Optimizing the Monitoring Path Design for Independent Dual Failures

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***Abstract*—This paper proposes a new monitoring path design paradigm for independent dual link failures. Specifically, the new approach exploits the sequential arrival and departure property of independent failure events to uniquely localize failed links. Such property, however, cannot be captured by the existing approach, which is built upon the notion of shared risk link groups. Consequently, we show via solution space comparison that the existing approach can result in overdesign in terms of monitoring resources required. Numerical results further indicate that the new approach outperforms the existing one in terms of monitoring cost and computational efficiency.**

I. Introduction

Optical transport networks are evolving from traditional opaque networks towards all-optical transparent networks. In transparent optical networks, traffic data traverses a lightpath without being electronically processed at the intermediate nodes. While signal transparency offers a cost-efficient solution to high-bit-rate data transmission, limited accessibility to digital processing makes the optical performance monitoring an essential block in physical-layer network planning to ensure the manageability and the efficiency of real-time network operations and maintenance that follow [1]. Specifically, optical performance monitoring is the basis for fast failure detection and localization, which are crucial steps in the fault management process [2]. Accurate identification of failed elements mitigates the range and duration of failure impacts, reduces the operational cost induced by on-site physical repairs, and justifies the capital cost to deploy the optical performance monitors [3]. In particular, the faulty-element information is required to initiate the connection recovery processes for any failure-dependent/failure-specific recovery schemes, such as p-cycle, link protection, restoration, etc.

To this end, various monitoring structures have been proposed as candidates [4]–[9], with each monitoring entity (a cycle, a path, a trail, etc.) associated with the deployment of one optical monitor. Many of these works (e.g., [4]–[6]) focus on unique localization of single link failures. Under single failure assumptions, a new link failure occurs only after the existing (single) one is physically repaired. As transport networks continue to grow in size and complexity, multiple links can fail concurrently (i.e., with overlaps in residence time), making single-link-failure coverage not sufficient. Therefore, as a step forward, in this paper, we consider the localization problem of independent dual link failures. More precisely, we assume that fiber links fail independently, and at most two failed links exist in the network [10]. In this context, links fail one at a time: the second failure occurs long enough after the first one to allow the completion of normal recovery processes but can occur before any physical repair is accomplished to the first one [11].

Some recent works have studied the dual-failure localization problem in the form of shared risk link groups (SRLGs) [7]–[9]. An SRLG refers to a set of links that shares a common network resource (e.g., cable, duct, conduit, etc.), and thus shares a risk of failure. In the event of an SRLG failure, all links in the SRLG fail simultaneously (i.e., at the same time). The current approach to localize dual failures [7]–[9] is to include all possible two-link failures in the SRLG set in addition to all single-link failures. The dual-failure localization problem is thus converted into the problem of uniquely identifying all single SRLG failures as discussed in [7]–[9]. As the notion of SRLG is originally proposed to model deterministically correlated (i.e., simultaneous) failures, it, however, cannot capture the dynamics of uncorrelated failure events [12]. Specifically, the SRLG-based representation of independent failures cannot characterize the sequential arrival and departure behaviors of link failures, thus losing the time-domain information in failure modeling.

Motivated by the above observation, in this paper, we propose a new design paradigm by exploring the sequential arrival and departure property of independent failures. We show that although the SRLG-based approach can be used in this context, the design principle, however, is not optimized for failures with independent arrivals and departures. Specifically, we unravel the relationship between the proposed approach and the SRLG-based approach in terms of solution space. We show that given the same search space, the solution space of the proposed approach is normally larger than that of the SRLG-based approach. This indicates that our approach can lead to better solutions in terms of monitor allocation and monitoring length. Additionally, we show that the proposed design model reduces the number of constraints by an order of $|\mathcal{L}|$, where $|\mathcal{L}|$ is the number of network links. This indicates that the new design paradigm has better scalability and tractability. Note that although we use the monitoring path as the monitoring structure under study, the proposed design paradigm can be applied to any other monitoring

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structures, and can be extended to cover higher-order failures. Also, the design principle proposed is applicable to multiple independent SRLG failures.

