem classes listed in the introduction: given a baseline set of system resources and level of cooperative connectivity, determine the impact on the achievable cooperative connectivity of incrementally adding or removing different system resources; and given a baseline set of system resources and level of cooperative connectivity, determine the possible (and lowest cost) sets of system resources that can be incrementally added or removed to achieve different levels of cooperative connectivity.

Fig. 17 shows the transitions between the different cooperative connectivity models when there are *K* channels available for various constraint changes.

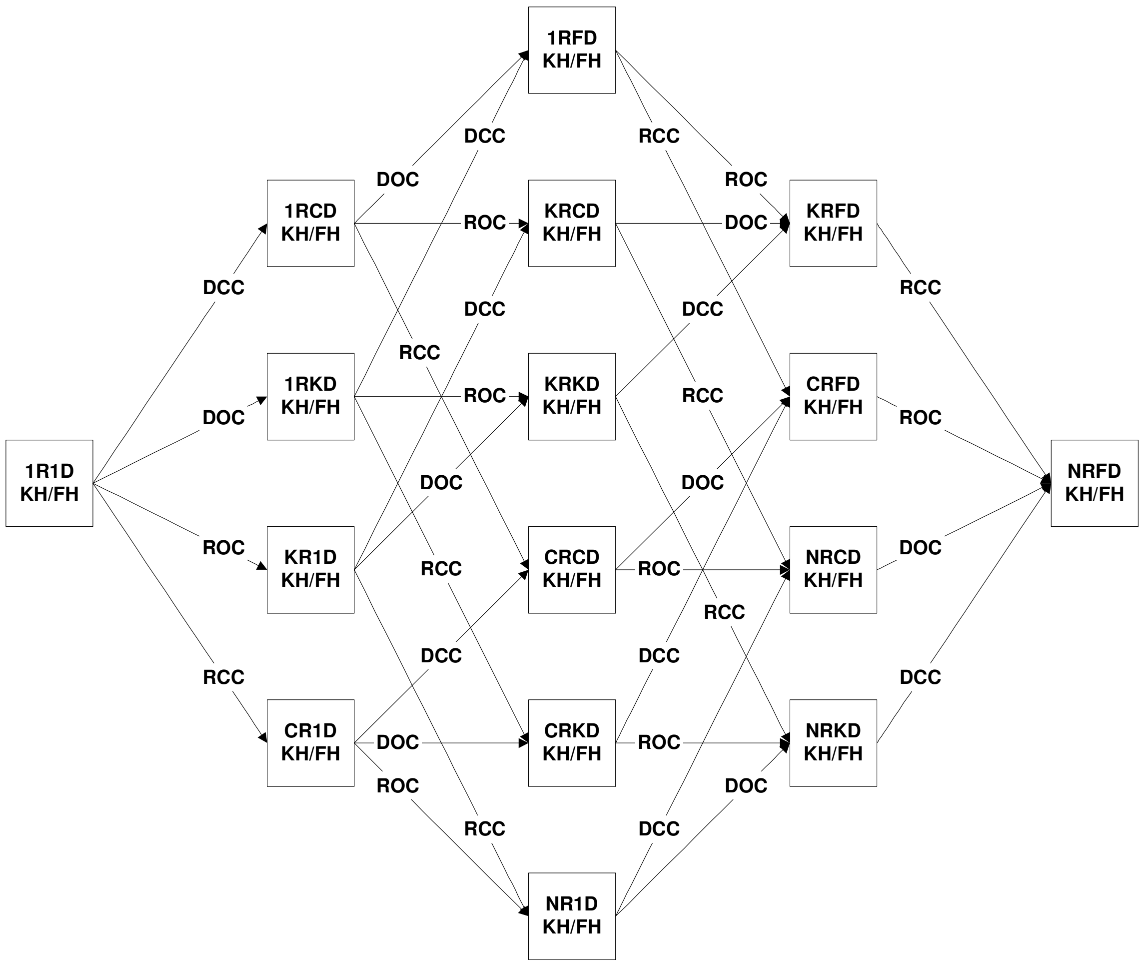

Fig. 17. Cooperative Connectivity Model Transitions for KCA

Fig. 18 shows the transitions between the different cooperative connectivity models when there are 2 channels available for various constraint changes.

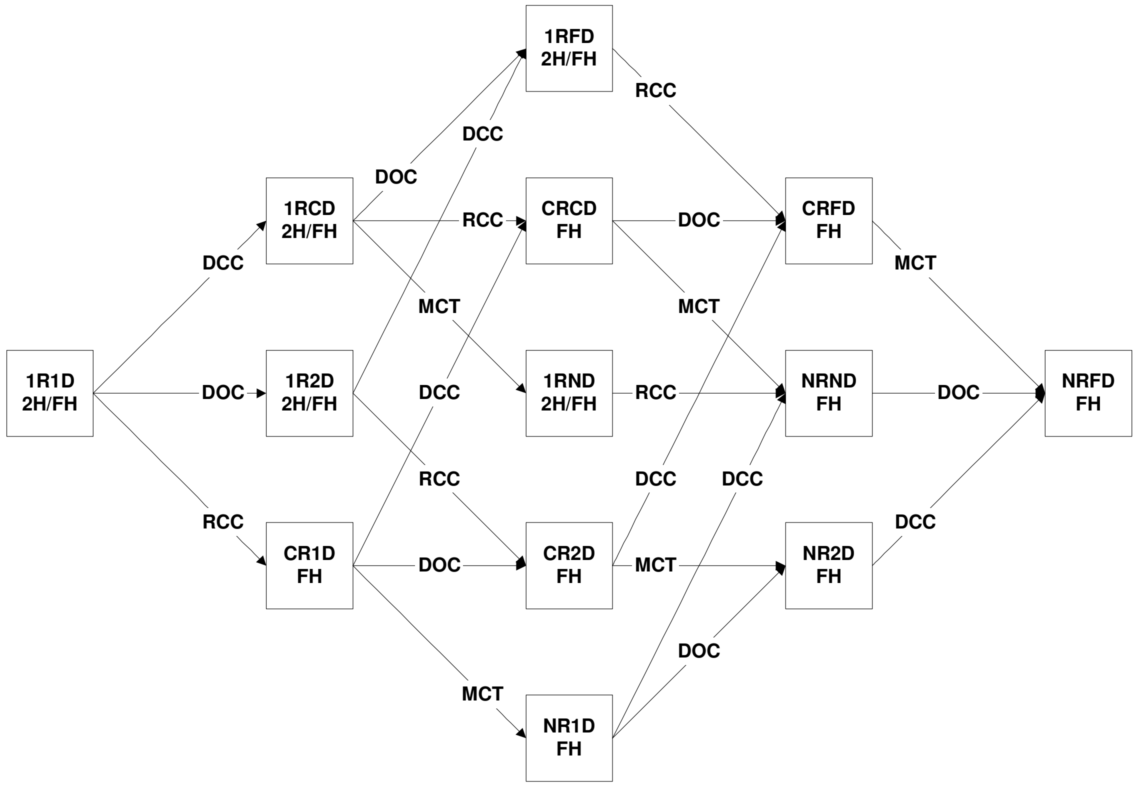

Fig. 18. Cooperative Connectivity Model Transitions for 2CA

Fig. 19 shows the transitions between the different cooperative connectivity models when there are *N* channels available for various constraint changes.

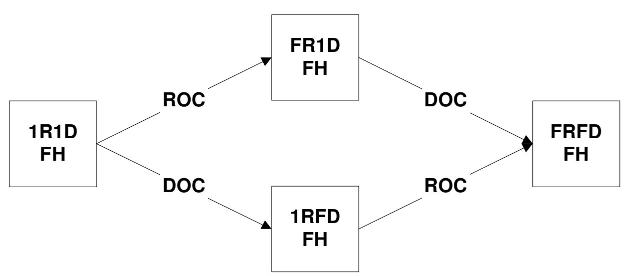

Fig. 19. Cooperative Connectivity Model Transitions for NCA

5.6 Mapping to the Literature

This section summarizes the mapping of the cooperative connectivity models to various distributed spatial diversity techniques presented in the literature. The list of presented references is not intended to be comprehensive, but is generally representative of the current state of the literature at the time of writing. Table 6 shows this mapping, indicating the name of the distributed spatial diversity technique, the corresponding references, the corresponding cooperative connectivity model of the developed framework, and the channel allocation assumed in the references. Those references that have an indicated channel allocation of "N/A" present information theoretic results that are independent of channel allocation.

This mapping highlights that although the literature published so far has only started to explore the many possible cooperative connectivity models, there is already a broad range of connectivity and channel allocation assumptions. It also illustrates the wide variation in terminology used in the literature. The developed framework allows us to compare the cooperative connectivity and system resource requirements of the proposed distributed spatial diversity techniques and could be further applied to analyze the efficiency, in comparison to the minimum cost constraint sets, of the available system resources assumed in the literature.

Distributed Spatial Diversity Technique	References	Model	# Chnls
2-hop multihop without diversity	[15]	1R1D2H	2CA
Conventional 2-hop relaying	[48]	1R1D2H	2CA
Relayed transmission	[46],[124]	1R1D2H	2CA
Multihop without diversity	[15]	1R1DFH	2CA
Multi-hop cooperation	[91]	1R1DFH	NCA
Multihop relaying	[45],[56],[84]	1R1DFH	NCA
Wireless network $O(\sqrt{n})$	[40]	1R1DFH	NCA
Multihop relaying	[57]	1R1DFH	N/A
2-hop multihop diversity	[14],[15]	1R2D2H	2CA
Coded cooperation	[51],[52],[53],[80]	1R2D2H	2CA
Cooperative coding	[71],[104]	1R2D2H	2CA
Cooperative diversity	[47],[61],[62],[64],[66]	1R2D2H	2CA
Cooperative protocols I, II, and III	[78]	1R2D2H	2CA
Cooperative space-time delay coding	[81]	1R2D2H	2CA
Cooperative superposition modulation	[67]	1R2D2H	2CA
Distributed turbo coded diversity	[132]	1R2D2H	2CA
Relayed block markov transmission	[21]	1R2D2H	2CA
MIMO relay channel	[112]	1R2D2H	N/A
Relay channel	[25]	1R2D2H	N/A
Multihop, multi-branch cooperation	[91]	1RKDKH	NCA
Non-interfering multi-path transmission	[88]	1RKDKH	NCA
2-hop MSE relaying	[58]	1RCD2H	2CA
Coherent cooperative transmission	[111]	1RCD2H	2CA
Interference relay network	[76]	1RCD2H	2CA
Linear relaying	[120]	1RCD2H	2CA
Parallel relays w space-time modulations	[50]	1RCD2H	2CA
User cooperation	[12],[59],[97]	1RCD2H	2CA
Virtual antenna array	[7],[27]	1RCD2H	2CA
Parallel relay network	[95]	1RCD2H	N/A
Interfering multi-path transmission	[88]	1RCDKH	KCA
2-hop cooperative relaying	[42],[43]	1RFD2H	2CA
2-hop relay network	[119]	1RFD2H	2CA
Collaborative coding	[74]	1RFD2H	2CA
Distributed Alamouti system	[5]	1RFD2H	2CA
Distributed space-time coding	[54],[63],[77]	1RFD2H	2CA
Dynamic decoded and forward	[6]	1RFD2H	2CA
Network path selection diversity	[11]	1RFD2H	2CA
Non-orthogonal amplify and forward	[6]	1RFD2H	2CA
Relay assisted MIMO channel	[89]	1RFD2H	2CA
Single-stage cooperative relaying	[84]	1RFD2H	2CA
Cooperative network	[4]	1RFD2H	NCA
Multi-branch cooperation	[91]	1RFD2H	NCA

Multi-user spatial diversity	[31]	1RFD2H	NCA
Repetition-based cooperative diversity	[63]	1RFD2H	NCA
C(*m*) cooperative diversity where *K*=*m*+1	[92]	KRFDFH	NCA
Cascaded (*K*-1)-hop cooperative diversity	[48]	KRKDFH	KCA
Two-level leapfrog scheme with *K*=2	[41]	KRKDFH	KCA
Amplify-and-forward MIMO tunnel	[88]	CRKDKH	KCA
Distributed MIMO multihop system	[28]	CRCDKH	KCA
Multi-stage cooperative relaying	[84]	CRCDKH	KCA
Opportunistic large array	[94],[102]	CRCDKH	KCA
C(*N*-1) cooperative diversity where *K*=*N*	[92]	FRFDFH	NCA
Full cooperative relaying	[48]	FRFDFH	NCA
Multihop diversity	[14],[15]	FRFDFH	NCA
Relay network	[35]	FRFDFH	NCA
Cooperative wireless system	[125],[126],[127]	FRFDFH	N/A
Decode / compress and forward	[60]	FRFDFH	N/A
Multiple level relay channel	[122]	FRFDFH	N/A
Wireless network *O(n)*	[40]	FRFDFH	N/A

Table 6. Mapping of Cooperative Connectivity Models to the Literature

5.7 Summary

This chapter has developed a framework for modeling cooperative connectivity that exposes the relationship between constraints on the available system resources and the achievable cooperative connectivity of wireless relay networks. The cooperative connectivity models resulting from the possible combinations of system resource constraints are derived and associated with their minimum cost constraint sets. The cooperative connectivity models are fully characterized according to three parameters: the achievable cooperative connectivity of the relays, the achievable cooperative connectivity of the destination, and the maximum achievable length of the longest multihop path of the network. Connectivity model equations are provided that specify the relationship between the system resource constraints and the characterizing parameters of the cooperative connectivity models. A set of baseline minimum cost constraint set equations are provided that specify the relationship between the characterizing parameters of the cooperative connectivity models and the presence of each system resource in the minimum cost constraint set.

Cooperative connectivity model transition diagrams that show the transitions between the different cooperative connectivity models for constraint changes following the minimum cost constraint sets are presented. These transition diagrams show the paths of increasing cooperative connectivity that can be followed without wasting any system resources, which would occur if a particular allocated system resource does not contribute to improving the cooperative connectivity. The cooperative connectivity models are mapped to the various distributed spatial diversity techniques presented in the literature in order to highlight the value of the developed framework and the general richness of the problem domain. Each mapping indicates the distributed spatial diversity technique and channel allocation from the literature along with the corresponding cooperative diversity model. Although the list of presented references is not comprehensive, it is representative of the current state of the literature. This mapping highlights that although the literature published so far has only started to explore the many possible cooperative connectivity models there is already a broad range of connectivity and channel allocation assumptions.

Chapter 6 - Cooperative Connectivity Model Simulations

This chapter applies the cooperative connectivity modeling framework in a series of simulations that illustrate the value of the framework as a modeling tool for wireless relay networks. The simulations provide a comparison with respect to probability of error and information theoretic probability of outage for some example relaying methods and network topologies, and allow the performance impact of the individual system resource constraints to be isolated. Section 6.1 describes the detailed structure and parameters of the simulations and provides supporting information on which cooperative connectivity models can be compared to isolate the impact of the various system resource constraints. Section 6.2 presents raw probability of error simulation results and a summary of the impact of the system resource constraints on the probability of error. Section 6.3 presents raw information theoretic probability of outage simulation results and a summary of the impact of the system resource constraints on the information theoretic probability of outage. Section 6.4 presents information theoretic probability of outage bound simulation results for common codebook generation, and compares these bounds with the information theoretic probability of outage results for the various considered relaying methods. Section 6.5 discusses some significant qualitative results, extrapolated from the simulations, which are worthy of more comprehensive study. Section 6.6 summarizes the contributions of the chapter.

6.1 Introduction

The cooperative connectivity models are applied in a series of simulations that provide a comparison with respect to probability of error and information theoretic probability of outage for various relaying methods and network topologies. The probability of error and probability of outage of the singlehop (direct source-destination) channel are respectively compared for reference. A BPSK modulation scheme is used for simplicity of exposition. The simulations use the equations provided in the system model presented in Chapter 3 with distance dependent attenuation, a propagation exponent of 4, flat slow Rayleigh fading, no shadowing, and equal power allocation for all transmitting terminals such that the total transmit power is constrained to the transmit power of the singlehop reference channel. Maximal ratio combining is assumed for the probability of error results and probability of outage results. Common codebook generation is assumed for the probability of outage results, as that is the relevant codebook generation scheme for the practical relaying methods under consideration.

One limitation of the probability of error analysis approach is that it does not explicitly take into account information rates, unlike the probability of outage analysis approach. However, it can still be used to provide a fair comparison when the systems being compared operate at the same rate. In the simulations provided, all of the cooperative connectivity models operate at the same rate. Additionally, all of the systems utilize the same channel resources except for the 1R1DKH, 1RFDKH, FR1DFH, and FRFDFH cooperative connectivity models, which are included to provide an explicit performance comparison when increasing the number of orthogonal channels that are used. We note again that the presented maximal ratio combining performance results are a lower bound on the achievable performance when using common channel combination techniques.

Simulations are presented for the three relaying methods described previously, as well as the information theoretic probability of outage bound with common codebook generation, for three network topologies. All of the simulated network topologies have a single source terminal, five relay terminals, and a single destination terminal. The *example network topology* has the example terminal distribution shown in Fig. 20 with network connectivity that optimizes the error performance for the respective cooperative connectivity model and relaying method combinations. The terminal distribution is symmetric with normalized link distances:

$$d_{S,D} \approx 3d_{S,R1} \approx 2d_{S,R3} \approx \tfrac{5}{4}d_{S,R4} \approx 3d_{R1,R2} \approx 2d_{R1,R4} \approx \tfrac{3}{2}d_{R1,R5}.$$

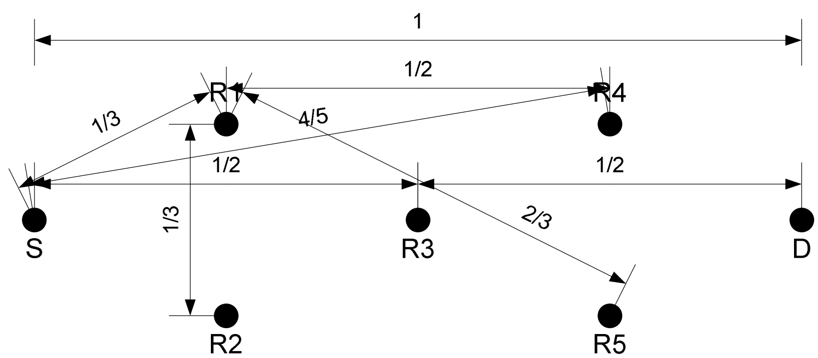

Fig. 20. Example Network Topology

The *linear network topology* has the linear terminal distribution shown in Fig. 21 (the relay terminals are fixed and collinear so that they divide the direct path between the source and destination terminals into six equal-length segments) with network connectivity that optimizes the error performance for the respective cooperative connectivity model and relaying method combinations. The terminal distribution is symmetric with normalized link distances:

$$d_{S,D} \approx \tfrac{6}{1}d_{S,R1} \approx \tfrac{6}{2}d_{S,R2} \approx \tfrac{6}{3}d_{S,R3} \approx \tfrac{6}{4}d_{S,R4} \approx \tfrac{6}{5}d_{S,R5}.$$

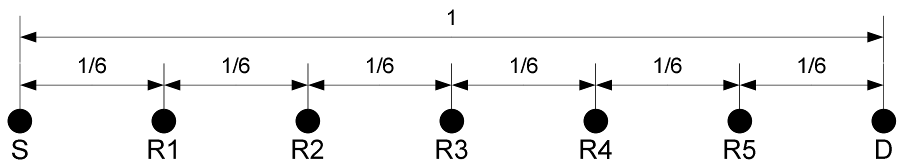

Fig. 21. Linear Network Topology

The *central network topology* has the central terminal distribution shown in Fig. 22 (the relay terminals are equidistant from the source and destination terminals and relatively close together) with network connectivity that optimizes the error performance for the respective cooperative connectivity model and relaying method combinations. Let T_I be the set of all intermediate relay terminals. The terminal distribution is symmetric with normalized link distances:

$$d_{S,D} \approx 2d_{S,Ri} \approx 2d_{Ri,D} \approx 8d_{Ri,Rj}, \forall i, j \in T_I.$$

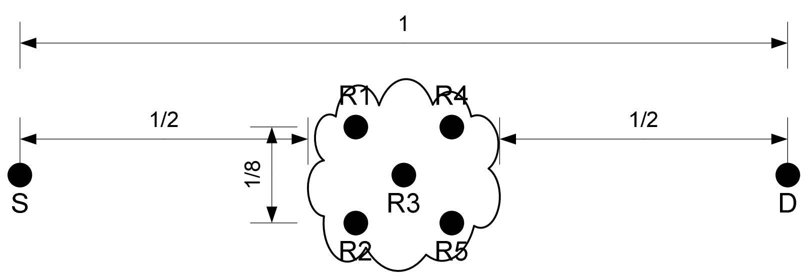

Fig. 22. Central Network Topology

All of the link distances are normalized with respect to the distance between the source and destination. The error optimization mentioned above is over the achievable network connectivity for each combination of cooperative connectivity model and relaying method and is in the large SNR regime such that the diversity order is maximized. Although this error performance is optimal for higher SNRs where the gains due to diversity dominate the gains due to reduced signal attenuation, it is not necessarily optimal for lower SNRs where the gains due to reduced signal attenuation may dominate.

The subset of cooperative connectivity models shown was chosen specifically to isolate the impact of the various constraints. Table 7 indicates which cooperative connectivity models can be compared in order to isolate each constraint. For example, comparison of the 1R1DFH and 1R2DFH models with the 1RCDFH and 1RFDFH models respectively isolates the impact of the destination common channel combination constraint.

Isolated Constraint	Model with Constraint Imposed	Model After Constraint Lifted
NCA: *N* Channels Available	NR1DFH	FR1DFH
(N = # Transmitters)	NRFDFH	FRFDFH
KCA: *K* Channels Available	1R1D2H	1R1DKH
($N > K > 2$)	1RFD2H	1RFDKH
RCC: Relay Common Channel	1R1DFH	CR1DFH
Combination	1RFDFH	CRFDFH
DCC: Destination Common	1R1DFH	1RCDFH
Channel Combination	1R2DFH	1RFDFH
ROC: Relay Orthogonal Channel	1R1DFH	KR1DFH
Combination	1RFDFH	KRFDFH
DOC: Destination Orthogonal	1R1D2H	1R2D2H
Channel Combination	1RCDFH	1RFDFH
MCT: Multiple Channel	1RCDFH	1RNDFH
Transmission	CR1DFH	NR1DFH
IIC: Interhop Interference	1R1D2H	1R1DFH
Cancellation	1RFD2H	1RFDFH

Table 7. Models Compared to Isolate Different System Resource Constraints

6.2 Probability of Error Simulation Results

Figs. 23-31 respectively compare the probability of error of the cooperative connectivity models for the described network topologies using amplified relaying, decoded relaying with error propagation, and decoded relaying without error propagation. These highlight the impact of the system resource constraints by providing a comparison of the cooperative connectivity models in terms of probability of error versus the signal to noise ratio of the singlehop reference channel. The probability of error results are plotted such that the total allocated power of each cooperative connectivity model is the same as that of the singlehop reference channel that achieves the annotated signal to noise ratio.

