Parameter Estimation Techniques for Ultrasound Phase Reconstruction

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Presentation Outline

- Motivation
- Thesis Objectives
- Background
- Simulation
- Quadrature Phase Measurement Method
- Phase Reconstruction Process (GN algorithm)
  - Simulation Results
  - Experimental (Phantom) Results
- Conclusion and Future Work

Motivation

- In human body, the musculature has a great deal to do with the wellbeing and health of an individual
  - *Athletic injury*
  - *Muscular disease*

- Quickly diagnosing and thus, preventing muscular disorders would help both patients and medical practitioners

Thesis Objectives

- Accurately measure the phase information of ultrasonic received (RF) signals
  - Define an algorithm to reconstruct the less accurate phase outcomes

- Implement the algorithm on the simulated and measured ultrasonic signals captured from experimental phantoms

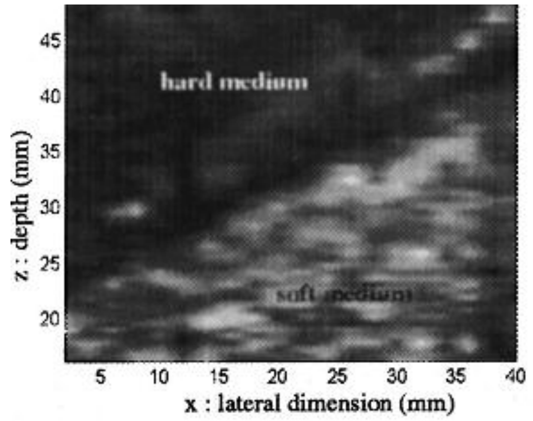
Background

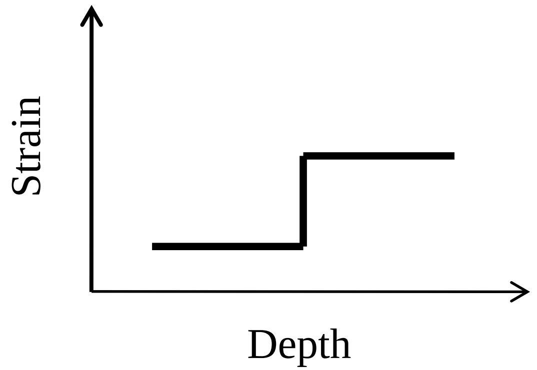
- Among several symptoms, stiffer muscle is a noticeable sign commonly found in different types of muscle disorders
  - *Muscle strain, muscle cramp, repetitive stress injuries*

- Elastography is an imaging technique that could measure tissues' stiffness

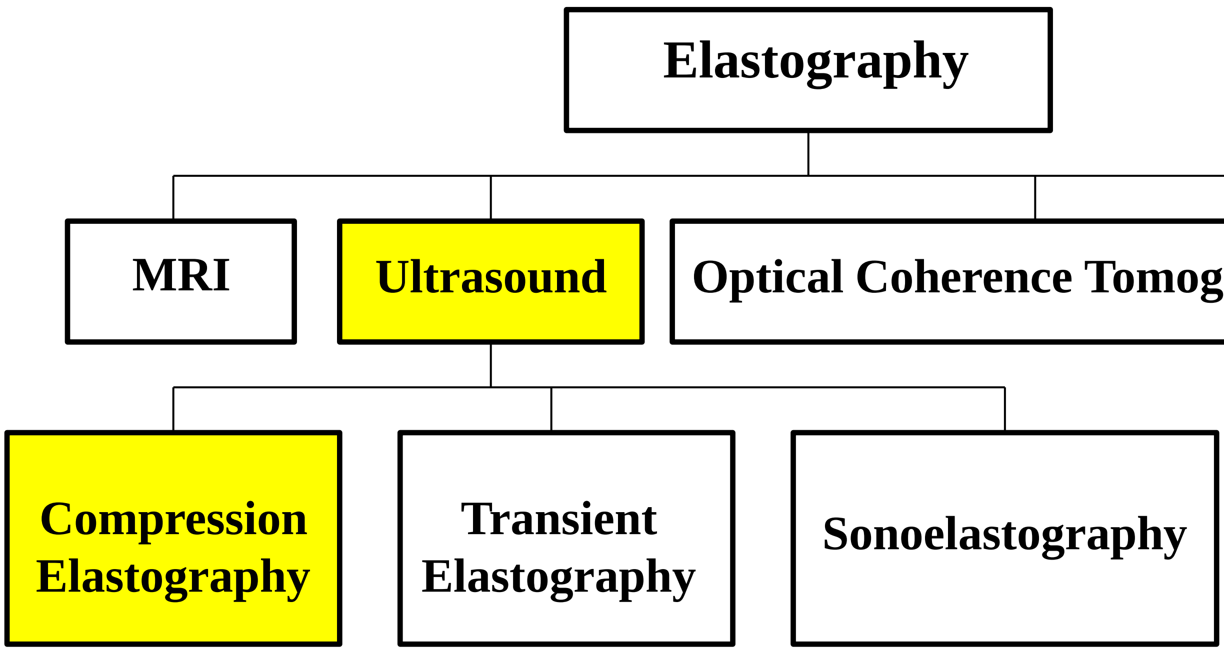
Elastography

- Elastography: a method in which stiffness or strain images of soft tissues are measured and used to detect or classify hard parts of the body such as tumors and injured muscles
- Strain: defined as the deformation of the tissue, normalized to its initial shape and is usually shown by “s”





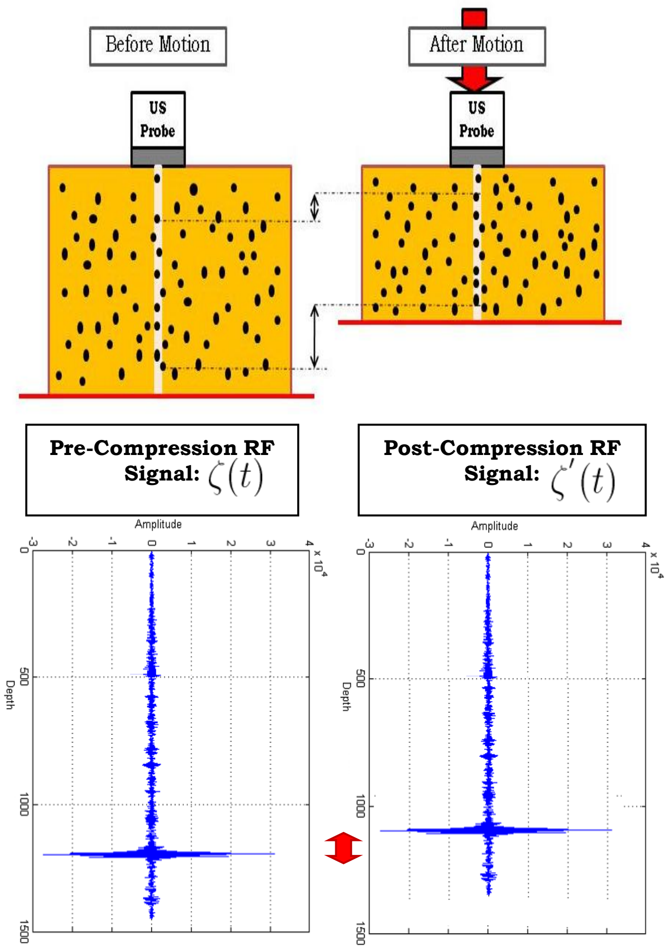
Current Imaging Techniques Used in Elastography



