BIOM5200 – Medical Imaging

Electrical Impedance Tomography: Image Algorithms and Applications

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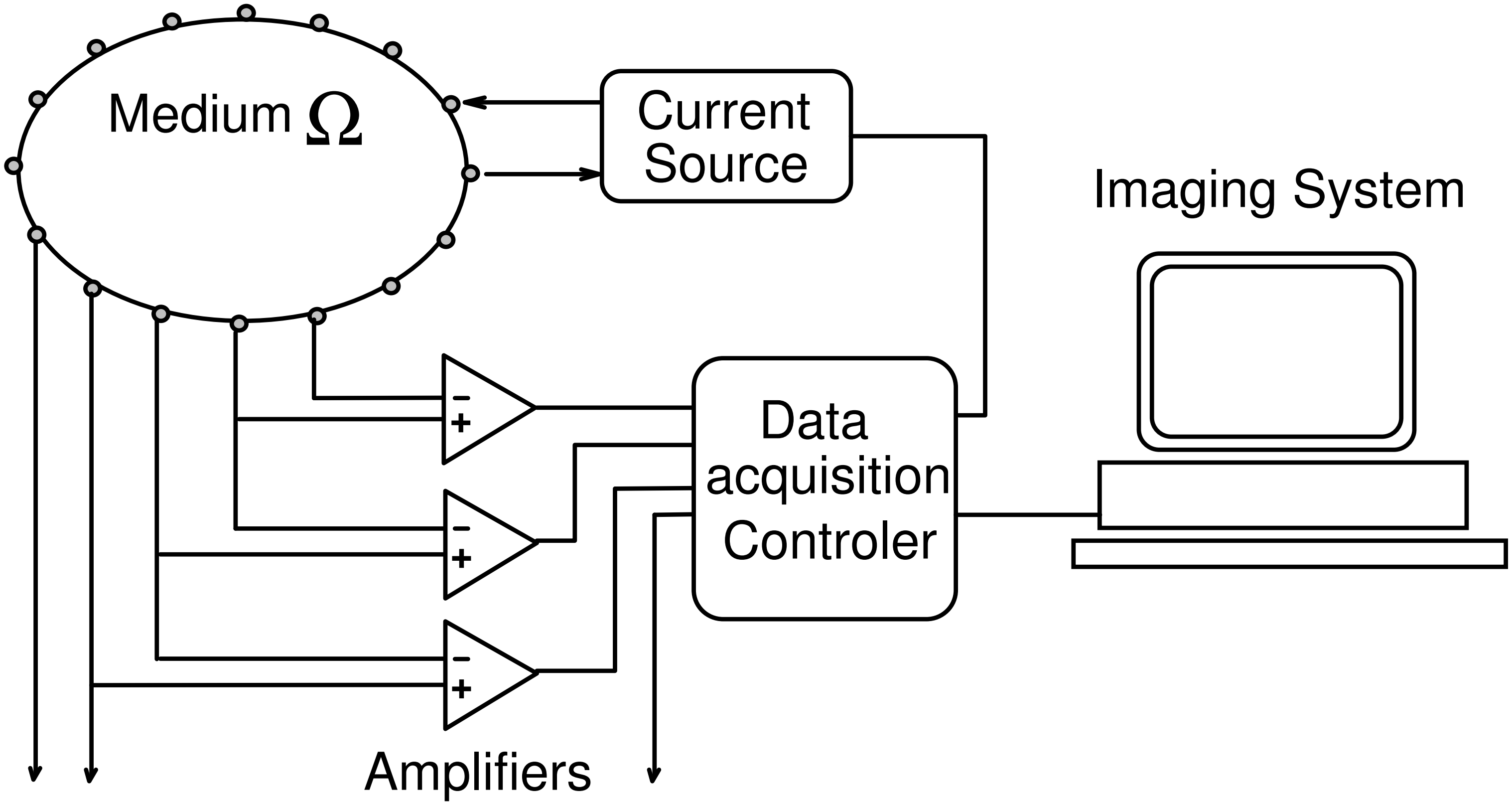
Outline

- Electrical Impedance Tomography
- Applications
- Physics
- Image Reconstruction
- Future Work

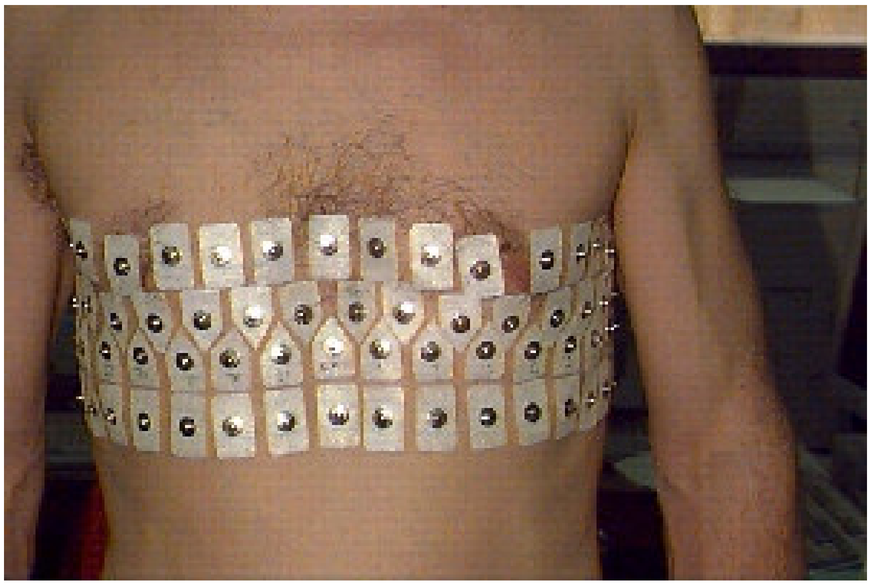
Electrical Impedance Tomography

- Relatively new medical imaging technique (early 1990’s)
- Body Surface Electrodes apply current patterns and measure the resulting voltages
- Distribution of conductivity is calculated

EIT: Block Diagram



Electrode placement to monitor the lungs and heart



Adult

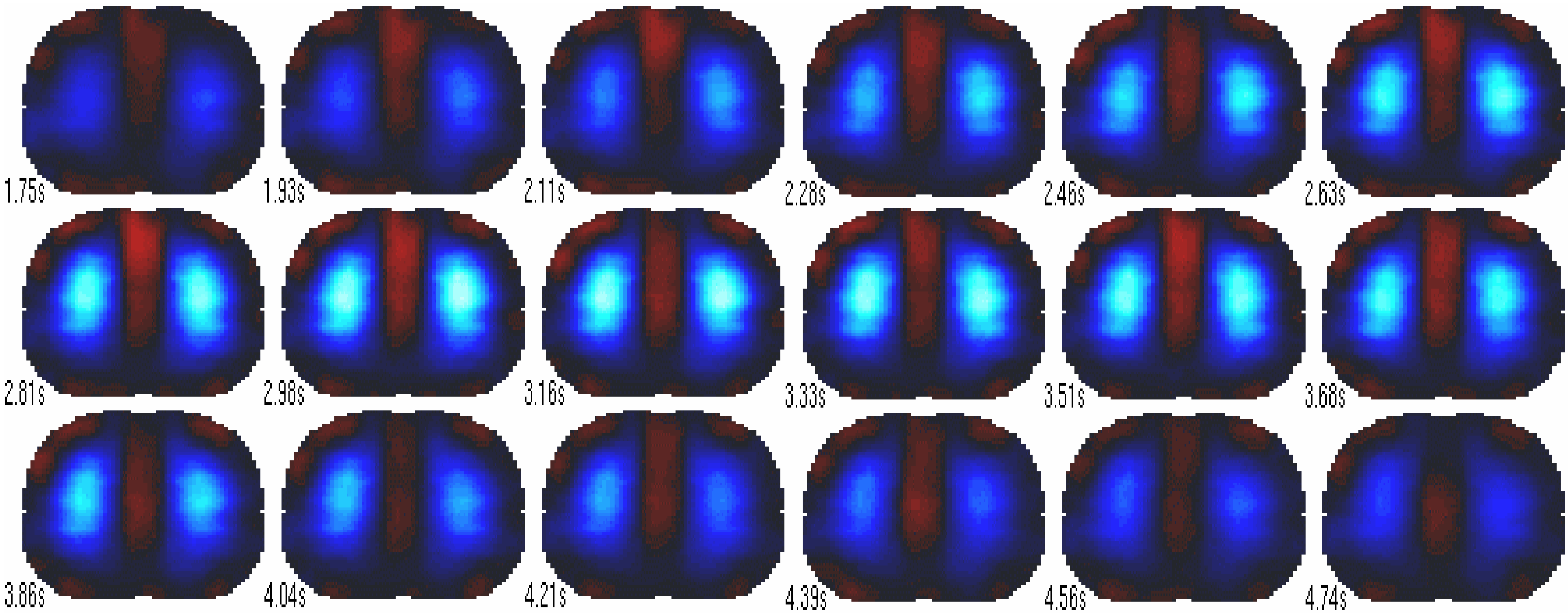
Preterm infant

EIT: Applications

EIT can image/monitor processes involving movement of conductive fluids and gasses

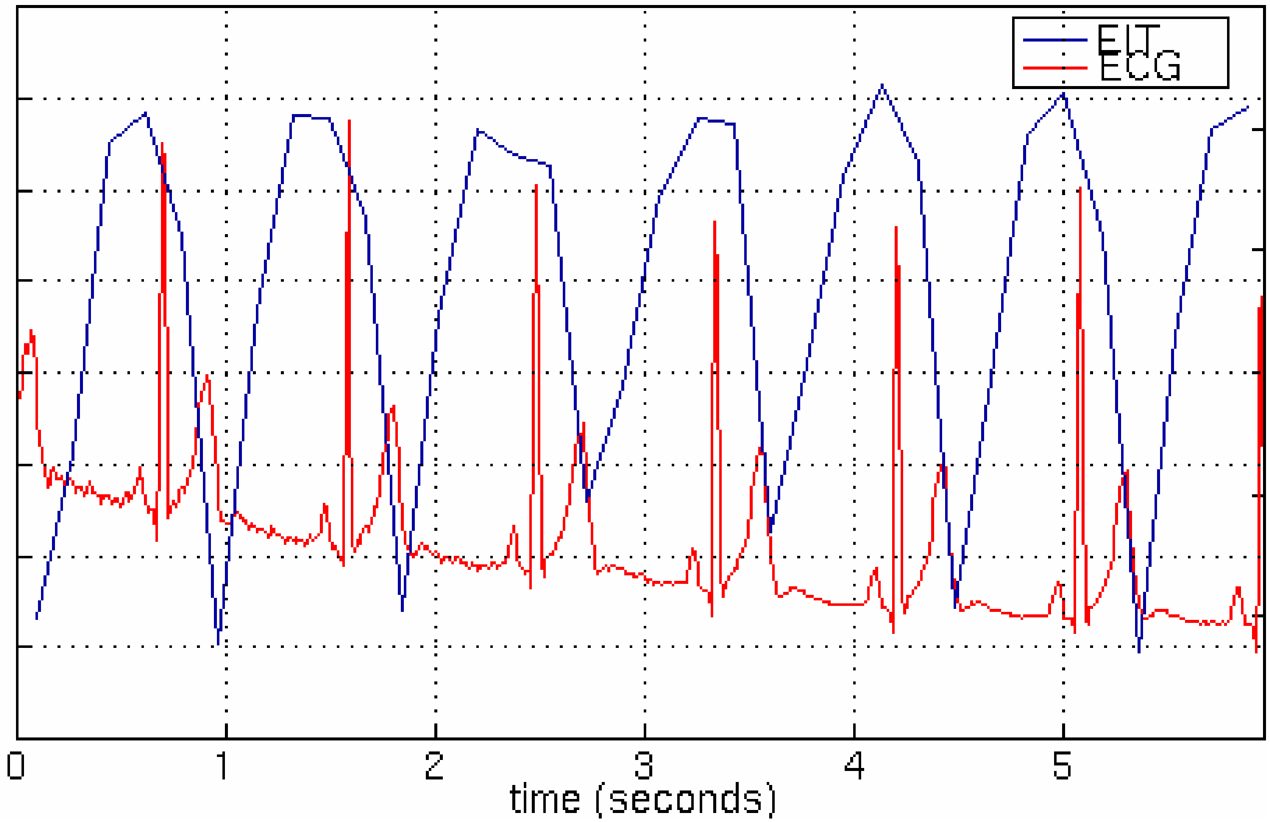
- Lungs
- Heart / perfusion (blood flow)
- GI tract
- Brain
- Breast

Application: Breathing



Chest images of tidal breathing in normal

Application: Heart Beat



EIT signal in ROI around heart and ECG

Why image lungs? Respiratory Failure

Inadequate gas exchange by the respiratory system.

Hypoxemia PaO2 < 60 mmHg or Hyercapnia PaCO2 > 45 mmHg

Causes

- Pulmonary dysfunction
  - Asthma ,Emphysema , Chronic obstructive airway disease, Pneumonia , Pneumothorax, Hemothorax, Acute Respiratory Distress Syndrome (ARDS), Cystic Fibrosis
- Cardiac dysfunction
  - Pulmonary edema, Arrhythmia, Congestive heart failure, Valve pathology

Treatment

- Emergency treatment: cardiopulmonary resuscitation.
- Treatment of the underlying cause is required.
- Mechanical ventilation may be required.

Mechanical Ventilation

used in acute settings (ICU). Often a life-saving technique, but has many complications

- pneumothorax,
- airway injury,
- alveolar damage,

Accordingly it is generally weaned off or to minimal settings as soon as possible.

Positive pressure in contrast to the more historically common negative pressure ventilators sucking air into the lungs.

Iron Lung

Modes of Ventilation

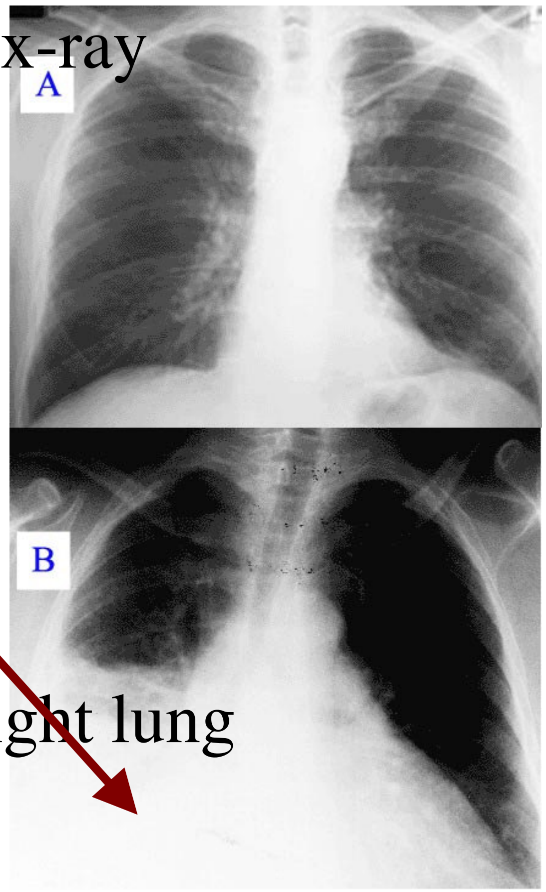
classifications based on how to control the ventilator breath.

