Conductivity Perturbations in EIT

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EIT Sensitivity . . .

What is sensitivity?

EIT Sensitivity . . .

EIT forward problem

$$v = F(\sigma)$$

If we are doing

- *difference imaging*,
- analyzing the sensitivity, or
- designing drive and measurement strategies

we want to know the difference signal.

$$\Delta v = F(\sigma + \Delta\sigma) - F(\sigma)$$

in a homogeneous (σ_h) medium, we want the sensitivity to perturbation for $\Delta\sigma$ in a ROI

$$\|\Delta v\| = F(\sigma_h + \Delta\sigma) - F(\sigma_h)$$

Two kinds of sensitivity!

Back in 1986 the first EIT meeting in Sheffield (Physiol Meas 1987 Suppl A)

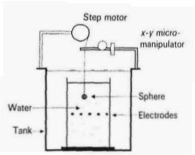


Figure 1. Principle of the *in vitro* study of electrode configurations

$$\frac{\partial Q_{mn}}{\partial \delta_k} = \int_D \delta_k \nabla \Phi_\sigma^{v_m} \cdot \nabla \Phi_\sigma^{v_n}$$

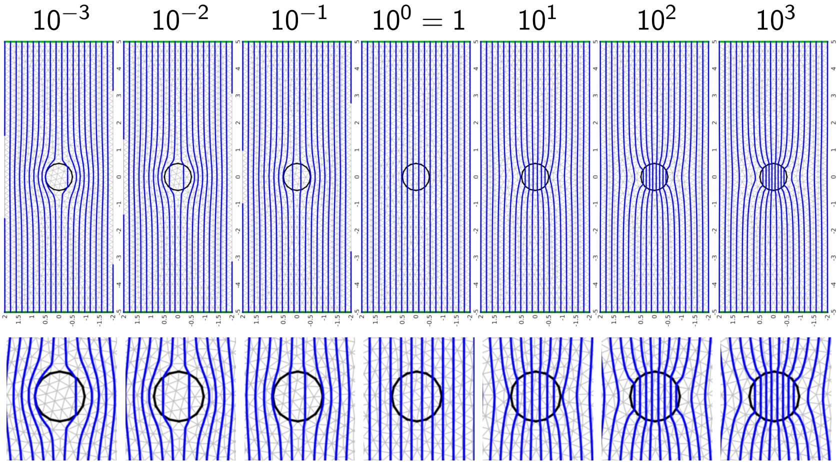
gives an expression for the Jacobian matrix of F.

- Jossinet and Kardous (pp33–37) showed an experimental determination of sensitivity using a ball and micro-manipulator while Seagar, Barber and Brown (pp13–31) gave the formulae for a offset circular anomaly
- Breckon and Pidcock (pp77–84) exhibited the Fréchet derivative formula. This is an 'infinitesimal' change, is limit as $\Delta\sigma$ tends to zero.

Sensitivity in 2D

Circular ball and the perturbation of streamlines

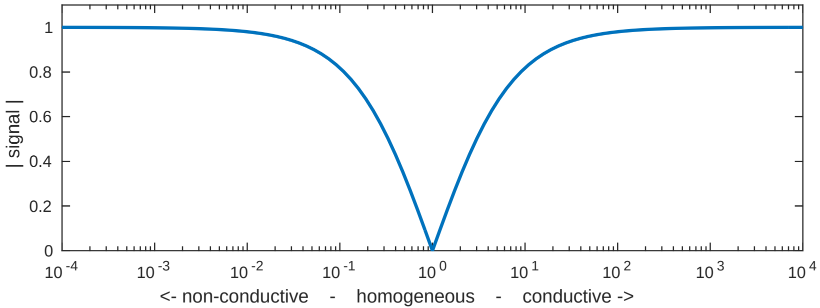
- Contrast $1 + \frac{\Delta\sigma}{\sigma_h}$



Sensitivity in 2D ...