- Ultrasound-based elastography is more commonly used in clinical elasticity imaging

Ultrasound Compression Elastography

- Relies on radio-frequency (RF) ultrasonic signals
- Based on statically compressing the tissue
  -Particles' movements toward or away from the probe will cause speckle echoes to experience a shift in time



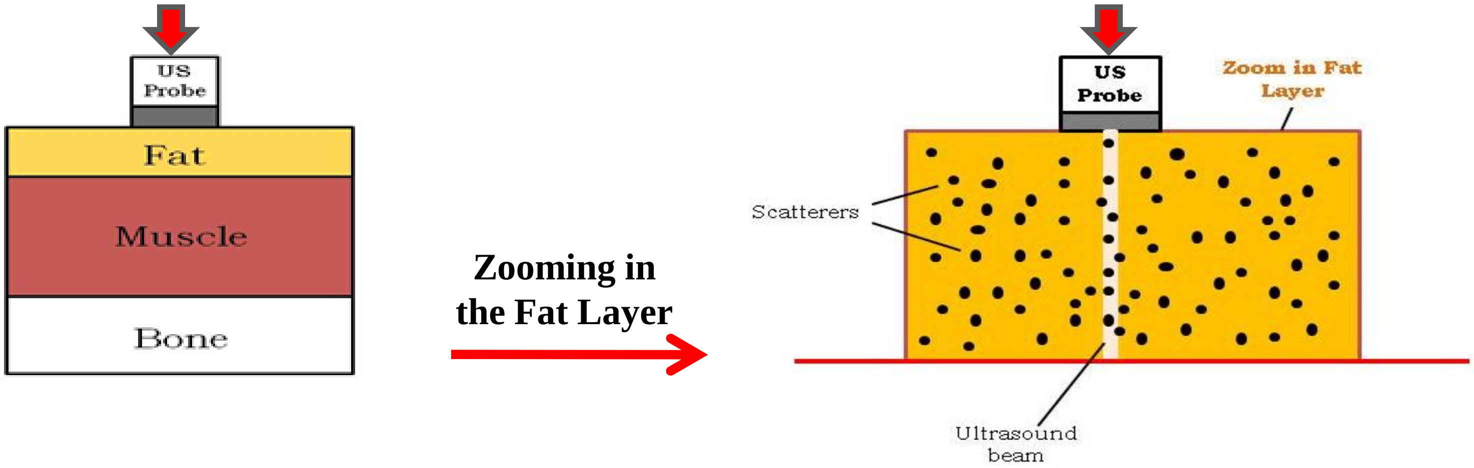
$$r(\hat{\tau}(t)) = \max_{\tau} \int_{-\infty}^{+\infty} \zeta(t_0+\tau)w(t_0-t+\tau).\zeta'(t_0)w(t_0-t)dt_0$$

$$\hat{d}(t) = \frac{c\hat{\tau}(t)}{2}$$

$$s = \frac{\partial \hat{d}(t)}{\partial z} = \frac{2}{c}\frac{\partial \hat{d}(t)}{\partial t}$$

Times shift $\longleftrightarrow$ Phase shift

Simulation



- Frame of reference: Ultrasonic probe reference frame
  - All displacements are relative to the surface of the probe
- Gaussian pulse-echo model
- Pre- and post-compression states
  - Pre-compression RF signal $$S_r(n) = \sum_{k=1}^{L} A_k(n)\cos(\omega_c n T_s - \phi_k(n))$$
  - Post-compression RF signal $$S_r'(n) = \sum_{k=1}^{L} A_k'(n)\cos(\omega_c n T_s - \phi_k'(n))$$

Quadrature Phase Detection Technique

- The phase information can be recovered by quadrature detection technique and used in estimation of displacement

Quadrature Detection

$$S_{ref}(n) = e^{-j\omega_{dem} n T_s}$$

$$S_r(n)$$

$$\otimes$$

LPF

$$u(n) = I(n) + jQ(n)$$

$$\angle u(n) = \varphi(n)$$

Displacement Estimation

$$\Delta\varphi(n) = \varphi_1(n) - \varphi_2(n)$$

$$\Delta d_z(n,m) = \frac{c\Delta\varphi_z(n,m)}{4\pi f_c} = \frac{c}{4\pi f_c}(\varphi_z(n,m+1) - \varphi_z(n,m))$$

$$d_z(n,m+1) = d_z(n,m) + \Delta d_z(n,m)$$

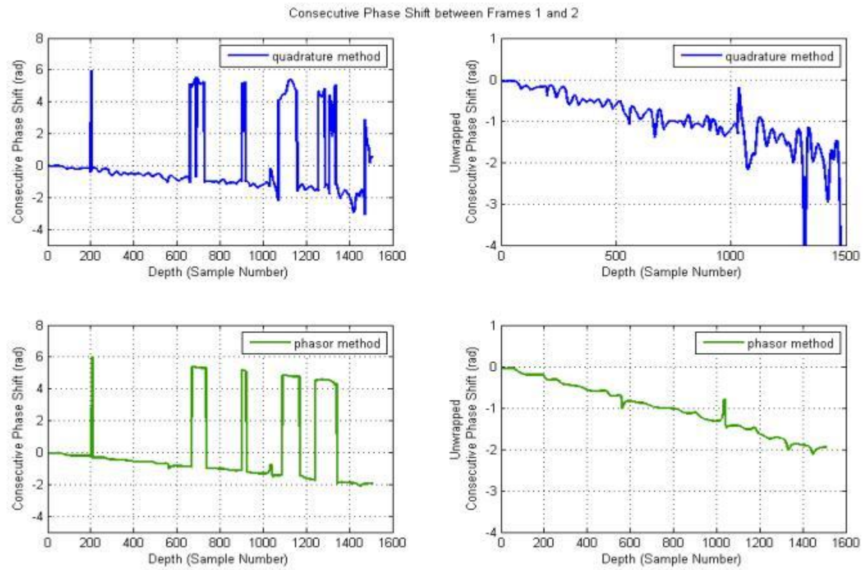
Quadrature Technique Performance

- This technique was compared with the phasor method (reference) in different conditions
  - The received RF signal is the sum of echoes (sinusoids) with the same frequency but different phasor parts
  - Phase of the received RF signal would be the angle of its phasor which is equal to the sum of the phasor parts of the constitutive echoes (*phasor addition theorem*)