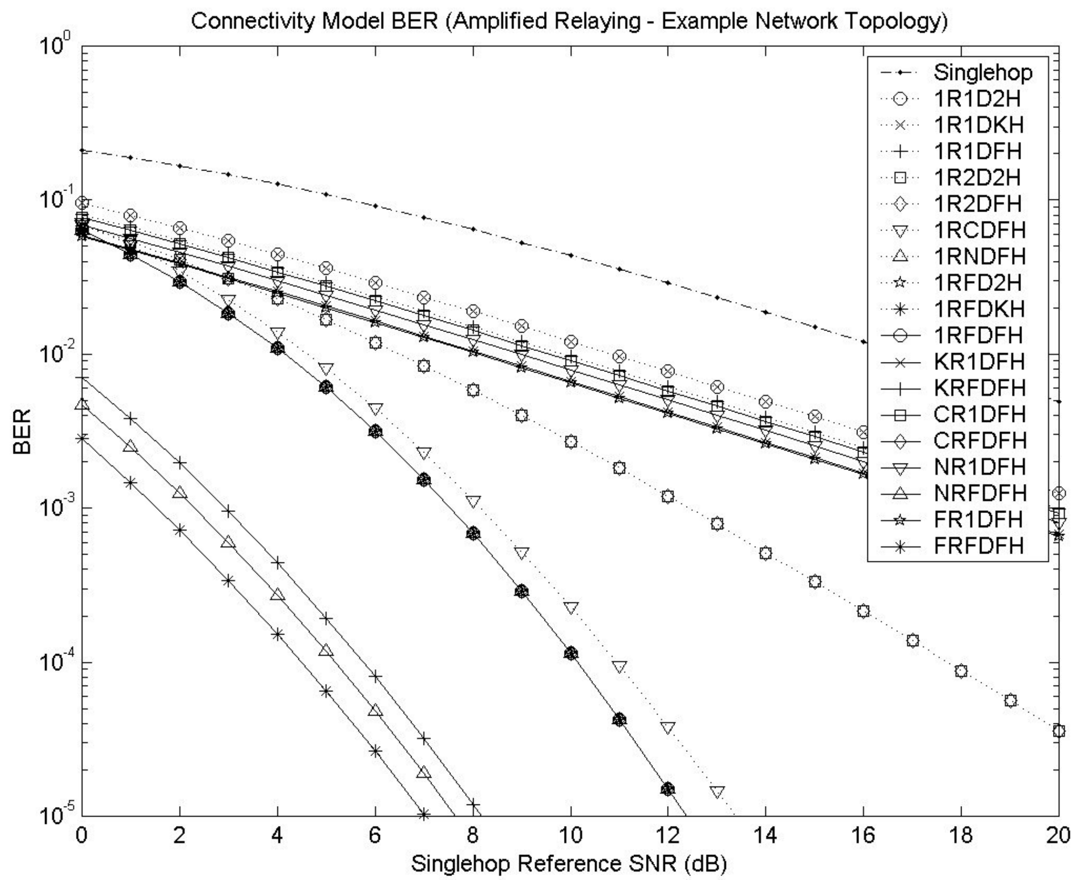

Fig. 23. Error of Amplified Relaying with Example Network Topology

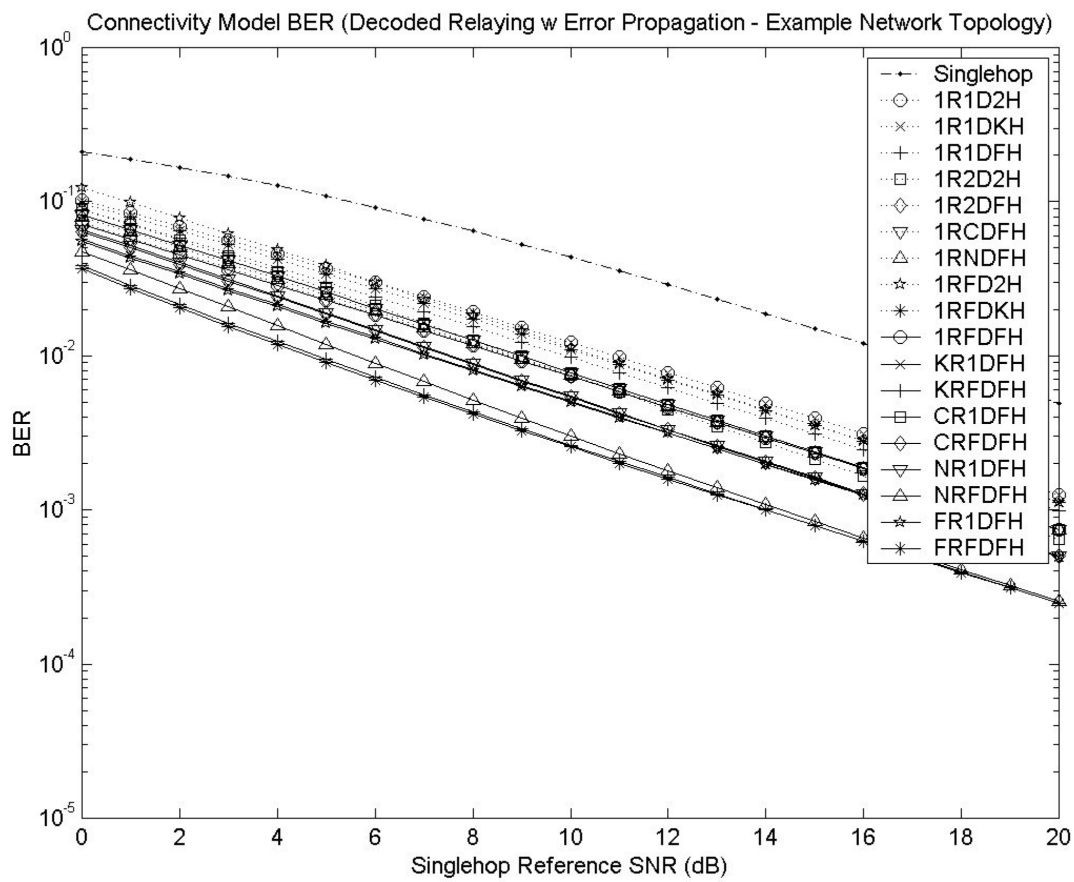

Fig. 24. Error of Decoded Relaying w Prop with Example Network Topology

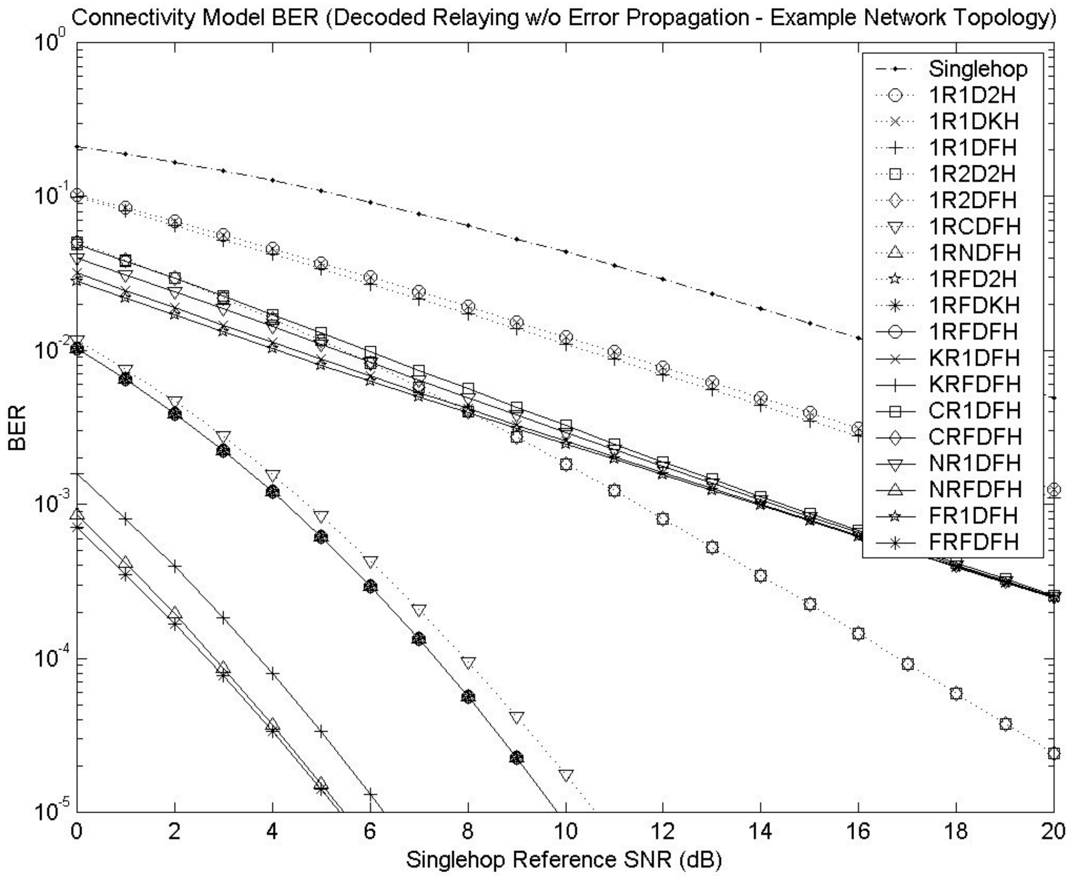

Fig. 25. Error of Decoded Relaying w/o Prop with Example Network Topology

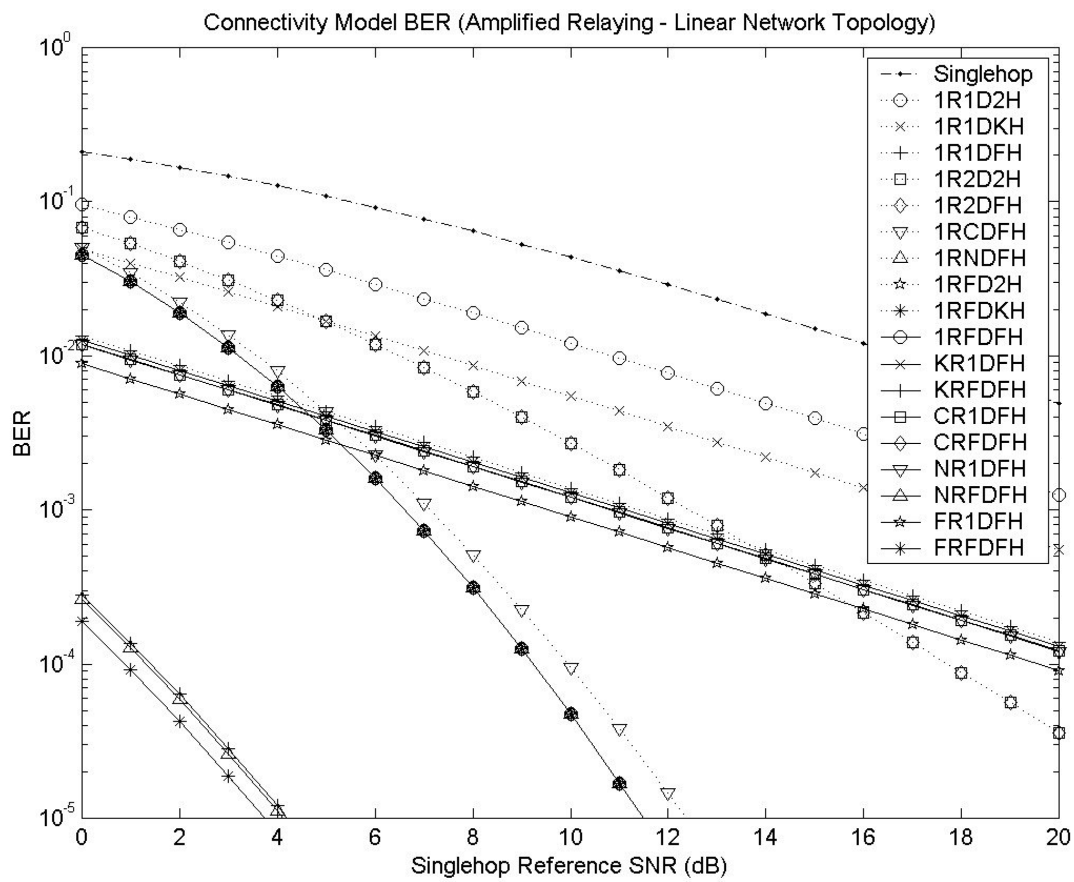

Fig. 26. Error of Amplified Relaying with Linear Network Topology

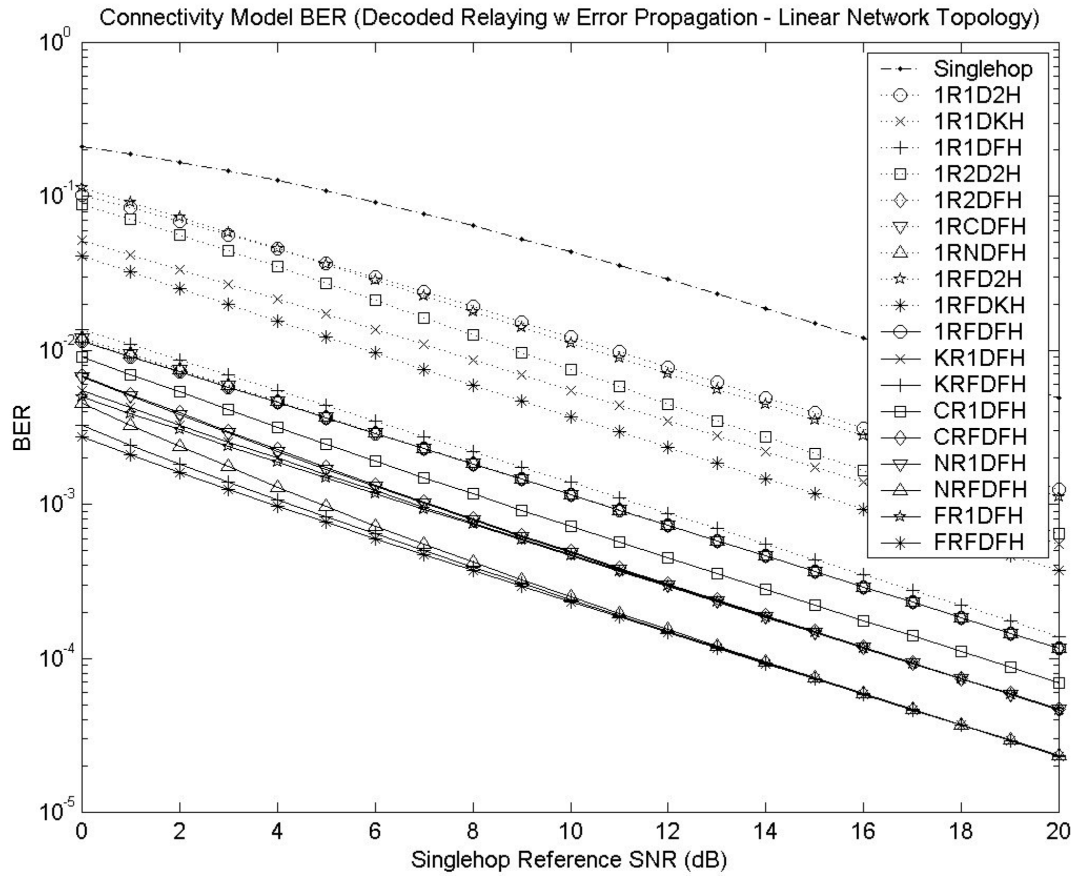

Fig. 27. Error of Decoded Relaying w Prop with Linear Network Topology

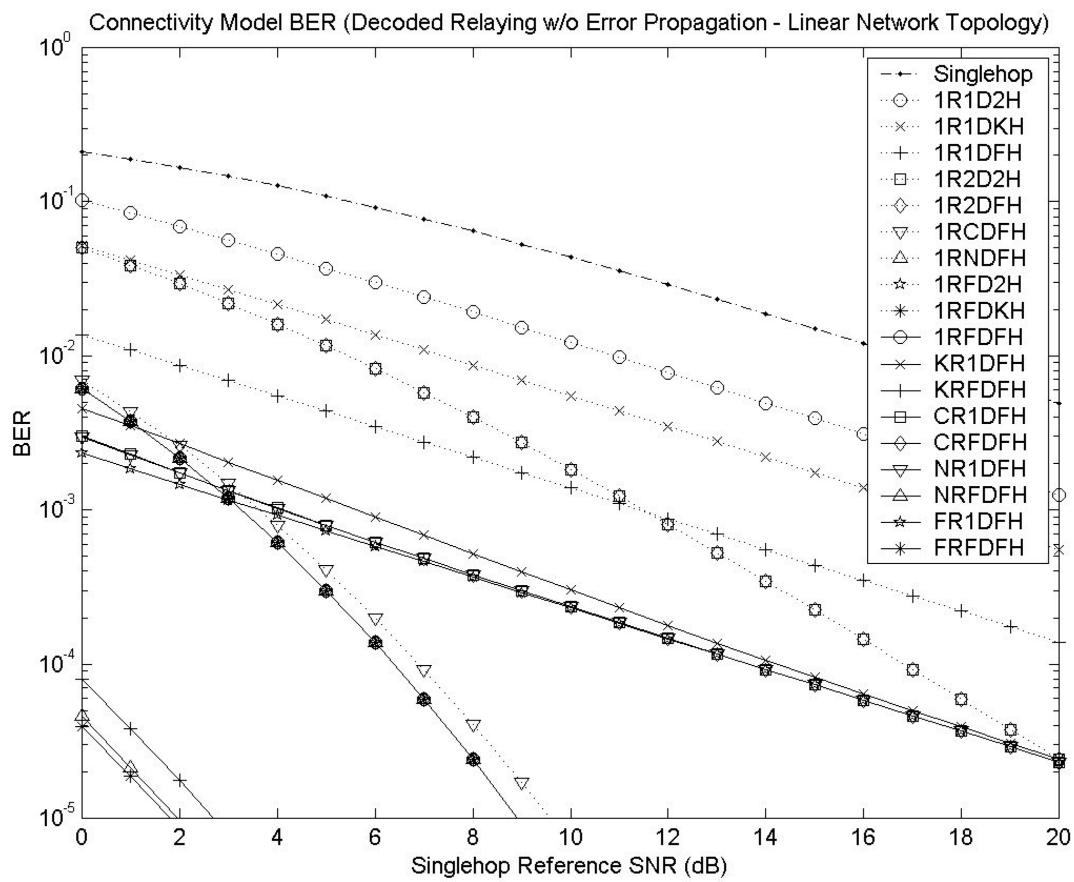

Fig. 28. Error of Decoded Relaying w/o Prop with Linear Network Topology

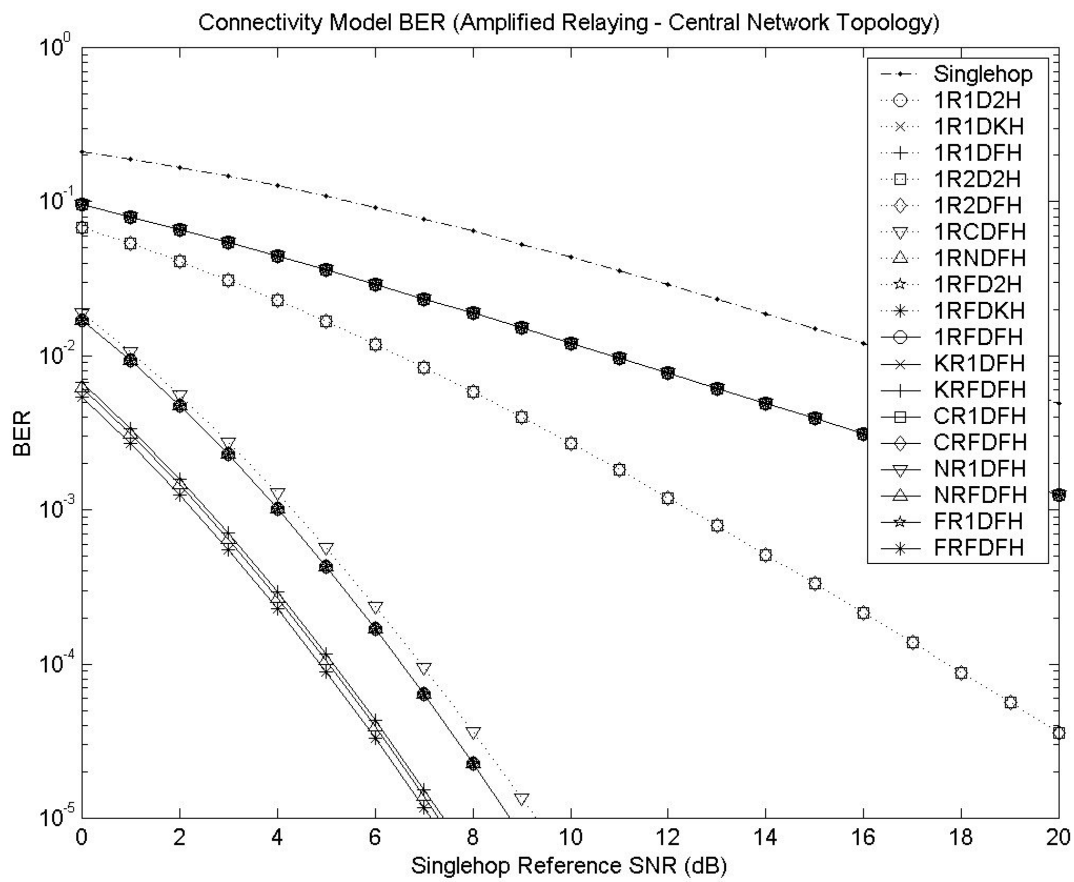

Fig. 29. Error of Amplified Relaying with Central Network Topology

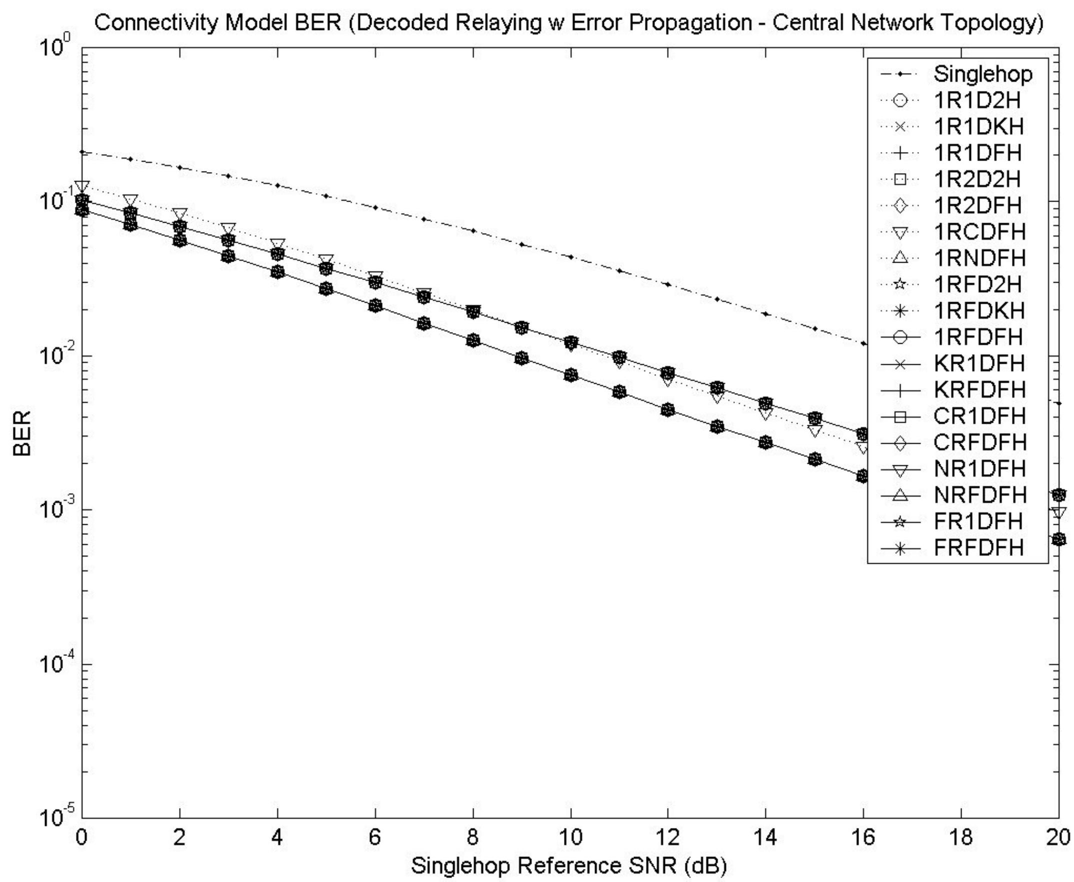

Fig. 30. Error of Decoded Relaying w Prop with Central Network Topology

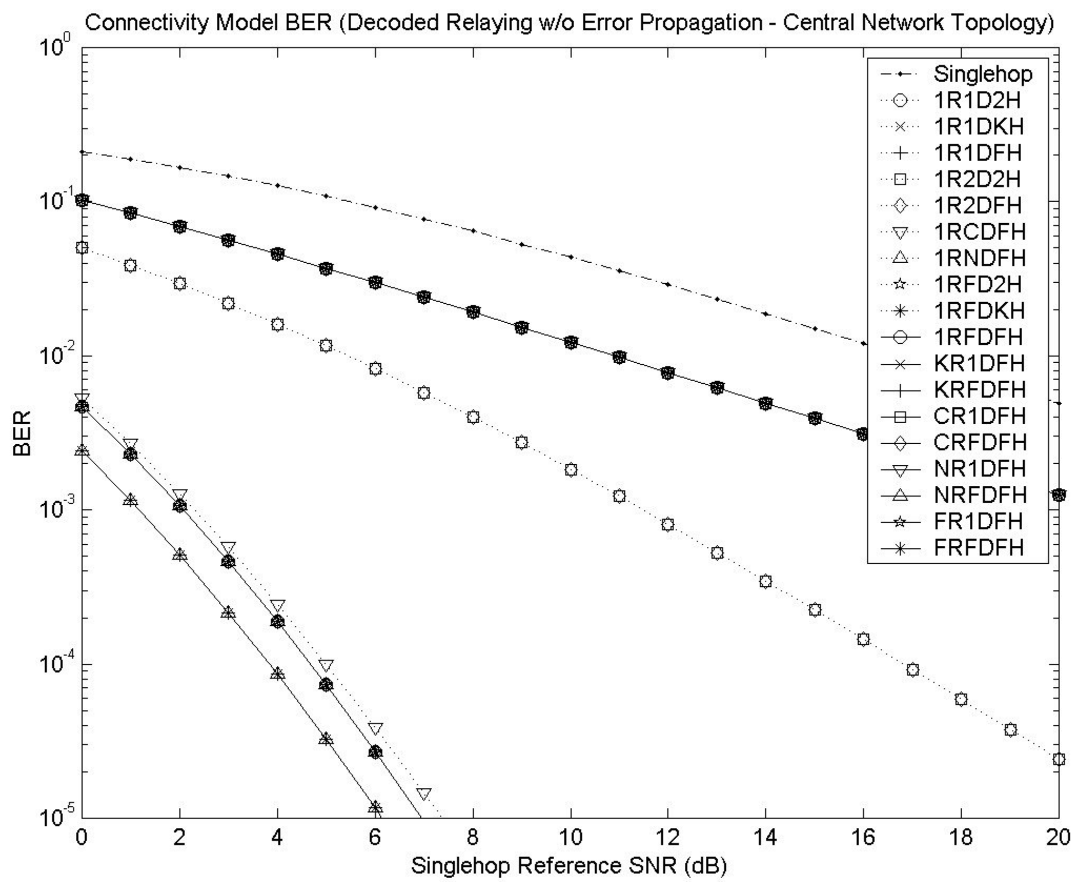

Fig. 31. Error of Decoded Relaying w/o Prop with Central Network Topology

Table 8 summarizes the diversity order achieved by each cooperative connectivity model for each relaying method. The achievable diversity order is the same for all network topologies. It can also be seen that the achievable diversity order is the same for amplified relaying and decoded relaying without error propagation for all cooperative connectivity models.