The remainder of the paper is organized as follows. Section II presents the new paradigm for monitoring path design. The comparative study with the SRLG-based design is given in Section III. Section IV provides numerical comparisons. Section V concludes the paper.

II. New Monitoring Design Paradigm

A. Mathematical Formulation

A network topology is represented by an undirected graph $G = (\mathcal{N}, \mathcal{L})$, where $\mathcal{N}$ is the node set, and $\mathcal{L}$ is the link set. The links are numbered from 1 to $|\mathcal{L}|$.

To uniquely localize single link failures, each link has to be traversed by a unique (but not empty) set of monitoring paths. This also applies to the localization of the first failure when dual failures can occur in a network. When the first failure occurs, the monitoring paths that do not traverse the first failed link will remain operational. The basic idea to detect and localize a second failure is to ensure that each operating link after a first failure carries a unique (but not empty) set of the remaining working monitoring paths.

Based on the above principle, the new monitoring path design is translated into the following integer linear programming (ILP) model. Let $\mathcal{P}$ denote the pre-calculated candidate monitoring path set. The paths are numbered from 1 to $|\mathcal{P}|$. Let $\boldsymbol{\alpha}^r = \{\alpha_i^r\}_{1\times|\mathcal{L}|}$ be a binary row vector indicating the r-th path in set $\mathcal{P}$. Parameter α_i^r takes the value of one if path r traverses link i ($i \in \mathcal{L}$); zero otherwise. Let x^r denote a binary decision variable, which takes the value of one if path r is selected as one monitoring path; zero otherwise.

ILP-SequDual:

Objective:

$$\min \quad \zeta \sum_{r=1}^{|\mathcal{P}|} x^r + \sum_{r=1}^{|\mathcal{P}|} \sum_{i=1}^{|\mathcal{L}|} x^r \alpha_i^r \tag{1}$$

Subject to:

$$\sum_{r=1}^{|\mathcal{P}|} x^r \alpha_i^r \geq 1, \qquad \forall\, 1 \leq i \leq |\mathcal{L}| \tag{2}$$

$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(\alpha_i^r \oplus \alpha_{i'}^r\right) \geq 1, \qquad \forall\, 1 \leq i, i' \leq |\mathcal{L}|\,,\, i < i' \tag{3}$$

$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(1 - \alpha_i^r\right) \alpha_j^r \geq 1, \qquad \forall\, 1 \leq i, j \leq |\mathcal{L}|\,,\, i \neq j \tag{4}$$

$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(1 - \alpha_i^r\right) \left(\alpha_j^r \oplus \alpha_{j'}^r\right) \geq 1, \quad \forall\, 1 \leq i, j, j' \leq |\mathcal{L}|\,,\, i \neq j,\, i \neq j',\, j < j' \tag{5}$$

The objective in (1) is to minimize the total number of monitors deployed (primary objective) as well as to minimize the total monitoring-path hop length (secondary objective). The optimization priority is enabled by setting weight parameter $\zeta \gg 1$. Note that the total number of monitors deployed is equal to the total number of chosen monitoring paths. Constraint (2) ensures that each link carries at least one chosen path. Constraint (3) ensures that no two links carry the same set of chosen paths. Constraints (2) and (3) guarantee the detection and the unique localization of any possible first failures. Constraint (4) ensures that every operating link carries at least one remaining working monitoring path after the first failure on link i ($\forall\, i \in \mathcal{L}$). Constraint (5) ensures that no two operating links carry the same set of the remaining working monitoring paths after the first failure on link i ($\forall\, i \in \mathcal{L}$). Constraints (4) and (5) guarantee the detection and the unique localization of any possible second failures. Note that the XOR operation in (3) and (5) is calculated as

$$\alpha_l^r \oplus \alpha_{l'}^r = \alpha_l^r \left(1 - \alpha_{l'}^r\right) + \left(1 - \alpha_l^r\right) \alpha_{l'}^r, \quad \forall\, 1 \leq l, l' \leq |\mathcal{L}|\,,\, l \neq l'. \tag{6}$$

The design solution is composed of all paths with $x^r = 1$. These paths are renumbered sequentially according to their indices in set $\mathcal{P}$ as $\sum_{k=1}^{r-1} x^k$, $\forall\, x^r = 1$.