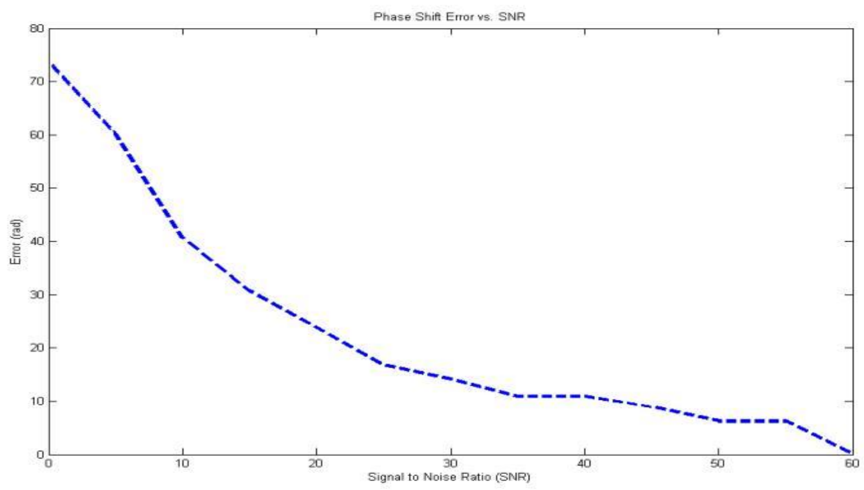
- Tested Parameters:
  - Signal to noise ratio (SNR) of the received RF signals
  - Bandwidth (B) of the received RF signals
  - Number of Scatterers (L)

Simulation Results

- SNR=3dB
  B=0.8 MHz
  L=111 (small)



- Phase shift error vs. the signal
  to noise ratio (SNR)
  - The error increases with decrease in SNR



Phase Reconstruction

- It is desired to find a technique/algorithm by which the less accurate phase outcomes of the quadrature technique can be reconstructed
  - *SNR parameter*

- Inverse problem techniques are able to somehow fix this problem

Inverse Problems

- The task where the values of some model parameters (m) must be obtained from the observed data (d)

$$\underbrace{\mathbf{G}(\mathbf{m}) = \mathbf{d}}_{\text{Mathematical model (system of equations)}} \ : \ \text{given d, m is aimed to be estimated}$$

- In case of having overdetermined/underdetermined systems of equations, least squares solution would be estimated

$$\underset{\mathbf{m}}{min} \quad \|\mathbf{d} - \mathbf{G}(\mathbf{m})\|_2 = \sqrt{\sum_{i=1}^{m} (d_i - \mathbf{G}(\mathbf{m})_i)^2}$$

Inverse Problems Cont.

- Inverse problems are often ill-posed (ill-conditioned)
  - Regularization techniques (Tikhonov)

$$\min_{\mathbf{m}} \quad \|\mathbf{G}(\mathbf{m}) - \mathbf{d}\|_2^2 + \mu^2 \|\mathbf{\Gamma m}\|_2^2$$

regularization parameter

- Nonlinear least squares problem can be solved by iterative algorithms such as Gauss-Newton (GN)

$$\begin{cases} (\mathbf{J}(\mathbf{m}^k)^T \mathbf{J}(\mathbf{m}^k) + \mu^2 \mathbf{\Gamma}^T \mathbf{\Gamma}) \Delta \mathbf{m} = -\mathbf{J}(\mathbf{m}^k)^T (\mathbf{G}(\mathbf{m}^k) - \mathbf{d}) - \mu^2 \mathbf{\Gamma}^T \mathbf{\Gamma} \mathbf{m}^k \\ \text{where} \quad \Delta \mathrm{m} = \mathrm{m}^{k+1} - \mathrm{m}^k \end{cases}$$

Inverse Problem Defined in This Work

- For each RF signal two mathematical models were defined

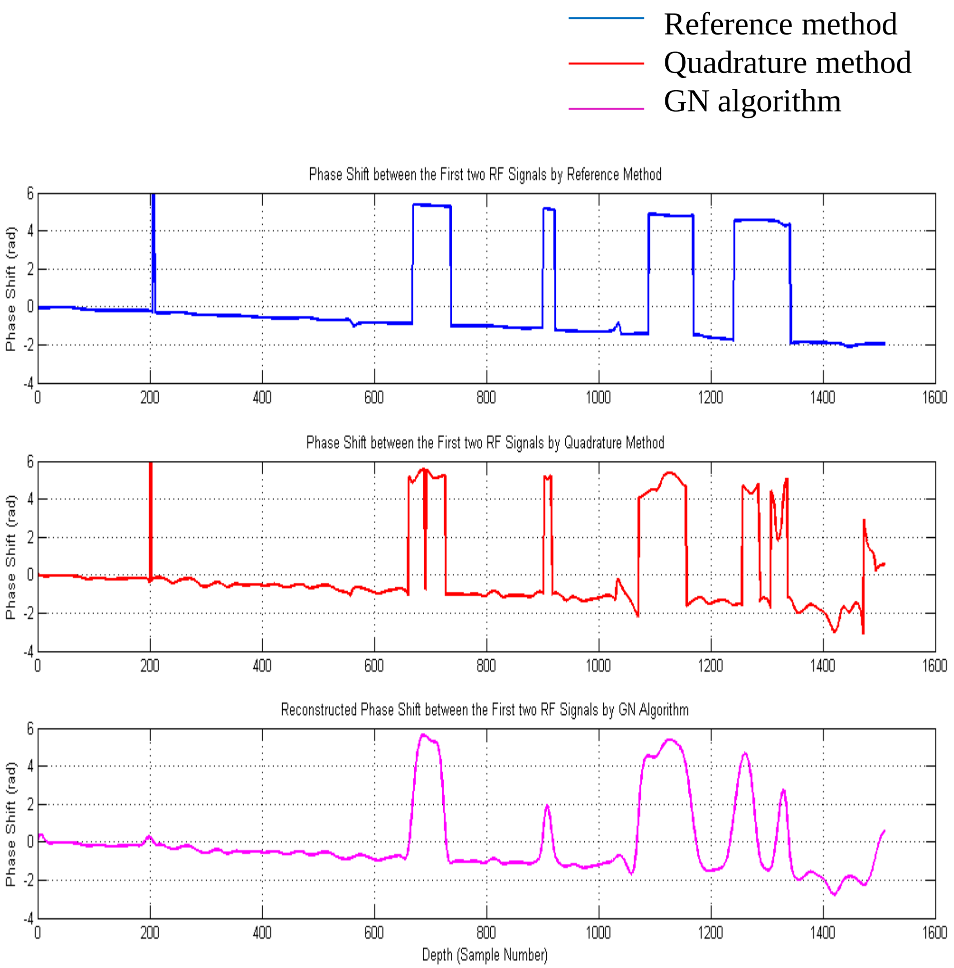
$$\begin{cases} \mathbf{G}_1(\mathbf{m}) = \mathbf{d}_1 \\ \mathbf{G}_2(\mathbf{m}) = \mathbf{d}_2 \end{cases}$$