Relaying Method	1R1D 2H	1R1D KH	1R1D FH	1R2D 2H	1R2D FH	1RCD FH	1RND FH	1RFD 2H	1RFD KH
AR	1	1	1	2	2	5	6	6	6
DR w Pr	1	1	1	1	1	1	1	1	1
DR w/o Pr	1	1	1	2	2	5	6	6	6
Relaying Method	**1RFD FH**	**KR1D FH**	**KRFD FH**	**CR1D FH**	**CRFD FH**	**NR1D FH**	**NRFD FH**	**FR1D FH**	**FRFD FH**
AR	6	1	6	1	6	1	6	1	6
DR w Pr	1	1	1	1	1	1	1	1	1
DR w/o Pr	6	1	6	1	6	1	6	1	6

Table 8. Diversity Order Summary from Probability of Error Simulations

Table 9 summarizes the probability of error performance impact of the constraints for each combination of relaying method and network topology. ‘AR’ denotes amplified relaying, ‘DR w Prop’ denotes decoded relaying with error propagation, and ‘DR w/o Prop’ denotes decoded relaying without error propagation. The symbol ‘•’ denotes that the diversity order is unaffected and the performance gain is less than 1 dB difference on average, the symbol ‘••’ denotes that the diversity order is unaffected and the performance gain is greater than 1 dB difference on average, and the symbol ‘•••’ denotes that the diversity order is increased. The column ‘NCA’ indicates the impact of making *N* channels available when all other constraints are lifted whereas the column ‘KCA’ indicates the impact of increasing the number of channels available by one (i.e., from two to three). This performance impact summary allows us to qualitatively compare the impact on the BER of the different system resource constraints relative to each other for different combinations of network topology and relaying method. It allows us to view trends in the BER impact of the different constraints as the network topology changes from more linear to more central terminal distributions and enables us to determine which relaying methods exhibit similar relative behavior with respect to the presence of the different constraints.

Network Topology	Relaying Method	NCA	KCA	RCC	DCC	ROC	DOC	MCT	IIC
Example	AR	•	•	•	•••	••	•••	•	•
	DR w Pr	•	•	•	•	••	•	••	••
	DR w/o Pr	•	•	•	•••	••	•••	•	•
Linear	AR	•	•	•	•••	••	•••	•	••
	DR w Pr	•	••	••	•	••	•	•	••
	DR w/o Pr	•	•	•	•••	••	•••	•	••
Central	AR	•	•	•	•••	•	••	•	•
	DR w Pr	•	•	•	••	•	••	••	•
	DR w/o Pr	•	•	•	•••	•	••	•	•

Table 9. Probability of Error Impact of System Resource Constraints

6.3 Probability of Outage Simulation Results

Figs. 32-40 respectively compare the information theoretic probability of outage of the cooperative connectivity models for the described network topologies using amplified relaying, decoded relaying with error propagation, and decoded relaying without error propagation. These highlight the impact of the system resource constraints by providing a comparison of the cooperative connectivity models in terms of probability of outage versus the signal to noise ratio of the singlehop reference channel. The probability of outage results are plotted such that the total allocated power of each cooperative connectivity model is the same as that of the singlehop reference channel that achieves the annotated signal to noise ratio. These probability of outage results allow us to make the same qualitative comparisons as the probability of error results, but independent of any particular coding or modulation scheme. It is of specific interest to determine whether the general trends extrapolated from the probability of error results hold true for the probability of outage results.