B. Alarm-State Transition Diagram

Based on the ILP design formulation, we propose an alarm-state transition diagram to capture the failure dynamics of a network.

Let state 0 be the normal state, where all the links are up. Let state i, $\forall\, i \in \mathcal{L}$ be the single-failure state, where link i is down. State i ($i > 0$) is specified by 2-tuple (n_i, u_i), where n_i is the total number of disrupted monitoring paths, calculated as $n_i = \sum_{r=1}^{|\mathcal{P}|} x^r \alpha_i^r > 0, \forall\, i \in \mathcal{L}$, and u_i is the decimal alarm code calculated according to the disrupted monitoring paths as $u_i = \sum_{r=1}^{|\mathcal{P}|} 2^{\sum_{k=1}^{r} x^k - 1} x^r \alpha_i^r, \forall\, i \in \mathcal{L}$. Transitions between state 0 and single-failure states are detected by the changes in the number of disrupted monitoring paths. The specific state/failed link i is identified by the decimal code u_i. Let state $j \mid i, \forall\, i, j \in \mathcal{L}, i \neq j$ be the dual-failure state, where link i and j are down, and link j fails after link i. State $j \mid i$ is specified by 2-tuple $\left(\Delta n_j^{(i)}, \Delta v_j^{(i)}\right)$, where $\Delta n_j^{(i)}$ is the total number of monitoring paths incrementally disrupted by link j, calculated as $\Delta n_j^{(i)} = \sum_{r=1}^{|\mathcal{P}|} x^r \left(1 - \alpha_i^r\right) \alpha_j^r > 0, \forall\, i, j \in \mathcal{L}, i \neq j$, and $\Delta v_j^{(i)}$ is the decimal alarm code calculated according to the incrementally disrupted monitoring paths as $\Delta v_j^{(i)} = \sum_{r=1}^{|\mathcal{P}|} 2^{\sum_{k=1}^{r} x^k - 1} x^r \left(1 - \alpha_i^r\right) \alpha_j^r, \forall\, i, j \in \mathcal{L}, i \neq j$. Transitions between single-failure state i and dual-failure state $j \mid i$ are detected by the changes in the number of disrupted monitoring paths. The second failed link j is identified by the incremental decimal alarm code $\Delta v_j^{(i)}$. Transitions from dual-failure state $j \mid i$ to single-failure state j are detected by the decrease in the number of disrupted monitoring paths, calculated as $n_i + \Delta n_j^{(i)} - n_j = \sum_{r=1}^{|\mathcal{P}|} x^r \alpha_i^r + \sum_{r=1}^{|\mathcal{P}|} x^r \left(1 - \alpha_i^r\right) \alpha_j^r - \sum_{r=1}^{|\mathcal{P}|} x^r \alpha_j^r = \sum_{r=1}^{|\mathcal{P}|} x^r \left(1 - \alpha_j^r\right) \alpha_i^r = \Delta n_i^{(j)} > 0, \forall\, i, j \in \mathcal{L}, i \neq j$. The specific state/failed link j is identified by the decimal code u_j, which is calculated according to the remaining disrupted monitoring paths in the network. The total number of states in the transition diagram is $1 + |\mathcal{L}| + |\mathcal{L}|\left(|\mathcal{L}| - 1\right)/2$.

As an illustrative example, consider the network topology in Fig. 1. The optimal design solution for the network is given in

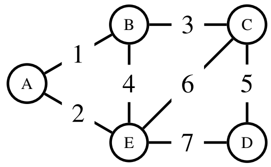

Fig. 1. Net 0 ($|\mathcal{N}| = 5, |\mathcal{L}| = 7$).