- **Breath termination**
- **Breath initiation**
- **High Frequency Ventilation (HFV)**

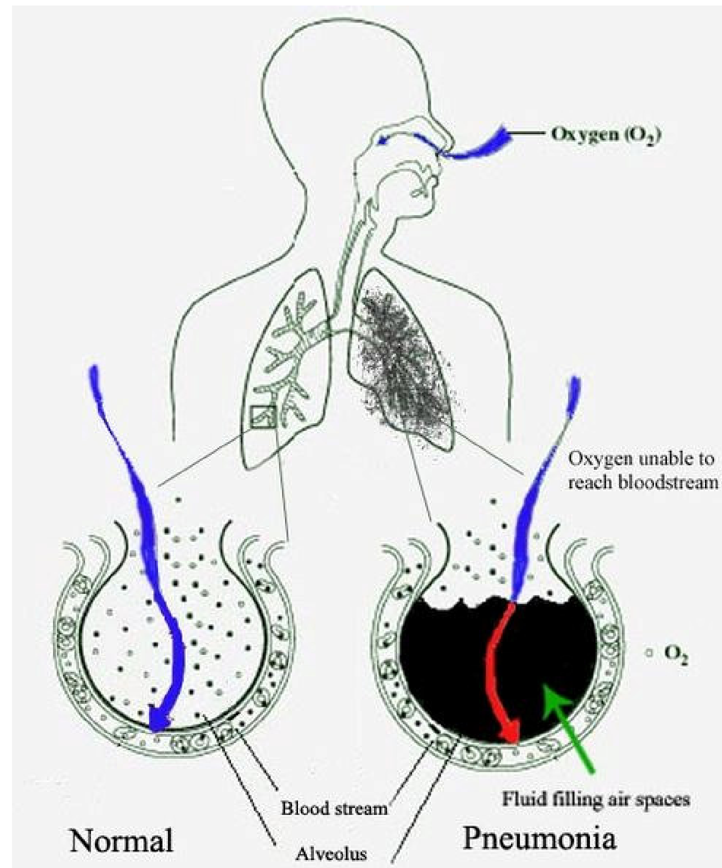
As microprocessors are incorporated into ventilator design, ventilators use combinations of all modes and flow-sensing

Why image lungs? Pneumonia

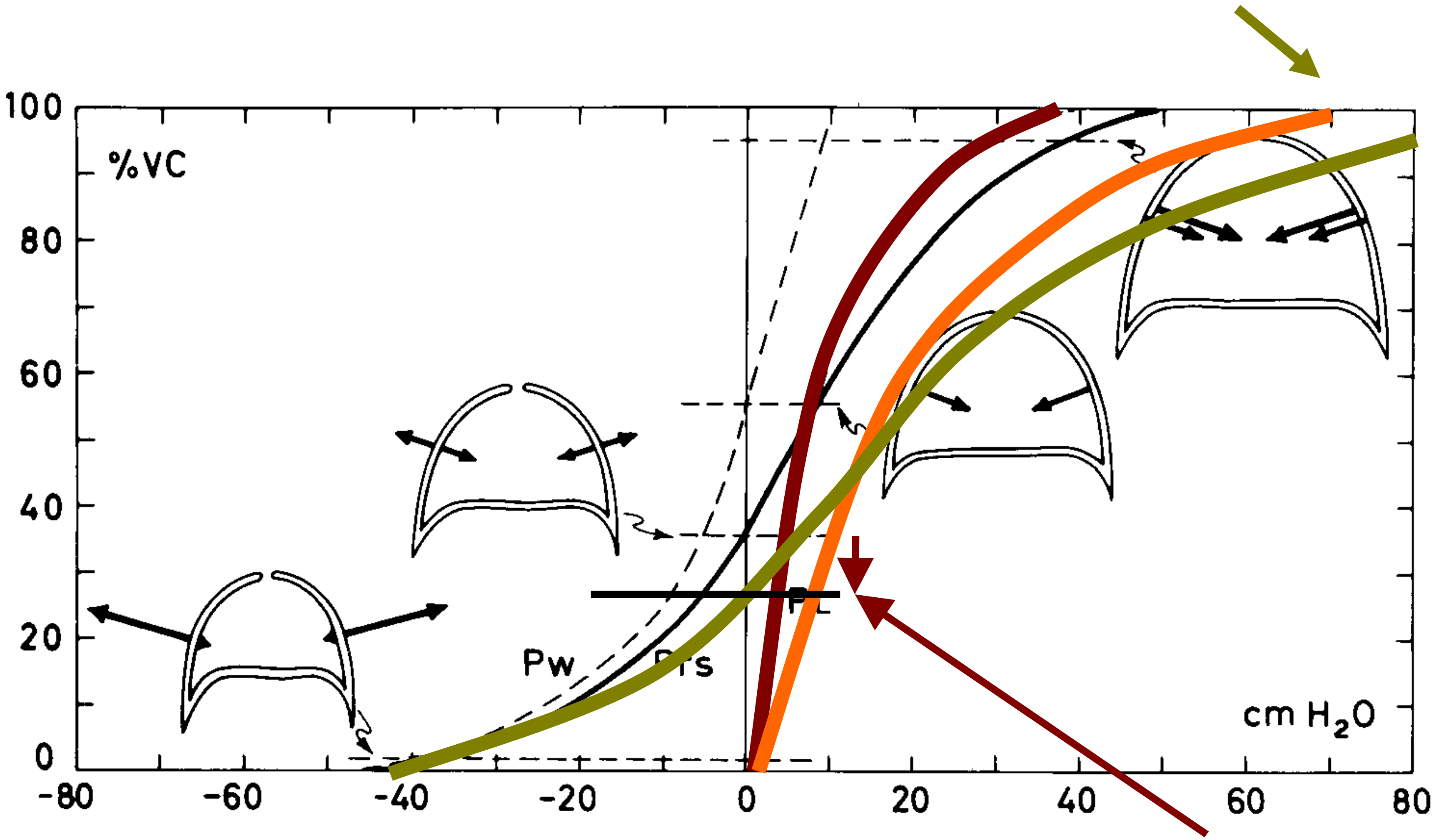
A: Normal chest x-ray.
B: Abnormal chest x-ray



shadowing from pneumonia in the right lung



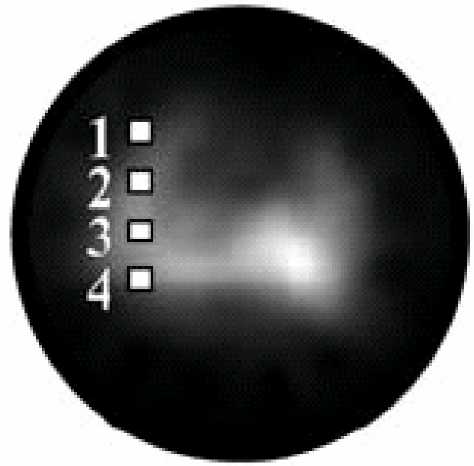
Static Mechanics: ventilation



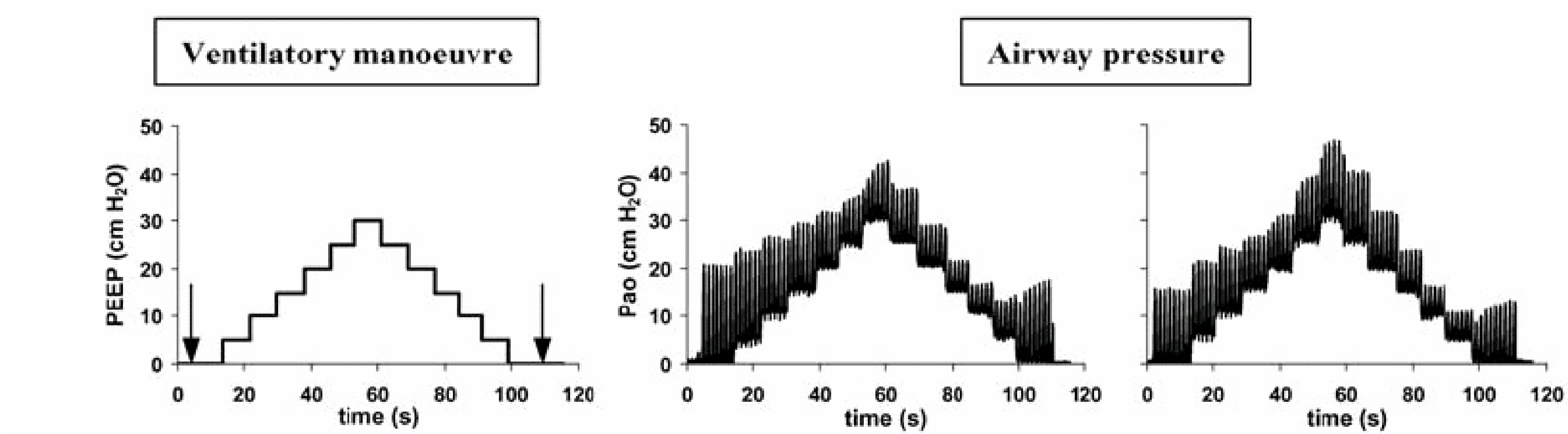
Stiffer lungs have decreased resting volume (FRC)

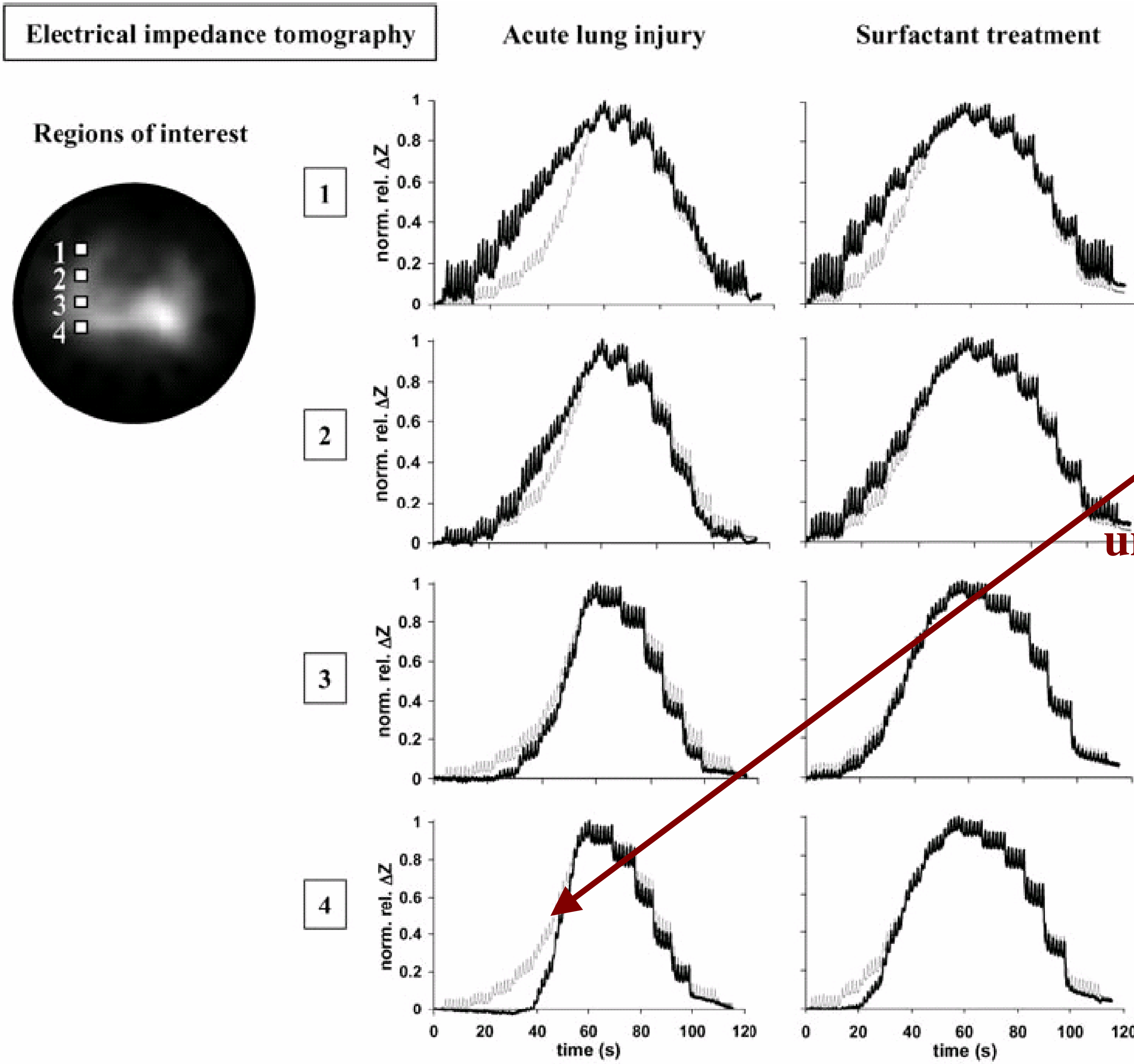
Regional ventilation

lung (*left top*) before and after surfactant treatment. An increase in local aeration is accompanied by an increase in electrical impedance; the small fluctuations in the impedance signal represent the individual breaths. For better comparison and identification of instanta-



Data from Frerichs *et al* (2003) *Intensive Care Med.*.