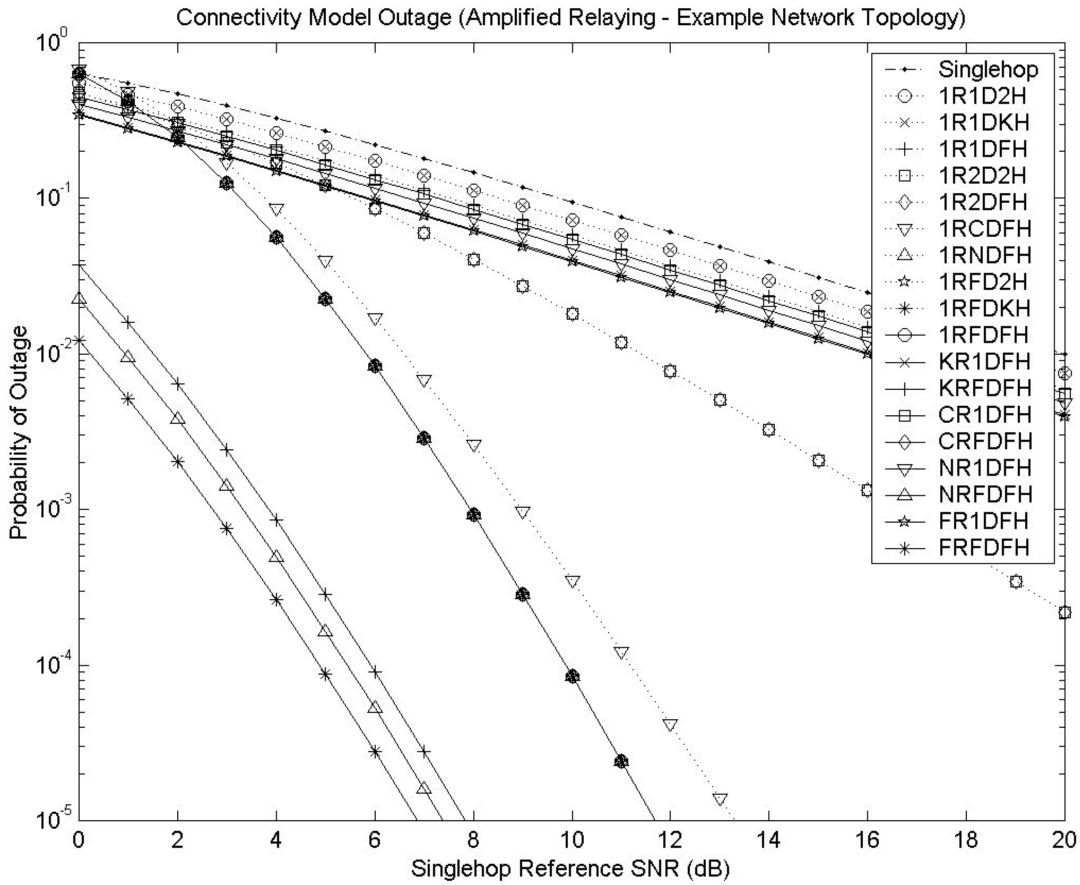

Fig. 32. Outage of Amplified Relaying with Example Network Topology

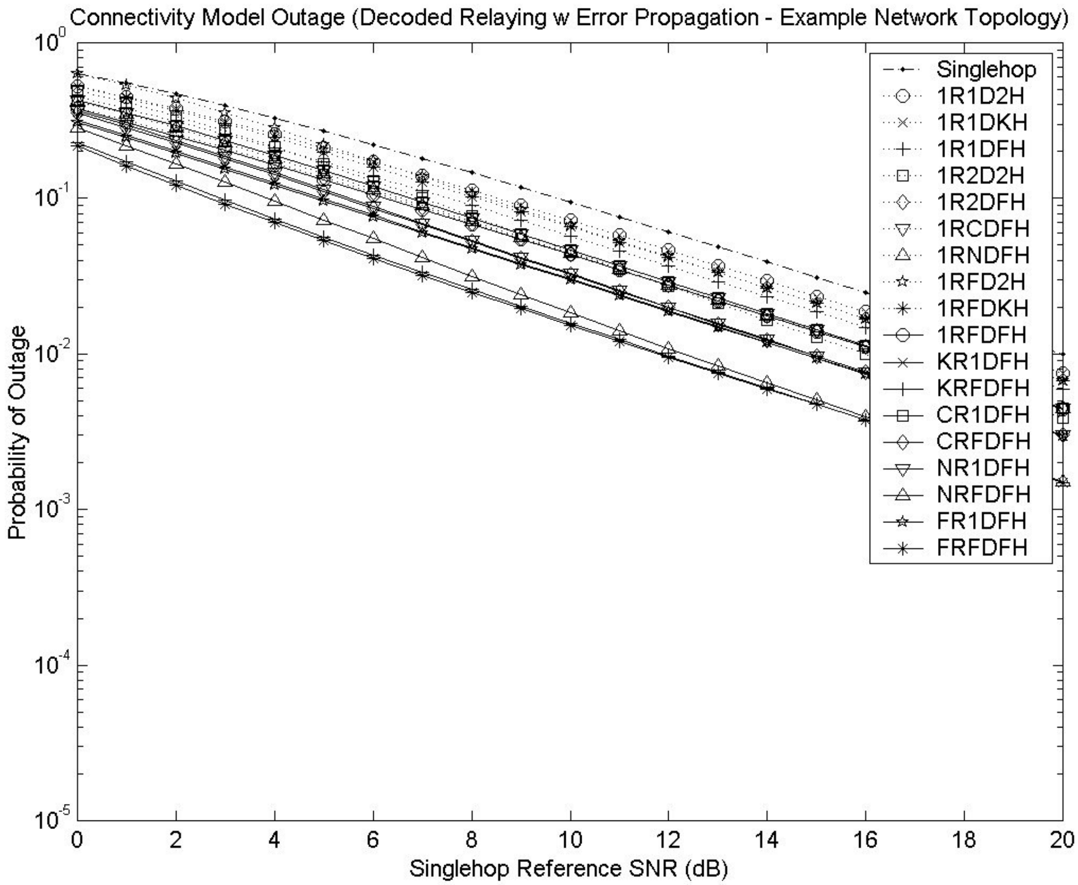

Fig. 33. Outage of Decoded Relaying w Prop with Example Network Topology

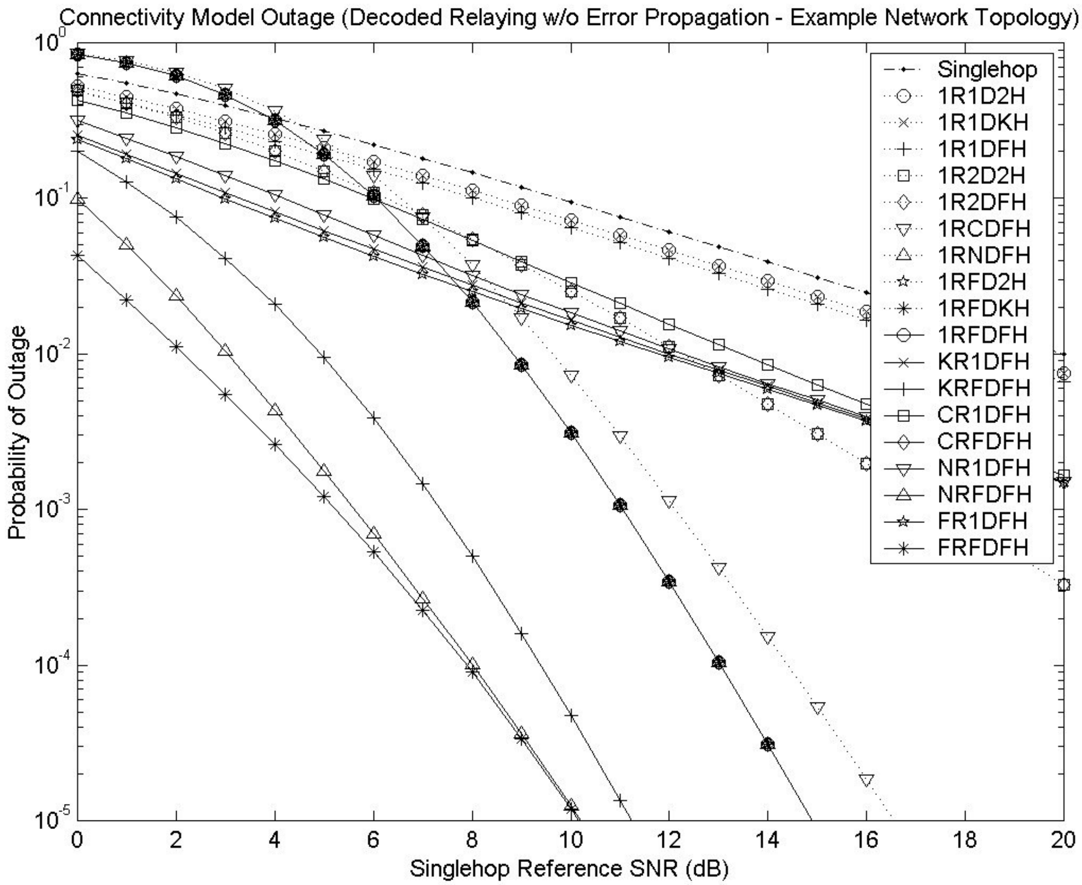

Fig. 34. Outage of Decoded Relaying w/o Prop with Example Network Topology

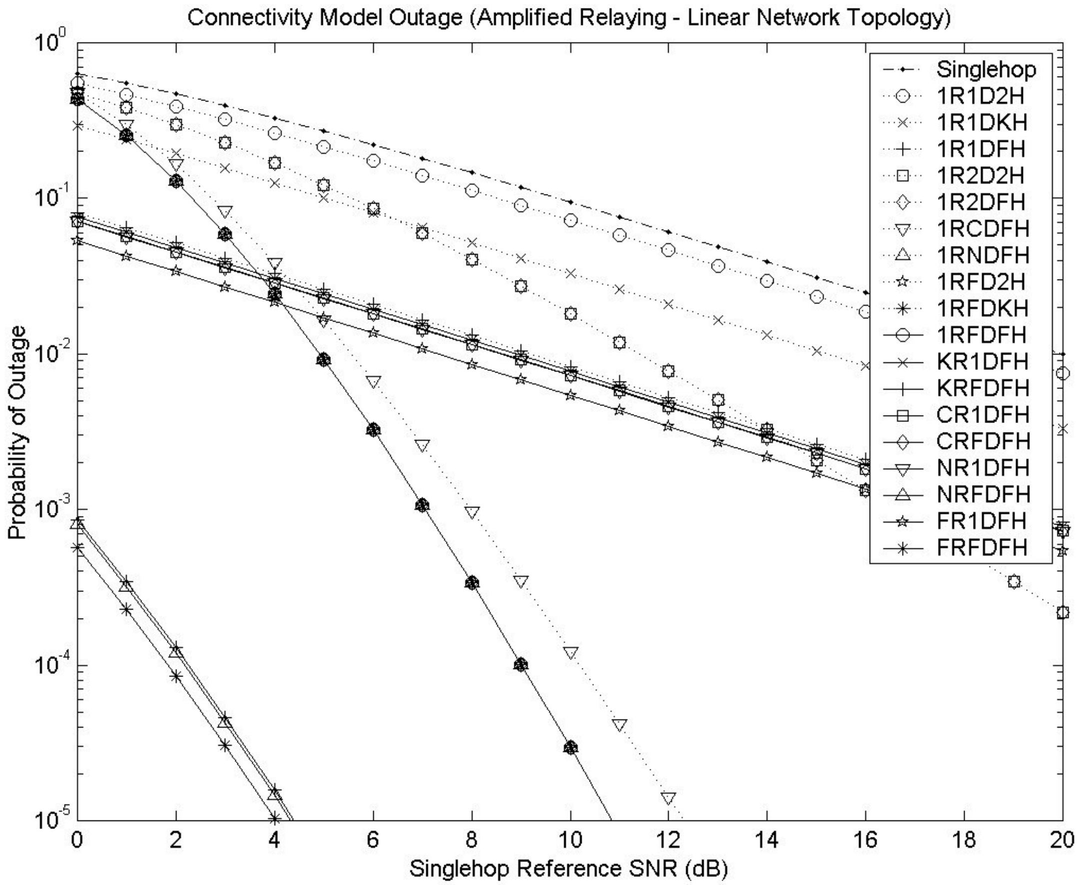

Fig. 35. Outage of Amplified Relaying with Linear Network Topology

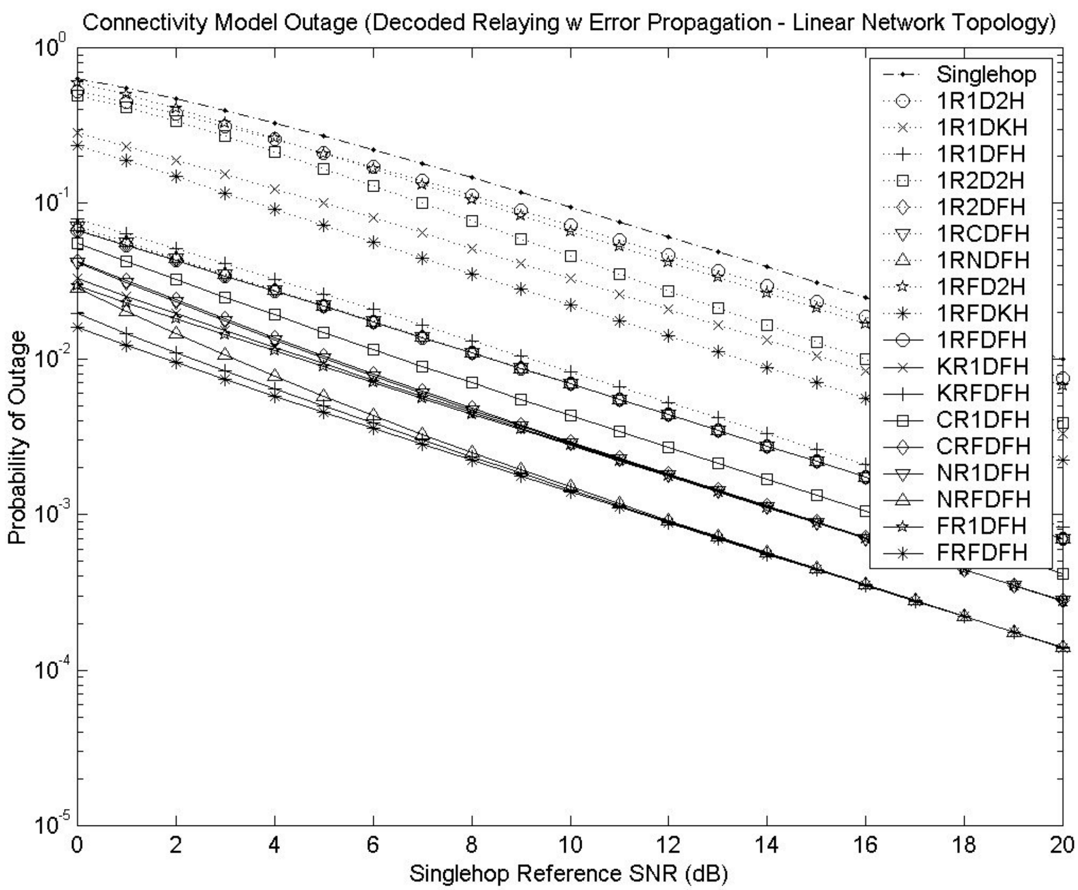

Fig. 36. Outage of Decoded Relaying w Prop with Linear Network Topology

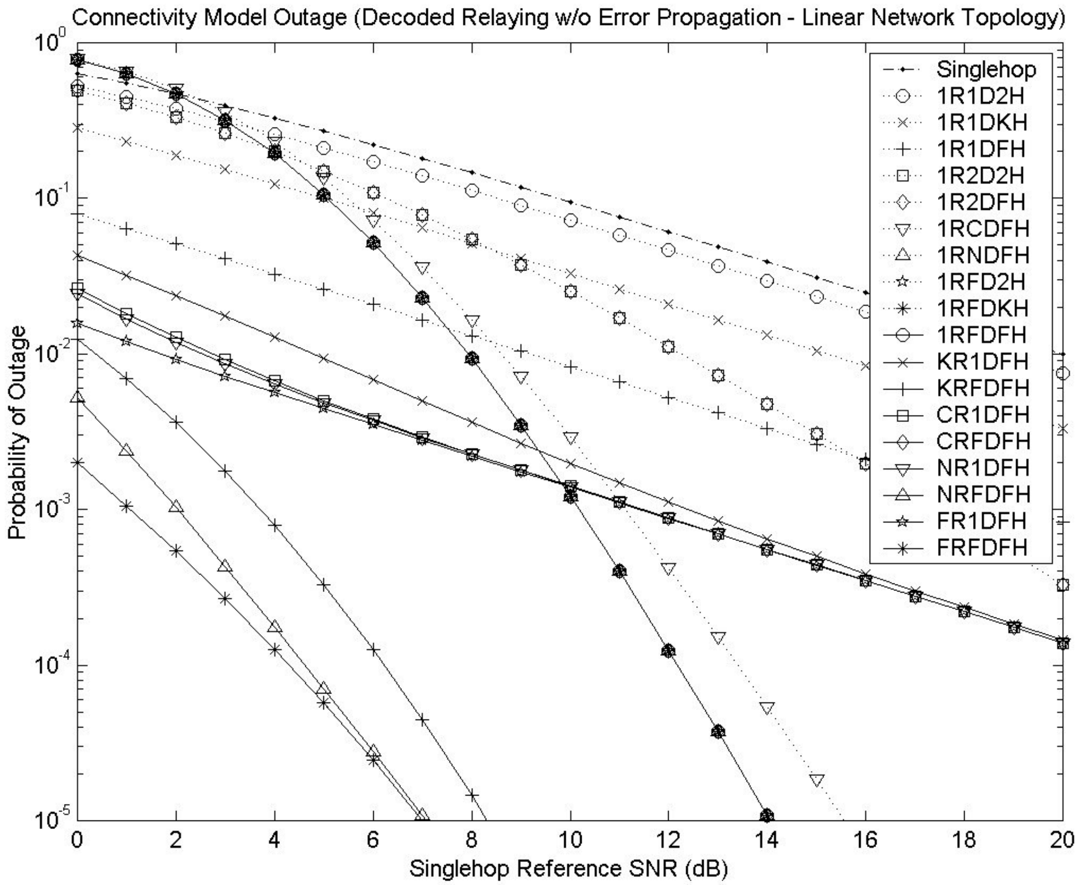

Fig. 37. Outage of Decoded Relaying w/o Prop with Linear Network Topology

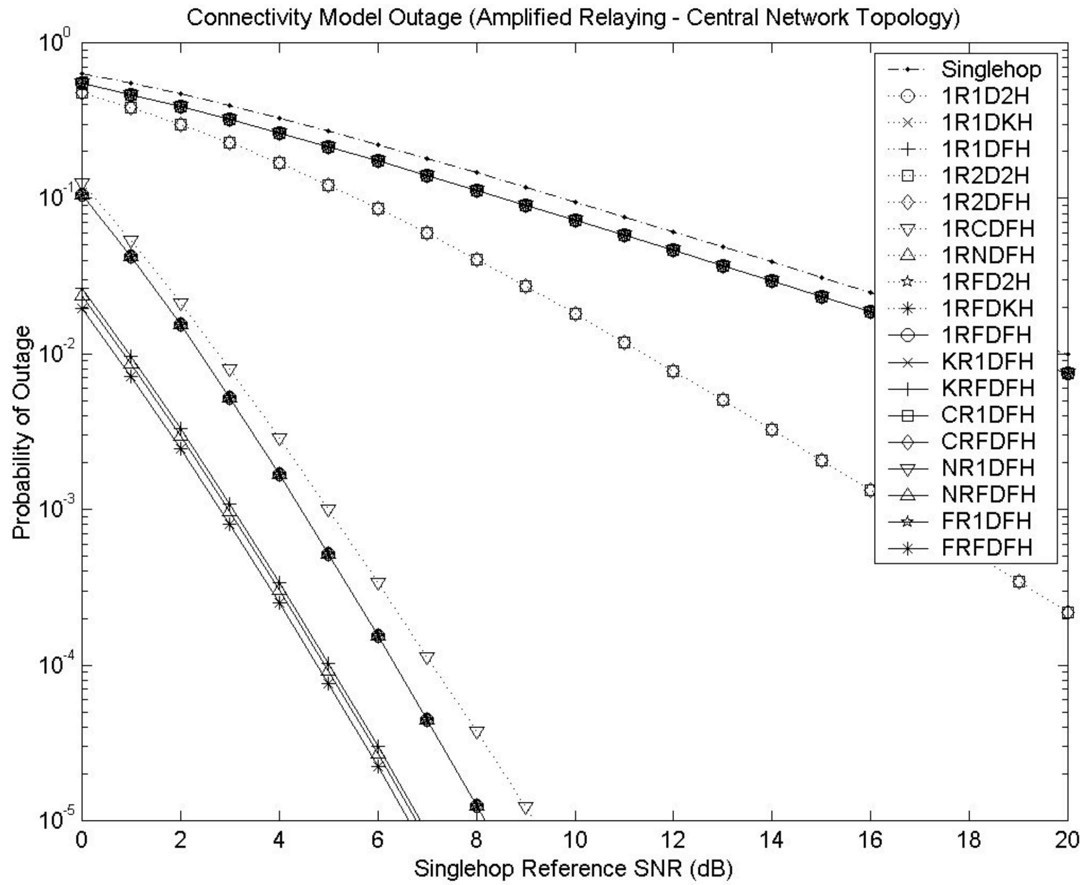

Fig. 38. Outage of Amplified Relaying with Central Network Topology

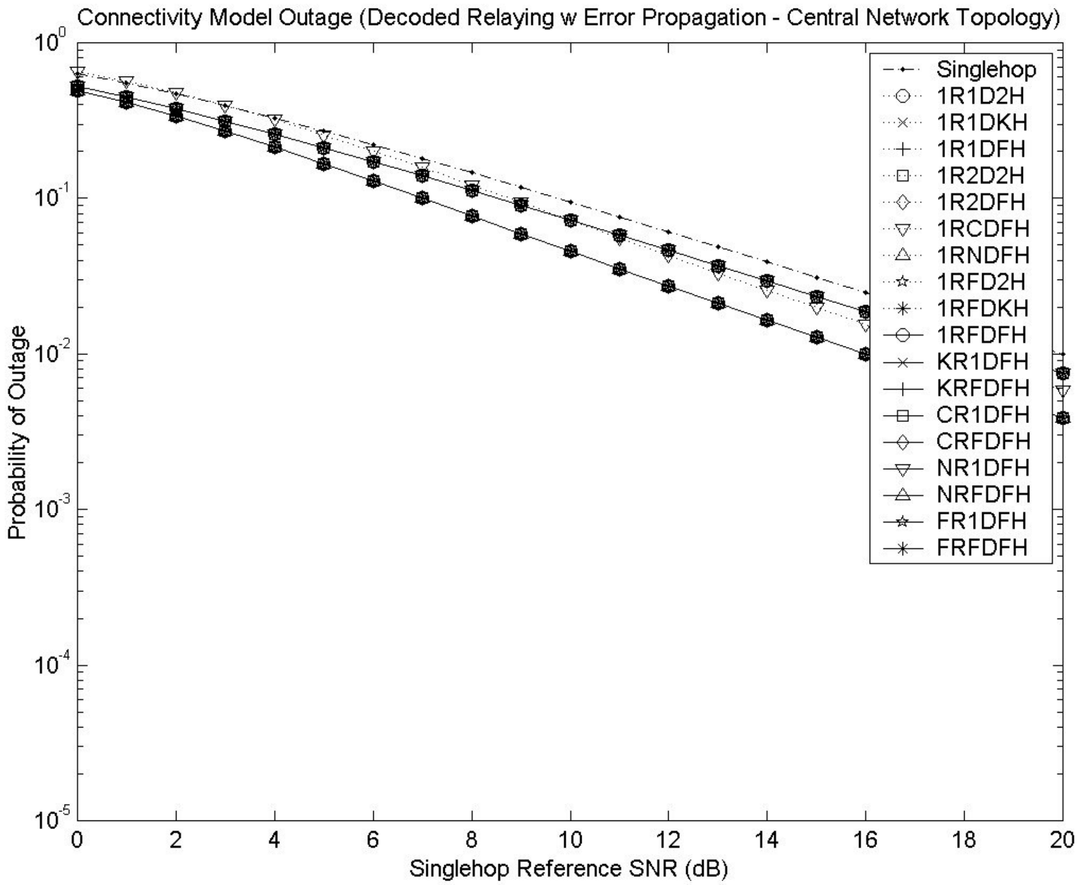

Fig. 39. Outage of Decoded Relaying w Prop with Central Network Topology

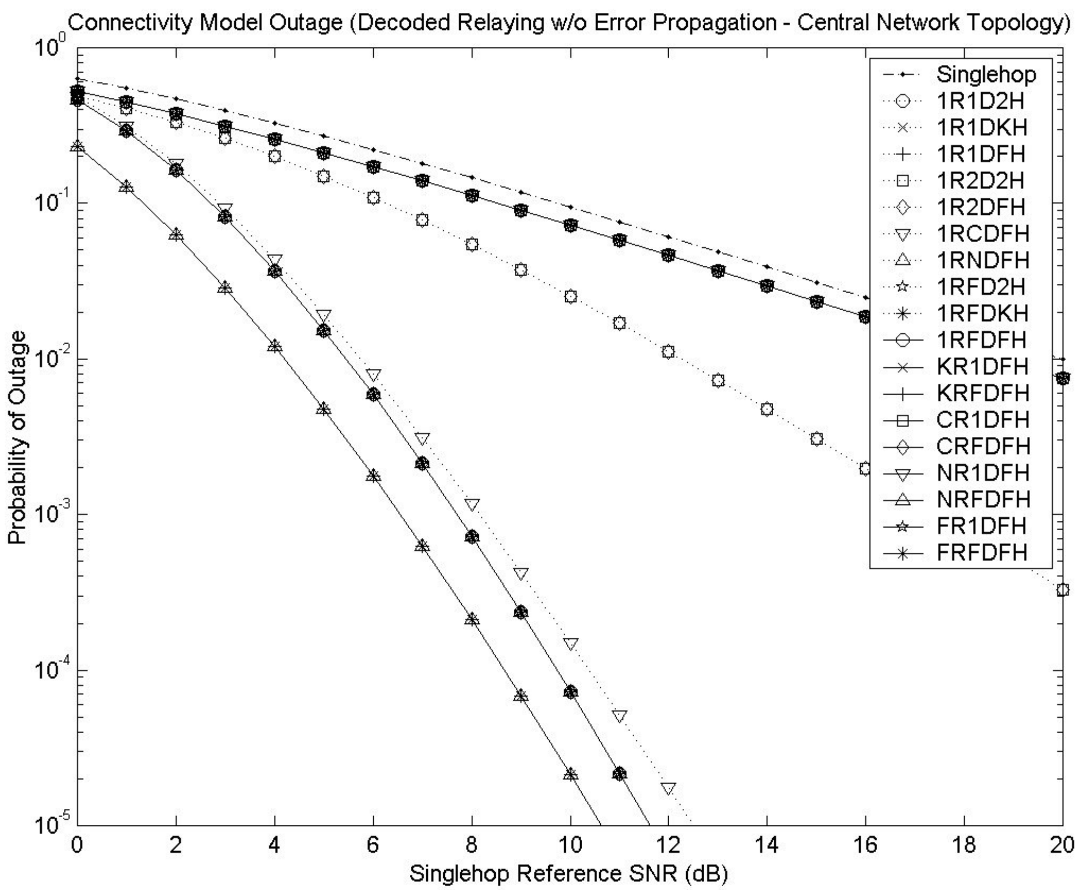

Fig. 40. Outage of Decoded Relaying w/o Prop with Central Network Topology

Table 10 summarizes the diversity order achieved by each cooperative connectivity model for each relaying method. Again, the achievable diversity order is the same for all network topologies. The achievable diversity orders shown by Table 10 are identical to those shown by Table 8. This indicates that the achievable diversity orders are not dependent on any particular coding or modulation scheme. The achievable diversity order is the again the same for amplified relaying and decoded relaying without error propagation for all cooperative connectivity models.

Relaying Method	1R1D 2H	1R1D KH	1R1D FH	1R2D 2H	1R2D FH	1RCD FH	1RND FH	1RFD 2H	1RFD KH
AR	1	1	1	2	2	5	6	6	6
DR w Pr	1	1	1	1	1	1	1	1	1
DR w/o Pr	1	1	1	2	2	5	6	6	6
Relaying Method	**1RFD FH**	**KR1D FH**	**KRFD FH**	**CR1D FH**	**CRFD FH**	**NR1D FH**	**NRFD FH**	**FR1D FH**	**FRFD FH**
AR	6	1	6	1	6	1	6	1	6
DR w Pr	1	1	1	1	1	1	1	1	1
DR w/o Pr	6	1	6	1	6	1	6	1	6

Table 10. Diversity Order Summary from Probability of Outage Simulations

Table 11 summarizes the probability of outage performance impact of the constraints for each combination of relaying method and network topology. Like Table 9, 'AR' denotes amplified relaying, 'DR w Prop' denotes decoded relaying with error propagation, 'DR w/o Prop' denotes decoded relaying without error propagation, the symbol '•' denotes that the diversity order is unaffected and the performance gain is less than 1 dB difference on average, the symbol '••' denotes that the diversity order is unaffected and the performance gain is greater than 1 dB difference on average, and the symbol '•••' denotes that the diversity order is increased. The relative performance impact of the various system resource constraints for each combination of relaying method and network topology shown by Table 11 is identical to that shown by Table 9. This indicates that the general trends extrapolated from the probability of error results hold true for the probability of outage results, and are therefore not dependent on any particular coding or modulation scheme.

Network Topology	Relaying Method	NCA	KCA	RCC	DCC	ROC	DOC	MCT	IIC
Example	AR	●	●	●	●●●	●●	●●●	●	●
	DR w Pr	●	●	●	●	●●	●	●●	●●
	DR w/o Pr	●	●	●	●●●	●●	●●●	●	●
Linear	AR	●	●	●	●●●	●●	●●●	●	●●
	DR w Pr	●	●●	●●	●	●●	●	●	●●
	DR w/o Pr	●	●	●	●●●	●●	●●●	●	●●
Central	AR	●	●	●	●●●	●	●●	●	●
	DR w Pr	●	●	●	●●	●	●●	●●	●
	DR w/o Pr	●	●	●	●●●	●	●●	●	●

Table 11. Probability of Outage Impact of System Resource Constraints

6.4 Probability of Outage Bound Simulation Results

Figs. 41-43 respectively compare the information theoretic probability of outage bounds of the cooperative connectivity models with common codebook generation for the described network topologies. These results highlight the impact of the system resource constraints independent of any particular relaying method, and allow comparison of these bounds with the probability of outage results from the previous section for the various considered relaying methods. The probability of outage results are again plotted such that the total allocated power of each cooperative connectivity model is the same as that of the singlehop reference channel that achieves the annotated signal to noise ratio.

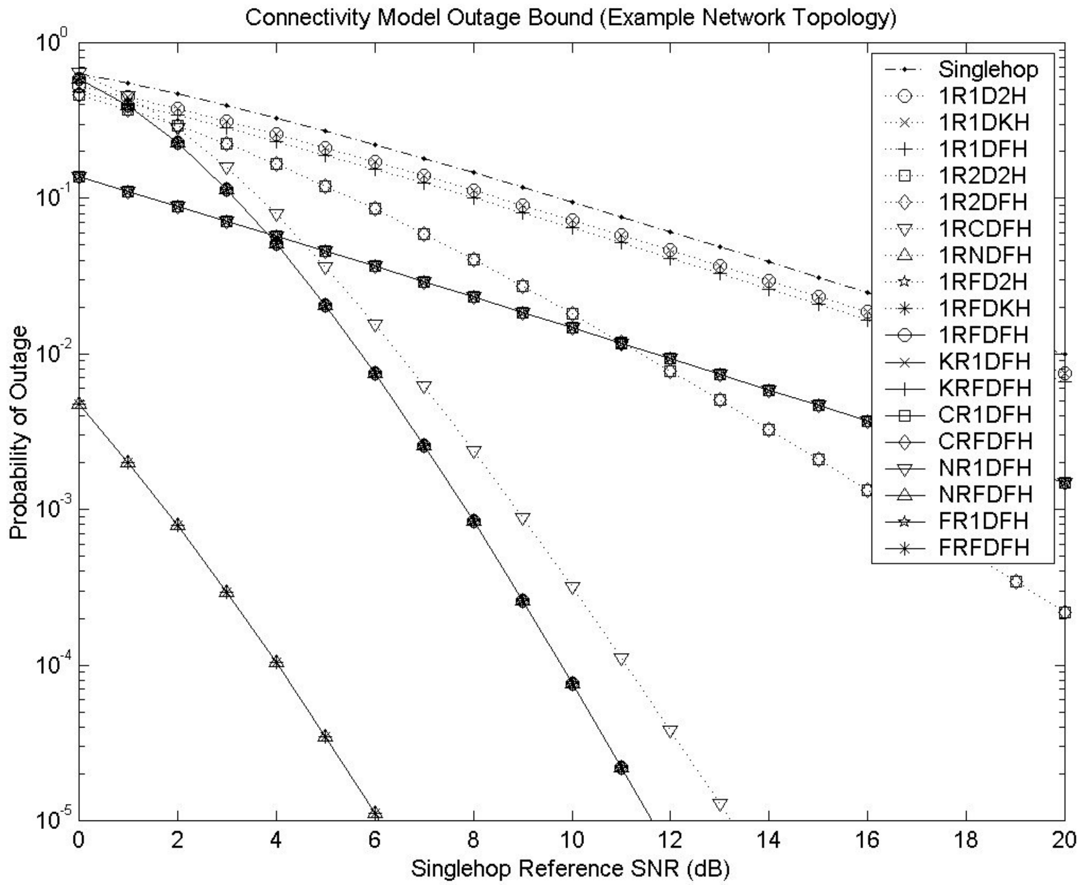

Fig. 41. Outage Bounds of Example Network Topology

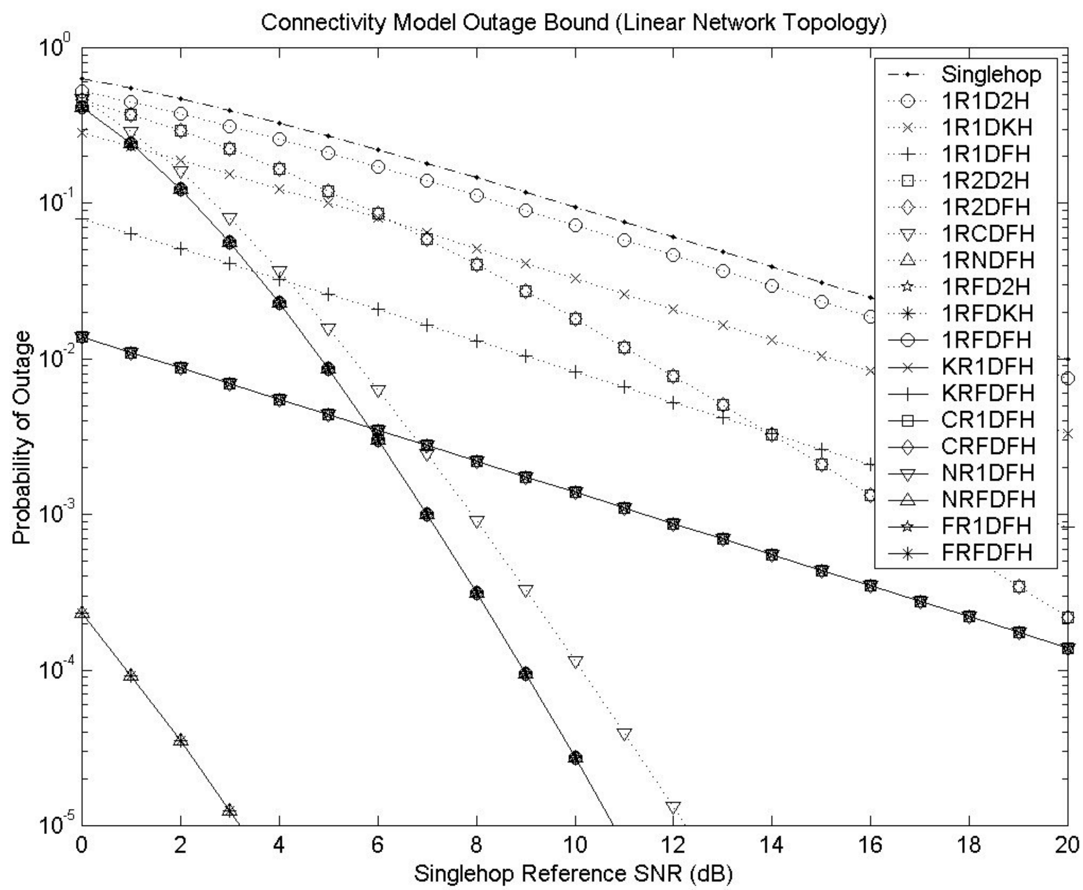

Fig. 42. Outage Bounds of Linear Network Topology

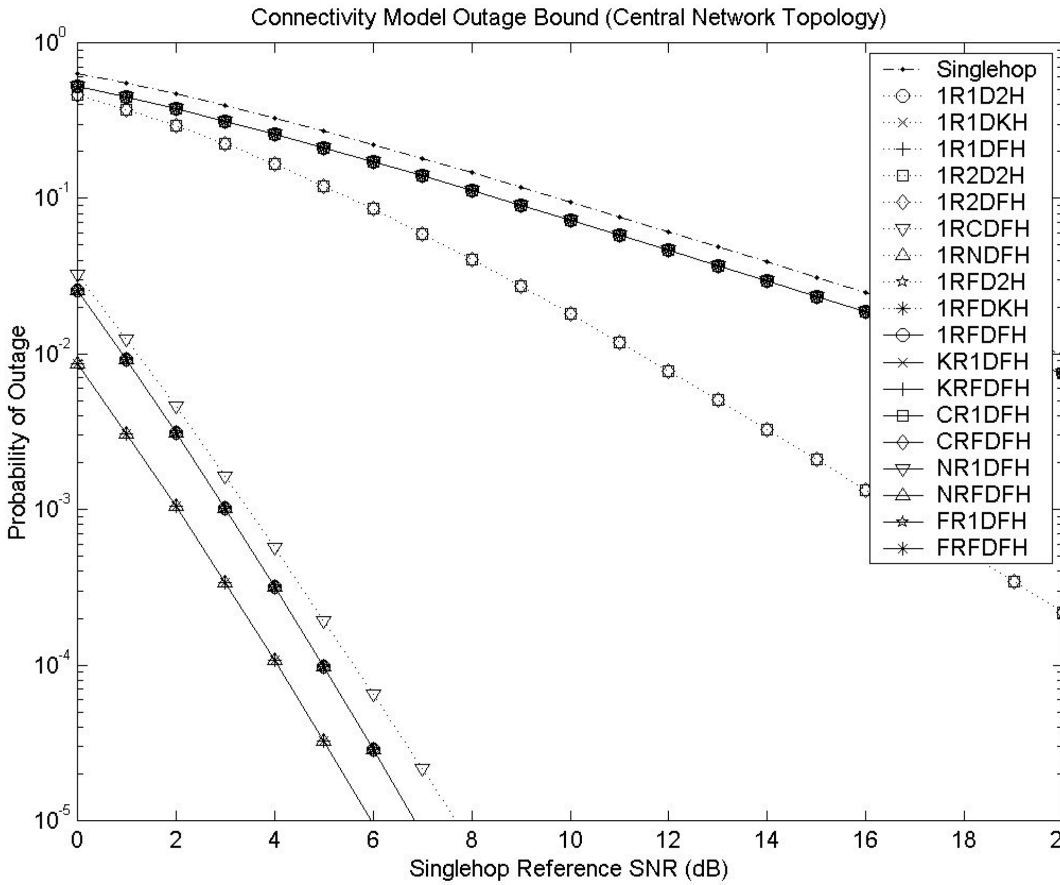

Fig. 43. Outage Bounds of Central Network Topology

These information theoretic probability of outage bound results indicate the same relative performance impact for the different system resource constraints as amplified relaying and decoded relaying without error propagation. This indicates that the general trends extrapolated from the probability of error and outage results are not dependent on any particular relaying method. Additionally, the probability of outage bounds indicate the same diversity order as amplified relaying and decoded relaying without error propagation for all cooperative connectivity models. This shows that amplified relaying and decoded relaying without error propagation are capable of attaining the maximum achievable diversity order for all network topologies.

6.5 Discussion

The simulations clearly illustrate the value of the presented cooperative connectivity modeling framework as a modeling tool for wireless relay networks. Although the simulation results are restricted to a specific system model and it is therefore not possible to extrapolate general conclusions with a high level of certainty, they highlight some significant qualitative trends that are worthy of more comprehensive study. Since the general trends extrapolated from the probability of error results hold true for the probability of outage results and the probability of outage bound results, the following discussion is not limited to any particular relaying method, coding scheme, or modulation scheme.

The order in which the constraints should be lifted and the resulting sequence of cooperative connectivity models and constraint transitions are more dependent on the capability of the relaying method to achieve full diversity order than on the physical layer relaying method (amplified or decoded). This is indicated in the simulations by the fact that the probability of outage results and probability of error results are very similar for amplified relaying and decoded relaying without error propagation, but very different for decoded relaying with error propagation, and is consistent for all network topologies. For amplified relaying and decoded relaying without error propagation the priority in general is to maximize the connectivity of the destination terminal. This is indicated by the relatively large impact of the system resource constraints that affect destination connectivity. For decoded relaying with error propagation the priority in general is to equalize the connectivity of the destination and relay terminals. This is indicated by the fact that neither system resources constraints that affect destination connectivity nor those that affect relay connectivity have significantly more relative impact across all network topologies.

The simulation results support the theoretically maximum diversity order results presented previously. The diversity orders shown in the simulation results and summarized in the diversity order summary tables for amplified relaying and decoded relaying without error propagation are aligned with the maximum diversity order analysis for networks with destination decoding. The corresponding results for decoded relaying with error propagation are aligned with the maximum diversity order analysis for networks with comprehensive decoding. For amplified relaying and decoded relaying without error propagation the diversity order of the system is dependent on the connectivity of both the destination and relays and is limited to the number of disjoint paths from the source to the destination through the relay network. For decoded relaying with error propagation the diversity order of the system is constrained by the connectivity of the minimally connected relay and is therefore limited to one.

For amplified relaying and decoded relaying without error propagation, the system resource constraint impact tables indicate that the relative impact of the constraints is independent of the network topology, with the destination combination constraints always more important. For decoded relaying with error propagation, the system resource constraint impact tables indicate that the relative impact of the constraints is dependent on the network topology, with the relay combination constraints more important for network topologies with more linear terminal distributions (where the relay terminals are distributed with dissimilar respective distances to the source and destination terminals) and the destination combination constraints more important for network topologies with more central terminal distributions (where the relay terminals are distributed with similar respective distances to the source and destination terminals). The impact of the number of channels available, relay common channel combination, relay orthogonal channel combination, destination orthogonal channel combination, and interhop interference cancellation is larger for network topologies with more linear terminal distributions that are conducive to relay terminals connected in serial. The impact of destination common channel combination and multiple channel transmission is larger for network topologies with more central terminal distributions that are conducive to relay terminals connected in parallel.

The process of optimizing the network connectivity to minimize the probability of error also yielded some interesting insight. For amplified relaying and decoded relaying without error propagation the optimal network connectivity tends to use all available relay terminals in order to maximize diversity improvements. For decoded relaying with error propagation the optimal network connectivity tends to use only a subset of available relay terminals to maximize attenuation improvements while minimizing the number of relay terminals that can propagate decoding errors. Optimizing the network connectivity for cooperative connectivity models with full or single relay or destination connectivity has less impact than optimizing the network connectivity for cooperative connectivity models with moderate relay or destination connectivity. For network topologies with more linear terminal distributions, optimization results in relay terminals connected in serial only at lower SNRs when the attenuation gains due to serial connection dominate the diversity gains due to parallel connection. For network topologies with more central terminal distributions, optimization always results in relay terminals connected in parallel.

From the system resource constraint impact tables it is also possible to extrapolate general guidance for the order in which constraints should be lifted to maximize connectivity and performance. The fundamental value of determining the order in which new capabilities are added is based on the importance of reusing existing investment in network infrastructure where possible. Due to the interdependence between system resources with respect to the cooperative connectivity they enable, the performance benefit of lifting a given system resource constraint is often relative to the system resources that are already available. Therefore, the problem of finding the order in which constraints should be lifted is closely related to finding the set of system resources that provide the greatest performance benefit when available.

Table 12 shows the recommended order in which the constraints should be lifted for each relaying method for network topologies with linear and central terminal distributions. Since the impact of lifting the NCA constraint when all other constraints are lifted is small, but increases the system cost significantly, it is not expected that having a separate channel available for each terminal will be implemented in practice. Full diversity order and minimal performance degradation can be achieved without requiring the use of a separate orthogonal channel per transmitter. This requires either the application of common channel combination techniques, or the application of orthogonal channel combination techniques with a separate orthogonal channel allocated for each layer of relay terminals between the source and destination. The results indicate that the most promising models from a performance efficiency perspective seem to be the

1R1DFH model (multihop relaying without diversity), 1R2D2H model (single relay cooperative diversity), 1RCD2H model (multiple relay cooperative diversity without direct source-destination connectivity), 1RFD2H model (multiple relay cooperative diversity with direct source-destination connectivity), and NRFDFH model (complete connectivity except between relays that transmit on identical subsets of channels). In particular, the NRFDFH model is highlighted as a promising, though relatively more complex, target that has not yet been studied in the literature. The NRFDFH model is very interesting because it achieves almost complete connectivity between all terminals without requiring the use of a separate orthogonal channel per transmitter.

Relaying Method and Network Topology Class	**Recommended Order to Lift Constraints**
Amp (Linear/Central)	DCC, DOC, IIC, RCC, MCT, KCA, ROC
Dec w Pr (Linear)	IIC, KCA, ROC, DOC, RCC, DCC, MCT
Dec w Pr (Central)	DCC, DOC, IIC, MCT, RCC, KCA, ROC
Dec w/o Pr (Linear/Central)	DCC, DOC, IIC, RCC, MCT, KCA, ROC

Table 12. Recommended Order to Lift System Resource Constraints

The central network topology probably of outage curves for the 1R2D2H connectivity model for the different relaying methods are aligned with the corresponding results presented in Fig. 5 of [64], adjusted for the fact that the current probability of outage curves have an additional distance-dependent factor not included in [64]. The central network topology probability of outage curves for the 1RFD2H connectivity model for the decoded relaying without error propagation relaying are aligned with the corresponding results presented in Fig. 6 of [63] for a network with six transmitting terminals, again adjusted to include network geometry.

Comparison of the amplified relaying and decoded relaying without error propagation probability of outage results with the probability of outage bounds indicates that amplified relaying achieves outage performance very close to the outage bound when the relays are connected in parallel, while decoded relaying without error propagation achieves outage performance very close to the outage bound when the relays are connected in serial. For cooperative connectivity models with high connectivity, when relays are commonly connected in both parallel and serial, neither relaying method achieves outage performance very close to the outage bound. The degradation of amplified relaying relative to the outage bound when relays are connected in serial is due to an increase of propagated noise over multiple hops. The degradation of decoded relaying without error propagation relative to the outage bound when relays are connected in parallel is due to an increase of independent decoding errors at relay terminals with low diversity orders.

In general, the probability of outage simulation results indicate that decoded relaying without error propagation performs better than amplified relaying when the overall diversity order of the network is low, while amplified relaying performs better than decoded relaying without error propagation when the overall diversity order of the network is high. This is different from the probability of error simulation results, where decoded relaying without error propagation always outperforms amplified relaying. This difference is due to the fact that the absolute values of the probability of outage metrics at low diversity orders or low SNRs are significantly greater than the corresponding probability of error metrics, increasing the relative likelihood of a decoding error at relay terminals when using the decoded relaying without error relaying method, and therefore resulting in a higher total probability of outage. Decoding error events at relay terminals occur more often in relation to decoding error events at the destination terminal, causing them to be a relatively more significant factor in the total probability of outage.