TABLE I
MONITORING PATH DESIGN SOLUTION TO DUAL FAILURES BASED ON THE PROPOSED APPROACH (NET 0 IN FIG. 1)

$\sum_{k=1}^{r} x^k$, $\forall x^r = 1$	$\alpha_i^r, \forall i =$ 1 2 3 4 5 6 7
1	0 1 0 0 1 0 1
2	1 0 1 0 0 1 0
3	0 0 1 0 1 0 0
4	0 0 0 1 0 0 0
5	1 1 0 0 0 0 0
6	0 0 0 0 0 1 1

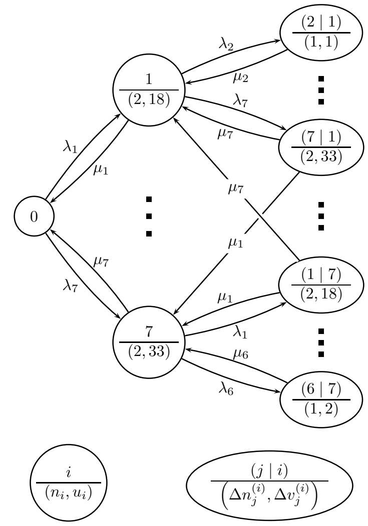

Fig. 2. Alarm-state transition diagram for Net 0 in Fig. 1 based on the proposed design approach.

TABLE II
NUMBER OF DISRUPTED AND INCREMENTALLY DISRUPTED MONITORING PATHS AT FAILURE ARRIVALS (NET 0 IN FIG. 1)

i	n_i	$\Delta n_j^{(i)}, \forall j =$ 1	2	3	4	5	6	7
1	2	-	1	1	1	2	1	2
2	2	1	-	2	1	1	2	1
3	2	1	2	-	1	1	1	2
4	1	2	2	2	-	2	2	2
5	2	2	1	1	1	-	2	1
6	2	1	2	1	1	2	-	1
7	2	2	1	2	1	1	1	-

TABLE III
DECIMAL ALARM CODE AND INCREMENTAL DECIMAL ALARM CODE TO IDENTIFY FAILED LINKS (NET 0 IN FIG. 1)

i	u_i	$\Delta v_j^{(i)}, \forall j =$ 1	2	3	4	5	6	7
1	18	-	1	4	8	5	32	33
2	17	2	-	6	8	4	34	32
3	6	16	17	-	8	1	32	33
4	8	18	17	6	-	5	34	33
5	5	18	16	2	8	-	34	32
6	34	16	17	4	8	5	-	1
7	33	18	16	6	8	4	2	-

Table I. Fig. 2 shows the corresponding alarm-state transition diagram, where λ_i and μ_i denote the mean failure rate and the mean repair rate of link i, respectively. Values for the complete set of parameters to construct the diagram is provided in Table II-III.

III. COMPARISON WITH THE SRLG-BASED APPROACH

In this section, we compare the proposed approach with the SRLG-based approach in terms of solution space. To this end, the SRLG-based design is presented first.

A. SRLG-Based Design

Let $\mathcal{F}$ denote the set of failure scenarios. The failure scenarios are numbered from 1 to $|\mathcal{F}|$. Let f be the failure scenario index. Each scenario corresponds to one SRLG failure. The complete SRLG information is captured by binary matrix $\boldsymbol{\Theta} = \{\boldsymbol{\theta}_f\}_{|\mathcal{F}|\times 1} = \{\theta_{fi}\}_{|\mathcal{F}|\times|\mathcal{L}|}$. The row vector $\boldsymbol{\theta}_f$ characterizes failure scenario f with binary element θ_{fi} equal to one if link i fails in scenario f; zero otherwise. The failure scenario matrix for a 7-link network as in Fig. 1 is illustrated in (10). The matrix enumerates all single-link and two-link failure scenarios.