  - d_1 and d_2 : in-phase and quadrature parts of the complex baseband signal obtained via quadrature method
  - m : vector of phase and amplitudes of the received RF signal at different depth sample numbers

- This problem is a nonlinear least squares problem which should be solved by the GN algorithm

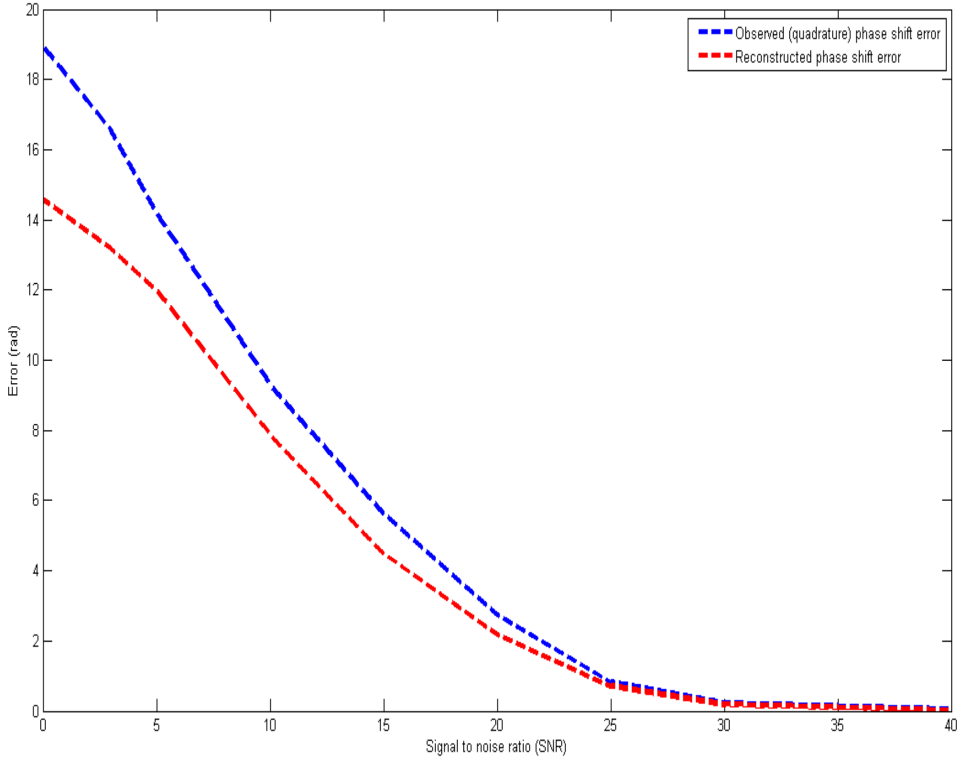
Phase Reconstruction by GN Algorithm

- Phase shift between the first two RF signals
- SNR= 3dB
- μ_1=24, μ_2=25
- Iteration numbers: 1, 8



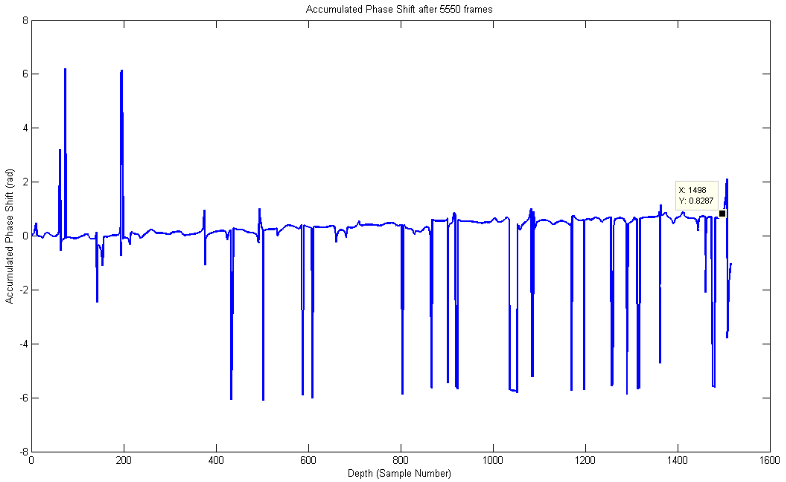
Error Reduction by GN Algorithm

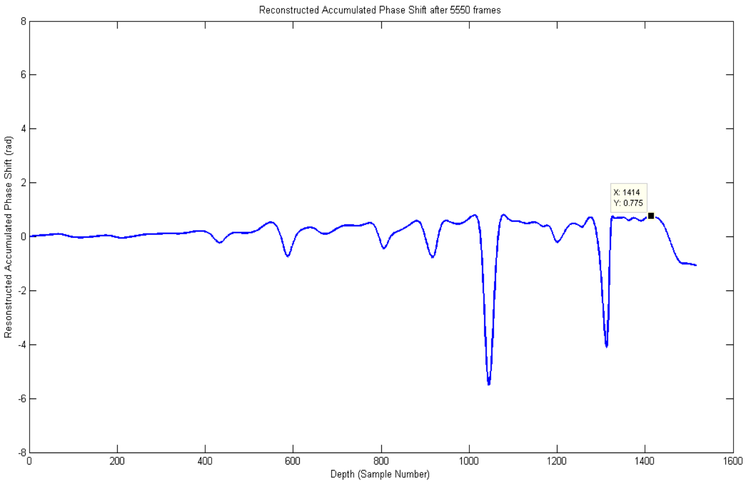
- Quadrature phase shift error (ε_1) and reconstructed phase shift error (ε_2) vs. the signal to noise ratio (SNR)