6.6 Summary

This chapter has presented cooperative connectivity model simulations for the purpose of illustrating the value of the cooperative connectivity framework as a modeling tool, comparing the various cooperative connectivity models, and analyzing the performance impact of the system resource constraints for various relaying methods and network topologies. The simulations highlight some significant qualitative trends that are worthy of more comprehensive study. These general trends are common across the probability of error results, the probability of outage results, and the probability of outage bound results, and are therefore independent of any particular relaying method, coding scheme, or modulation scheme. For all network topologies, the diversity orders shown in the simulation results for amplified relaying and decoded relaying without error propagation are aligned with the maximum diversity order analysis for networks with destination decoding, while the corresponding results for decoded relaying with error propagation are aligned with the maximum diversity order analysis for networks with comprehensive decoding.

The order in which the constraints should be lifted and the resulting sequence of cooperative connectivity models and constraint transitions is more dependent on the capability of the relaying method to achieve full diversity order than on the physical layer relaying method (amplified or decoded), and the relative impact of the constraints is dependent on both the relaying method and network topology. For relaying methods with the capability to achieve full diversity order (amplified relaying and decoded relaying without error propagation) it is indicated that the network connectivity should be chosen to maximize the number of disjoint paths from the source to the destination. For relaying methods without the capability to achieve full diversity order (decoded relaying with error propagation) it is indicated that the network connectivity should be chosen to maximize the performance of the minimally connected relay. General guidance is given for the order in which constraints should be lifted to maximize performance and some promising cooperative connectivity models are highlighted for further study. Finally, the results indicate that full diversity order and minimal performance degradation can in theory be achieved without requiring the use of a separate orthogonal channel per transmitter.

Chapter 7 - Concluding Remarks

This chapter summarizes the content of this dissertation, describing the scope, objectives, contributions, and main results of the developed system model, maximum diversity order analysis, and cooperative connectivity framework. Section 7.1 provides a high-level summary of the primary areas of research covered in the dissertation. Section 7.2 reviews the key contributions of the dissertation. Section 7.3 reviews the main qualitative results of the dissertation. Section 7.4 discusses some suggestions for further research.

7.1 Summary

This dissertation has addressed three primary areas of research for wireless relay networks with an arbitrary number of cooperating wireless terminals and arbitrary sets of communication links between pairs of cooperating terminals. First, we developed a general system model for arbitrarily connected wireless relay networks employing various relaying methods. The developed system model incorporated multipath propagation, distance dependent path loss, shadowing, fading, Gaussian noise, and other forms of interference, and can easily be extended to include other factors. Probability of error and information theoretic probability of outage results were presented for a number of practical relaying methods. The developed expressions are applicable for a given set of source, destination, and relaying terminals with any number of antennas, link connectivity, link attenuation, transmit power, and receiver noise, and can be used to extend many of the traditional two-hop cooperative diversity results to more than two hops and arbitrary connectivity. The results were extended to the most general case where there may be more than one physical antenna at each terminal. The developed expressions provide a method for analyzing the impact of varying the link connectivity or power allocation for a given set of terminals, and were further applied in the maximum diversity order analysis and cooperative connectivity model simulations.

Second, we derived bounds on the maximum achievable diversity order and diversity-multiplexing tradeoff of wireless relay networks with arbitrary link connectivity between cooperating terminals. Two classes of relaying method were considered, those requiring all cooperating terminals to correctly decode the transmitted information signal in order for the destination to correctly decode (comprehensive decoding), and those requiring only a subset of cooperating terminals to correctly decode the transmitted information signal in order for the destination to correctly decode (destination decoding). Two general schemes for how codebooks are generated by and partitioned among terminals in the relay network were considered and compared: common codebook generation and independent codebook generation. The high signal to noise ratio probability of outage and diversity order results of these two relaying method classes and two codebook generation schemes were compared. The results were extended to the most general case where there may be more than one physical antenna at each terminal. The complexity of the relevant algorithms for minimization across all terminals or cut sets in the network was analyzed.

Third, we developed a framework for modeling cooperative connectivity that exposes the relationship between constraints on the available system resources and the achievable cooperative connectivity of wireless relay networks. The system resource constraints considered were the available number of orthogonal relaying channels, the ability of terminals to diversity combine signals on a single common channel, the ability of terminals to diversity combine signals on orthogonal channels, the ability of terminals to transmit signals on multiple orthogonal channels, and the ability of terminals to cancel the effects of interhop interference. The cooperative connectivity models resulting from the possible combinations of system resource constraints were derived and associated with their minimum cost constraint sets. Connectivity model equations and minimum cost constraint set equations were provided that further specified the relationship between the system resource constraints and the characterizing parameters of the cooperative connectivity models. Cooperative connectivity model transition diagrams that show the transitions between the different cooperative connectivity models for constraint changes following the minimum cost constraint sets were presented. The cooperative connectivity models were mapped to the various distributed spatial diversity techniques presented in the literature. Finally, a number of cooperative connectivity model simulations were presented for the purpose of illustrating the value of the cooperative connectivity framework as a modeling tool, comparing the various cooperative connectivity models, and analyzing the performance impact of the system resource constraints for various relaying methods and network topologies.

7.2 Contributions

This section summarizes the key contributions of the dissertation for each of the three primary areas of research: the general system model, the maximum diversity order analysis, and the cooperative connectivity modeling framework for arbitrarily connected wireless relay networks.

7.2.1 System Model

The key contributions of the general system model for arbitrarily connected wireless relay networks are the following:

- Development of system model expressions for wireless relay channels with an arbitrary set of source, destination, and relay terminals, inter-terminal link connectivity, link attenuation, transmit powers, and receiver noises and that incorporate wireless channel environmental factors such as multipath propagation, distance dependent path loss, shadowing, fading, Gaussian noise, and other forms of interference.
- Development of probability of error results and information theoretic probability of outage results for wireless relay networks that employ decoded relaying with error propagation.
- Development of probability of error results and information theoretic probability of outage results for wireless relay networks that employ decoded relaying without error propagation.
- Development of aggregate signal to noise ratio results, probability of error results, and information theoretic probability of outage results for wireless relay networks that employ amplified relaying.
- Extension of the basic single-antenna results to wireless relay networks composed of terminals with multiple antennas.

7.2.2 Maximum Diversity Order Analysis

The key contributions of the maximum diversity order analysis for arbitrarily connected wireless relay networks are the following:

- Development of the maximum achievable diversity order and high SNR probability of outage for wireless relay networks that employ:
    - Common codebook generation and comprehensive decoding,
    - Common codebook generation and destination decoding,
    - Independent codebook generation and comprehensive decoding, and
    - Independent codebook generation and destination decoding.
- Extension of the basic single-antenna results to wireless relay networks composed of terminals with multiple antennas.
- Development of upper bounds on the diversity-multiplexing tradeoff for wireless relay networks that employ comprehensive decoding and destination decoding.
- Analysis of the complexity of algorithms for minimization across all terminals or cut sets in a network.

7.2.3 Cooperative Connectivity Modeling Framework

The key contributions of the cooperative connectivity modeling framework for arbitrarily connected wireless relay networks are the following:

- Definition of a number of system resource constraints that limit the connectivity of individual wireless relay terminals and therefore the cooperative diversity techniques that can be applied.
- Development of the cooperative connectivity models resulting from the possible combinations of system resource constraints and specification of connectivity model equations that indicate the relationship between the system resource constraints and the characterizing parameters of the cooperative connectivity models.

- Development of the minimum cost constraint sets associated with the cooperative connectivity models and specification of minimum cost constraint set equations that indicate the relationship between the characterizing parameters of the cooperative connectivity models and the presence of each system resource in the minimum cost constraint set.
- Development of cooperative connectivity model transition diagrams that show the transitions between the different cooperative connectivity models for constraint changes following the minimum cost constraint sets.
- Mapping of the cooperative connectivity models to the various cooperative diversity techniques presented in the literature in order to highlight the value of the developed framework and the general richness of the problem domain.
- Simulation of the probability of error for the cooperative connectivity models for various relaying methods and network topologies, along with associated analysis. This allows the performance impact of the individual system resource constraints to be isolated and the diversity order of the different cooperative connectivity models to be determined.
- Simulation of the probability of outage for the cooperative connectivity models for various relaying methods and network topologies, along with associated analysis. This highlights the impact of the system resource constraints independent of any particular coding or modulation scheme.
- Simulation of the probability of outage bounds for the cooperative connectivity models for various network topologies. This highlights the impact of the system resource constraints independent of any particular relaying method, and allows

comparison of these bounds with the probability of outage results for the various considered relaying methods.

- General guidance for the order in which the constraints should be lifted to maximize the performance for various relaying methods and network topologies.
- Presentation of some especially promising cooperative connectivity models for further investigation, including at least one that has not yet been studied in the literature.

7.3 Main Qualitative Results

This section summarizes the main qualitative results of the dissertation for each of the three primary areas of research: the general system model, the maximum diversity order analysis, and the cooperative connectivity modeling framework for arbitrarily connected wireless relay networks. The main qualitative results of the dissertation are the following:

- When there is a single antenna per terminal and all cooperating terminals must correctly decode, it is shown that the maximum achievable diversity order is constrained by the minimum number of immediately preceding terminals across all receiving terminals in the network, which is one. Furthermore, inter-terminal links that are not associated with terminals with the minimum number of immediately preceding terminals do not asymptotically (at high SNR) affect the probability of outage.
- When there is a single antenna per terminal and only the destination terminal must correctly decode, the maximum achievable diversity order is constrained by the minimum number of inter-terminal links across all cut sets in the network, which is

the number of disjoint paths through the network. Furthermore, inter-terminal links that are not associated with cut sets with the minimum number of inter-terminal links do not asymptotically (at high SNR) affect the probability of outage.

- When there are an arbitrary number of antennas per terminal and all cooperating terminals must correctly decode, it is shown that the maximum achievable diversity order is constrained by the minimum number of incident inter-terminal antenna links across all receiving terminals in the network.
- When there are an arbitrary number of antennas per terminal and all cooperating terminals must correctly decode, it is shown that the maximum achievable multiplexing gain is constrained by the minimum number of transmit or receive antennas for the incident multiple antenna channels across all terminals, divided by the number of orthogonal channels required to operate the given transmission scheme.
- When there are an arbitrary number of antennas per terminal and only the destination terminal must correctly decode, it is shown that the maximum achievable diversity order is constrained by the minimum number of associated inter-terminal antenna links across all cut sets in the network.
- When there are an arbitrary number of antennas per terminal and only the destination terminal must correctly decode, it is shown that the achievable multiplexing gain is constrained by the minimum number of transmit or receive antennas for the incident multiple antenna channels across all cut sets, divided by the number of orthogonal channels required to operate the given transmission scheme.
- Independent codebook generation does not offer any diversity gain over common codebook generation.

- Independent codebook generation can offer a significant probability of outage improvement over common codebook generation under the conditions that the minimum number of inter-terminal or inter-terminal antenna links across all cut sets in the network is high and the number of channels required for independent codebook generation is not significantly larger than the number of channels required for common codebook generation. This has a higher likelihood of being the case when there are multiple antennas per terminal.
- In wireless relay networks with arbitrary link connectivity between cooperating terminals, the terminals or cut sets that limit the maximum achievable diversity order may not necessarily be the same terminals or cut sets that limit the maximum achievable multiplexing gain.
- The complexity of practical algorithms to minimize the diversity order or diversity-multiplexing tradeoff across all terminals when there are N terminals is linear, of order $O(N)$. The complexity of practical algorithms to minimize the diversity order or diversity-multiplexing tradeoff across all cut sets when there are N terminals is polynomial, of order $O(N^3)$.
- Cooperative connectivity models are fully characterized according to three parameters: the achievable cooperative connectivity of the relays, the achievable cooperative connectivity of the destination, and the maximum achievable length of the longest multihop path of the network.
- The cooperative connectivity models derived when there are 2 channels available are a subset of the models derived when there are K ($2 < K < N$) channels available, with

the reduction resulting from additional system resource constraints and intersection between models.

- The majority of cooperative connectivity models result from constraint combinations with less than *N* channels available. Only the models with full relay connectivity, or full relay and destination connectivity, are exclusive to constraint combinations with *N* channels available.
- Minimum cost constraint sets are not very sensitive to small changes in the system cost weights of the different system resource constraints, but are sensitive to significant reordering of the system resource constraints with respect to system cost.
- The only minimum cost constraint sets that are not at all sensitive to reordering of the system resource constraints with respect to system cost are those where only a single combination of system resource constraints results in the corresponding cooperative connectivity model, or those where a single combination of system resource constraints is a subset of all other combinations that result in the corresponding cooperative connectivity model.
- Mapping of the cooperative connectivity models to the various distributed spatial diversity techniques presented in the literature highlights that although the literature published so far has only started to explore the many possible cooperative connectivity models there is already a broad range of connectivity and channel allocation assumptions and a wide variation in terminology used.
- The general trends indicated by the simulations are common across the probability of error results, the probability of outage results, and the information theoretic

probability of outage bound results, and are therefore independent of any particular relaying method, coding scheme, or modulation scheme.

- For all network topologies, the diversity orders shown in the simulation results for amplified relaying and decoded relaying without error propagation are aligned with the maximum diversity order analysis for networks with destination decoding, while the corresponding results for decoded relaying with error propagation are aligned with the maximum diversity order analysis for networks with comprehensive decoding.
- The diversity order of a wireless relay network can not be improved beyond that achievable with relays in parallel between the source and destination. Additional inter-terminal links between the relays does not increase the diversity order of the network.
- The order in which the constraints should be lifted and the resulting sequence of cooperative connectivity models and constraint transitions is more dependent on the capability of the relaying method to achieve full diversity order than on the physical layer relaying method (amplified or decoded), and the relative impact of the constraints is generally dependent on both the relaying method and network topology.
- For relaying methods with the capability to achieve full diversity order (amplified relaying and decoded relaying without error propagation) it is indicated that the network connectivity should be chosen to maximize the number of disjoint paths from the source to the destination.

- For relaying methods without the capability to achieve full diversity order (decoded relaying with error propagation) it is indicated that the network connectivity should be chosen to maximize the performance of the minimally connected relay.
- The most promising models from a performance efficiency perspective seem to be the 1R1DFH model (multihop relaying without diversity), 1R2D2H model (single relay cooperative diversity), 1RCD2H model (multiple relay cooperative diversity without direct source-destination connectivity), 1RFD2H model (multiple relay cooperative diversity with direct source-destination connectivity), and NRFDFH model (complete connectivity except between relays that transmit on identical subsets of channels). In particular, the NRFDFH model is highlighted as a promising, though relatively more complex, target that has not yet been studied in the literature.
- Full diversity order and minimal performance degradation can be achieved without requiring the use of a separate orthogonal channel per transmitter.
- Amplified relaying achieves outage performance very close to the outage bound when the relays are connected in parallel. Decoded relaying without error propagation achieves outage performance very close to the outage bound when the relays are connected in serial. For cooperative connectivity models with high connectivity, when relays are commonly connected in both parallel and serial, neither relaying method achieves outage performance very close to the outage bound.

7.4 Suggestions for Further Research

There are a number of interesting areas for future research on cooperative wireless relay networks associated with various extensions of the work presented in this dissertation. In this section we summarize some of the more significant areas.

7.4.1 System Model Extensions

The system model developed in this dissertation can be extended by applying the same analysis process to different channel models, fading distributions, transmit power optimizations, and relaying methods. The specific system model we have considered was used because it results in many tractable and easily understandable analytic characterizations, but does not reflect all possible wireless channel environments or transmission schemes. Following are some areas for future research in the context of extensions to the system model:

- Extension to more general channel models and more general fading distributions. Addition of alternative performance measures and other combination techniques such as equal gain combining or selection combining.
- Incorporation of power control (requiring channel state information at transmitters) and power optimization across all transmitting terminals, as opposed to an assumption of equal transmit power at all transmitting terminals. Investigation of distributed power control algorithms for optimization of performance metrics for different cooperative connectivity models.
- Extension to additional relaying methods such as combinations of amplify-and-forward and decoded-and-forward, non-orthogonal methods, and methods involving only partial decoding, compression, or quantization at relays.

7.4.2 Maximum Diversity Order Analysis Extension

The maximum diversity order analysis developed in this dissertation focused mainly on derivation of the maximum diversity order with some fairly simple results related to the diversity-multiplexing tradeoff. These presented diversity-multiplexing tradeoff results can be tightened and extended to more general communications scenarios including cooperative broadcast and multiple user networks. Following are some areas for future research in the context of extensions to the maximum diversity order analysis:

- Development of simpler results for the diversity-multiplexing tradeoff of networks with destination decoding. Specifically, this will involve tightening of the presented upper bounds in cases where not all terminals communicating across each given cut set are fully connected.
- Derivation of the diversity-multiplexing tradeoff of more general cooperative broadcast scenarios, where some part of the message transmitted by the source terminal is addressed to all receivers and some parts of the message are addressed uniquely to individual receivers or groups of receivers.
- Derivation of the diversity-multiplexing tradeoff of general multicast scenarios, where the intended receivers of a given message are a subset of the cooperating terminals in the network.
- Derivation of the diversity-multiplexing tradeoff of general networks with multiple source-destination pairs operating in parallel, including derivation of the scaling behavior of diversity order and multiplexing gain per source-destination pair as the number of users in the network increases.

7.4.3 Cooperative Connectivity Modeling Framework Extensions

The cooperative connectivity modeling framework developed in this dissertation can be extended by incorporating additional system resource constraints, network topologies, and different network deployment considerations, and by considering the practical application of the system resource constraints required by different cooperative connectivity models. These extensions would address the practical implementation of the different cooperative connectivity models in different network contexts and therefore address the practical feasibility of the theoretical results we have presented. Following are some areas for future research in the context of extensions to the cooperative connectivity modeling framework:

- Extension to include additional system resource constraints and assignment of cost metrics for the system resource constraints applicable to different practical contexts.
- Analysis of the efficiency, in comparison to the minimum cost constraint sets presented in the dissertation, of the available system resources assumed in the literature for the different proposed distributed spatial diversity techniques.
- Analysis of the practical application of common channel combination techniques and achievable orders of diversity of different techniques when there are different levels of correlation and different incidence time distributions between multiple paths.
- Analysis of the practical application of interhop interference cancellation techniques and achievable levels of mitigation.
- Simulation of additional network topologies to confirm that the results extrapolated from the network topologies considered in the dissertation hold in general. Execution of a series of simulations where the physical placement of the relay terminals is randomly chosen, each network topology is associated with a metric indicating how linear or central the terminal distribution is, an aggregate metric for the impact of each system resource constraint on the performance of the network topology is calculated, and the relationship of these metrics is analyzed.

- Analysis of the relation to and implications of the network coding techniques considered in [2] and [70].

7.4.4 General Network Extensions

Perhaps the most interesting area for future research is the extension of the results presented in this dissertation to general networks composed of multiple source-destination pairs operating in parallel. The application of cooperative diversity in multiple user networks will have a number of implications in the areas of multiple access, routing, interference, and spatial reuse that need to be analyzed. Following are some areas for future research in the context of general network extensions:

- Extension to multiple source-destination pairs operating in parallel, including analysis of multiple access protocols and multihop routing protocols for cooperative wireless relay networks, consideration of orthogonal interleaving, superposition of multiple users with successive decoding, and packet collision and/or capture behavior. For example, some of the issues related to the impact of cooperative diversity techniques on the traditional link abstraction are introduced in [93].
- Analysis of interference between multiple source-destination pairs, including comparison of interference distributions caused by cooperative diversity techniques compared to traditional direct transmissions or relaying without cooperative diversity.
- Analysis of the impact of cooperative diversity techniques on spatial reuse and overall network capacity, considering that although cooperative diversity techniques improve the diversity and general performance, they generally require longer individual inter-terminal links than relaying without cooperative diversity, and so reduce the opportunities for spatial reuse. This fundamental tradeoff has a significant impact on

overall network capacity and raises the question of determining the optimal amount of cooperative diversity to apply in different contexts.

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Appendix A – Independent Codebook Generation Outage

This appendix presents a detailed derivation of the high SNR approximation of the probability of outage using the results from [65]. The main result of [65] is the following: Let u_s and v_s be two independent random variables with the properties that $\lim_{s\to\infty} s \cdot \Pr[u_s < t] = f(t)$ and $\lim_{s\to\infty} s^d \cdot \Pr[v_s < t] = g(t)$, where $f(t)$ and $g(t)$ are monotonically increasing and integrable, and $f'(t)$ is integrable; then

$$\lim_{s\to\infty} s^{d+1} \cdot \Pr[u_s + v_s < t] = \int_0^t g(t-x) f'(x) \partial x \,. \tag{A.1}$$

If $u_k = \log(1 + s\mu_{k,i} |a_{k,i}|^2)$, since $\mu_{k,i} |a_{k,i}|^2$ is exponential with parameter $1 / \mu_{k,i}\sigma_{k,i}^2$,

$$\lim_{s\to\infty} s \cdot \Pr[u_k < t] = \frac{1}{\mu_{k,i}\sigma_{k,i}^2} g_1(t) \,, \tag{A.2}$$

where $g_1(t) = (2^t - 1)$. Repeatedly applying (A.1) results in

$$\lim_{s\to\infty} s^n \cdot \Pr[\sum_{k=1}^n u_k < t] = (\prod_{k=1}^n \frac{1}{\mu_{k,i}\sigma_{k,i}^2}) g_n(t) \,, \tag{A.3}$$

where $g_n(t) = \int_0^t g_{n-1}(t-x) f'(x) \partial x$ can be recursively expanded from $g_1(t) = f(t) = (2^t - 1)$ to result in

$$g_n(t) = \sum_{j=2}^n \left((-1)^{n-j} (\frac{2^t}{j-1})(t \ln 2)^{j-1} \right) + (-1)^{n-1}(2^t) + (-1)^n \,. \tag{A.4}$$

Applying (A.3) and (A.4) to (18) results in (19).

Appendix B – Aggregate Signal to Noise Ratio

This appendix presents intermediate results for the aggregate signal to noise ratio derived for amplified relaying channels.

Theorem 1 – Serial Amplified Relaying Channels: The aggregate SNR at terminal T_i for a set of preceding amplified relaying terminals in serial is given by

$$\gamma_{P(i),i} = (\psi_{k,i}^{-1} + \gamma_{P(k),k}^{-1} + \psi_{k,i}^{-1}\gamma_{P(k),k}^{-1})^{-1}, T_k \in T_{P(i)}, \tag{B.1}$$

where $\gamma_{P(k),k}$ is the aggregate SNR of the immediately preceding terminal T_k. Note that each terminal has only a single immediately preceding terminal so that the cardinality of $T_{P(i)}$ is one. These recursive terms can be expanded to result in a sum of products form given by

$$\begin{aligned}\gamma_{P(i),i} = (&\sum_{T_j \in T_{R(i)}} \psi_{P(j),j}^{-1} + \sum_{\substack{T_j, T_k \in T_{R(i)} \\ T_j \neq T_k}} \psi_{P(j),j}^{-1}\psi_{P(k),k}^{-1} \\ &+ \sum_{\substack{T_j, T_k, T_l \in T_{R(i)} \\ T_j \neq T_k \neq T_l}} \psi_{P(j),j}^{-1}\psi_{P(k),k}^{-1}\psi_{P(l),l}^{-1} + \ldots)^{-1},\end{aligned} \tag{B.2}$$

where there is one multiplicative term for each possible unordered combination of serial links.

Proof: Consider an amplified relaying channel with n links in serial with source terminal T_1, intermediate terminals T_2 through T_n, and destination terminal T_{n+1}. Note that $T_{P(k)} = T_{k-1}$ since each receiving terminal has a single immediately preceding terminal. Selecting any intermediate terminal, the signal received by terminal T_k is given by

$$r_{P(k),k} = a_{P(k),k}\sqrt{\varepsilon_{P(k)}}(\alpha_{P(k)} + \beta_{P(k)}) + z_{P(k),k}, \tag{B.3}$$

and the aggregate SNR at terminal T_k is given by

$$\gamma_{P(k),k} = \frac{|\alpha_{P(k)}|^2}{E[|\beta_{P(k)}|^2] + N_{P(k),k} \big/ |a_{P(k),k}|^2 \varepsilon_{P(k)}}. \tag{B.4}$$

If this signal is amplified according to (26) then the signal transmitted by terminal T_k is given by

$$\begin{aligned} s_k &= (a_{P(k),k}\sqrt{\varepsilon_{P(k)}}(\alpha_{P(k)} + \beta_{P(k)}) + z_{P(k),k}) \\ &\quad \times (\sqrt{\varepsilon_k} \Big/ \sqrt{|a_{P(k),k}|^2 \varepsilon_{P(k)} + N_{P(k),k}}), \end{aligned} \tag{B.5}$$

the signal received by terminal T_i, $T_k \in T_{P(i)}$ is given by

$$\begin{aligned} r_{k,i} &= a_{k,i}(a_{P(k),k}\sqrt{\varepsilon_{P(k)}}(\alpha_{P(k)} + \beta_{P(k)}) + z_{P(k),k}) \\ &\quad \times (\sqrt{\varepsilon_k} \Big/ \sqrt{|a_{P(k),k}|^2 \varepsilon_{P(k)} + N_{P(k),k}}) + z_{k,i}, \end{aligned} \tag{B.6}$$

and the aggregate SNR at terminal T_i is given by

$$\begin{aligned} \gamma_{k,i} &= |\alpha_{P(k)}|^2 \begin{pmatrix} E[|\beta_{P(k)}|^2] + N_{P(k),k} \big/ (|a_{P(k),k}|^2 \varepsilon_{P(k)}) \\ + (1 + N_{P(k),k} \big/ (|a_{P(k),k}|^2 \varepsilon_{P(k)}))(N_{k,i} \big/ |a_{k,i}|^2 \varepsilon_k) \end{pmatrix}^{-1} \\ &= \begin{pmatrix} \dfrac{E[|\beta_{P(k)}|^2] + N_{P(k),k} \big/ (|a_{P(k),k}|^2 \varepsilon_{P(k)})}{|\alpha_{P(k)}|^2} + (\dfrac{N_{k,i}}{|a_{k,i}|^2 \varepsilon_k}) \\ + (\dfrac{E[|\beta_{P(k)}|^2] + N_{P(k),k} \big/ (|a_{P(k),k}|^2 \varepsilon_{P(k)})}{|\alpha_{P(k)}|^2})(\dfrac{N_{k,i}}{|a_{k,i}|^2 \varepsilon_k}) \end{pmatrix}^{-1} \\ &= (\gamma_{P(k),k}^{-1} + \psi_{k,i}^{-1} + \gamma_{P(k),k}^{-1}\psi_{k,i}^{-1})^{-1}. \end{aligned} \tag{B.7}$$

Note the use of the normalization $|\alpha_i|^2 + E[|\beta_i|^2] = 1$ in the derivation. Rearranging the order of terms results in the given theorem.