No change in lower lungs until pressures get really big

Applications: Brain

Newborn with EIT electrode cap on head

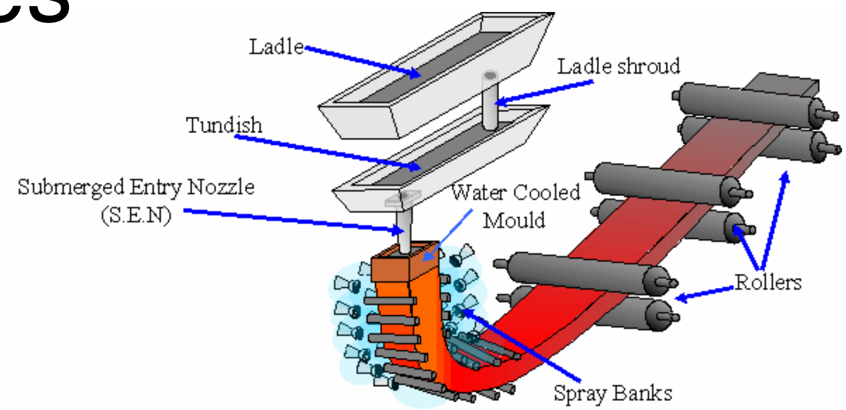
Applications

- Hemorrhage
- Localization of epileptic foci

Industrial Applications

Process Tomography

- Fluid/gas flow in pipes
- Metal Castings



Geophysics

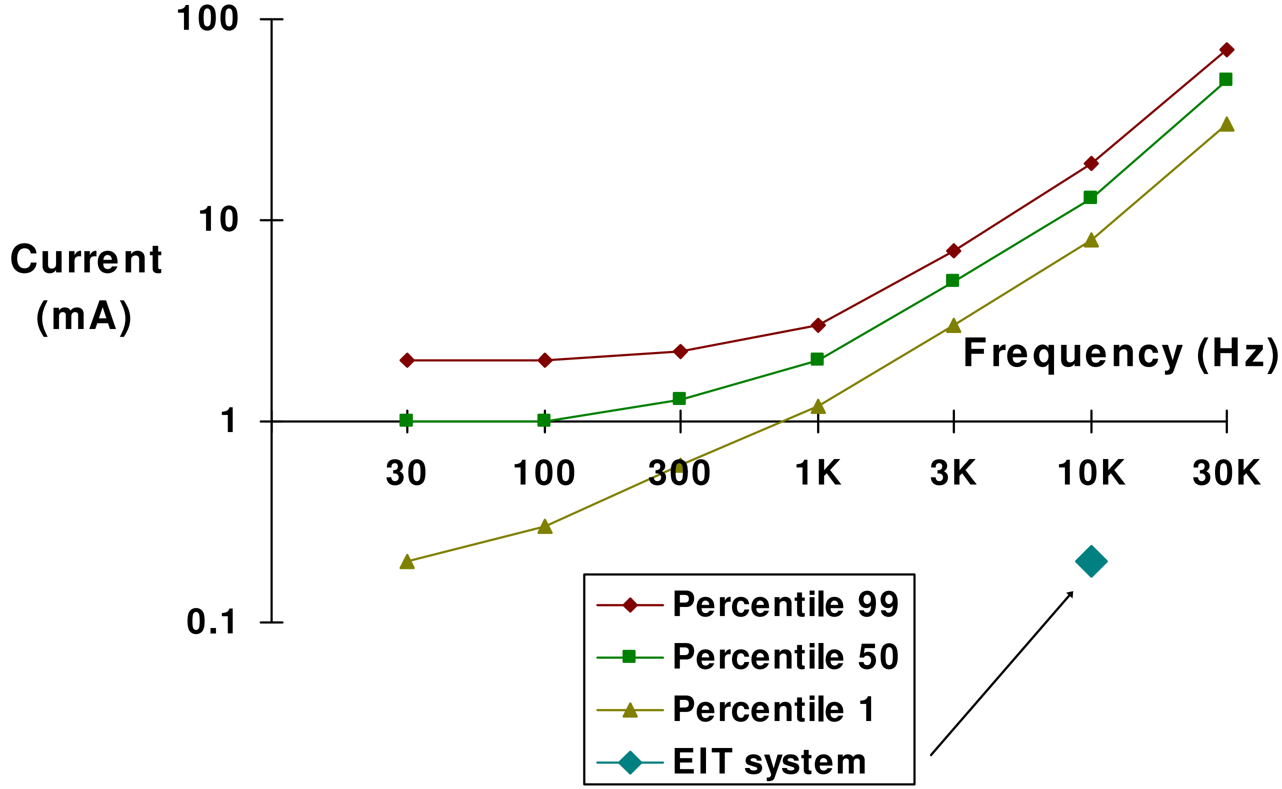
- Undersurface geology
- Mine detection

EIT: Advantages

EIT is a relatively low resolution imaging modality, *with several advantages*

- Non-invasive
- Non-cumbersome
- Suitable for monitoring
- Underlying technology is low cost

Non-invasive



Thresholds for cutaneous perception of electric current vs. frequency and EIT system

Hardware: Electrodes

- Current stimulation is better than voltage, because it accounts for electrode contact impedance
- Traditionally EIT uses adjacent current drive.
- Some systems separate drive and measurement electrodes, using adaptive current patterns

EIT: Physics

- Within medium Ω there is **E** and **J**.
- $$J_c = \sigma E$$
  $$J_d = \varepsilon\varepsilon_o \frac{dE}{dt}$$
  $$\mathbf{J} = \left(\sigma - j\omega\varepsilon\varepsilon_o\right)\mathbf{E}$$

EIT: Physics

In the absence of magnetic fields

$$\mathrm{E} = -\nabla V$$

No charge build up in conductive medium

We have $$\nabla \bullet \mathrm{J} = -\frac{\partial \rho}{\partial t} = 0$$

$$\nabla \bullet \left(\sigma - j\omega\varepsilon\varepsilon_o\right)\nabla V = 0 \qquad \text{in } \Omega$$

EIT: Physics

Current is applied at electrodes

$$\nabla \bullet \mathbf{J} = -\frac{\partial \rho}{\partial t} = I_e$$

Body need to be grounded, somewhere

$$V = 0 \qquad \text{at some point}$$

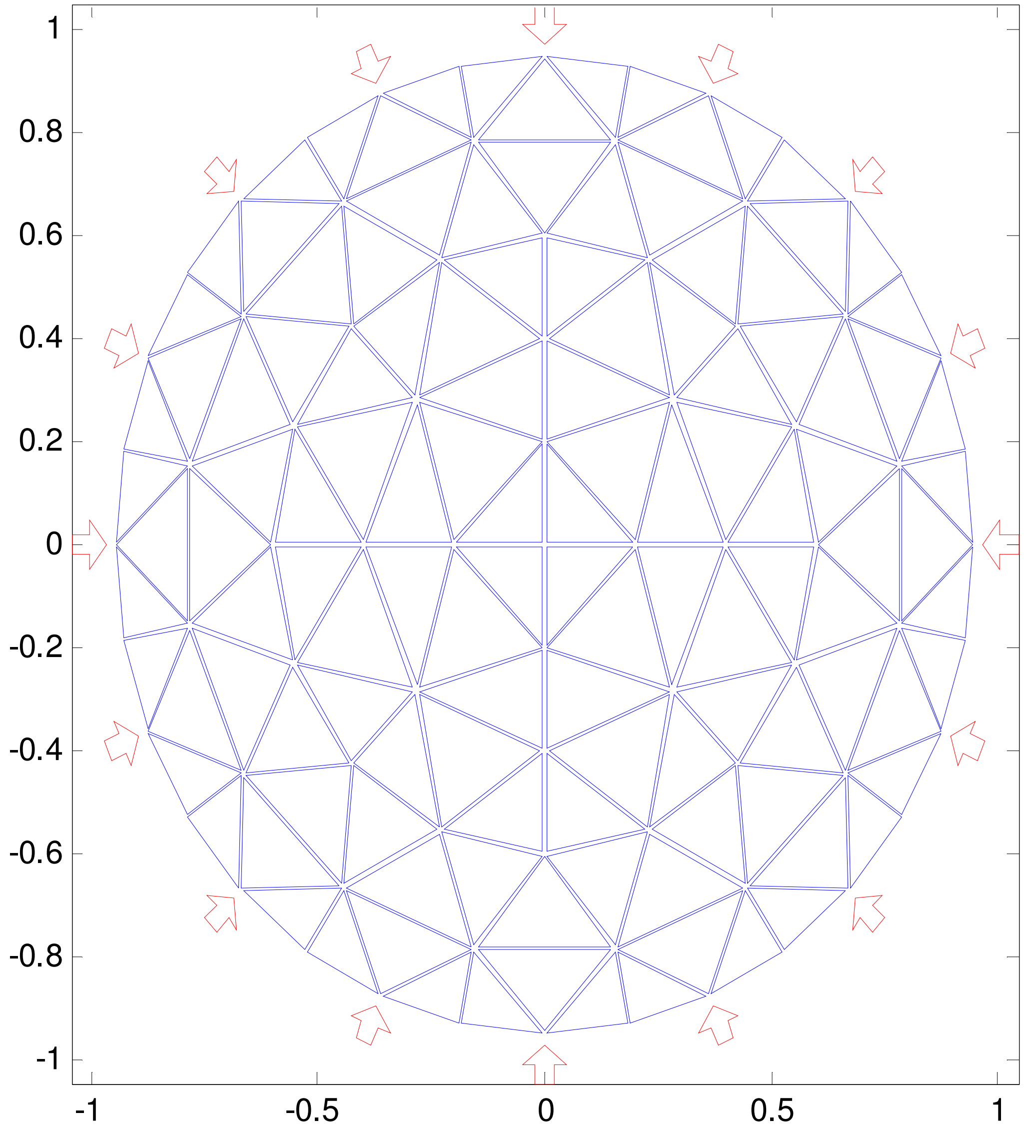
EIT: Numerical Models

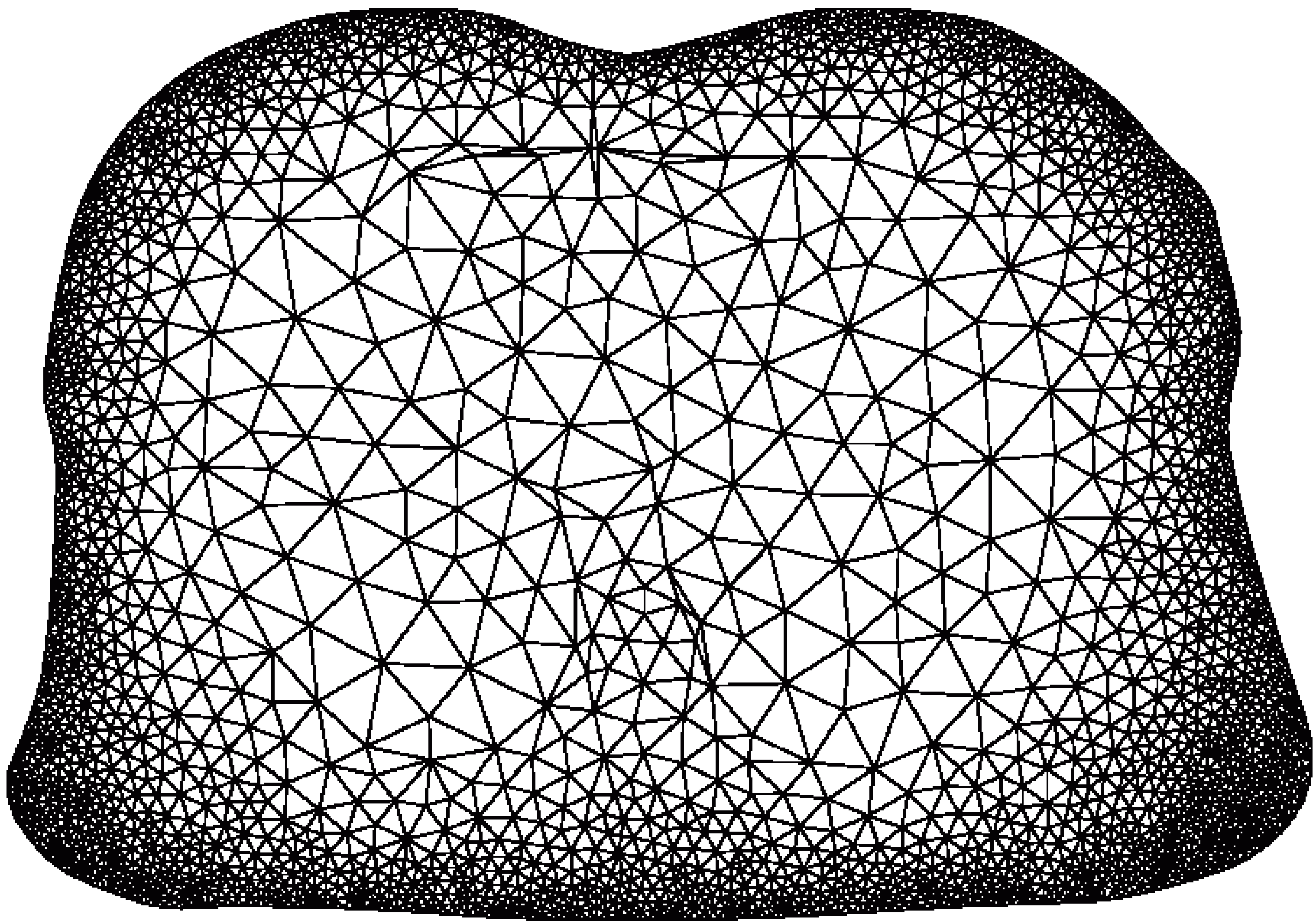
In order to calculate measurements from conductivities, we can use:

- Analytic Techniques
  – Analytic models exist for elliptic 3D media; however, numerical approximations of sums required
- Numerical Models
  – Finite Element Techniques, main method