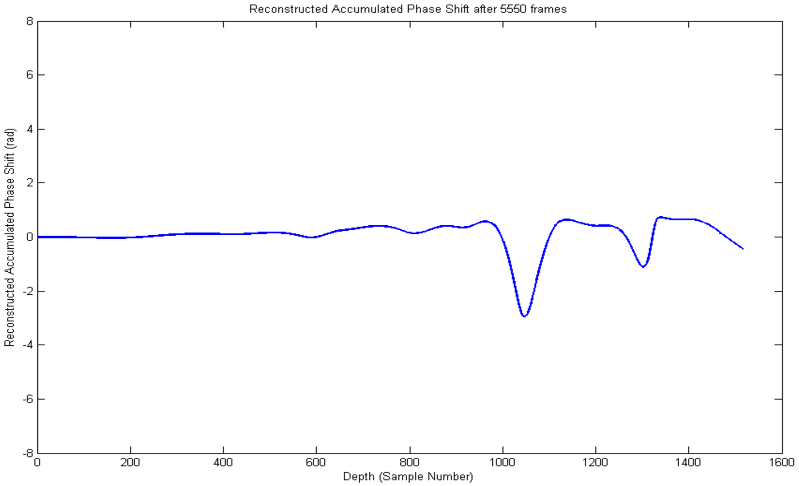
Phantoms and Results

- Accumulated phase shift after generation of 5550 RF signals (frames) in M-mode operation





Reconstructed accumulated phase shift, μ=100



Reconstructed accumulated phase shift, μ=1000

Conclusion and Future Work

Conclusion

- Defined a novel approach for ultrasound phase reconstruction by means of GN algorithm
  - The algorithm acted as a filter and was able to remove noise

- In designing the algorithm, optimized regularization parameters were selected
  - Extremely small or large values reduced the effectiveness of the algorithm

Future Works

- Finding a general approach for regularization parameter selection, improving the algorithm to test greater number of iterations and testing other parameters affecting the quadrature method are left as the future work

Thank You

Questions ?

Time or Phase Shift Measurement

- Tissue compression causes the speckle echoes and in turn, the received RF ultrasonic signals to experience a shift in time. This time shift changes the phase information of corresponding signals resulting in a phase shift between them.

- Resulted time or phase shift can be estimated and used for displacement estimation in one of the two approaches of:
  - Estimating the time shift between small windows by cross correlation function

$$\boxed{\hat{d}(t) = \frac{c\hat{\tau}(t)}{2}}$$

  - Estimating the phase shift between RF signals by means of a phase measurement technique such as quadrature phase detection technique

$$\boxed{\Delta d_z(n,m) = \frac{c\Delta\varphi_z(n,m)}{4\pi f_c}}$$

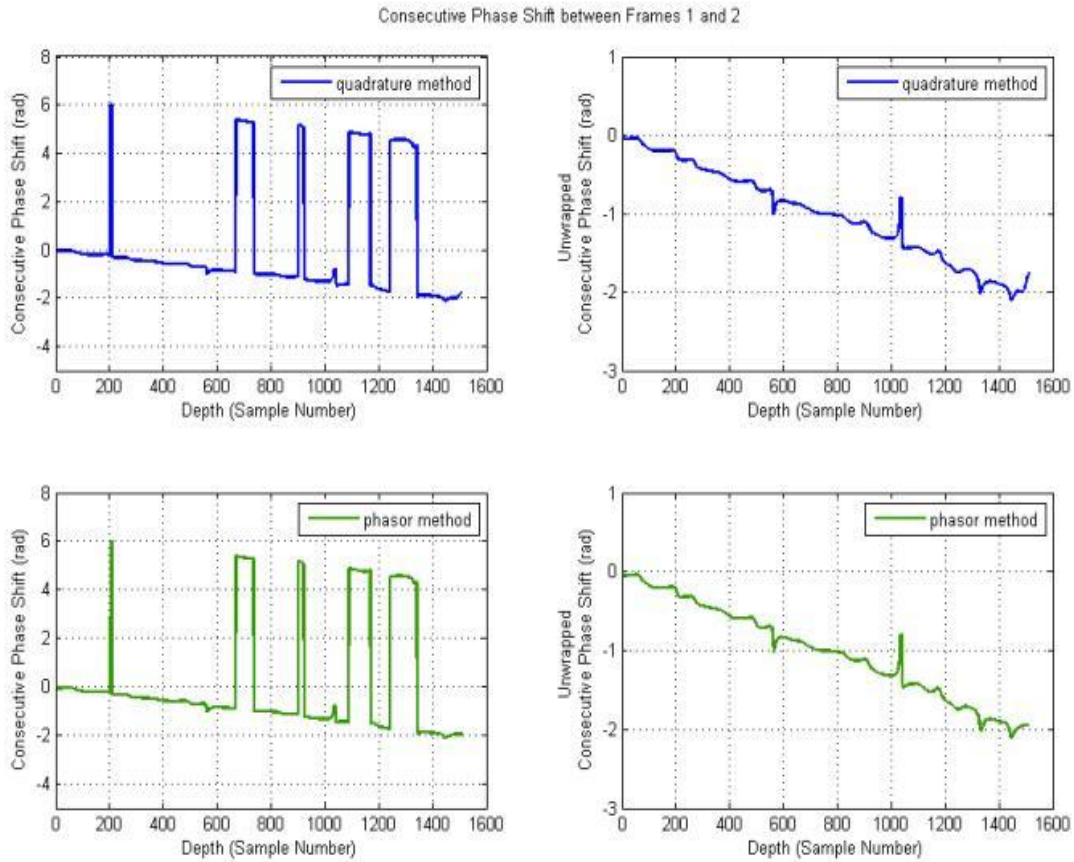
Simulation Results

- Initial setting:

  SNR=40dB

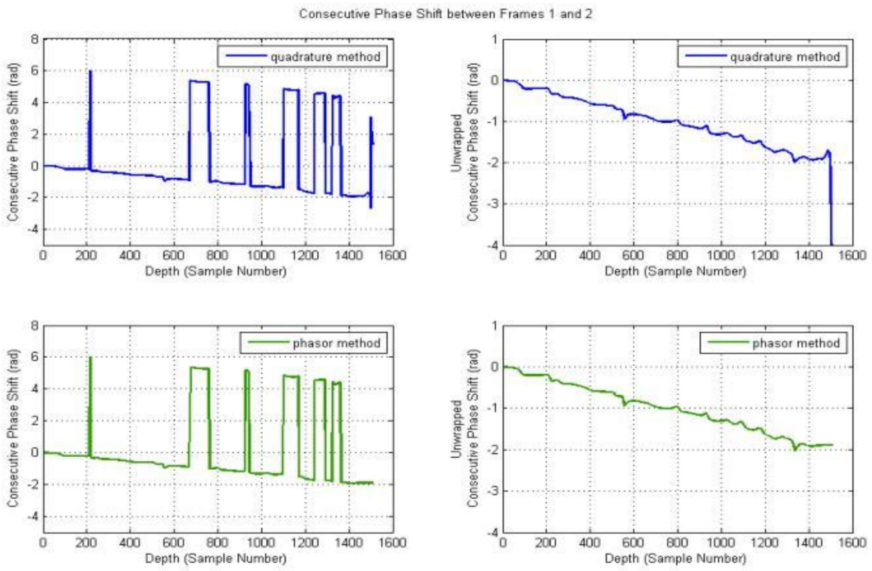
  B=0.8 MHz

  L=111 (small)

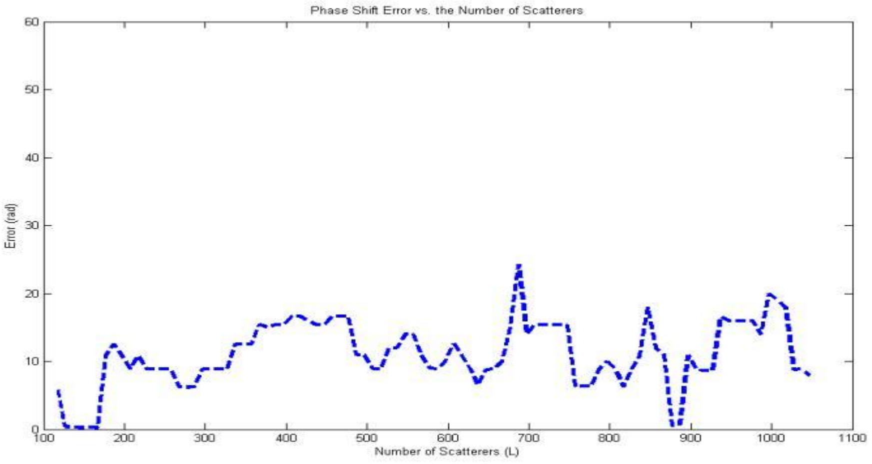


Simulation Results

- SNR=40dB
  B=0.8 MHz
  L=1055 (large)

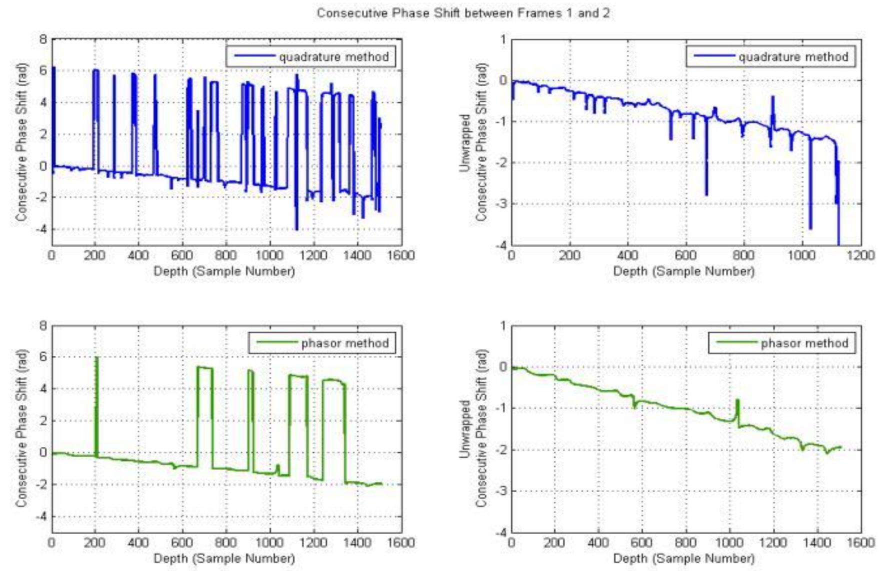


- Phase shift error vs. the number of scatterers (L)
  - By changing the number of scatterers, the error remains fix in a range between 0 to 15 rad.

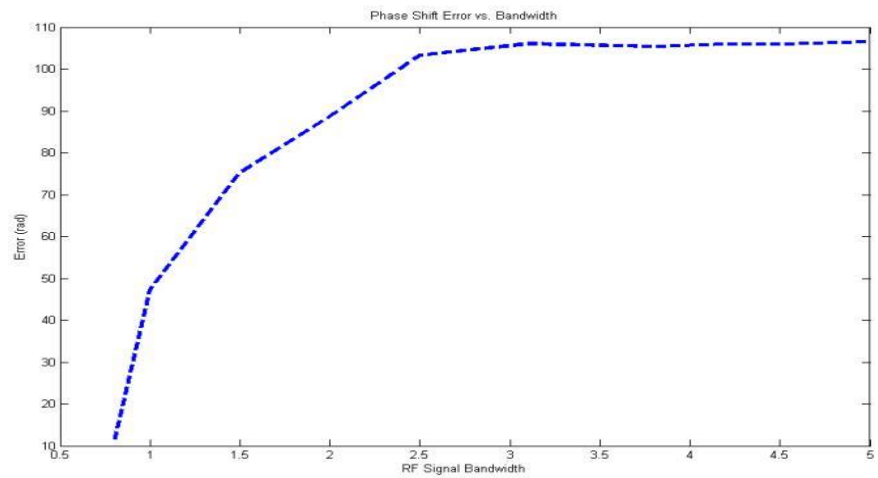


Simulation Results

- SNR=40dB
  B=2.25 MHz
  L=111 (small)