The SRLG-based approach is formulated by model *ILP-SRLG*. Set $\mathcal{P}$, parameter α_i^r, and variable x^r are the same as *ILP-SequDual*. Additionally, let parameter β_f^r be a binary path-failure indicator for path r in set $\mathcal{P}$. The indicator takes the value of one if the corresponding path is disrupted in scenario f. Given θ_{fi} and α_i^r, the value of β_f^r is computed through

$$\beta_f^r = \begin{cases} \alpha_i^r & \text{if } (\theta_{fi} = 1) \wedge (\theta_{fk} = 0, \forall\, k \neq i)\,, \\ \alpha_i^r \sqcup \alpha_j^r & \text{if } (\theta_{fi} = 1) \wedge (\theta_{fj} = 1, \forall\, j \neq i) \\ & \quad \wedge (\theta_{fk} = 0, \forall\, k \neq i, j)\,, \end{cases}$$
$$\forall 1 \leq r \leq \mathcal{P}, 1 \leq f \leq \mathcal{F}, \quad (7)$$

where "$\sqcup$" is the binary disjunction operator.

ILP-SRLG:

Objective: (1)

Subject to:

$$\sum\nolimits_{r=1}^{|\mathcal{P}|} x^r \beta_f^r \geq 1, \qquad \forall 1 \leq f \leq |\mathcal{F}| \qquad (8)$$

$$\sum\nolimits_{r=1}^{|\mathcal{P}|} x^r \left(\beta_f^r \oplus \beta_{f'}^r\right) \geq 1, \quad \forall 1 \leq f, f' \leq |\mathcal{F}|\,,\, f < f' \quad (9)$$

Constraints (8) and (9) ensure the detection and the unique localization of all failure scenarios, respectively.

B. Comparison between Two Design Approaches

Theorem 1: Given the same search space (i.e., candidate path set $\mathcal{P}$), if failure scenario set $\mathcal{F}$ includes all single-link failures, i.e.,

$$\forall\, i{:}1 \leq i \leq |\mathcal{L}|\,, \exists f{:}\,(1 \leq f \leq |\mathcal{F}|) \wedge (\theta_{fi} = 1)$$
$$\wedge\, (\theta_{fk} = 0, \forall 1 \leq k \leq |\mathcal{L}|\,, k \neq i)\,, \quad (11)$$

and all two-link failures, i.e.,

$$\forall\, i, j{:}1 \leq i, j \leq |\mathcal{L}|\,, i \neq j,\, \exists f{:}\,(1 \leq f \leq |\mathcal{F}|) \wedge (\theta_{fi} = 1)$$
$$\wedge\, (\theta_{fj} = 1) \wedge (\theta_{fk} = 0, \forall 1 \leq k \leq |\mathcal{L}|\,, k \neq i, j)\,, \quad (12)$$

then a solution to *ILP-SRLG* is also a feasible solution to *ILP-SequDual*.

Proof: We prove that if (11) and (12) are satisfied, constraint set (2)-(5) for *ILP-SequDual* can be derived from constraints of *ILP-SRLG*.

First, (11) and case 1 in (7) guarantee that for every link i, there exists failure scenario f such that

$$\alpha_i^r = \beta_f^r, \qquad \forall 1 \leq r \leq |\mathcal{P}|. \qquad (13)$$

Consequently, we have

$$\sum\nolimits_{r=1}^{|\mathcal{P}|} x^r \beta_f^r \geq 1, \qquad \forall 1 \leq f \leq |\mathcal{F}|$$
$$\xRightarrow{(11),\ \text{case 1 in (7)}}$$
$$\sum\nolimits_{r=1}^{|\mathcal{P}|} x^r \alpha_i^r \geq 1, \qquad \forall 1 \leq i \leq |\mathcal{L}|\,, \quad (14)$$