Finite Element Models

- Simple Model with 64 elements
- Used for inverse solution





Model of Borsic, *Physiol Meas, 22:77-83, 2002*

Finite Element Models

- "Simple" 3D Model with 768 elements
- Used for inverse solution

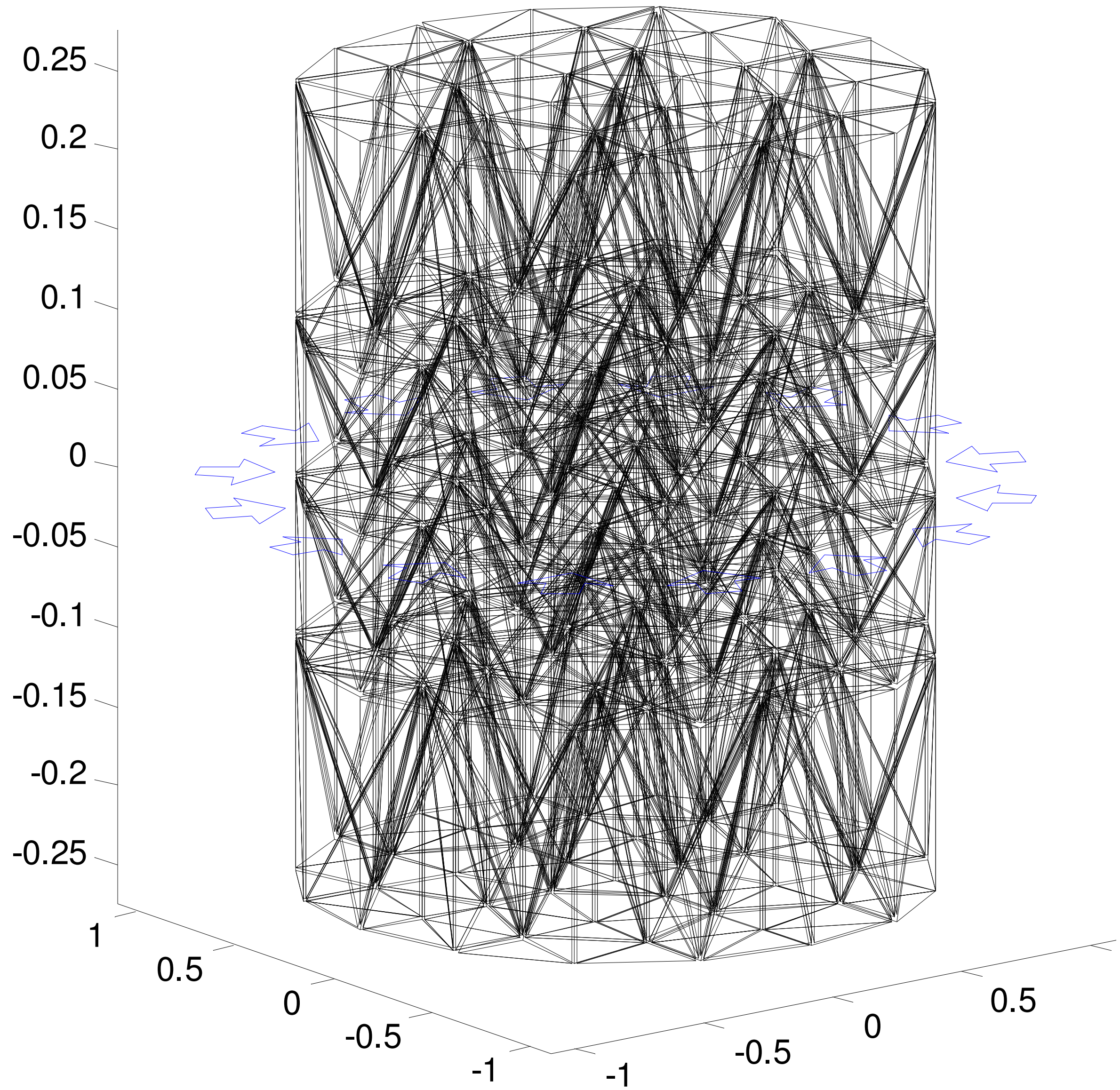


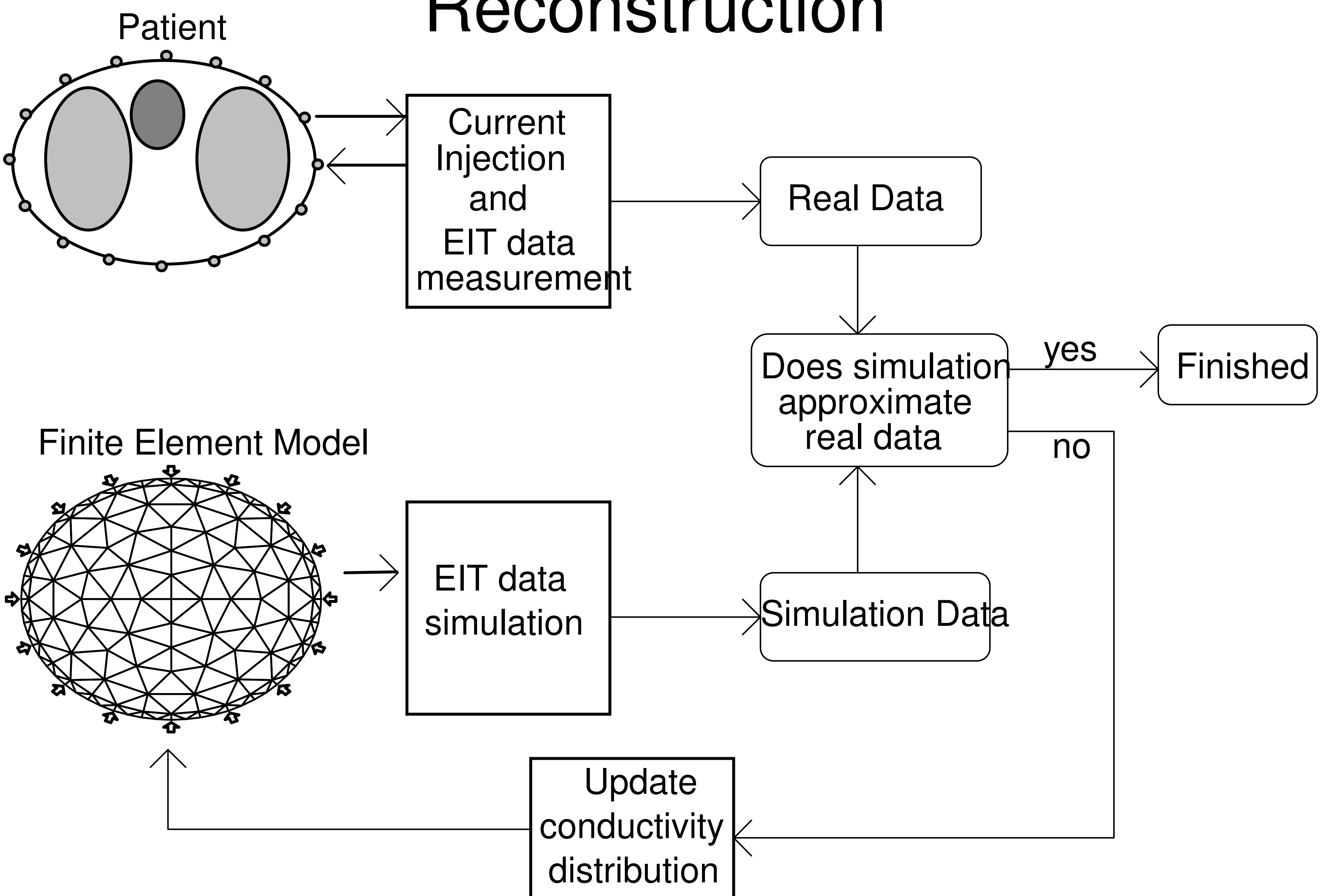
Image Reconstruction: Static Imaging

Static imaging reconstructs the absolute conductivity from measurements.

Algorithms:

- Iterative (Newton-Raphson)
- Layer Stripping

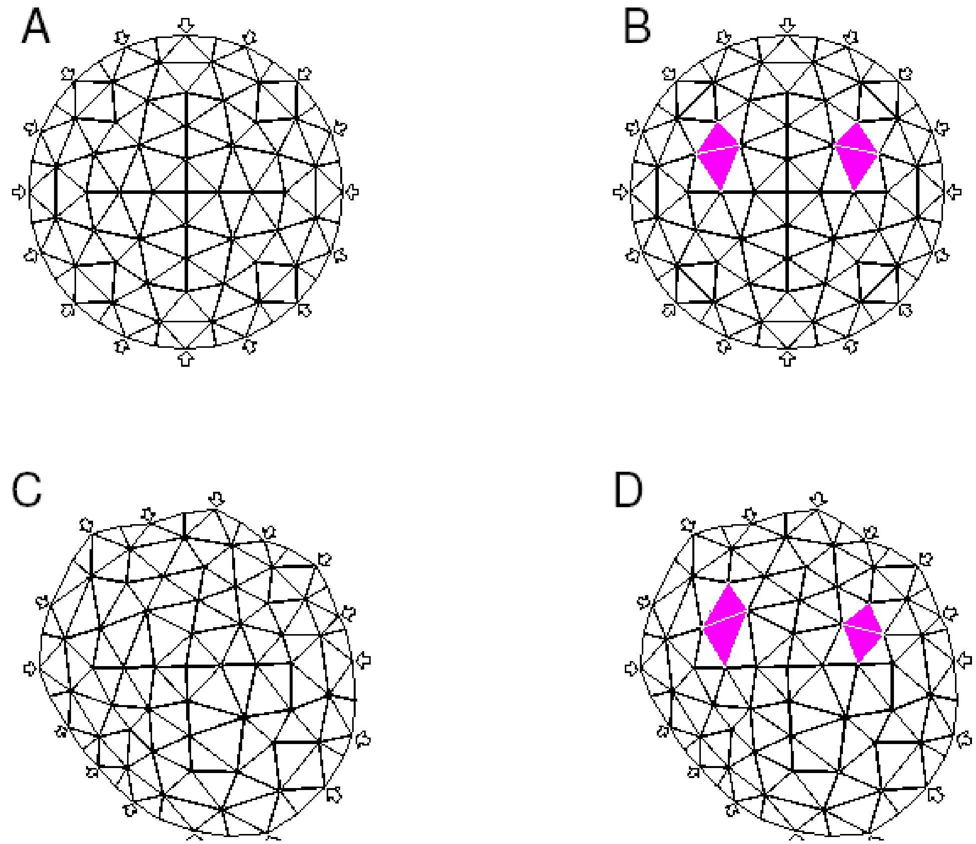
Iterative (Absolute) Image Reconstruction



Absolute Imaging Difficulties

- Extremely sensitive to uncertainties in electrode position
  - Need to know where electrodes are to and electrode shape to 1mm
  - "Absolutely" must do 3D
- Numerical instability
- Slow reconstructions
- Is muscle in chest isotropic?

Difference Imaging: Example



Difference Imaging

- Calculate Δ conductivity
  from Δ measurements

- Inverse problem *linearized*
- reduced sensitivity to electrode and hardware errors.
- Suitable for physiological imaging: lung, heart, GI

Inverse Techniques

- We can pose dynamic imaging as linear inverse, using a *sensitivity matrix*

$$\mathbf{z}_j = \frac{\mathbf{z}(\sigma_h) - \mathbf{z}(\sigma_h + \delta_j)}{\delta_j}$$

$$\mathbf{z} = \mathbf{H}\Delta\sigma$$

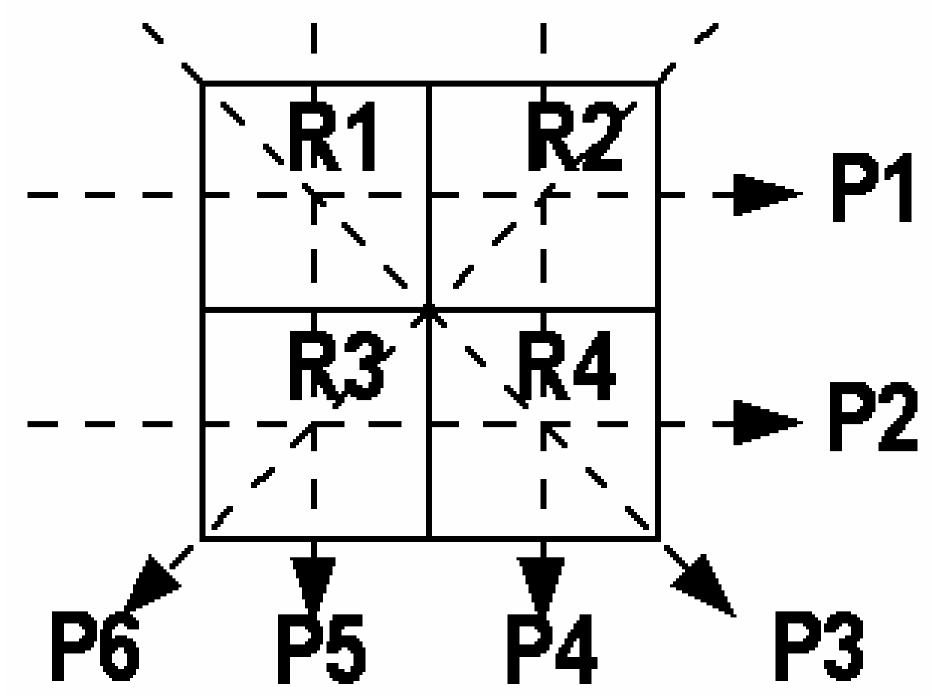
Inverse Techniques

- Classic least-squares inverse

$$\mathbf{z} = \mathbf{H}\mathbf{x}$$

$$\hat{\mathbf{x}} = \left(\mathbf{H}^t\mathbf{H}\right)^{-1}\mathbf{H}^t\mathbf{z}$$

Model based matrix inverses



$$\begin{bmatrix} P1 \\ P2 \\ P3 \\ P4 \\ P5 \\ P6 \end{bmatrix} = \mathbf{H} \begin{bmatrix} R1 \\ R2 \\ R3 \\ R4 \end{bmatrix} + \mathbf{b}$$

Matrix Techniques

However, problem is:

- ill-conditioned: measurements depend much more on data near electrodes than in centre
- ill-formed: more unknowns than measurements

Regularized Imaging

Handwaving argument for regularization:
used for ill-posed and ill-formed problems to find a solution with:

- Low error: small ($\mathbf{z} - \mathbf{Hx}$)
- Stable: small change in $\mathbf{x}$ for small $\mathbf{\Delta z}$
- Good looking:
  - Somewhat hard to define, but includes smoothness, clean edges, etc.

MAP estimates

- MAP approach says choose **x** such that *f(***x**|**z**) is maximized
  - In other words, choose the image that is most likely, considering the measured data
- Bayes Rule

$$f(\mathbf{x}|\mathbf{z}) = \frac{f(\mathbf{z}|\mathbf{x})\, f(\mathbf{x})}{f(\mathbf{z})}$$

MAP estimates

$f(\mathbf{z}|\mathbf{x})$ the distribution of measurements given an image

- Based on forward model and noise properties

$f(\mathbf{z})$ distribution of measurements

- Not a parameter of MAP estimate

$f(\mathbf{x})$ distribution of image

- Based on *a priori* knowledge of physically possible and likely images distributions

Regularized Imaging

Given Linear Model:

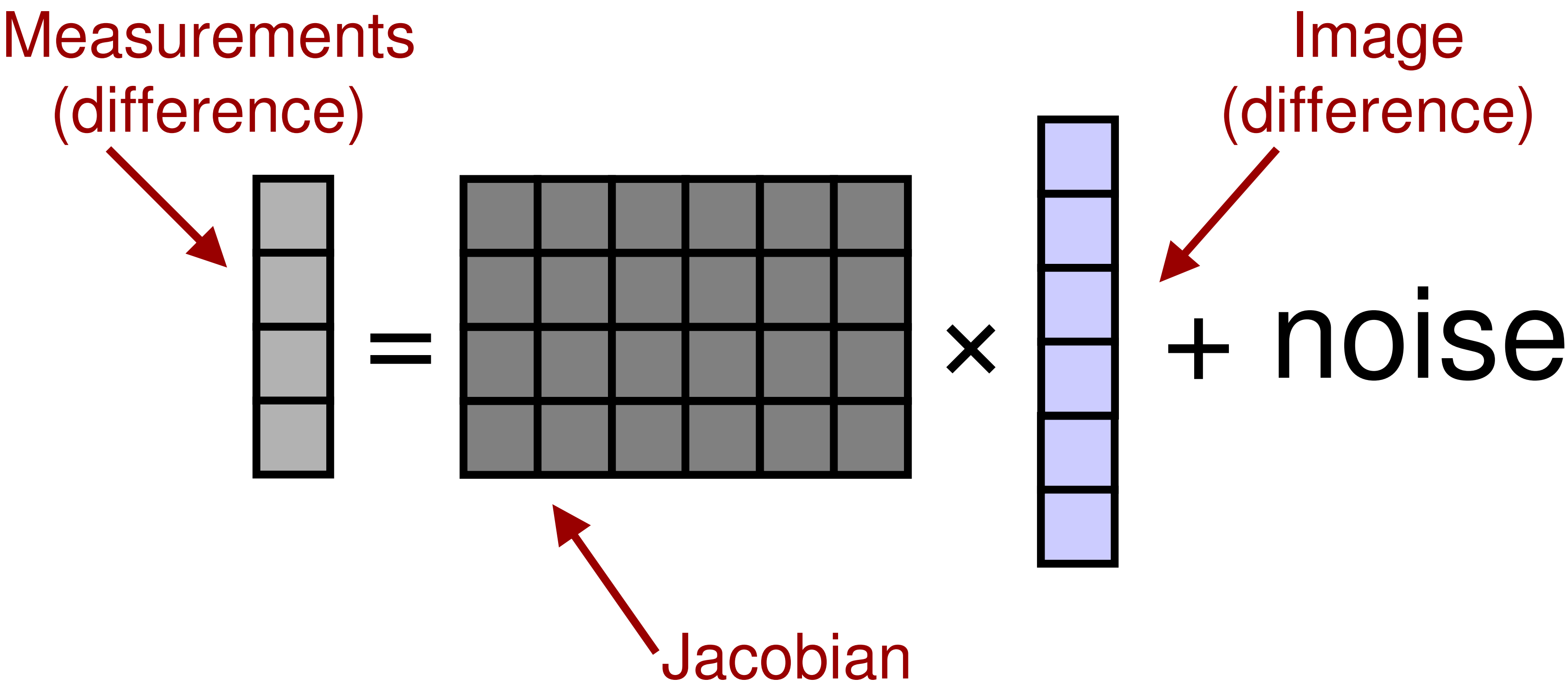
$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