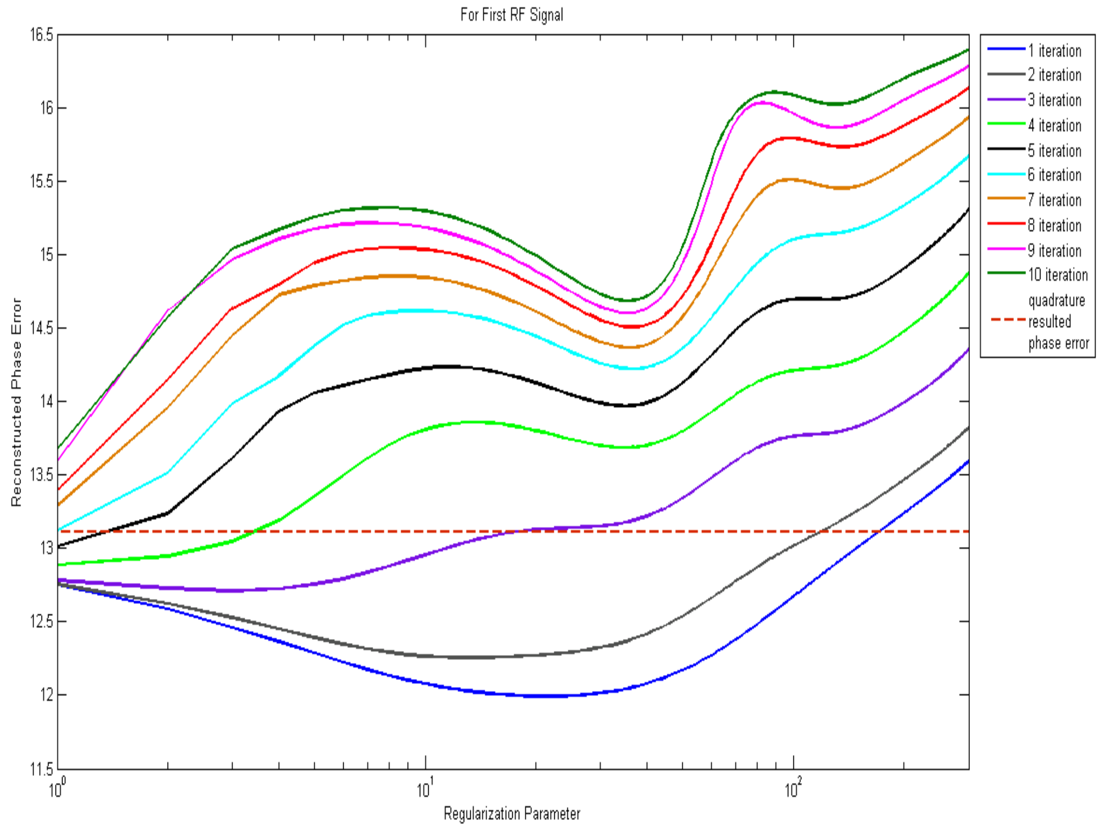


- Phase shift error vs. the RF signal's bandwidth (B)
  - As the bandwidth of echoes increases, the error increases



Regularization Parameter Selection

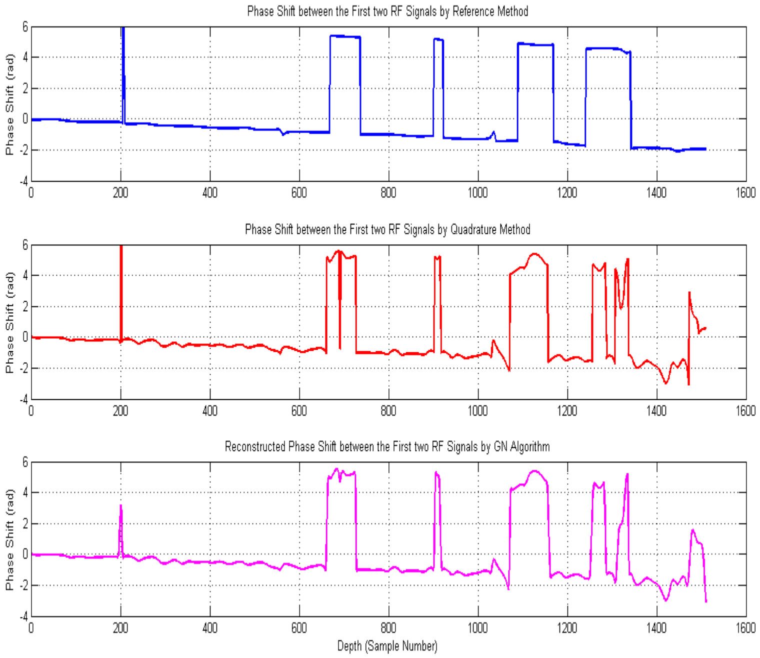
- First RF signal
- SNR= 3dB
- Reconstructed phase error (ε_2) vs. the regularization parameter and iteration number.