TABLE IV
DECIMAL ALARM CODE BASED ON THE SOLUTION IN TABLE I FOR FAILURE SCENARIOS IN (10)

f	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
m_f	18	17	6	8	5	34	33	19	22	26	$\underline{23}$	50	$\utilde{51}$	$\underline{23}$	25	21	$\utilde{51}$	49	14	7	38	$\underline{\underline{\underline{39}}}$	13	42	41	$\underline{\underline{\underline{39}}}$	37	35

$$\boldsymbol{\Theta}^T = \begin{array}{l} \left(\begin{array}{cccccccccccccccccccccccccccc} 1&0&0&0&0&0&0&1&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&1&0&0&0&0&0&1&1&1&1&1&0&0&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&1&1&1&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0&0&1&0&0&0&0&1&0&0&0&1&0&0&0&1&1&1&0&0&0\\ 0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&1&0&0&0&1&0&0&1&0&0&1&1&0\\ 0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&1&0&0&0&1&0&0&1&0&1&0&1\\ 0&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&1&0&0&0&1&0&0&1&0&1&1 \end{array}\right) \begin{array}{c} 1\\2\\3\\4\\5\\6\\7 \end{array} \\ \quad\;\begin{array}{cccccccccccccccccccccccccccc} 1&2&3&4&5&6&7&8&9&10&11&12&13&14&15&16&17&18&19&20&21&22&23&24&25&26&27&28 \end{array} \end{array} \quad (10)$$

and

$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(\beta_f^r \oplus \beta_{f'}^r\right) \geq 1, \quad \forall 1 \leq f, f' \leq |\mathcal{F}|, \, f < f'$$
$$\xRightarrow{\text{(11), case 1 in (7)}}$$
$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(\alpha_i^r \oplus \alpha_{i'}^r\right) \geq 1, \quad \forall 1 \leq i, i' \leq |\mathcal{L}|, \, i < i'. \quad (15)$$

The derived inequalities in (14) and (15) correspond to constraints (2) and (3), respectively.

Next, (11), (12) and (7) ensure that for any link i and link $j \neq i$, there exist failure scenarios f and $f' \neq f$ such that

$$\alpha_i^r = \beta_f^r, \; \alpha_i^r \sqcup \alpha_j^r = \beta_{f'}^r, \qquad \forall 1 \leq r \leq |\mathcal{P}|. \quad (16)$$

Thus, we get

$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(\beta_f^r \oplus \beta_{f'}^r\right) \geq 1, \quad \forall 1 \leq f, f' \leq |\mathcal{F}|, \, f < f'$$
$$\xRightarrow{(11),(12),(7)}$$
$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(\alpha_i^r \oplus \left(\alpha_i^r \sqcup \alpha_j^r\right)\right) \geq 1, \qquad \forall 1 \leq i, j \leq |\mathcal{L}|, \, i \neq j. \quad (17)$$

Boolean expression $\alpha_i^r \oplus \left(\alpha_i^r \sqcup \alpha_j^r\right)$ in (17) can be tabulated as

α_i^r	$\alpha_i^r \oplus \left(\alpha_i^r \sqcup \alpha_j^r\right)$
1	0
0	α_j^r

It immediately follows that

$$\alpha_i^r \oplus \left(\alpha_i^r \sqcup \alpha_j^r\right) = \left(1 - \alpha_i^r\right) \alpha_j^r. \quad (18)$$

Introducing (18) in (17) yields

$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(\beta_f^r \oplus \beta_{f'}^r\right) \geq 1, \quad \forall 1 \leq f, f' \leq |\mathcal{F}|, \, f < f'$$
$$\xRightarrow{(11),(12),(7),(18)}$$
$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(1 - \alpha_i^r\right) \alpha_j^r \geq 1, \quad \forall 1 \leq i, j \leq |\mathcal{L}|, \, i \neq j. \quad (19)$$

(19) shows the satisfaction of constraint (4).