Maximum A Posteriori (MAP) estimate is:

$$\hat{\mathbf{x}} = \left(\mathbf{H}^t \mathbf{R}_n^{-1} \mathbf{H} + \mathbf{R}_x^{-1}\right)^{-1} \left(\mathbf{H}^t \mathbf{R}_n^{-1} z + \mathbf{R}_x^{-1} \mathbf{x}_\infty\right)$$

Image Reconstruction

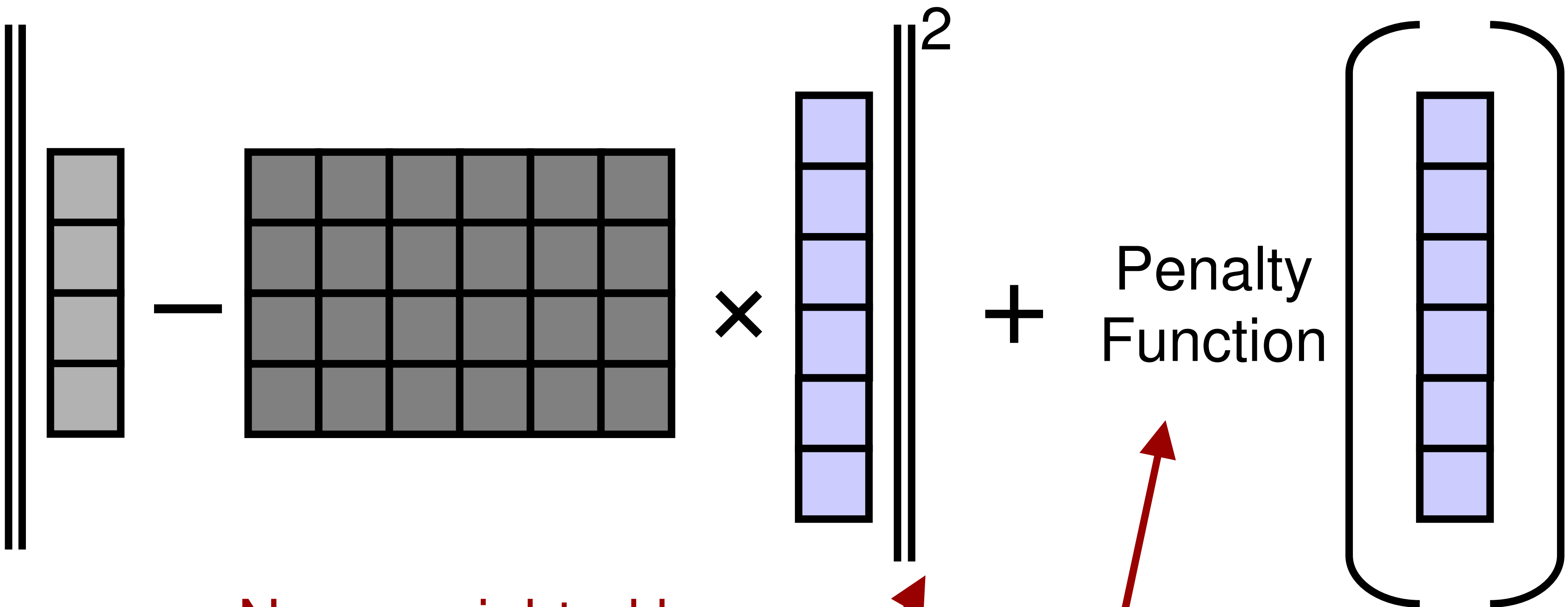
- Forward Model (linearized)



System is underdetermined

Image Reconstruction

Regularized linear Inverse Model



Norm weighted by measurement accuracy

Way to introduce "prior knowledge" into solution

Regularized Imaging

- Parameters $\mathbf{R_x}$, $\mathbf{R_n}$, x_∞ , represent *a priori* statistical knowledge of problem

$$\mathbf{x}_\infty = E[\mathbf{x}]$$

$$\mathbf{R_x} = E\left[(\mathbf{x} - \mathbf{x}_\infty)^t(\mathbf{x} - \mathbf{x}_\infty)\right] = E\left[\mathbf{x}^t\mathbf{x}\right] - \mathbf{x}_\infty{}^t\mathbf{x}_\infty$$

$$\mathbf{R_n} = E\left[\mathbf{n}^t\mathbf{n}\right] = \begin{bmatrix} \sigma_1^2 & 0 & \cdots \\ 0 & \sigma_2^2 & \\ \vdots & & \ddots \end{bmatrix}$$

Choice of parameter $\mathbf{R_x}$

- Parameter is a “penalty function”
- Many regularization approaches use a diagonal matrix
  - Tikhonov regularization uses the scaled identity matrix
  - This will penalize large amplitude pixels in image
- We choose a dense matrix
  - Penalize image frequency content above maximum possible with measurements

Regularization: Hyperparameters

Regularizations techniques must finally introduce a “hyperparameter” (μ)

$$\hat{\mathbf{x}} = \left(\mathbf{H}^t \mathbf{W} \mathbf{H} + \mu \mathbf{Q}\right)^{-1} \left(\mathbf{H}^t \mathbf{W} \mathbf{z} + \mu \mathbf{Q} \mathbf{x}_\infty\right)$$

where

$$\mathbf{W} = \frac{1}{\sigma_n^2} \mathbf{R}_{\mathbf{n}}^{-1}$$, ie. the relative noise amplitudes

$$\mathbf{Q} = \frac{1}{\sigma_x^2} \mathbf{R}_{\mathbf{x}}^{-1}$$, ie. the relative image correlations

Regularization: Hyperparameters

μ is thus the ratio of image and noise amplitudes,

$$\mu = \frac{\sigma_x^2}{\sigma_y^2}$$

it can be interpreted as a filter noise figure

Regularized Inverse

Parameters:

- **W**: models measurement noise
- **Q**: penalizes image features which are greater than data supports
- $\mathbf{x}_\infty$: represents the background conductivity distribution (heart,lungs,etc)
- **μ**: "hyper-parameter" amount of regularization

Advantages of Regularization

- Stabilizes ill-conditioned inverse
- Introduction of *a priori* information
- Control of *resolution-noise* performance trade-off
- MAP inverse justifies the formulation in terms of Bayesian statistics

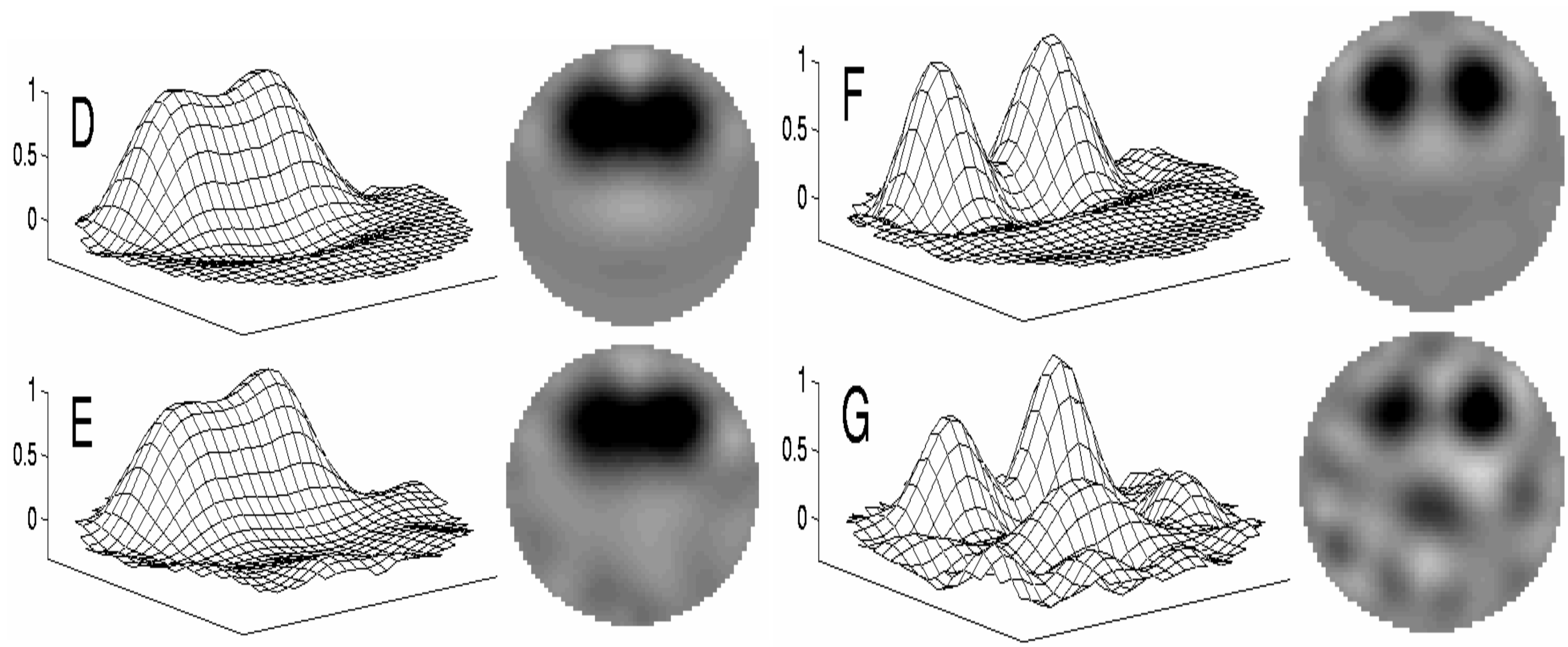
Noise – Resolution Tradeoff

Lots of Regularization (large penalty)

Little Regularization (small penalty)

No Noise

-3dB SNR



Applications …

- Electrode Errors
- Electrode Movement
- 3D Imaging / Electrode Placement
- Temporal Filtering

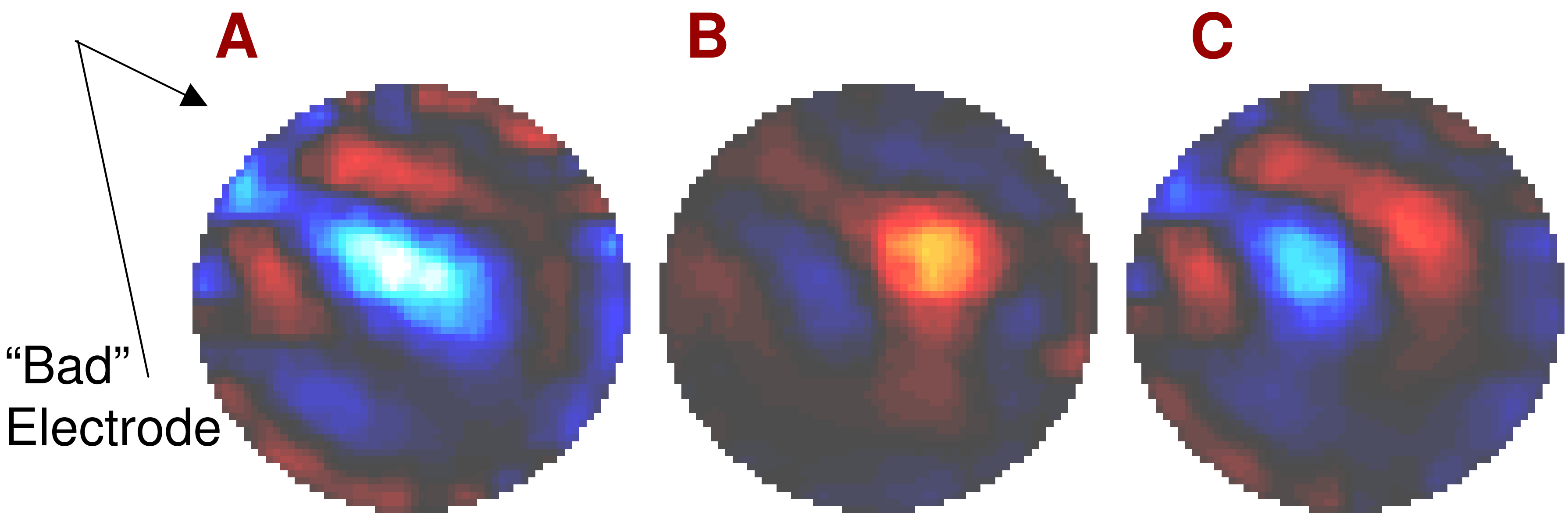
Electrode Measurement Errors

Experimental measurements with EIT quite often show large errors from one electrode

Causes aren’t always clear

- Electrode Detaching
- Skin movement
- Sweat changes contact impedance
- Electronics Drift?