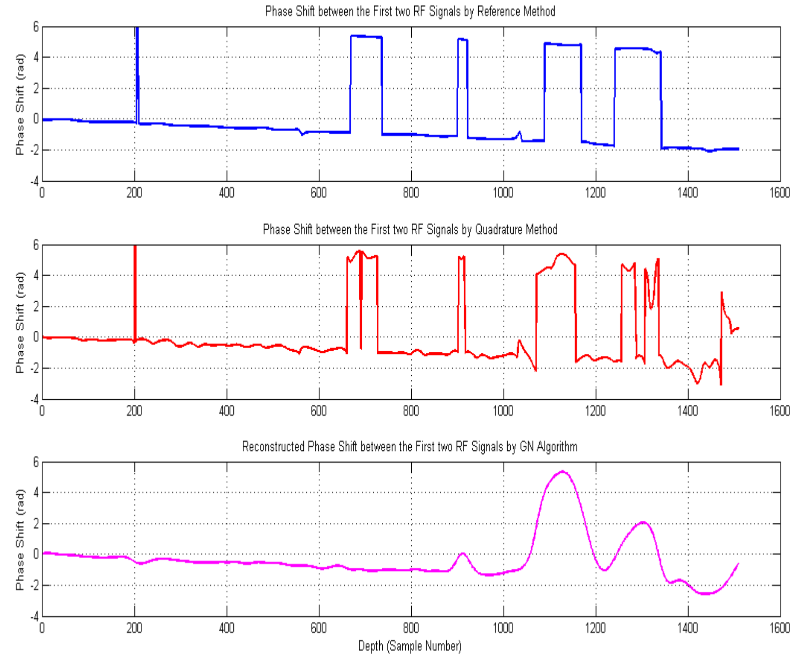
Regularization Parameter Effect

- Phase shift between the first two RF signals
- SNR= 3 dB

Reference method
Quadrature method
GN algorithm



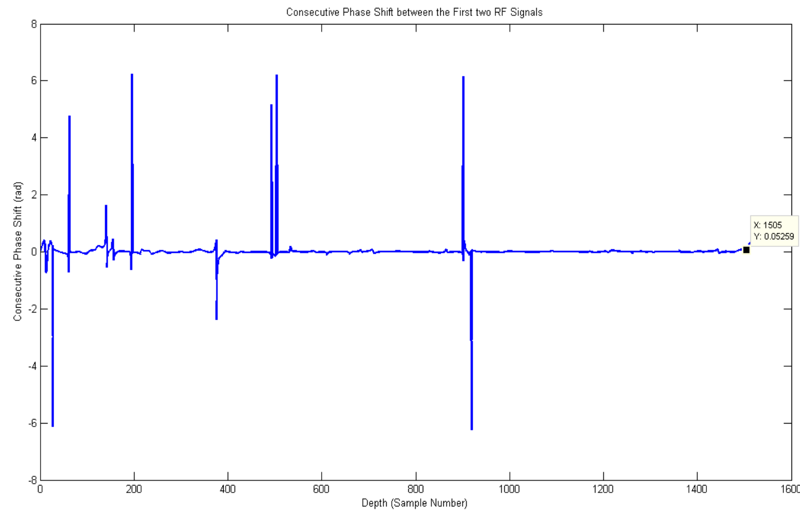
μ_1=1, μ_2=1

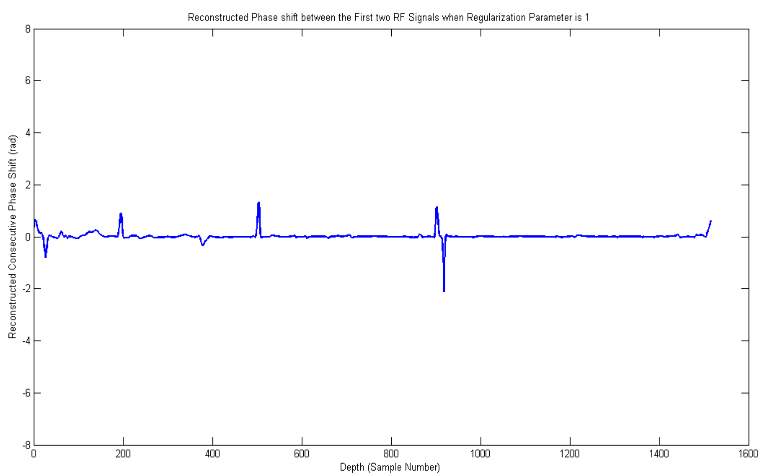


μ_1=166, μ_2=165

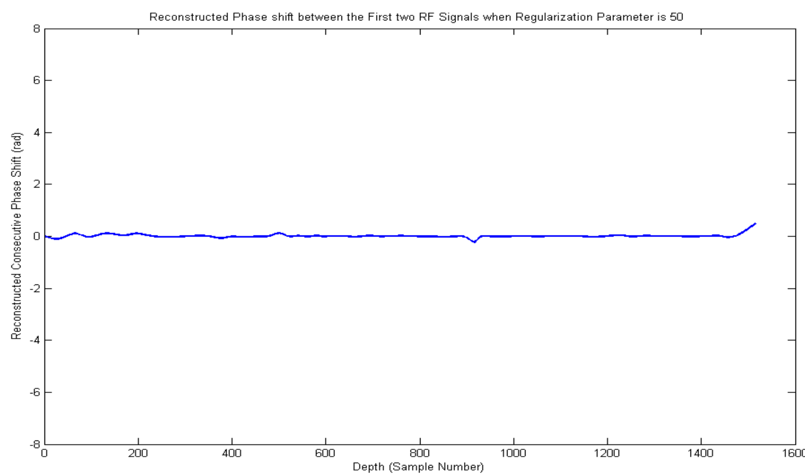
Phantoms and Results

- Phase shift between the first two consecutive RF signals (frames) in M-mode operation





Reconstructed accumulated phase shift, $\mu = 1$



Reconstructed accumulated phase shift, $\mu = 50$

Pulse-Echo Mathematical Representation

$$P(t) = \cos(\omega_c t)$$

Transmitted Pulse

$$S_k(\tau_k : t) = \alpha_k e^{-\beta(t-\tau_k)^2} \cdot P(t - \tau_k = \overbrace{\alpha_k e^{-\beta(t-\tau_k)^2}}^{A_k} \cdot \cos(\omega_c t - \overbrace{\omega_c \tau_k}^{\phi_k})$$

Echo Signal

$$S_r(n) = \sum_{k=1}^{L} A_k(n) \cos(\omega_c n T_s - \phi_k(n))$$

Received Signal

In-phase and Quadrature Signals

$$I(n) = LPF\{S_r(n).\cos(\omega_{dem} nT_s)\}$$

$$= LPF\{[\sum_{k=1}^{L} A_k(n)\cos(\omega_c nT_s - \phi_k(n))]\cos(\omega_{dem} nT_s)\}$$

$$= \sum_{k=1}^{L} \frac{A_k(n)}{2}\cos[\Delta\omega nT_s - \phi_k(n)] \longrightarrow$$ In-phase Signal

$$Q(n) = LPF\{S_r(n).(-\sin(\omega_{dem} nT_s))\}$$

$$= LPF\{[\sum_{k=1}^{L} A_k(n)\cos(\omega_c nT_s - \phi_k(n))](-\sin(\omega_{dem} nT_s))\}$$

$$= \sum_{k=1}^{L} \frac{A_k(n)}{2}\sin[\Delta\omega nT_s - \phi_k(n)] \longrightarrow$$ Quadrature Signal

$$\underbrace{u(n) = I(n) + jQ(n),}_{\text{Complex Baseband Signal}} \qquad \underset{\text{Phase Information}}{\angle u(n)} = \arctan\left(\frac{Q(n)}{I(n)}\right) = \arctan\left(\frac{\sum_{k=1}^{L} \frac{A_k(n)}{2}\sin[\Delta\omega nT_s - \phi_k(n)]}{\sum_{k=1}^{L} \frac{A_k(n)}{2}\cos[\Delta\omega nT_s - \phi_k(n)]}\right)$$

RF Signal Waveform Shape & Phase Signal