Lemma 1 – Parallel Amplified Relaying Channels: The aggregate SNR at terminal T_i for a set of preceding amplified relaying terminals in parallel is lower bounded by

$$\gamma_{P(i),i} \geq \sum_{T_k \in T_{P(i)}} \gamma_{k,i}. \tag{B.8}$$

Note that this form implies diversity combining of the multiple input signal links using a maximal ratio combiner.

Proof of Lemma 1: First consider the case where the propagated noise components of the signals from the parallel preceding terminals are mutually independent. It is well known that the optimal combiner for a set of input signals with noise components that are mutually independent is a maximal ratio combiner with output signal to noise ratio equal to the sum of the input branch signal to noise ratios [96]. Therefore, when the propagated noise components from the parallel preceding terminals are mutually independent the optimal combiner is a maximal ratio combiner and the received signal to noise ratio is equal to the sum of the input branch signal to noise ratios.

Now consider the case where the propagated noise components of the signals from the parallel preceding terminals are correlated (but the information bearing components are uncorrelated). Although the derivation of the optimal combiner that leverages this correlation and resultant output signal to noise ratio are beyond the scope of this manuscript, the output signal to noise ratio of the optimal combiner can be lower bounded in the following fashion. It is well known that the lowest output signal to noise ratio of an optimal combiner occurs when the noise components for the set of input signals are mutually independent [96]. Therefore, the sum of the input signal to noise ratios is a lower bound on the received signal to noise ratio. Generalizing the described equality for mutually independent input signal links and lower bound for correlated input signal links results in the given lemma.

Corollary 1 – General Amplified Relaying Channels: The aggregate SNR at terminal T_i for a general set of preceding amplified relaying terminals is lower bounded by

$$\gamma_{P(i),i} \geq \sum_{T_k \in T_{P(i)}} (\psi_{k,i}^{-1} + \gamma_{P(k),k}^{-1} + \psi_{k,i}^{-1}\gamma_{P(k),k}^{-1})^{-1}. \tag{B.9}$$

Proof of Corollary 1: Combining the results of Theorem 1 and Lemma 1, the aggregate SNR of general amplified relaying channels can be considered in the light of resistance theory for electrical circuits. Signal links in serial are analogous to resistors in parallel (with additional multiplicative terms). Signal links in parallel are analogous to resistors in serial. Deriving the aggregate SNR in a recursive fashion by employing Theorem 1 and Lemma 1 results in the given corollary.

Some interesting qualitative statements can be extrapolated from the form of the developed results. The performance of serial amplified relaying channels is sensitive to the performance of the single weakest link. Therefore, the performance will be improved when more power is allocated to the weakest link relative to the other links. The performance of parallel-amplified relaying channels is insensitive to the performance of the single weakest link. Therefore, the performance will be improved when less power is allocated to the weakest link relative to the other links. This can be applied to general amplified relaying channels where the performance will be improved by allocating relatively more power to weak links that are not parallel to strong links and relatively less power to weak links that are parallel to strong links.

When a serial amplified relaying channel is composed of strong links the lower order multiplicative terms dominate the aggregate SNR and the performance is approximately linear with respect to the component link SNRs. When a serial amplified relaying channel is composed of weak links the higher order multiplicative terms dominate the aggregate SNR and the performance is less than linear with respect to the component link SNRs. This means that for amplified relaying channels where there are many weak links in serial the performance will be significantly degraded with respect to a linear relation.

Appendix C – System Resource Constraint Combinations

This appendix comprehensively summarizes all possible combinations of system resource constraints and the resultant cooperative connectivity models. The combinations with different numbers of channels available are presented separately in order to improve the readability of the tables. Tables 13, 14, and 15, respectively summarize the constraint combinations and resultant cooperative connectivity models with K, 2, and N channels available.

Note that although not practical in mobile relay networks, constraint combinations with relay diversity combination but not destination diversity combination are of interest for fixed relay networks where the fixed relays will have less resource constraints than the mobile destination. Constraint combinations with destination or relay common channel diversity combination but not orthogonal channel diversity combination or multiple channel transmission can actually achieve less connectivity when each relay transmits on a separate orthogonal channel than when the source and all relays transmit on the same two channels. Intelligent reuse of the available channels can result in improved connectivity.

NCA	RCC	DCC	ROC	DOC	MCT	IIC	Model
KCA	RCC	DCC	ROC	DOC	MCT	IIC	NRFDFH
KCA	RCC	DCC	ROC	DOC	MCT	NIIC	NRFDKH
KCA	RCC	DCC	ROC	DOC	NMCT	IIC	NRFDFH
KCA	RCC	DCC	ROC	DOC	NMCT	NIIC	NRFDKH
KCA	RCC	DCC	ROC	NDOC	MCT	IIC	NRFDFH
KCA	RCC	DCC	ROC	NDOC	MCT	NIIC	NRFDKH
KCA	RCC	DCC	ROC	NDOC	NMCT	IIC	NRCDFH
KCA	RCC	DCC	ROC	NDOC	NMCT	NIIC	NRCDKH
KCA	RCC	DCC	NROC	DOC	MCT	IIC	NRFDFH
KCA	RCC	DCC	NROC	DOC	MCT	NIIC	NRFDKH
KCA	RCC	DCC	NROC	DOC	NMCT	IIC	CRFDFH
KCA	RCC	DCC	NROC	DOC	NMCT	NIIC	CRFDKH
KCA	RCC	DCC	NROC	NDOC	MCT	IIC	NRFDFH
KCA	RCC	DCC	NROC	NDOC	MCT	NIIC	NRFDKH
KCA	RCC	DCC	NROC	NDOC	NMCT	IIC	CRCDFH
KCA	RCC	DCC	NROC	NDOC	NMCT	NIIC	CRCDKH
KCA	RCC	NDCC	ROC	DOC	MCT	IIC	NRKDFH
KCA	RCC	NDCC	ROC	DOC	MCT	NIIC	NRKDKH
KCA	RCC	NDCC	ROC	DOC	NMCT	IIC	NRKDFH
KCA	RCC	NDCC	ROC	DOC	NMCT	NIIC	NRKDKH
KCA	RCC	NDCC	ROC	NDOC	MCT	IIC	NR1DFH
KCA	RCC	NDCC	ROC	NDOC	MCT	NIIC	NR1DKH
KCA	RCC	NDCC	ROC	NDOC	NMCT	IIC	NR1DFH
KCA	RCC	NDCC	ROC	NDOC	NMCT	NIIC	NR1DKH
KCA	RCC	NDCC	NROC	DOC	MCT	IIC	NRKDFH
KCA	RCC	NDCC	NROC	DOC	MCT	NIIC	NRKDKH
KCA	RCC	NDCC	NROC	DOC	NMCT	IIC	CRKDFH
KCA	RCC	NDCC	NROC	DOC	NMCT	NIIC	CRKDKH
KCA	RCC	NDCC	NROC	NDOC	MCT	IIC	NR1DFH
KCA	RCC	NDCC	NROC	NDOC	MCT	NIIC	NR1DKH
KCA	RCC	NDCC	NROC	NDOC	NMCT	IIC	CR1DFH
KCA	RCC	NDCC	NROC	NDOC	NMCT	NIIC	CR1DKH
KCA	NRCC	DCC	ROC	DOC	MCT	IIC	KRFDFH
KCA	NRCC	DCC	ROC	DOC	MCT	NIIC	KRFDKH
KCA	NRCC	DCC	ROC	DOC	NMCT	IIC	KRFDFH
KCA	NRCC	DCC	ROC	DOC	NMCT	NIIC	KRFDKH
KCA	NRCC	DCC	ROC	NDOC	MCT	IIC	KRFDFH
KCA	NRCC	DCC	ROC	NDOC	MCT	NIIC	KRFDKH
KCA	NRCC	DCC	ROC	NDOC	NMCT	IIC	KRCDFH
KCA	NRCC	DCC	ROC	NDOC	NMCT	NIIC	KRCDKH
KCA	NRCC	DCC	NROC	DOC	MCT	IIC	1RFDFH
KCA	NRCC	DCC	NROC	DOC	MCT	NIIC	1RFDKH

KCA	NRCC	DCC	NROC	DOC	NMCT	IIC	1RFDFH
KCA	NRCC	DCC	NROC	DOC	NMCT	NIIC	1RFDKH
KCA	NRCC	DCC	NROC	NDOC	MCT	IIC	1RFDFH
KCA	NRCC	DCC	NROC	NDOC	MCT	NIIC	1RFDKH
KCA	NRCC	DCC	NROC	NDOC	NMCT	IIC	1RCDFH
KCA	NRCC	DCC	NROC	NDOC	NMCT	NIIC	1RCDKH
KCA	NRCC	NDCC	ROC	DOC	MCT	IIC	KRKDFH
KCA	NRCC	NDCC	ROC	DOC	MCT	NIIC	KRKDKH
KCA	NRCC	NDCC	ROC	DOC	NMCT	IIC	KRKDFH
KCA	NRCC	NDCC	ROC	DOC	NMCT	NIIC	KRKDKH
KCA	NRCC	NDCC	ROC	NDOC	MCT	IIC	KR1DFH
KCA	NRCC	NDCC	ROC	NDOC	MCT	NIIC	KR1DKH
KCA	NRCC	NDCC	ROC	NDOC	NMCT	IIC	KR1DFH
KCA	NRCC	NDCC	ROC	NDOC	NMCT	NIIC	KR1DKH
KCA	NRCC	NDCC	NROC	DOC	MCT	IIC	1RKDFH
KCA	NRCC	NDCC	NROC	DOC	MCT	NIIC	1RKDKH
KCA	NRCC	NDCC	NROC	DOC	NMCT	IIC	1RKDFH
KCA	NRCC	NDCC	NROC	DOC	NMCT	NIIC	1RKDKH
KCA	NRCC	NDCC	NROC	NDOC	MCT	IIC	1R1DFH
KCA	NRCC	NDCC	NROC	NDOC	MCT	NIIC	1R1DKH
KCA	NRCC	NDCC	NROC	NDOC	NMCT	IIC	1R1DFH
KCA	NRCC	NDCC	NROC	NDOC	NMCT	NIIC	1R1DKH

Table 13. System Resource Constraint Combinations with KCA

NCA	RCC	DCC	ROC	DOC	MCT	IIC	Model
2CA	RCC	DCC	ROC	DOC	MCT	IIC	NRFDFH
2CA	RCC	DCC	ROC	DOC	MCT	NIIC	1RFD2H
2CA	RCC	DCC	ROC	DOC	NMCT	IIC	CRFDFH
2CA	RCC	DCC	ROC	DOC	NMCT	NIIC	1RFD2H
2CA	RCC	DCC	ROC	NDOC	MCT	IIC	NRNDFH
2CA	RCC	DCC	ROC	NDOC	MCT	NIIC	1RND2H
2CA	RCC	DCC	ROC	NDOC	NMCT	IIC	CRCDFH
2CA	RCC	DCC	ROC	NDOC	NMCT	NIIC	1RCD2H
2CA	RCC	DCC	NROC	DOC	MCT	IIC	NRFDFH
2CA	RCC	DCC	NROC	DOC	MCT	NIIC	1RFD2H
2CA	RCC	DCC	NROC	DOC	NMCT	IIC	CRFDFH
2CA	RCC	DCC	NROC	DOC	NMCT	NIIC	1RFD2H
2CA	RCC	DCC	NROC	NDOC	MCT	IIC	NRNDFH
2CA	RCC	DCC	NROC	NDOC	MCT	NIIC	1RND2H
2CA	RCC	DCC	NROC	NDOC	NMCT	IIC	CRCDFH
2CA	RCC	DCC	NROC	NDOC	NMCT	NIIC	1RCD2H
2CA	RCC	NDCC	ROC	DOC	MCT	IIC	NR2DFH
2CA	RCC	NDCC	ROC	DOC	MCT	NIIC	1R2D2H
2CA	RCC	NDCC	ROC	DOC	NMCT	IIC	CR2DFH
2CA	RCC	NDCC	ROC	DOC	NMCT	NIIC	1R2D2H
2CA	RCC	NDCC	ROC	NDOC	MCT	IIC	NR1DFH
2CA	RCC	NDCC	ROC	NDOC	MCT	NIIC	1R1D2H
2CA	RCC	NDCC	ROC	NDOC	NMCT	IIC	CR1DFH
2CA	RCC	NDCC	ROC	NDOC	NMCT	NIIC	1R1D2H
2CA	RCC	NDCC	NROC	DOC	MCT	IIC	NR2DFH
2CA	RCC	NDCC	NROC	DOC	MCT	NIIC	1R2D2H
2CA	RCC	NDCC	NROC	DOC	NMCT	IIC	CR2DFH
2CA	RCC	NDCC	NROC	DOC	NMCT	NIIC	1R2D2H
2CA	RCC	NDCC	NROC	NDOC	MCT	IIC	NR1DFH
2CA	RCC	NDCC	NROC	NDOC	MCT	NIIC	1R1D2H
2CA	RCC	NDCC	NROC	NDOC	NMCT	IIC	CR1DFH
2CA	RCC	NDCC	NROC	NDOC	NMCT	NIIC	1R1D2H
2CA	NRCC	DCC	ROC	DOC	MCT	IIC	1RFDFH
2CA	NRCC	DCC	ROC	DOC	MCT	NIIC	1RFD2H
2CA	NRCC	DCC	ROC	DOC	NMCT	IIC	1RFDFH
2CA	NRCC	DCC	ROC	DOC	NMCT	NIIC	1RFD2H
2CA	NRCC	DCC	ROC	NDOC	MCT	IIC	1RNDFH
2CA	NRCC	DCC	ROC	NDOC	MCT	NIIC	1RND2H
2CA	NRCC	DCC	ROC	NDOC	NMCT	IIC	1RCDFH
2CA	NRCC	DCC	ROC	NDOC	NMCT	NIIC	1RCD2H
2CA	NRCC	DCC	NROC	DOC	MCT	IIC	1RFDFH
2CA	NRCC	DCC	NROC	DOC	MCT	NIIC	1RFD2H

2CA	NRCC	DCC	NROC	DOC	NMCT	IIC	1RFDFH
2CA	NRCC	DCC	NROC	DOC	NMCT	NIIC	1RFD2H
2CA	NRCC	DCC	NROC	NDOC	MCT	IIC	1RNDFH
2CA	NRCC	DCC	NROC	NDOC	MCT	NIIC	1RND2H
2CA	NRCC	DCC	NROC	NDOC	NMCT	IIC	1RCDFH
2CA	NRCC	DCC	NROC	NDOC	NMCT	NIIC	1RCD2H
2CA	NRCC	NDCC	ROC	DOC	MCT	IIC	1R2DFH
2CA	NRCC	NDCC	ROC	DOC	MCT	NIIC	1R2D2H
2CA	NRCC	NDCC	ROC	DOC	NMCT	IIC	1R2DFH
2CA	NRCC	NDCC	ROC	DOC	NMCT	NIIC	1R2D2H
2CA	NRCC	NDCC	ROC	NDOC	MCT	IIC	1R1DFH
2CA	NRCC	NDCC	ROC	NDOC	MCT	NIIC	1R1D2H
2CA	NRCC	NDCC	ROC	NDOC	NMCT	IIC	1R1DFH
2CA	NRCC	NDCC	ROC	NDOC	NMCT	NIIC	1R1D2H
2CA	NRCC	NDCC	NROC	DOC	MCT	IIC	1R2DFH
2CA	NRCC	NDCC	NROC	DOC	MCT	NIIC	1R2D2H
2CA	NRCC	NDCC	NROC	DOC	NMCT	IIC	1R2DFH
2CA	NRCC	NDCC	NROC	DOC	NMCT	NIIC	1R2D2H
2CA	NRCC	NDCC	NROC	NDOC	MCT	IIC	1R1DFH
2CA	NRCC	NDCC	NROC	NDOC	MCT	NIIC	1R1D2H
2CA	NRCC	NDCC	NROC	NDOC	NMCT	IIC	1R1DFH
2CA	NRCC	NDCC	NROC	NDOC	NMCT	NIIC	1R1D2H

Table 14. System Resource Constraint Combinations with 2CA

NCA	RCC	DCC	ROC	DOC	MCT	IIC	Model
NCA	RCC	DCC	ROC	DOC	MCT	IIC	FRFDFH
NCA	RCC	DCC	ROC	DOC	MCT	NIIC	FRFDFH
NCA	RCC	DCC	ROC	DOC	NMCT	IIC	FRFDFH
NCA	RCC	DCC	ROC	DOC	NMCT	NIIC	FRFDFH
NCA	RCC	DCC	ROC	NDOC	MCT	IIC	FRFDFH
NCA	RCC	DCC	ROC	NDOC	MCT	NIIC	FRFDFH
NCA	RCC	DCC	ROC	NDOC	NMCT	IIC	FR1DFH
NCA	RCC	DCC	ROC	NDOC	NMCT	NIIC	FR1DFH
NCA	RCC	DCC	NROC	DOC	MCT	IIC	FRFDFH
NCA	RCC	DCC	NROC	DOC	MCT	NIIC	FRFDFH
NCA	RCC	DCC	NROC	DOC	NMCT	IIC	1RFDFH
NCA	RCC	DCC	NROC	DOC	NMCT	NIIC	1RFDFH
NCA	RCC	DCC	NROC	NDOC	MCT	IIC	FRFDFH
NCA	RCC	DCC	NROC	NDOC	MCT	NIIC	FRFDFH
NCA	RCC	DCC	NROC	NDOC	NMCT	IIC	1R1DFH
NCA	RCC	DCC	NROC	NDOC	NMCT	NIIC	1R1DFH
NCA	RCC	NDCC	ROC	DOC	MCT	IIC	FRFDFH
NCA	RCC	NDCC	ROC	DOC	MCT	NIIC	FRFDFH
NCA	RCC	NDCC	ROC	DOC	NMCT	IIC	FRFDFH
NCA	RCC	NDCC	ROC	DOC	NMCT	NIIC	FRFDFH
NCA	RCC	NDCC	ROC	NDOC	MCT	IIC	FR1DFH
NCA	RCC	NDCC	ROC	NDOC	MCT	NIIC	FR1DFH
NCA	RCC	NDCC	ROC	NDOC	NMCT	IIC	FR1DFH
NCA	RCC	NDCC	ROC	NDOC	NMCT	NIIC	FR1DFH
NCA	RCC	NDCC	NROC	DOC	MCT	IIC	FRFDFH
NCA	RCC	NDCC	NROC	DOC	MCT	NIIC	FRFDFH
NCA	RCC	NDCC	NROC	DOC	NMCT	IIC	1RFDFH
NCA	RCC	NDCC	NROC	DOC	NMCT	NIIC	1RFDFH
NCA	RCC	NDCC	NROC	NDOC	MCT	IIC	FR1DFH
NCA	RCC	NDCC	NROC	NDOC	MCT	NIIC	FR1DFH
NCA	RCC	NDCC	NROC	NDOC	NMCT	IIC	1R1DFH
NCA	RCC	NDCC	NROC	NDOC	NMCT	NIIC	1R1DFH
NCA	NRCC	DCC	ROC	DOC	MCT	IIC	FRFDFH
NCA	NRCC	DCC	ROC	DOC	MCT	NIIC	FRFDFH
NCA	NRCC	DCC	ROC	DOC	NMCT	IIC	FRFDFH
NCA	NRCC	DCC	ROC	DOC	NMCT	NIIC	FRFDFH
NCA	NRCC	DCC	ROC	NDOC	MCT	IIC	FRFDFH
NCA	NRCC	DCC	ROC	NDOC	MCT	NIIC	FRFDFH
NCA	NRCC	DCC	ROC	NDOC	NMCT	IIC	FR1DFH
NCA	NRCC	DCC	ROC	NDOC	NMCT	NIIC	FR1DFH
NCA	NRCC	DCC	NROC	DOC	MCT	IIC	1RFDFH
NCA	NRCC	DCC	NROC	DOC	MCT	NIIC	1RFDFH

NCA	NRCC	DCC	NROC	DOC	NMCT	IIC	1RFDFH
NCA	NRCC	DCC	NROC	DOC	NMCT	NIIC	1RFDFH
NCA	NRCC	DCC	NROC	NDOC	MCT	IIC	1RFDFH
NCA	NRCC	DCC	NROC	NDOC	MCT	NIIC	1RFDFH
NCA	NRCC	DCC	NROC	NDOC	NMCT	IIC	1R1DFH
NCA	NRCC	DCC	NROC	NDOC	NMCT	NIIC	1R1DFH
NCA	NRCC	NDCC	ROC	DOC	MCT	IIC	FRFDFH
NCA	NRCC	NDCC	ROC	DOC	MCT	NIIC	FRFDFH
NCA	NRCC	NDCC	ROC	DOC	NMCT	IIC	FRFDFH
NCA	NRCC	NDCC	ROC	DOC	NMCT	NIIC	FRFDFH
NCA	NRCC	NDCC	ROC	NDOC	MCT	IIC	FR1DFH
NCA	NRCC	NDCC	ROC	NDOC	MCT	NIIC	FR1DFH
NCA	NRCC	NDCC	ROC	NDOC	NMCT	IIC	FR1DFH
NCA	NRCC	NDCC	ROC	NDOC	NMCT	NIIC	FR1DFH
NCA	NRCC	NDCC	NROC	DOC	MCT	IIC	1RFDFH
NCA	NRCC	NDCC	NROC	DOC	MCT	NIIC	1RFDFH
NCA	NRCC	NDCC	NROC	DOC	NMCT	IIC	1RFDFH
NCA	NRCC	NDCC	NROC	DOC	NMCT	NIIC	1RFDFH
NCA	NRCC	NDCC	NROC	NDOC	MCT	IIC	1R1DFH
NCA	NRCC	NDCC	NROC	NDOC	MCT	NIIC	1R1DFH
NCA	NRCC	NDCC	NROC	NDOC	NMCT	IIC	1R1DFH
NCA	NRCC	NDCC	NROC	NDOC	NMCT	NIIC	1R1DFH

Table 15. System Resource Constraint Combinations with NCA

Appendix D – Connectivity Model Examples with KCA

This appendix presents graphical examples illustrating the connectivity of each cooperative connectivity model with *K* channels available.

1R1DKH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to one transmitter, and the network has *K* hops in the longest multihop path.

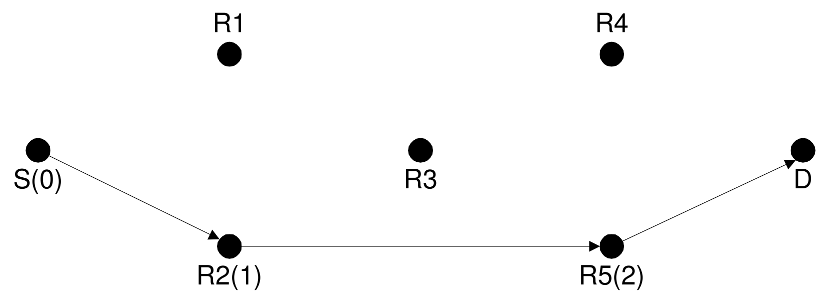

Fig. 44. Example 1R1DKH Model with KCA

1R1DFH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to one transmitter, and the network has *N* hops in the longest multihop path.

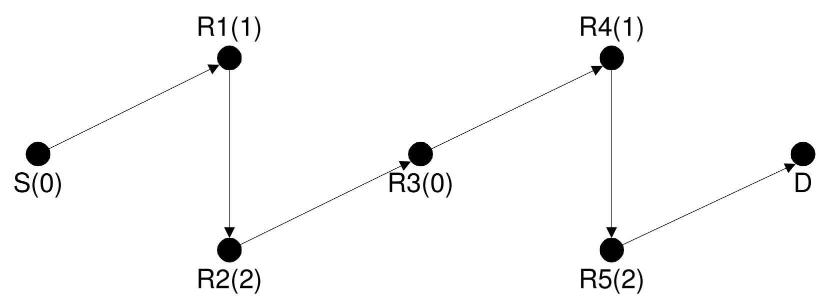

Fig. 45. Example 1R1DFH Model with KCA

1RKDKH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to one transmitter on each channel, and the network has *K* hops in the longest multihop path.

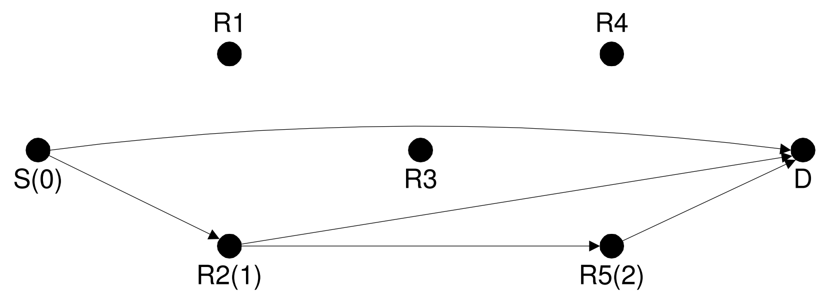

Fig. 46. Example 1RKDKH Model with KCA

1RKDFH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to one transmitter on each channel, and the network has *N* hops in the longest multihop path.