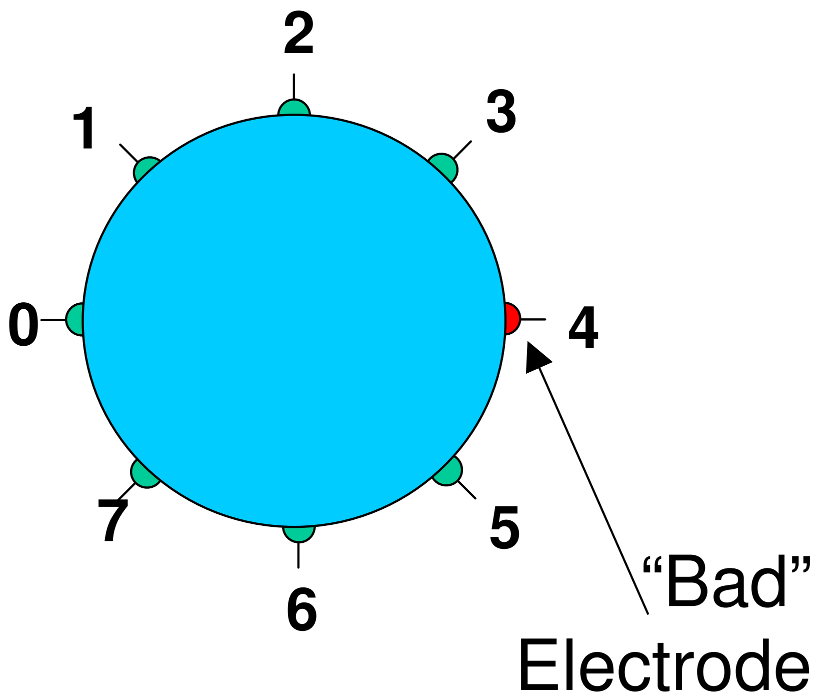
Example of electrode errors



Images measured in anaesthetised, ventilated dog

A. Image of 700 ml ventilation
B. Image of 100 ml saline instillation in right lung
C. Image of 700 ml ventilation and 100 ml saline

Measurements with “bad” electrode



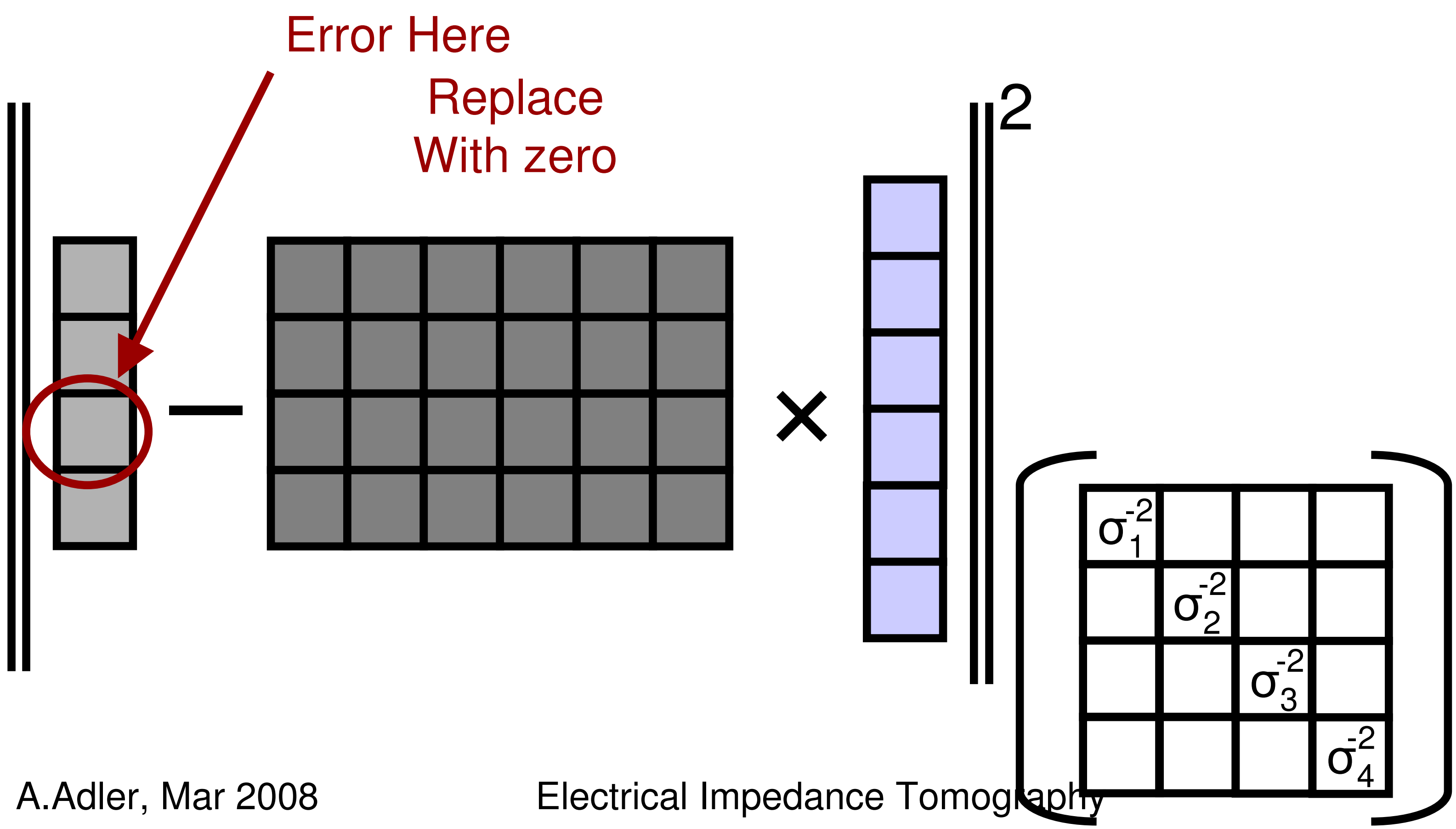
	01	12	23	34	45	56	67	70
01	X	X		*	*			X
12	X	X	X	*	*			
23		X	X	X	*			
34	*	*	X	X	X	*	*	*
45	*	*	*	X	X	X	*	*
56				*	X	X	X	
67				*	*	X	X	X
70	X			*	*		X	X

* “bad” measurement

X measurement at current injection

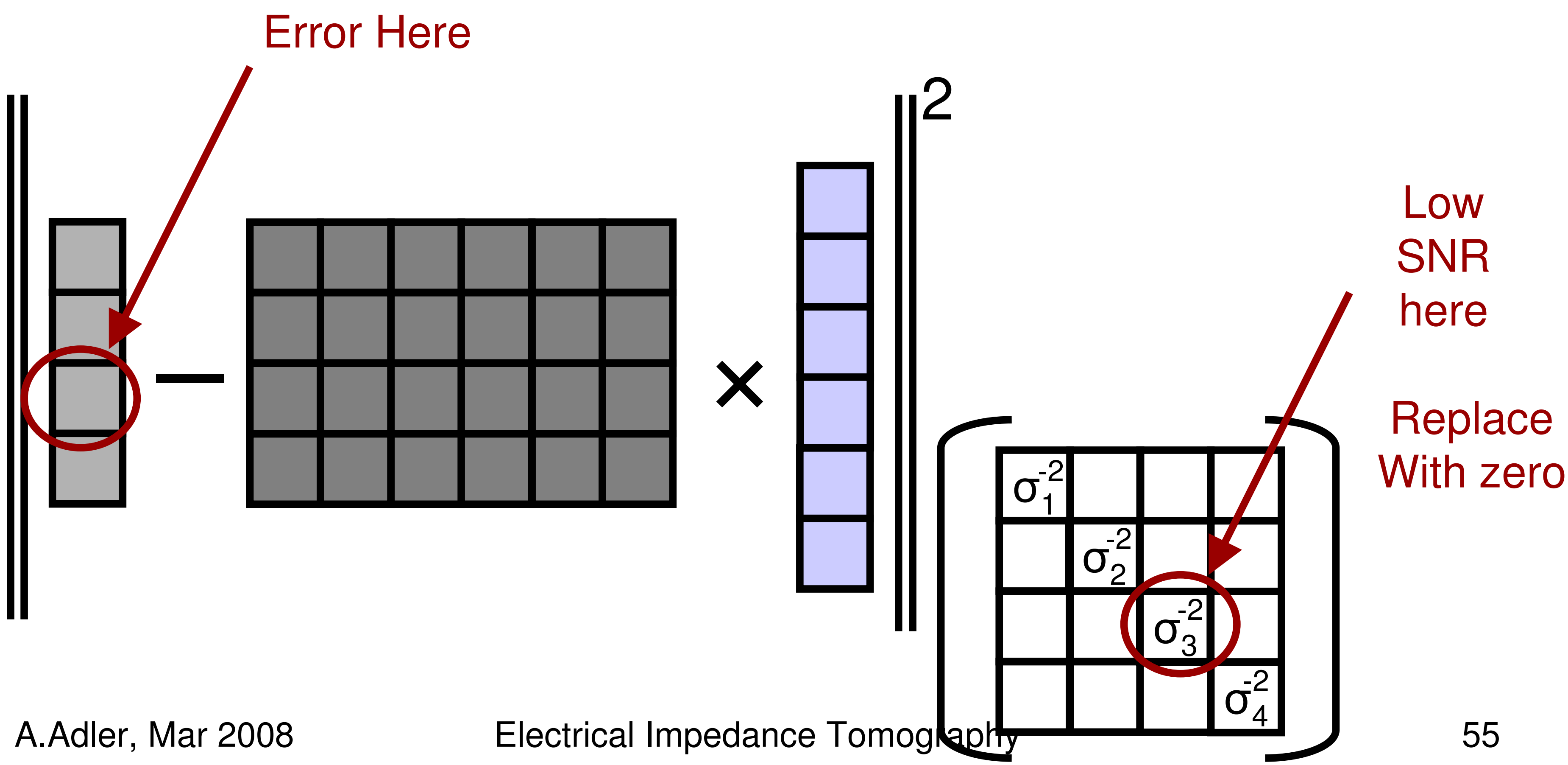
“Zero bad data” solution

“Traditional solution” (in the sense that I’ve done this)

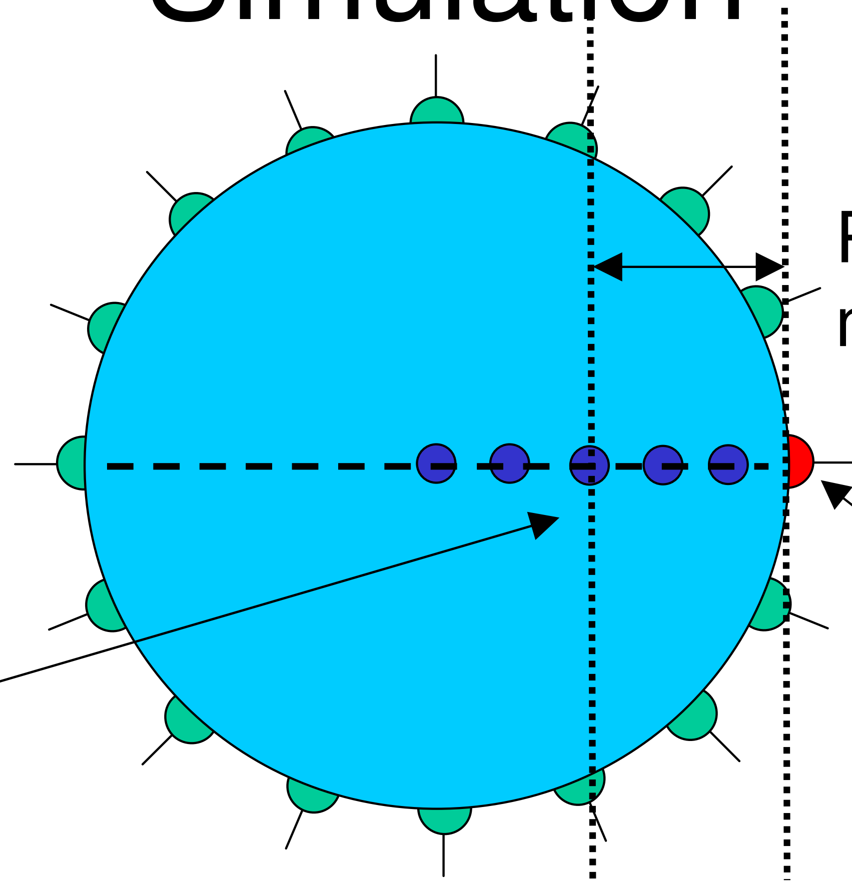


Regularized imaging solution

Electrode errors are large measurement noise on affected electrode



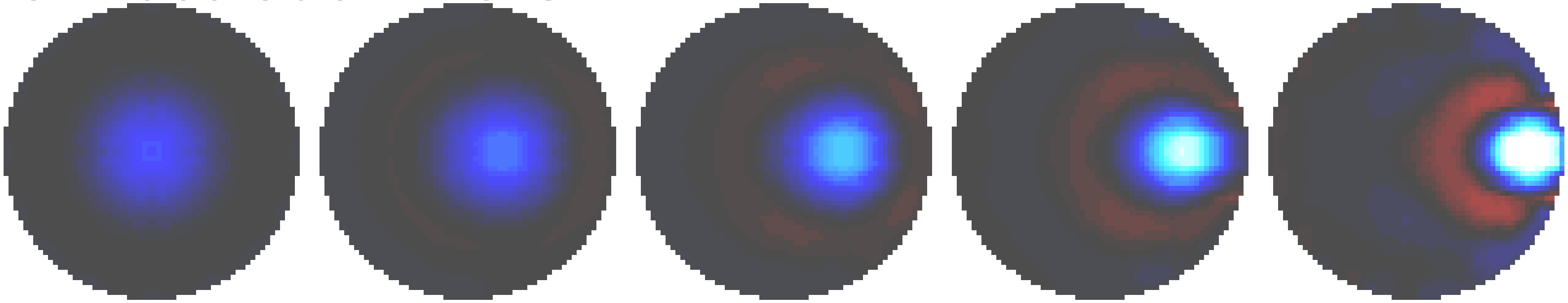
Simulation



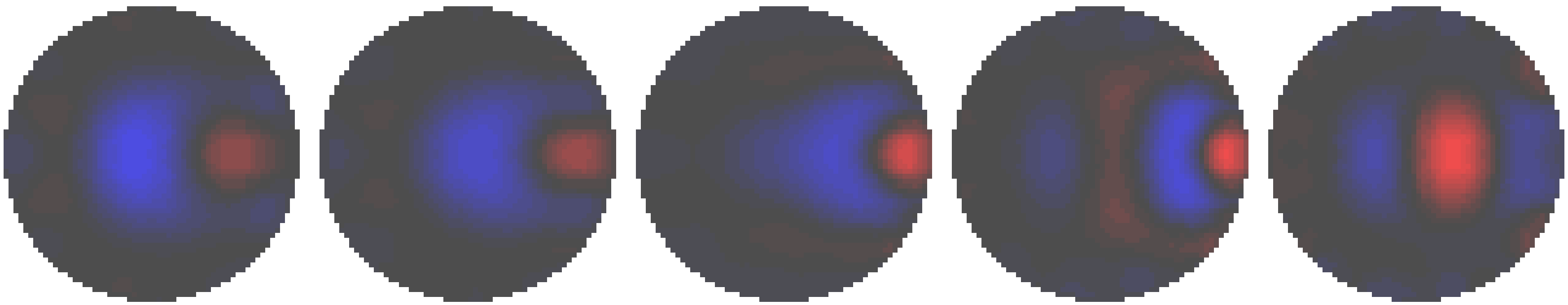
Data simulated with 2D FEM with 1024 elements
– not same as inverse model