Last, (12) and case 2 in (7) guarantee that for any link i, link $j \neq i$ and link $j'\colon (j' \neq i) \wedge (j' \neq j)$, there exist failure scenarios f and $f' \neq f$ such that

$$\alpha_i^r \sqcup \alpha_j^r = \beta_f^r, \; \alpha_i^r \sqcup \alpha_{j'}^r = \beta_{f'}^r, \qquad \forall 1 \leq r \leq |\mathcal{P}|. \quad (20)$$

Accordingly, we have

$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(\beta_f^r \oplus \beta_{f'}^r\right) \geq 1, \quad \forall 1 \leq f, f' \leq |\mathcal{F}|, \, f < f'$$
$$\xRightarrow{\text{(12), case 2 in (7)}}$$
$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(\alpha_i^r \sqcup \alpha_j^r\right) \oplus \left(\alpha_i^r \sqcup \alpha_{j'}^r\right) \geq 1, \qquad \forall 1 \leq i, j, j' \leq |\mathcal{L}|, \, i \neq j, \, i \neq j', \, j < j'. \quad (21)$$

Boolean expression $\left(\alpha_i^r \sqcup \alpha_j^r\right) \oplus \left(\alpha_i^r \sqcup \alpha_{j'}^r\right)$ in (21) can be tabulated as follows.

α_i^r	$\left(\alpha_i^r \sqcup \alpha_j^r\right) \oplus \left(\alpha_i^r \sqcup \alpha_{j'}^r\right)$
1	0
0	$\alpha_j^r \oplus \alpha_{j'}^r$

Thus,

$$\left(\alpha_i^r \sqcup \alpha_j^r\right) \oplus \left(\alpha_i^r \sqcup \alpha_{j'}^r\right) = \left(1 - \alpha_i^r\right) \left(\alpha_j^r \oplus \alpha_{j'}^r\right). \quad (22)$$

Introducing (22) in (21) gives

$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(\beta_f^r \oplus \beta_{f'}^r\right) \geq 1, \quad \forall 1 \leq f, f' \leq |\mathcal{F}|, \, f < f'$$
$$\xRightarrow{\text{(12), case 2 in (7), (22)}}$$
$$\sum_{r=1}^{|\mathcal{P}|} x^r \left(1 - \alpha_i^r\right) \left(\alpha_j^r \oplus \alpha_{j'}^r\right) \geq 1, \qquad \forall 1 \leq i, j, j' \leq |\mathcal{L}|, \, i \neq j, \, i \neq j', \, j < j'. \quad (23)$$

(23) completes the proof by showing that constraint (5) in *ILP-SequDual* is also satisfied. ■

Theorem 1 states that given the same search space, the solution space of *ILP-SequDual* is no smaller than that of *ILP-SRLG*. It is more interesting to further ask whether the solution space can be larger as a larger solution space can indicate the existence of better solutions in terms of optimization goal.

As an example, we check the feasibility of the optimal design solution in Table I to model *ILP-SRLG*. Table IV gives the decimal code for each failure scenario in (10). The decimal code for scenario f is calculated as $m_f = \sum_{r=1}^{|\mathcal{P}|} 2^{\sum_{k=1}^{r} x^k - 1} x^r \beta_f^r, \forall 1 \leq f \leq |\mathcal{F}|$, where β_f^r is given by (7). It is easy to note that scenarios 11 and 14, scenarios 13 and 17, and scenarios 22 and 26 share the same codes, respectively. Consequently, failures within each pair cannot be distinguished. This leads to the following observation:

Observation 1: Given the same search space (i.e., candidate path set $\mathcal{P}$), solutions to *ILP-SequDual* can be infeasible to the

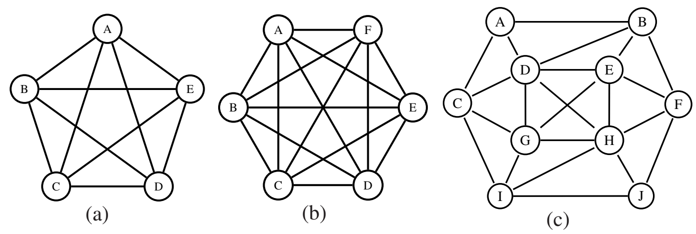

Fig. 3. Network topology (a) Net 1 ($|\mathcal{L}| = 10$); (b) Net 2 ($|\mathcal{L}| = 15$); (c) Net 3 ($|\mathcal{L}| = 22$).