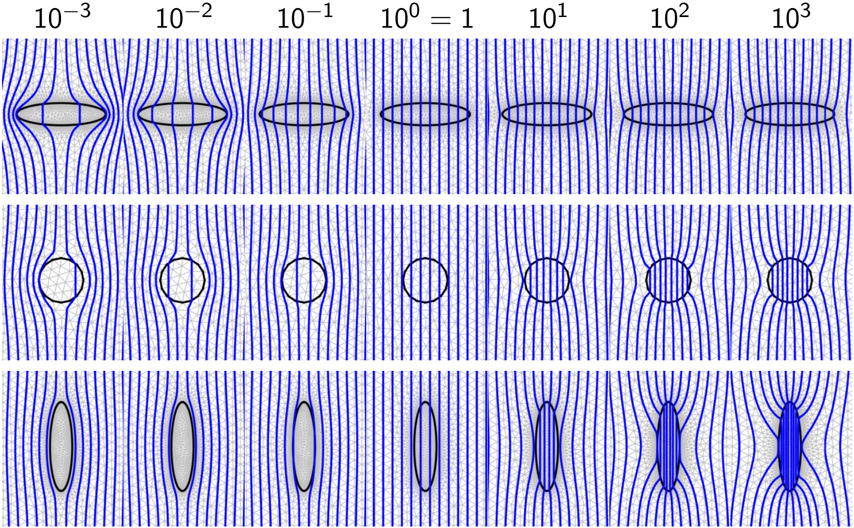
For a circle in 2D,

$$\text{Signal}: \ \|\Delta v\| \propto \frac{\sigma_h - \Delta\sigma}{\sigma_h + \Delta\sigma}$$



So, non-conductive contrasts are as easy to see as conductive ones?

Sensitivity in 2D: Non-circular shapes

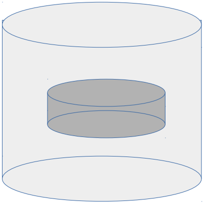


Sensitivity . . .

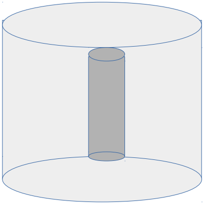
We see that

- Non-conductive object most visible when *against the streamlines*
- Conductive object most visible when *with the streamlines*

But in 3D EIT, there are two directions with, and one against.

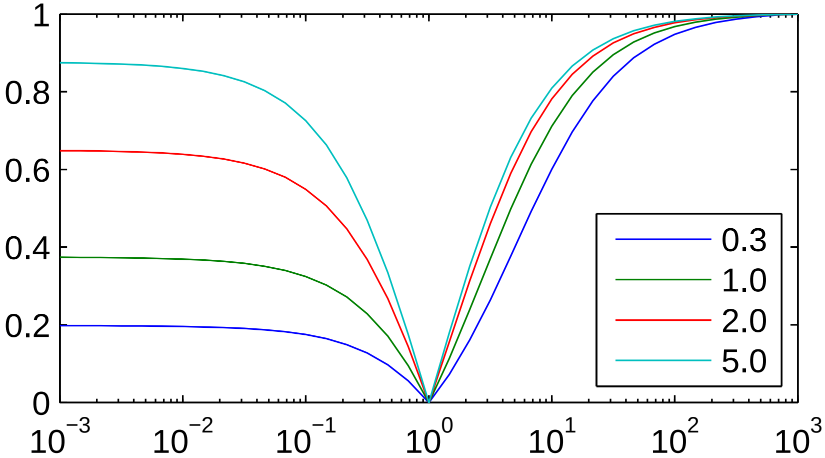


Conductive



Non-conductive

Sensitivity in 3D



Normalized signal vs. conductivity contrast, for different ratios of height to radius of ROI.

Polarization tensor

- The so called polarization or polarizability tensor of Pólya and Szegö gives an expression for the dipole moment of the perturbation in potential due to an object with a finite conductivity contrast.
- For an ellipsoid there is an explicit formula that includes the *saturation* so it is better than than the linear approximation in this respect
- It is a good approximation for well separated objects distant from the boundary, independent of boundary shape.

What does direction sensitivity tell us?

- Reconstruction algorithms have been based on linear sensitivity. Should we use non-linear sensitivites?
- Conductive contrast agents can be seen; no-one has ever succeeded with non-conductive contrasts. Does this explain why?