Simulation results for opposite drive

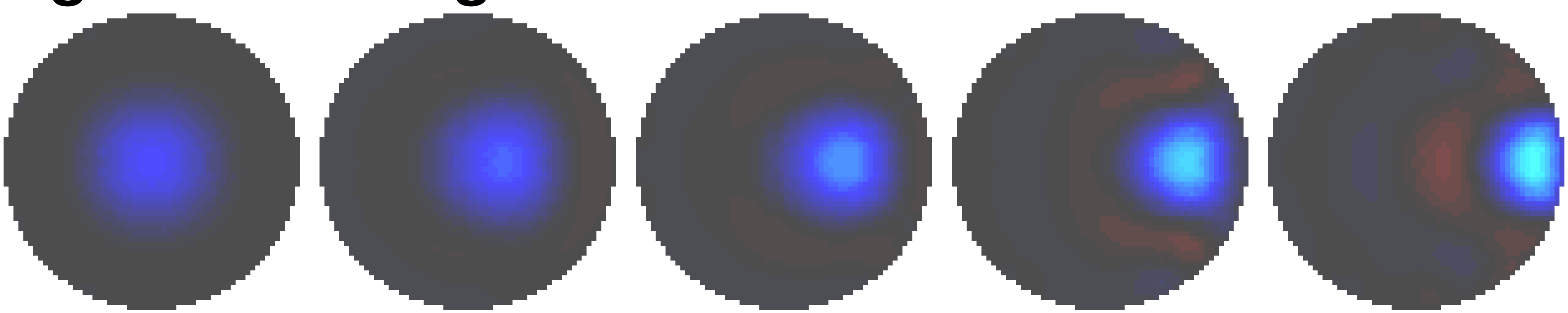
No Electrode Errors



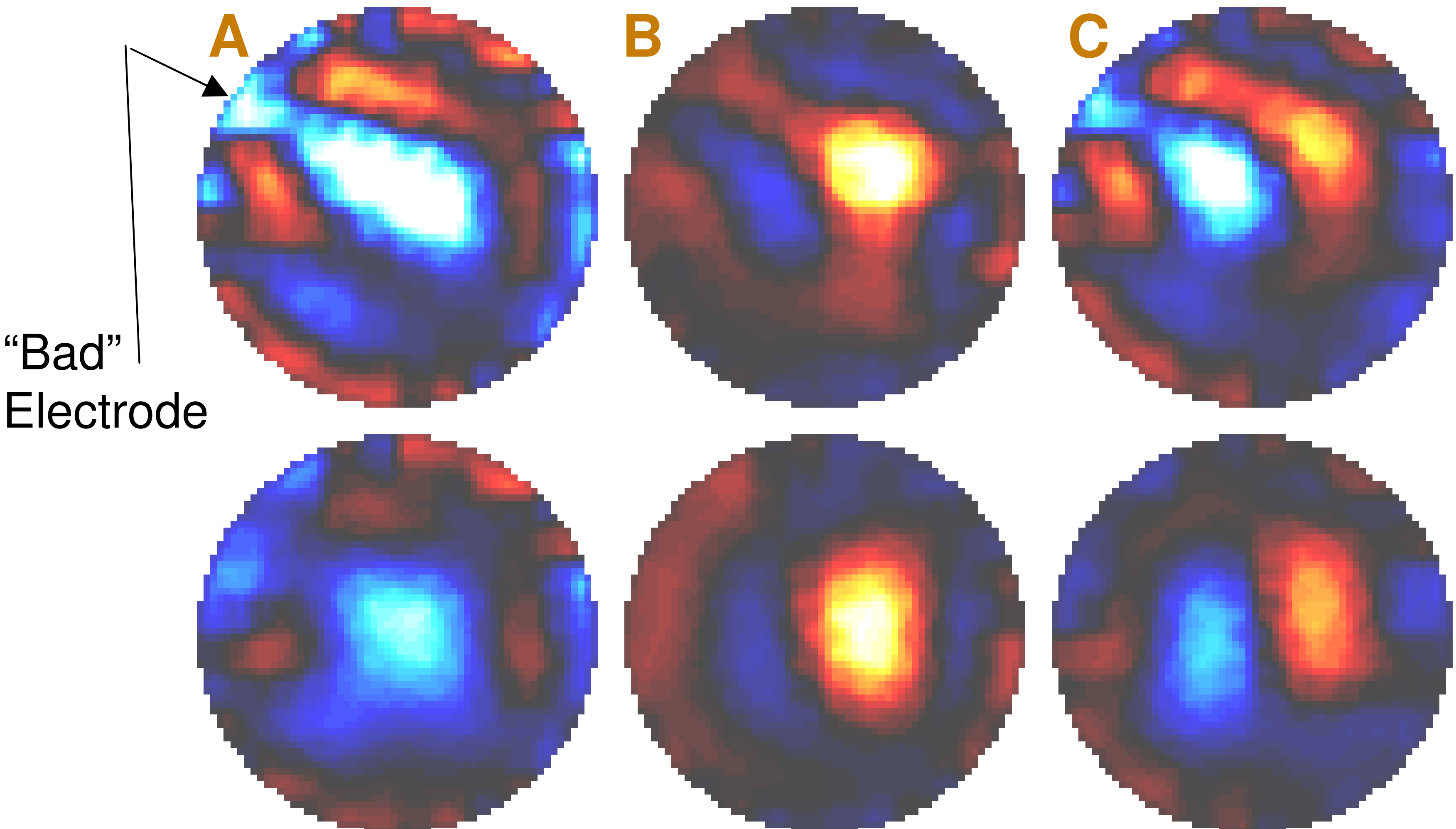
Zero Affected Measurements



Regularized Image

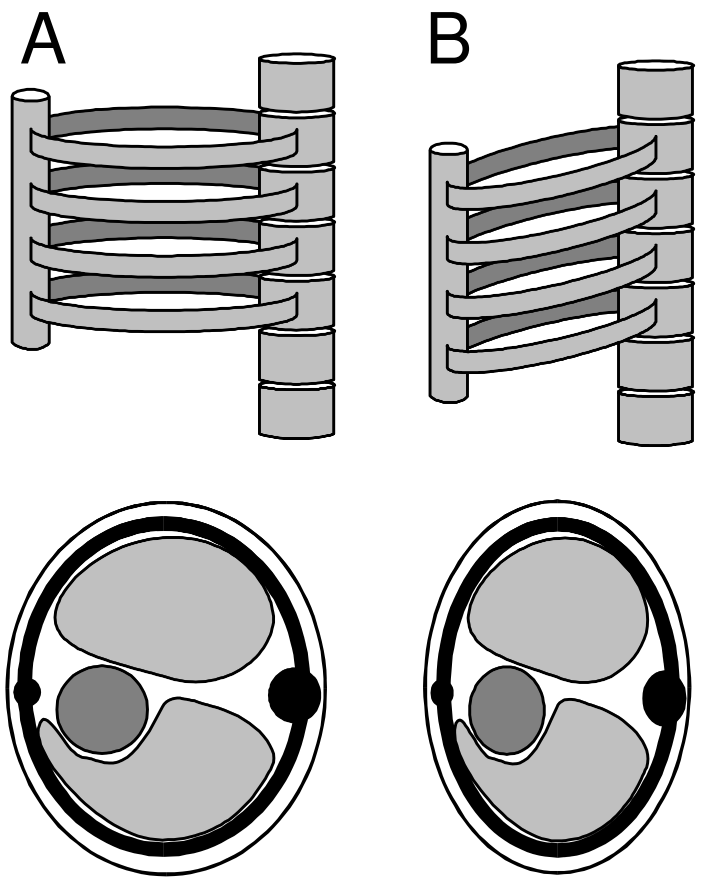


How does this work with real data?



A. Image of 700 ml ventilation
B. Image of 100 ml saline instillation in right lung
C. Image of 700 ml ventilation and 100 ml saline

Electrode Movement



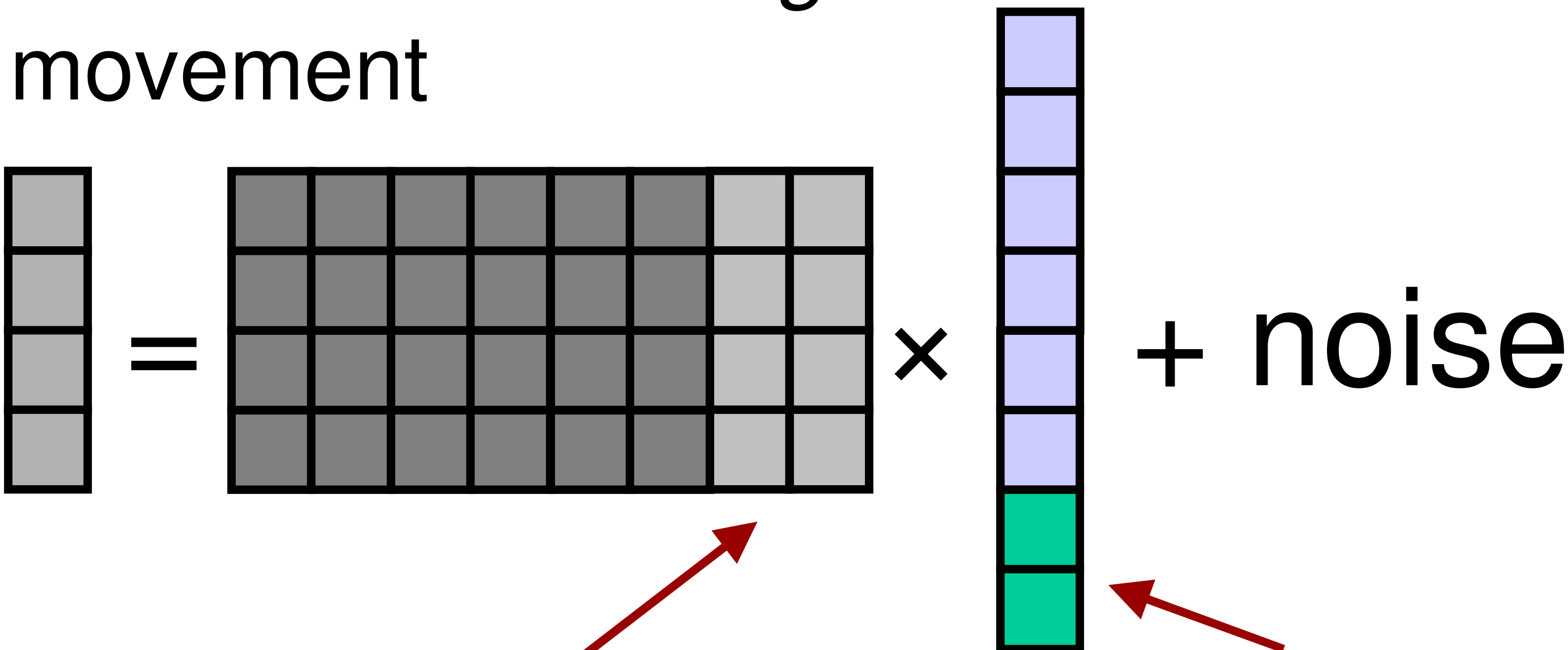
Electrodes move

- with breathing
- with posture change

Simulations show broad central artefact in images

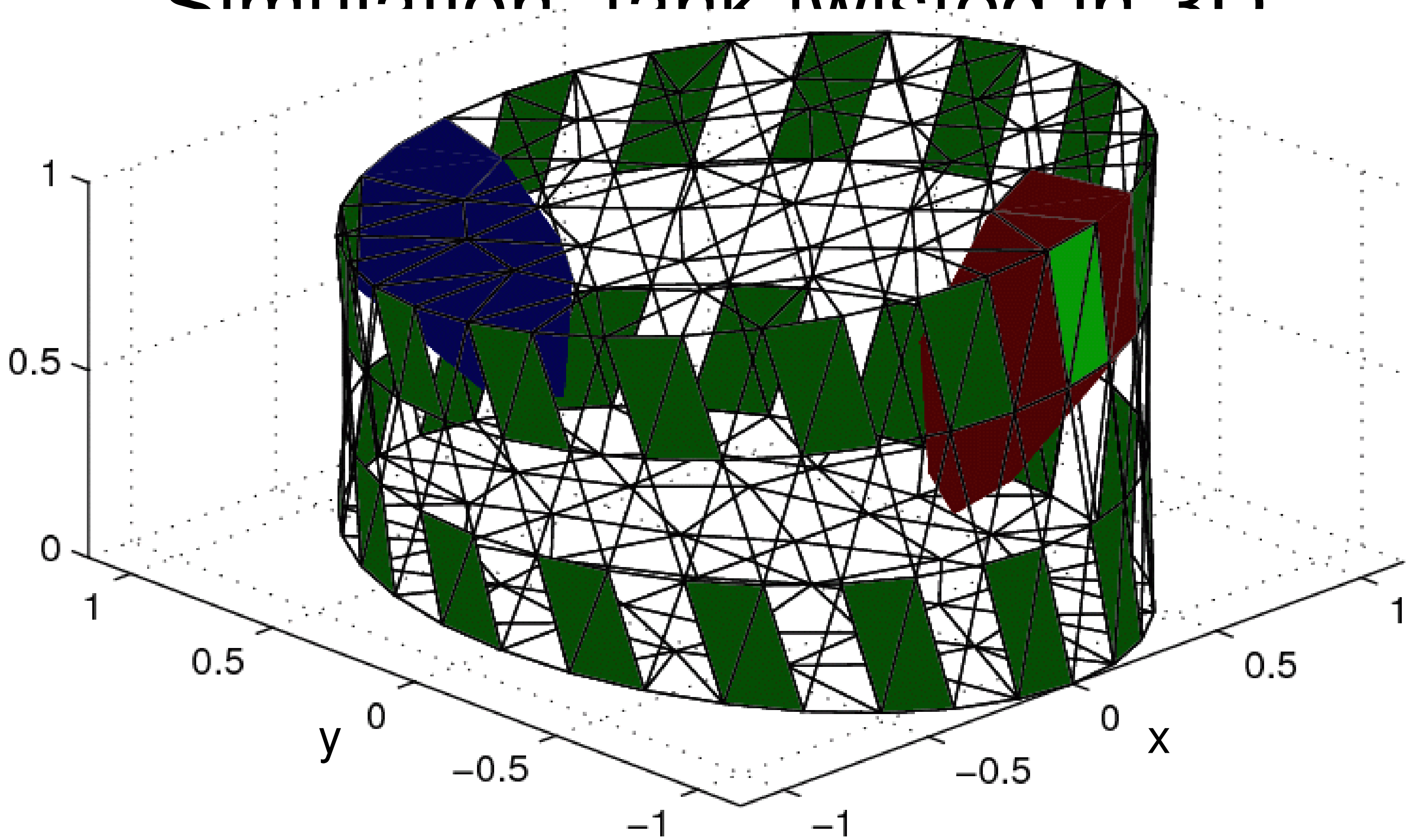
Imaging Electrode Movement

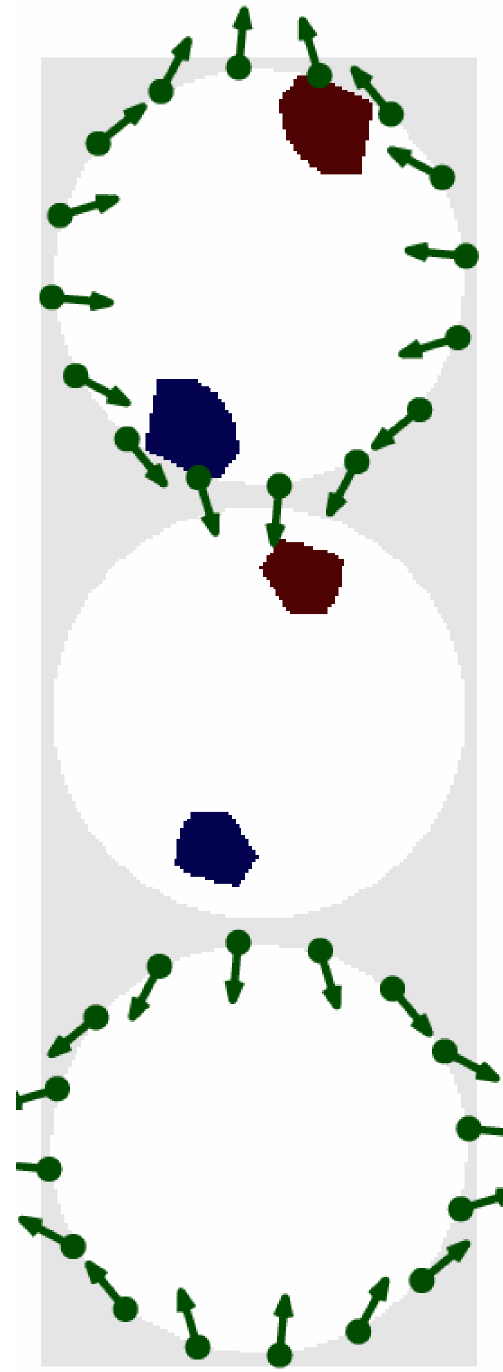
- Forward model *image* includes movement