ILP-SRLG counterpart, which considers (only) single-link and two-link failure scenarios.

Theorem 1 and Observation 1 indicate that the SRLG-based approach can lead to overdesign in terms of monitor amount and monitoring length for independent multiple failures.

IV. Numerical Results

ILP-based studies are conducted for the topology instances shown in Fig. 3. The candidate monitoring path set includes all possible paths for net 1 and 2. For net 3, 20 paths per node pair are pre-calculated by using the K shortest path algorithm based on the hop count metric. The choice of limited path set is to reduce the size of the search space, and hence the computational complexity of the model. Weight parameter ζ is set to 10000. Optimization instances are solved by CPLEX 11.0, which is installed on a computer with an Intel QuadCore processor (3.00 GHz) and 8 GB RAM. The run-time limit of net 1, 2, and 3 is 1 day, 1 day, and 5 days, respectively.

Table V summarizes the design solutions with both approaches. We find that the proposed approach requires fewer monitors and less monitoring length than the SRLG-based approach for all design instances. This accords with our discussions in Section III that the SRLG-based design principle is not optimized for independent failures while the proposed approach is better suited in this context. In a nutshell, this is because the proposed approach explores the time-domain information of failure events, more specifically, the sequential arrival and departure property of independent failures, to identify the exact failed links. This information, however, cannot be captured by the SRLG-based approach.

Table VI compares the size of the ILP models in terms of total number of variables and constraints. We observe that the proposed design approach has the same number of variables as the SRLG-based approach, but requires much fewer constraints than the SRLG-based approach. It is easy to find that the number of constraints produced by the proposed and the SRLG-based approach is $O(|\mathcal{L}|^3)$ and $O(|\mathcal{F}|^2) = O(|\mathcal{L}|^4)$, respectively. In other words, the proposed model reduces the number of constraints by $O(|\mathcal{L}|)$. The explicit benefit of this is that for net 1, the proposed model delivers the optimal solution in 959.2 seconds while the SRLG-based model cannot find the optimal solution within the 1-day time limit. This indicates that the new model is more flexible, scalable and tractable than the SRLG-based model to facilitate the monitoring path design.

TABLE V
Monitoring Path Design Solution Based on ILP

Net	$\|\mathcal{P}\|$	*ILP-SequDual*		*ILP-SRLG*	
		$\sum_r x^r$	$\sum_r \sum_i x^r \alpha_i^r$	$\sum_r x^r$	$\sum_r \sum_i x^r \alpha_i^r$
1	160	$7^\dagger$	$19^\dagger$	8	20
2	975	8	30	10	36
3	900	11	44	13	51

$\sum_r x^r$: Total number of monitors deployed. $\sum_r \sum_i x^r \alpha_i^r$: Total monitoring-path hop length. †: Optimal solution.

TABLE VI
Variable and Constraint Numbers of the ILP Models

Net	$\|\mathcal{P}\|$	*ILP-SequDual*		*ILP-SRLG*	
		Variables	Constraints	Variables	Constraints
1	160	160	505	160	1540
2	975	975	1695	975	7260
3	900	900	5335	900	32131

V. Conclusions

We proposed a new paradigm to design monitoring paths for the unique localization of independent dual link failures. In particular, the new design paradigm exploits the sequential arrival and departure property of failure events to optimize the design process. Such property, however, can not be explored under the existing SRLG-based design framework. We show via solution space comparison that as a result, the existing SRLG-based approach can lead to overdesign in terms of monitoring costs. Numerical results further indicate that the new design approach is more efficient than the SRLG-based approach in terms of the monitoring resources required as well as the computational complexity.

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