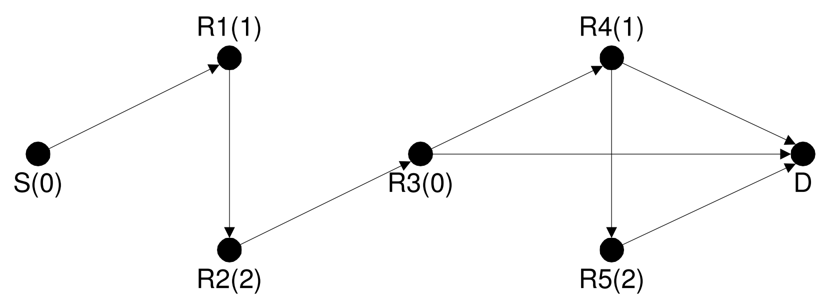

Fig. 47. Example 1RKDFH Model with KCA

1RCDKH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to a subset of transmitters on one channel, and the network has *K* hops in the longest multihop path.

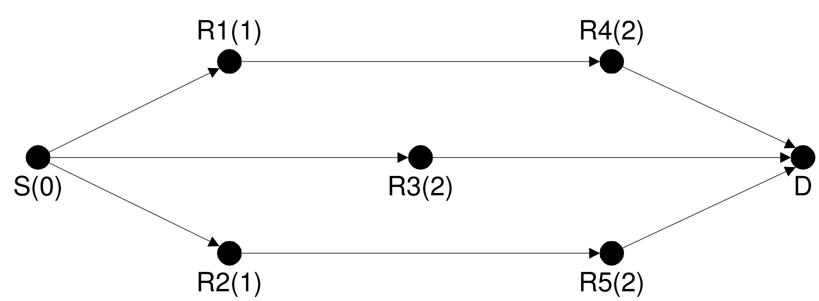

Fig. 48. Example 1RCDKH Model with KCA

1RCDFH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to a subset of transmitters on one channel, and the network has *N* hops in the longest multihop path.

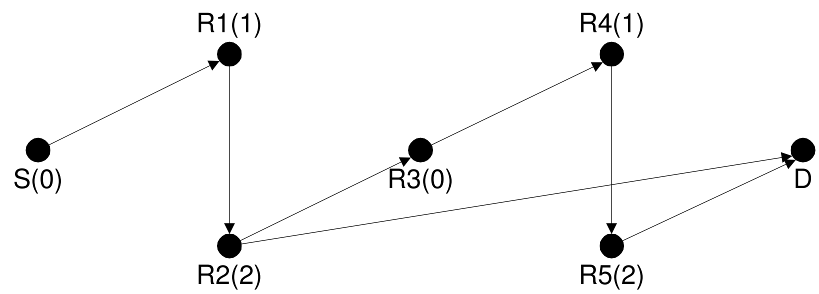

Fig. 49. Example 1RCDFH Model with KCA

1RND2H Connectivity Model: Each relay is connected to the source and destination, the source and destination are connected to each other, and the network has 2 hops in the longest multihop path.

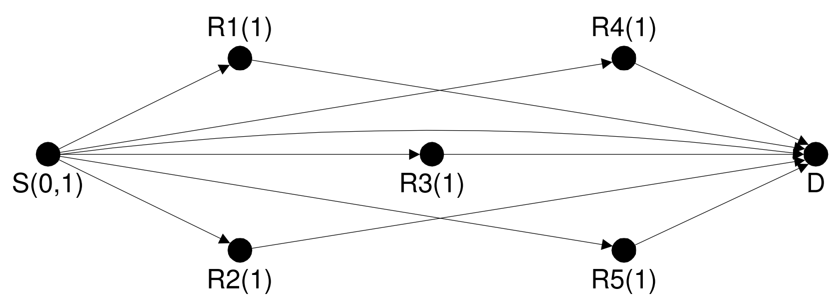

Fig. 50. Example 1RND2H Model with KCA

1RFDKH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to all transmitters, and the network has *K* hops in the longest multihop path.

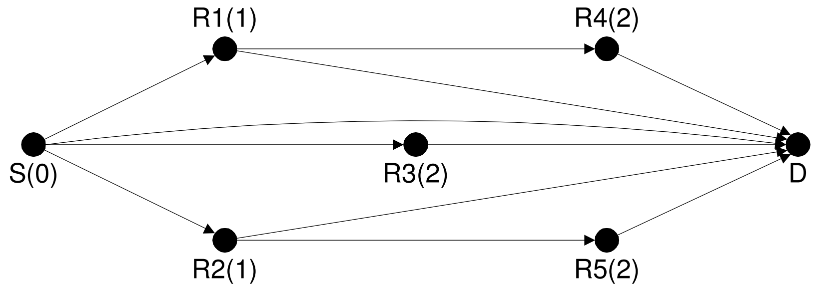

Fig. 51. Example 1RFDKH Model with KCA

1RFDFH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to all transmitters, and the network has N hops in the longest multihop path.

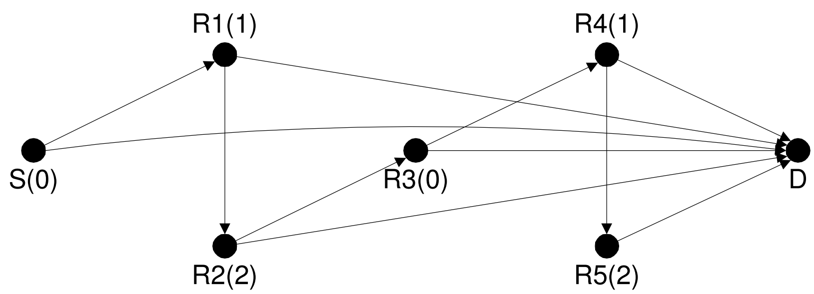

Fig. 52. Example 1RFDFH Model with KCA

KR1DKH Connectivity Model: Each relay is connected to one transmitter on each channel, the destination is connected to one transmitter, and the network has K hops in the longest multihop path.

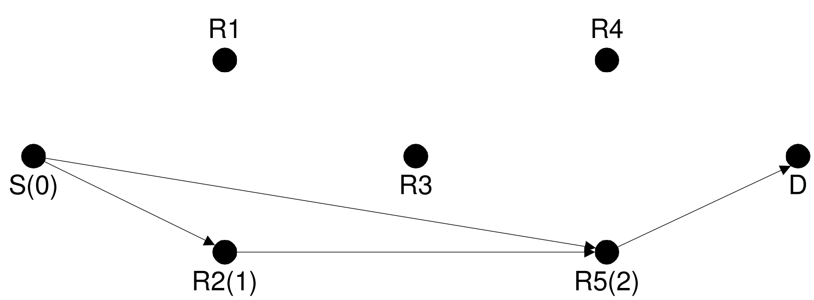

Fig. 53. Example KR1DKH Model with KCA

KR1DFH Connectivity Model: Each relay is connected to one transmitter on each channel, the destination is connected to one transmitter, and the network has *N* hops in the longest multihop path.

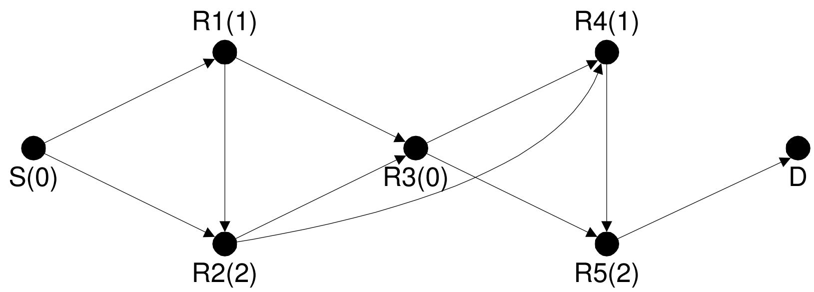

Fig. 54. Example KR1DFH Model with KCA

KRKDKH Connectivity Model: Each relay is connected to one transmitter on each channel, the destination is connected to one transmitter on each channel, and the network has *K* hops in the longest multihop path.

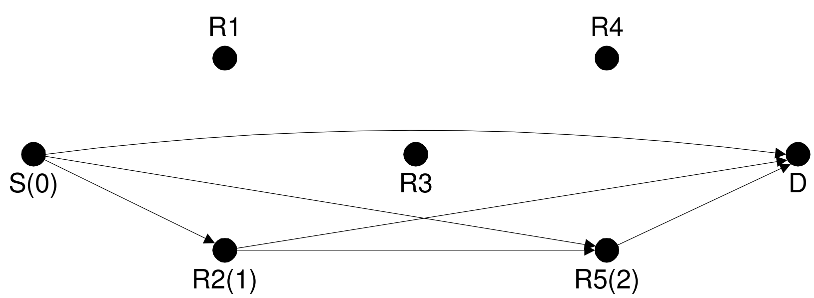

Fig. 55. Example KRKDKH Model with KCA

KRKDFH Connectivity Model: Each relay is connected to one transmitter on each channel, the destination is connected to one transmitter, and the network has *N* hops in the longest multihop path.

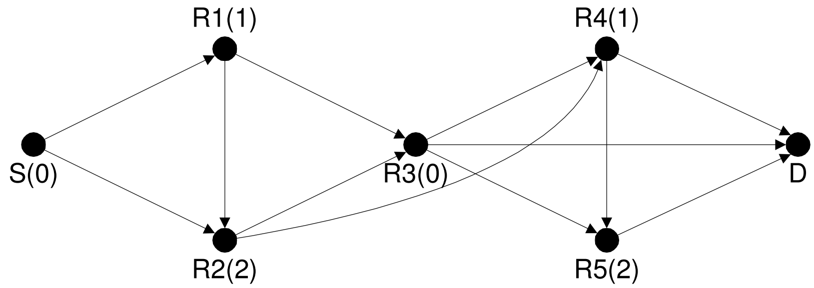

Fig. 56. Example KRKDFH Model with KCA

KRCDKH Connectivity Model: Each relay is connected to one transmitter on each channel, the destination is connected to a subset of transmitters on one channel, and the network has *K* hops in the longest multihop path.

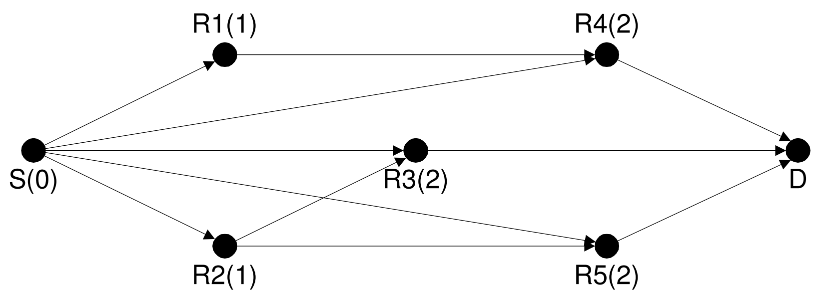

Fig. 57. Example KRCDKH Model with KCA

KRCDFH Connectivity Model: Each relay is connected to one transmitter on each channel, the destination is connected to a subset of transmitters on one channel, and the network has *N* hops in the longest multihop path.

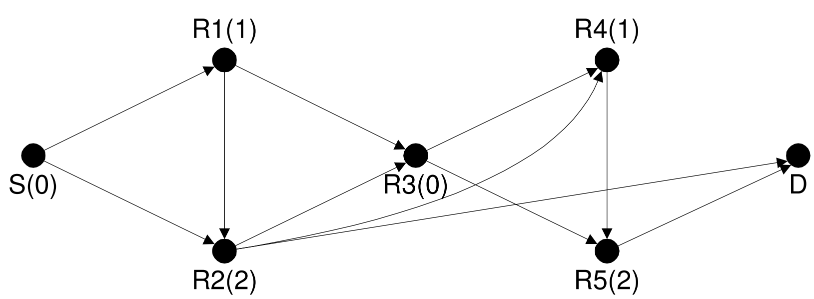

Fig. 58. Example KRCDFH Model with KCA

KRFDKH Connectivity Model: Each relay is connected to one transmitter on each channel, the destination is connected to all transmitters, and the network has *K* hops in the longest multihop path.

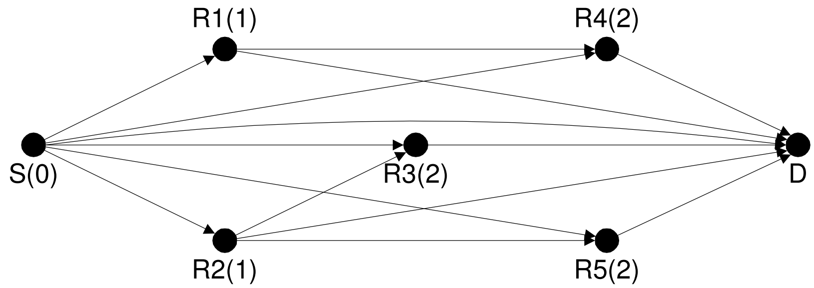

Fig. 59. Example KRFDKH Model with KCA

KRFDFH Connectivity Model: Each relay is connected to one transmitter on each channel, the destination is connected to all transmitters, and the network has *N* hops in the longest multihop path.

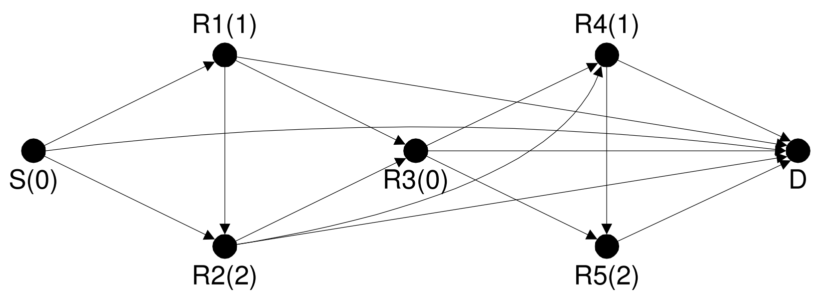

Fig. 60. Example KRFDFH Model with KCA

CR1DKH Connectivity Model: Each relay is connected to a subset of transmitters on one channel, the destination is connected to one transmitter, and the network has *K* hops in the longest multihop path.

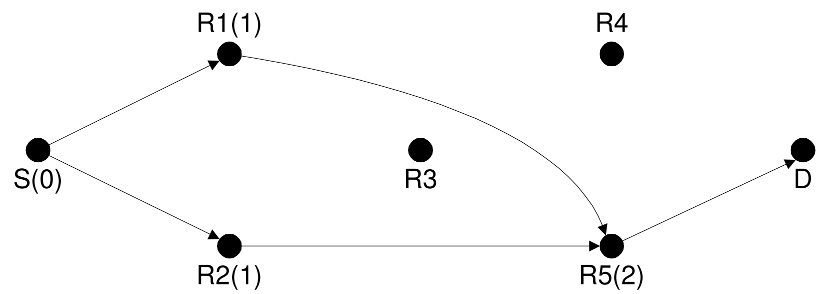

Fig. 61. Example CR1DKH Model with KCA

CR1DFH Connectivity Model: Each relay is connected to a subset of transmitters on one channel, the destination is connected to one transmitter, and the network has *N* hops in the longest multihop path.

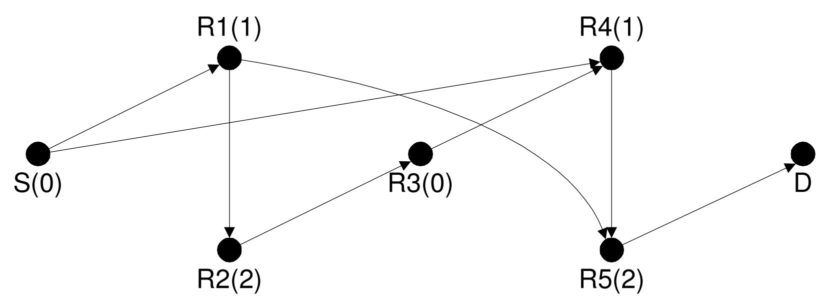

Fig. 62. Example CR1DFH Model with KCA

CRKDKH Connectivity Model: Each relay is connected to a subset of transmitters on one channel, the destination is connected to one transmitter on each channel, and the network has *K* hops in the longest multihop path.

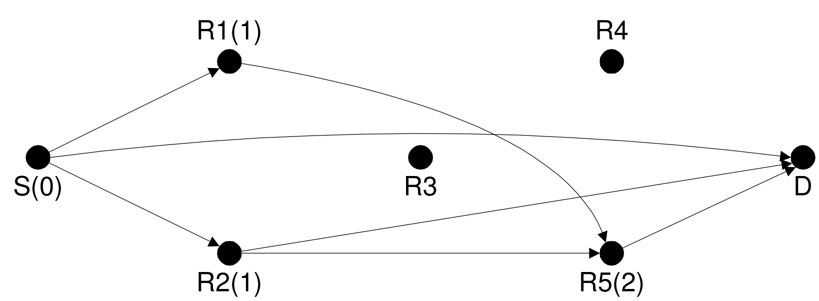

Fig. 63. Example CRKDKH Model with KCA

CRKDFH Connectivity Model: Each relay is connected to a subset of transmitters on one channel, the destination is connected to one transmitter on each channel, and the network has *N* hops in the longest multihop path..

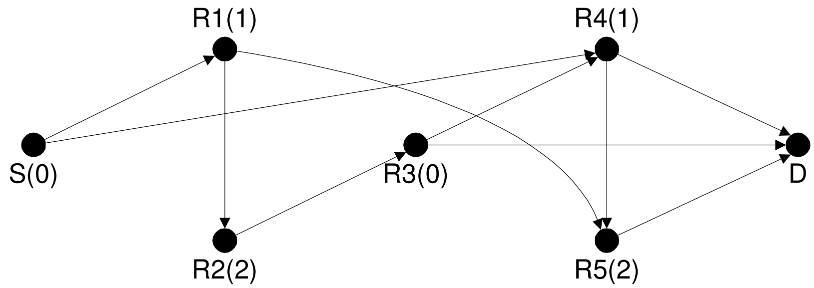

Fig. 64. Example CRKDFH Model with KCA

CRCDKH Connectivity Model: Each relay is connected to a subset of transmitters on one channel, the destination is connected to a subset of transmitters on one channel, and the network has *K* hops in the longest multihop path.

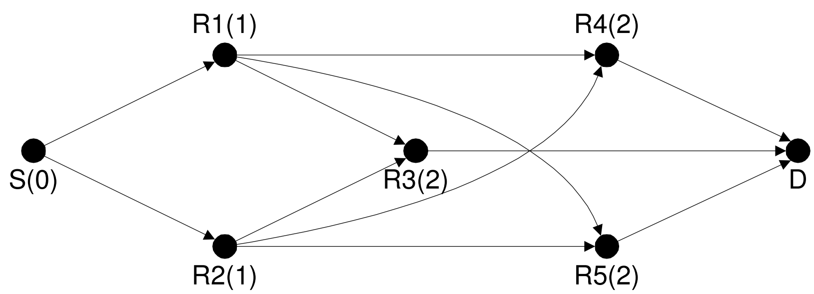

Fig. 65. Example CRCDKH Model with KCA

CRCDFH Connectivity Model: Each relay is connected to a subset of transmitters on one channel, the destination is connected to a subset of transmitters on one channel, and the network has *N* hops in the longest multihop path.

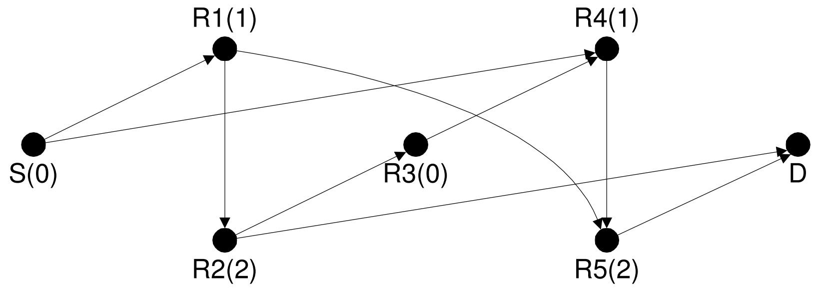

Fig. 66. Example CRCDFH Model with KCA

CRFDKH Connectivity Model: Each relay is connected to a subset of transmitters on one channel, the destination is connected to all transmitters, and the network has K hops in the longest multihop path.

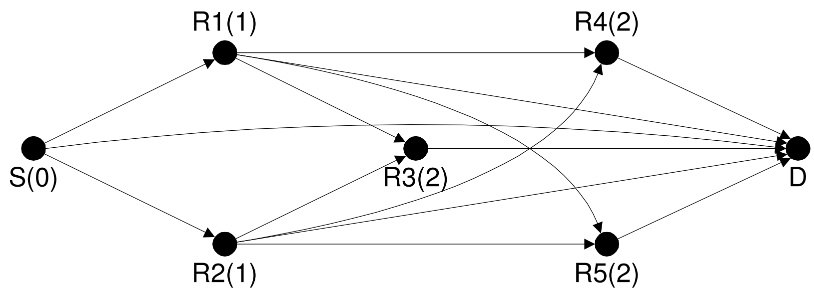

Fig. 67. Example CRFDKH Model with KCA

CRFDFH Connectivity Model: Each relay is connected to a subset of transmitters on one channel, the destination is connected to all transmitters, and the network has N hops in the longest multihop path.

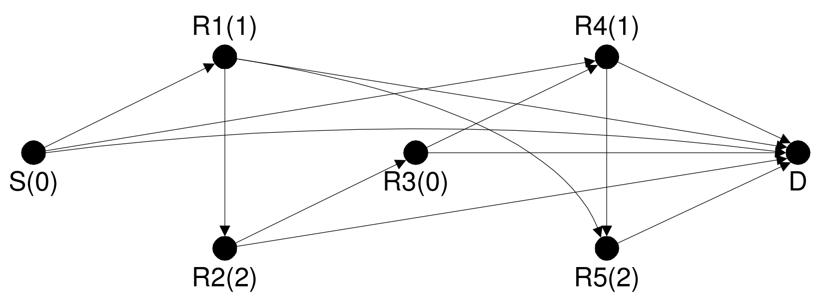

Fig. 68. Example CRFDFH Model with KCA

NR1DKH Connectivity Model: Each relay is connected to all previous transmitters that do not transmit or receive on an identical subset of channels, the destination is connected to one transmitter, and the network has *K* hops in the longest multihop path.

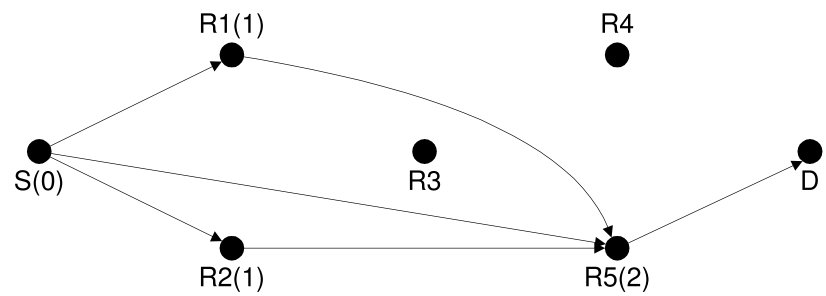

Fig. 69. Example NR1DKH Model with KCA

NR1DFH Connectivity Model: Each relay is connected to all previous transmitters that do not transmit or receive on an identical subset of channels, the destination is connected to one transmitter, and the network has *N* hops in the longest multihop path.

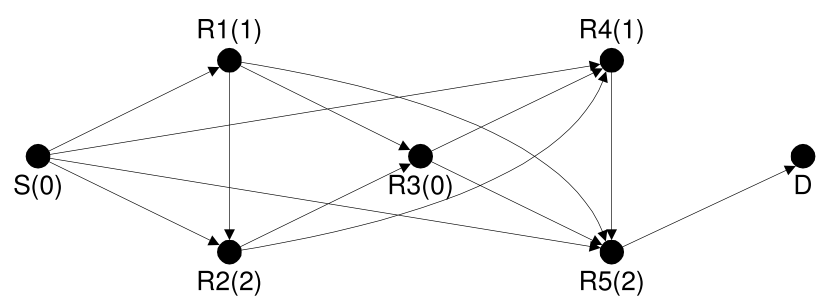

Fig. 70. Example NR1DFH Model with KCA

NRKDKH Connectivity Model: Each relay is connected to all previous transmitters that do not transmit or receive on an identical subset of channels, the destination is connected to one transmitter on each channel, and the network has *K* hops in the longest multihop path.

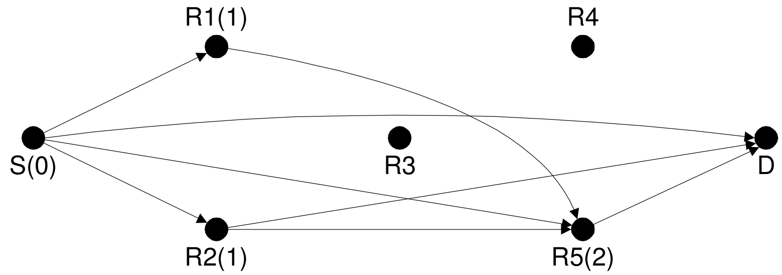

Fig. 71. Example NRKDKH Model with KCA

NRKDFH Connectivity Model: Each relay is connected to all previous transmitters that do not transmit or receive on an identical subset of channels, the destination is connected to one transmitter on each channel, and the network has *N* hops in the longest multihop path.

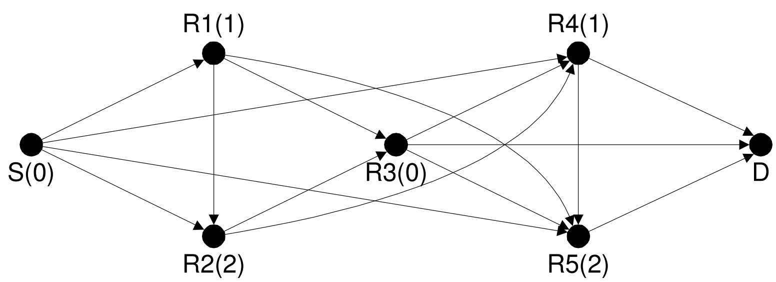

Fig. 72. Example NRKDFH Model with KCA

NRCDKH Connectivity Model: Each relay is connected to all previous transmitters that do not transmit or receive on an identical subset of channels, the destination is connected to a subset of transmitters on one channel, and the network has *K* hops in the longest multihop path.