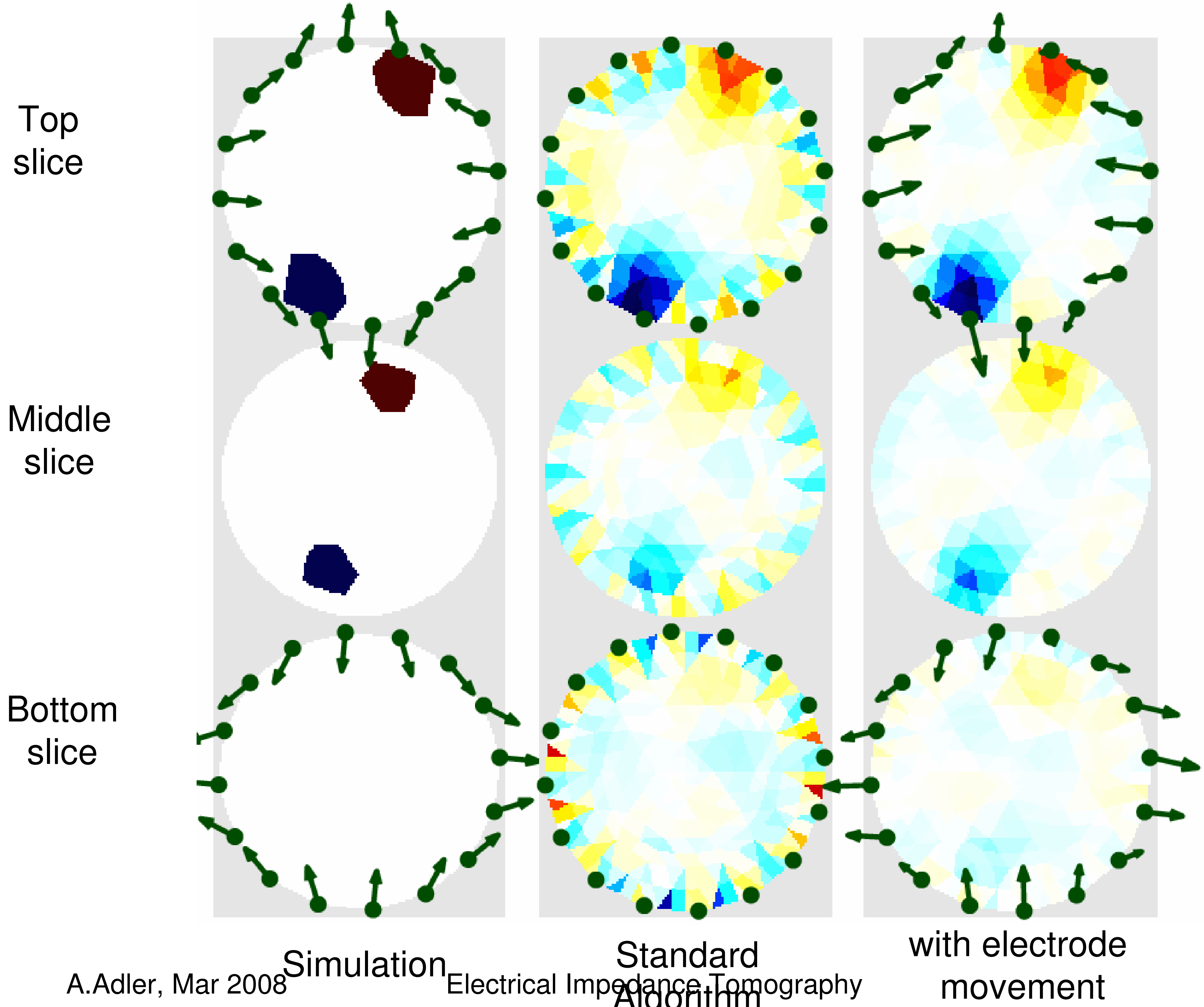


Images of electrode movement

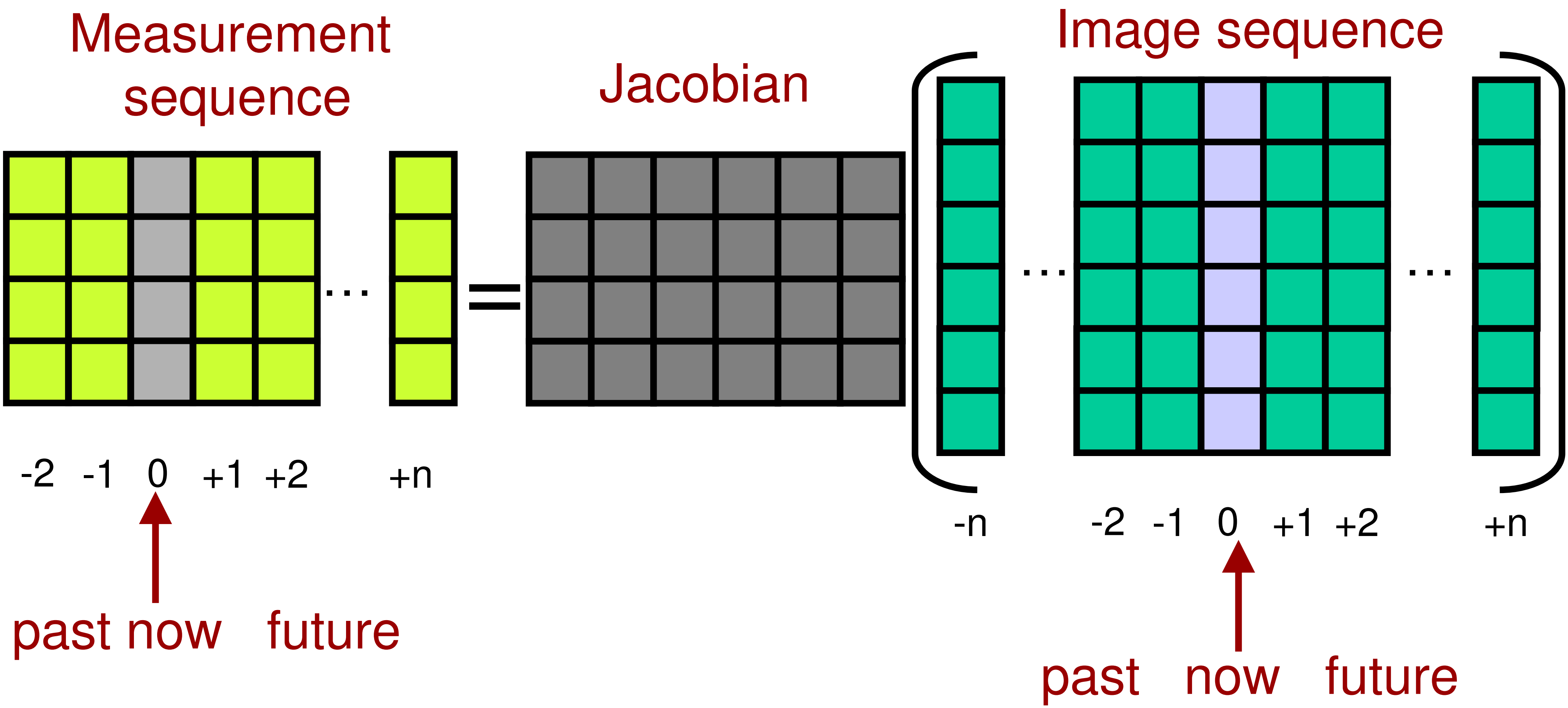
Simulation: tank twisted in 3D





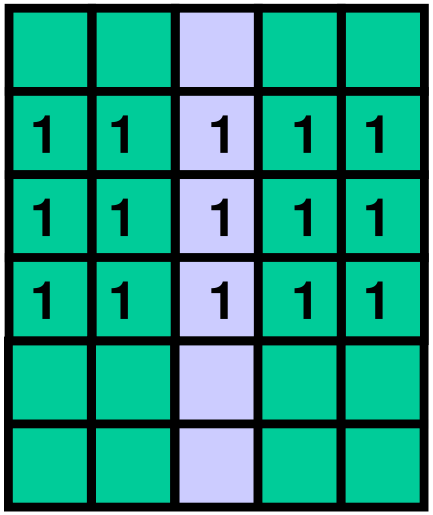

Top
slice
Middle
slice
Bottom
slice
Simulation
Standard
Algorithm
with electrode
movement

EIT makes fast measurements. Can we use this fact?

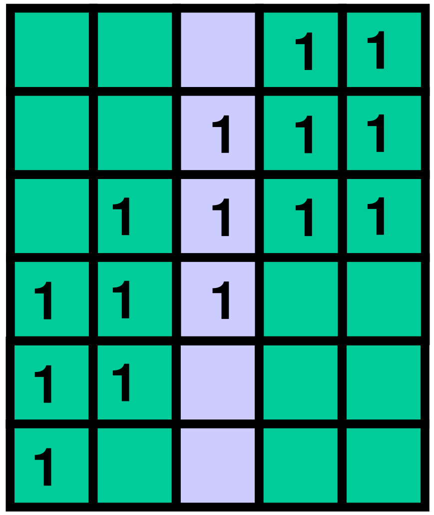


Temporal Reconstruction

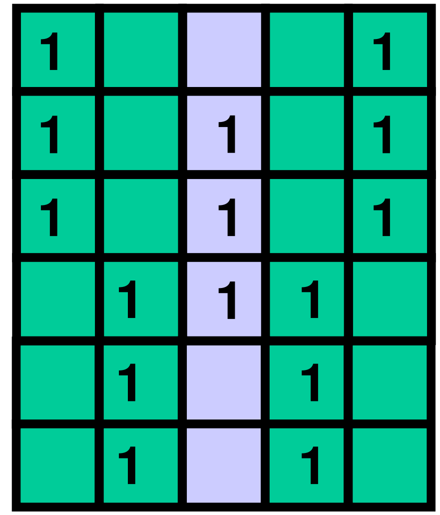
Temporal Penalty Functions



likely



quite likely



unlikely

Standard EIT approaches to not take this into account

GN vs. Temporal Inverse

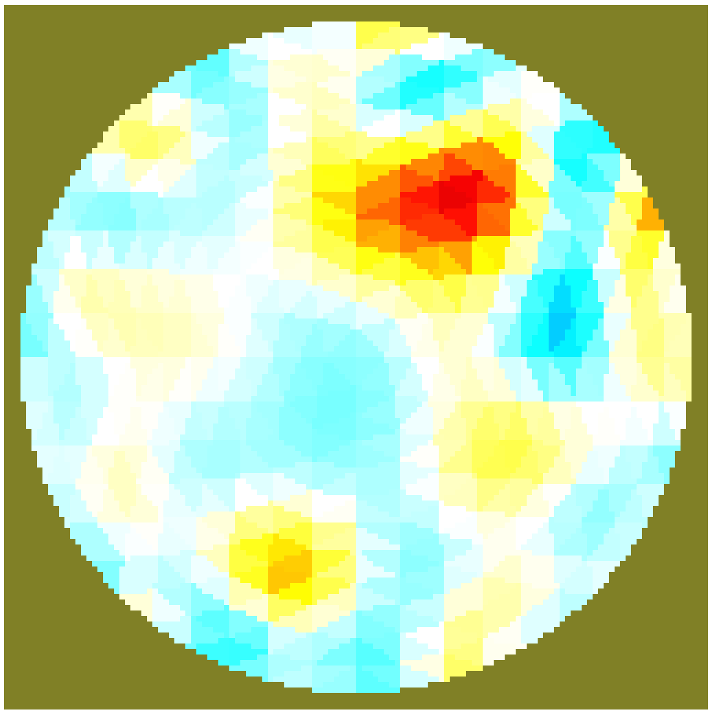
1. Noise free data (IIRC tank)
2. Data with added 6dB SNR noise

Gauss-Newton solver

Solve time = 5.33 s
(with caching) = 0.22 s

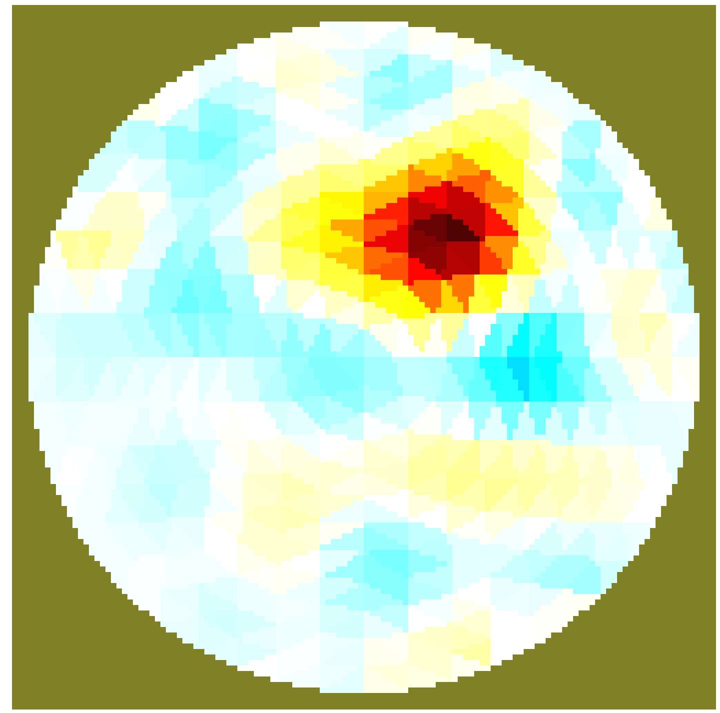
Temporal solver
(4 time steps)
Solve time = 34.81 s
(with caching) = 0.60 s

Gauss Newton vs. Temporal Inverse (6db SNR)



Gauss-Newton solver

Solve time = 5.33 s
(with caching) = 0.22 s



Temporal solver
(4 time steps)
Solve time = 34.81 s
(with caching) = 0.60 s