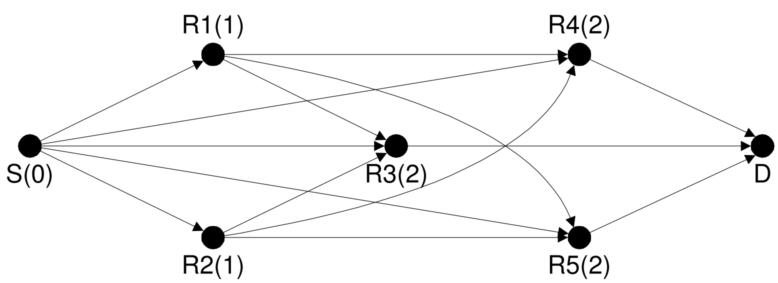

Fig. 73. Example NRCDKH Model with KCA

NRCDFH Connectivity Model: Each relay is connected to all previous transmitters that do not transmit or receive on an identical subset of channels, the destination is connected to a subset of transmitters on one channel, and the network has *N* hops in the longest multihop path.

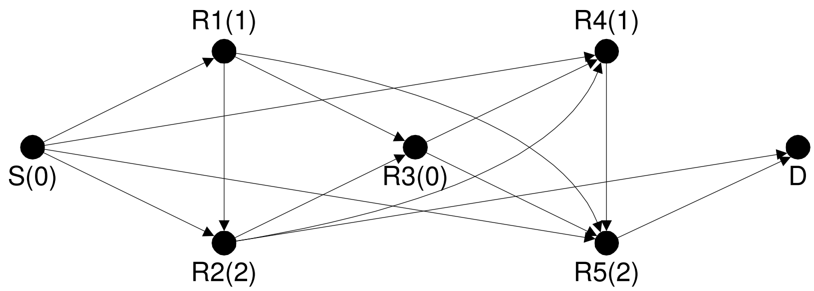

Fig. 74. Example NRCDFH Model with KCA

NRFDKH Connectivity Model: Each relay is connected to all previous transmitters that do not transmit or receive on an identical subset of channels, the destination is connected to all transmitters, and the network has *K* hops in the longest multihop path.

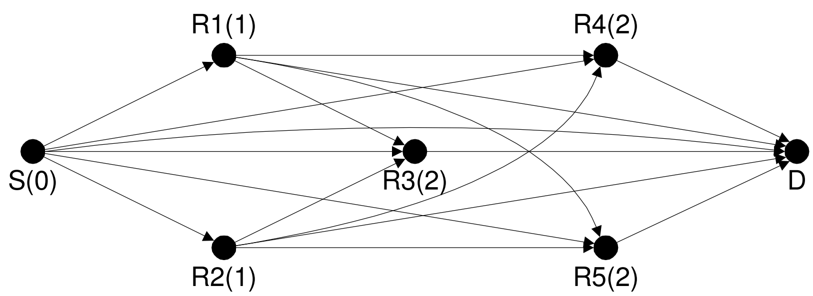

Fig. 75. Example NRFDKH Model with KCA

NRFDFH Connectivity Model: Each relay is connected to all previous transmitters that do not transmit or receive on an identical subset of channels, the destination is connected to all transmitters, and the network has *N* hops in the longest multihop path.

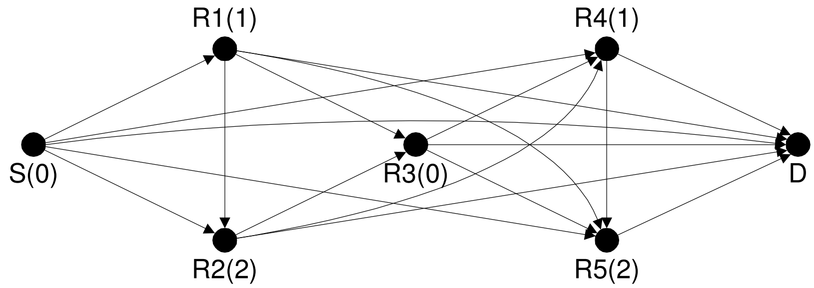

Fig. 76. Example NRFDFH Model with KCA

Appendix E – Connectivity Model Examples with 2CA

This appendix presents graphical examples illustrating the connectivity of each cooperative connectivity model with 2 channels available.

1R1D2H Connectivity Model: One relay is connected to the source and destination, and the network has *N* hops in the longest multihop path.

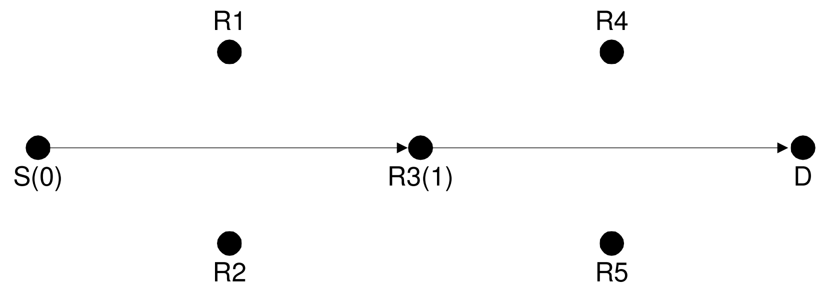

Fig. 77. Example 1R1D2H Model with 2CA

1R1DFH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to one transmitter, and the network has *N* hops in the longest multihop path.

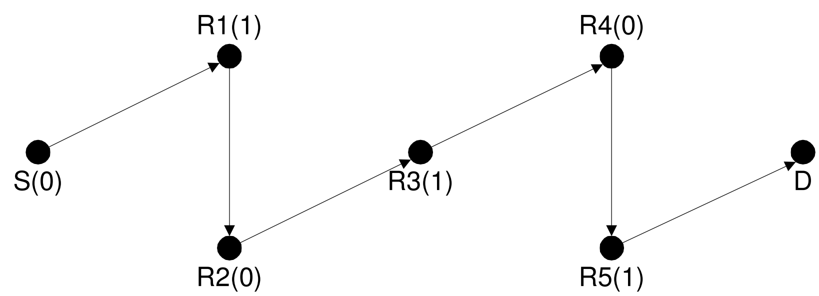

Fig. 78. Example 1R1DFH Model with 2CA

1R2D2H Connectivity Model: One relay is connected to the source, the destination is connected to the source and relay, and the network has 2 hops in the longest multihop path.

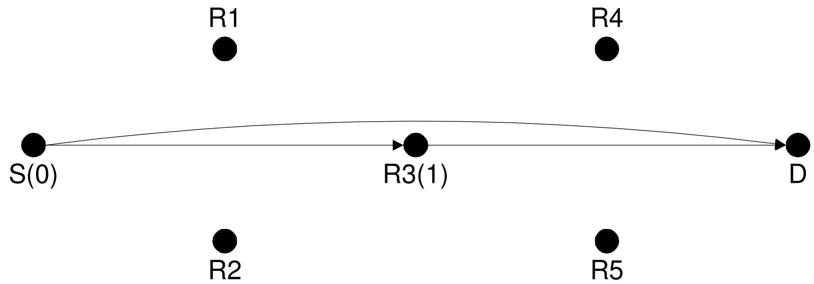

Fig. 79. Example 1R2D2H Model with 2CA

1R2DFH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to one transmitter on each channel, and the network has *N* hops in the longest multihop path.

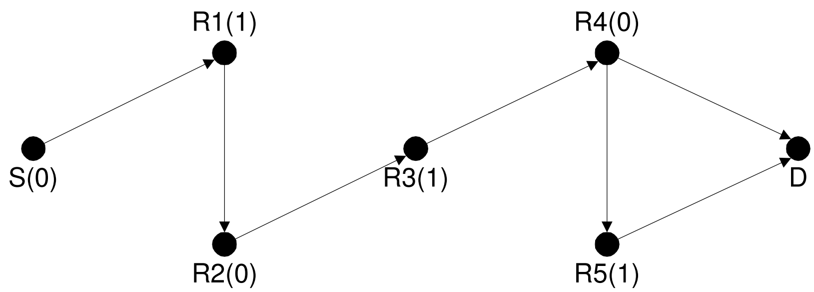

Fig. 80. Example 1R2DFH Model with 2CA

1RCD2H Connectivity Model: Each relay is connected to the source and destination, and the network has 2 hops in the longest multihop path.

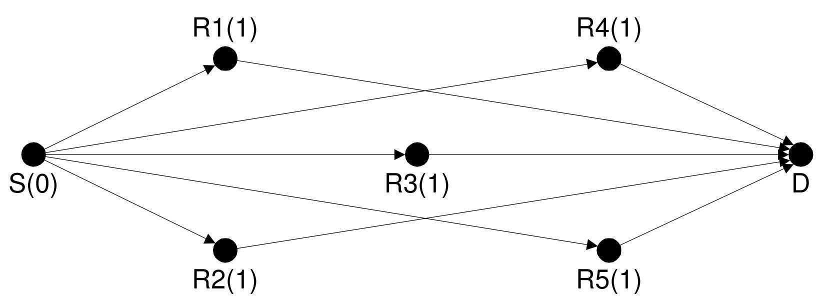

Fig. 81. Example 1RCD2H Model with 2CA

1RCDFH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to a subset of transmitters on one channel, and the network has *N* hops in the longest multihop path.

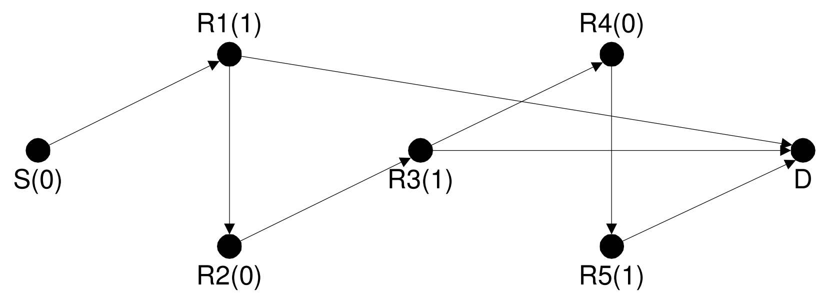

Fig. 82. Example 1RCDFH Model with 2CA

1RNDFH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to all previous transmitters that do not receive on an identical subset of channels, and the network has *N* hops in the longest multihop path.

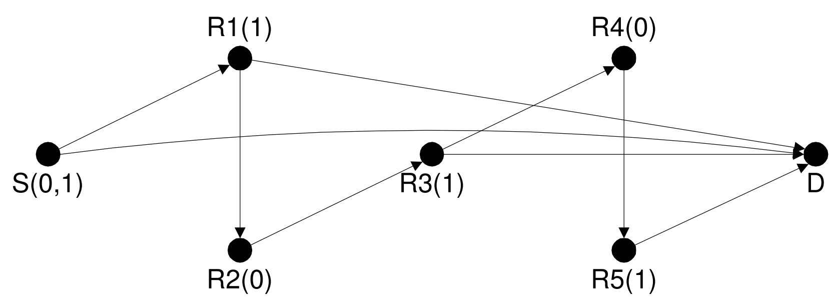

Fig. 83. Example 1RNDFH Model with 2CA

1RFD2H Connectivity Model: Each relay is connected to the source and destination, the source and destination are connected to each other, and the network has 2 hops in the longest multihop path.

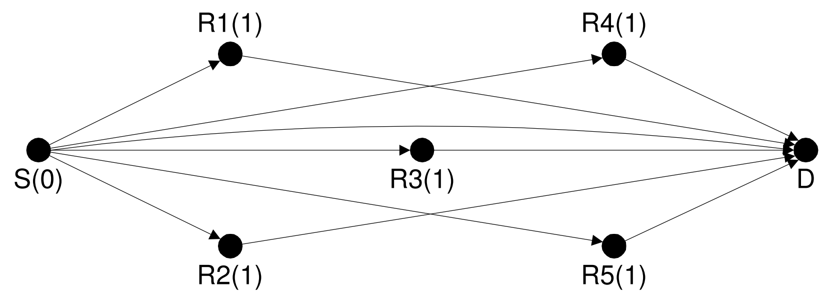

Fig. 84. Example 1RFD2H Model with 2CA

1RFDFH Connectivity Model: Each relay is connected to one transmitter, the destination is connected to all transmitters, and the network has *N* hops in the longest multihop path.

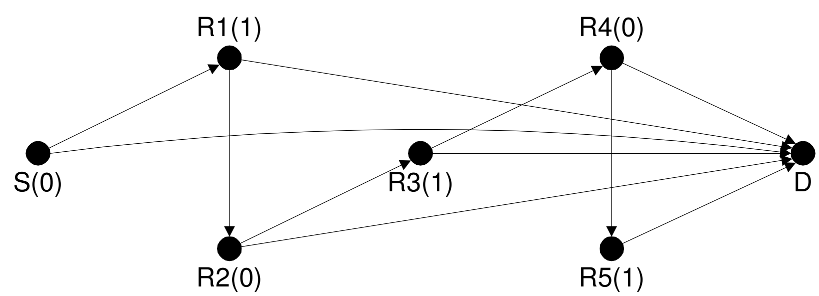

Fig. 85. Example 1RFDFH Model with 2CA

CR1DFH Connectivity Model: Each relay is connected to a subset of transmitters on one channel, the destination is connected to one transmitter, and the network has *N* hops in the longest multihop path.

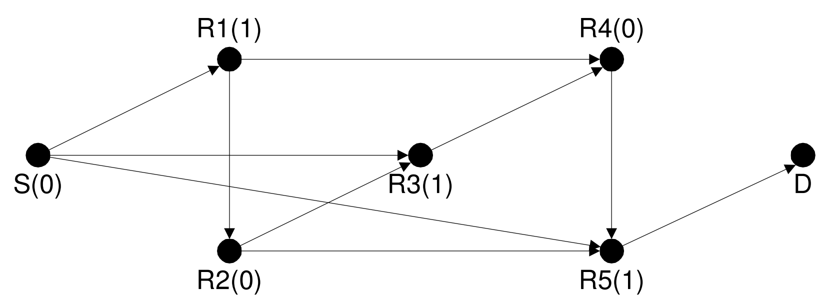

Fig. 86. Example CR1DFH Model with 2CA

CR2DFH Connectivity Model: Each relay is connected to a subset of transmitters on one channel, the destination is connected to one transmitter on each channel, and the network has *N* hops in the longest multihop path.

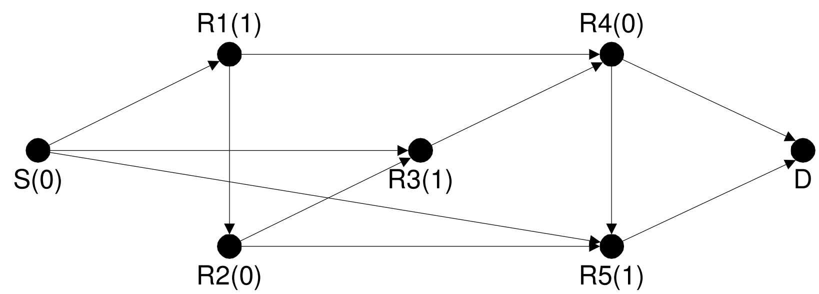

Fig. 87. Example CR2DFH Model with 2CA

CRCDFH Connectivity Model: Each relay is connected to a subset of transmitters on one channel, the destination is connected to a subset of transmitters on one channel, and the network has *N* hops in the longest multihop path.

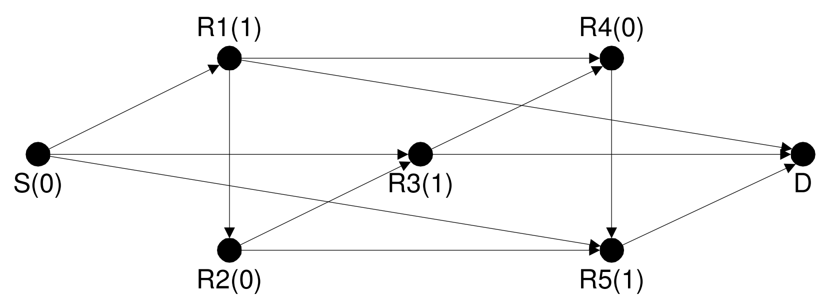

Fig. 88. Example CRCDFH Model with 2CA

CRFDFH Connectivity Model: Each relay is connected to a subset of transmitters on one channel, the destination is connected to all transmitters, and the network has *N* hops in the longest multihop path.

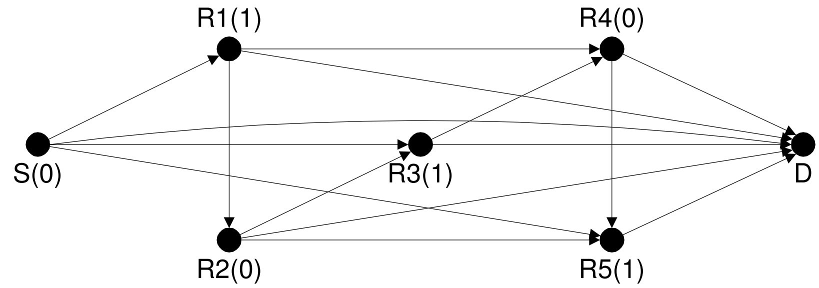

Fig. 89. Example CRFDFH Model with 2CA

NR1DFH Connectivity Model: Each relay is connected to all previous transmitters that do not transmit or receive on an identical subset of channels, the destination is connected to one transmitter, and the network has *N* hops in the longest multihop path.

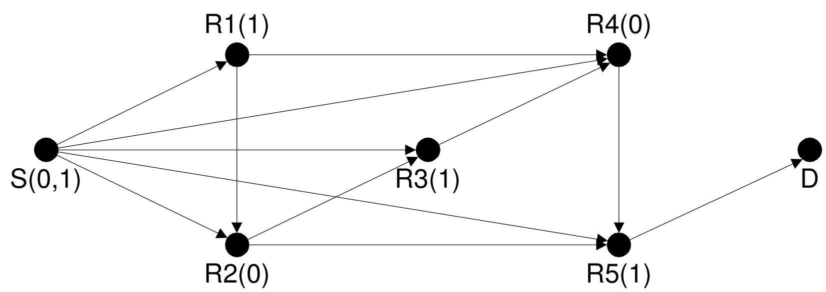

Fig. 90. Example NR1DFH Model with 2CA

NR2DFH Connectivity Model: Each relay is connected to all previous transmitters that do not transmit or receive on an identical subset of channels, the destination is connected to one transmitter on each channel, and the network has *N* hops in the longest multihop path.

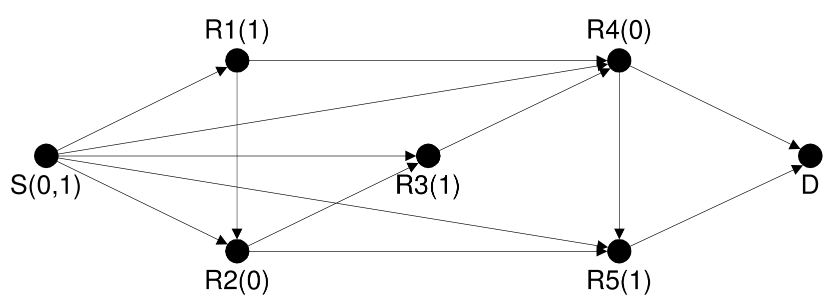

Fig. 91. Example NR2DFH Model with 2CA

NRNDFH Connectivity Model: Each relay is connected to all previous transmitters that do not transmit or receive on an identical subset of channels, the destination is connected to all previous transmitters that do not receive on an identical subset of channels, and the network has *N* hops in the longest multihop path.

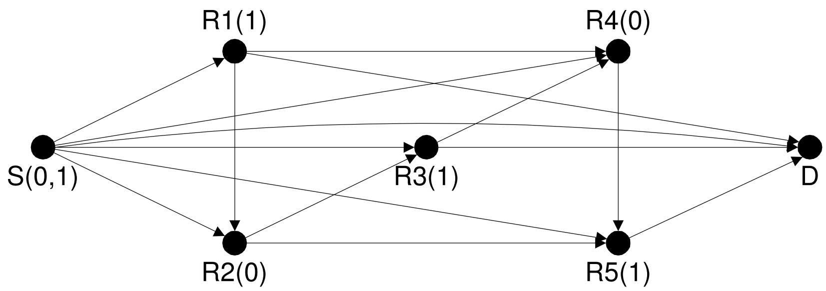

Fig. 92. Example NRNDFH Model with 2CA

NRFDFH Connectivity Model: Each relay is connected to all previous transmitters that do not transmit or receive on an identical subset of channels, the destination is connected to all transmitters, and the network has *N* hops in the longest multihop path.

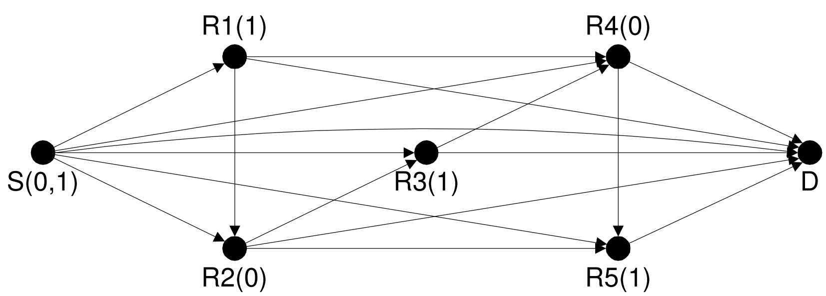

Fig. 93. Example NRFDFH Model with 2CA

Appendix F – Connectivity Model Examples with NCA

This appendix presents graphical examples illustrating the connectivity of each cooperative connectivity model with N channels available.

1R1DFH Connectivity Model: Each relay is connected to one transmitter and the destination is connected to one transmitter, and the network has N hops in the longest multihop path.

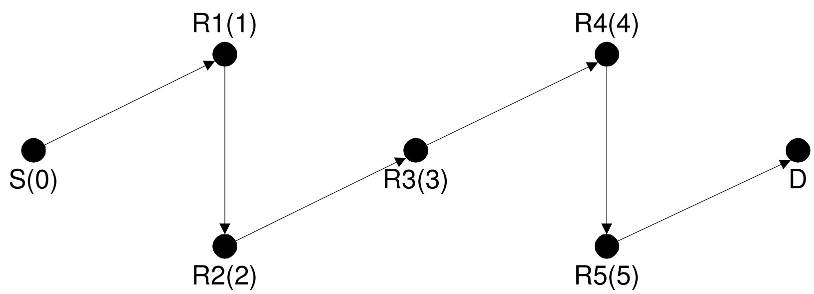

Fig. 94. Example 1R1DFH Model with NCA

1RFDFH Connectivity Model: Each relay is connected to one transmitter and the destination is connected to all transmitters, and the network has N hops in the longest multihop path.

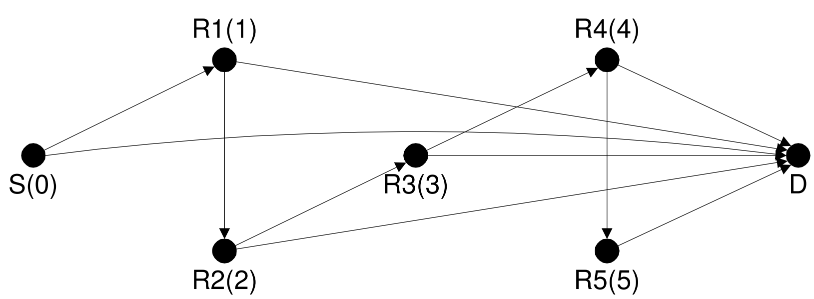

Fig. 95. Example 1RFDFH Model with NCA

FR1DFH Connectivity Model: Each relay is connected to all transmitters and the destination is connected to one transmitter, and the network has N hops in the longest multihop path.

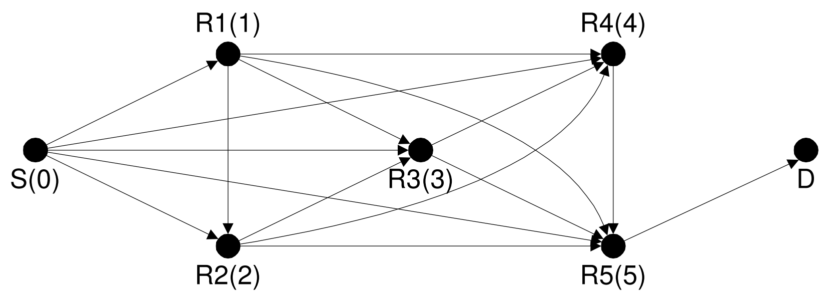

Fig. 96. Example FR1DFH Model with NCA

FRFDFH Connectivity Model: Each relay is connected to all transmitters previous along the transmission path and the destination is connected to all transmitters, and the network has *N* hops in the longest multihop path.

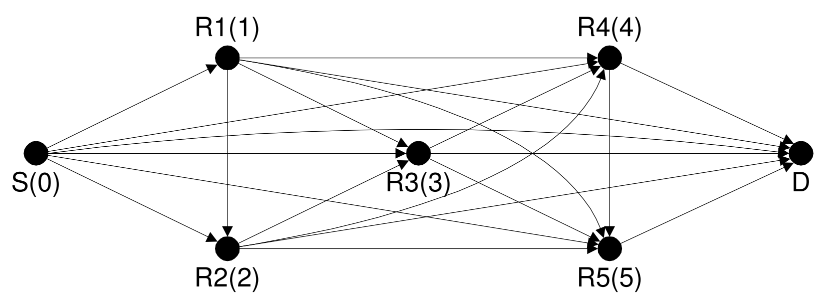

Fig. 97. Example FRFDFH Model with NCA