Electrical Impedance Tomography: *recent advances in Image Reconstruction*

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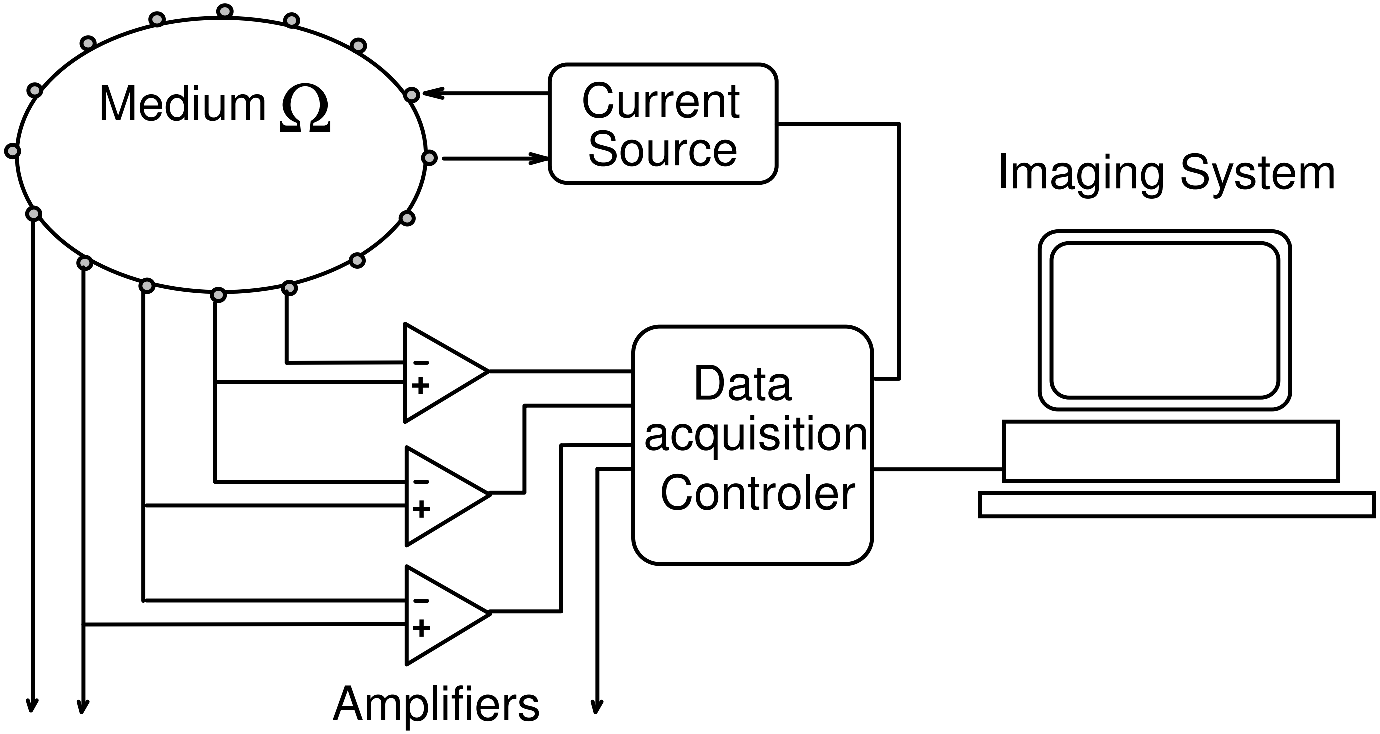
Outline

- Electrical Impedance Tomography
- Physics and Image Reconstruction
- Measurement Difficulties and solutions
  - Electrode Errors
  - Electrode Movement
  - Temporal Filtering
- EIDORS Project

Electrical Impedance Tomography: Block Diagram



EIT: Applications

EIT can image physiological processes involving movement of conductive fluids and gasses

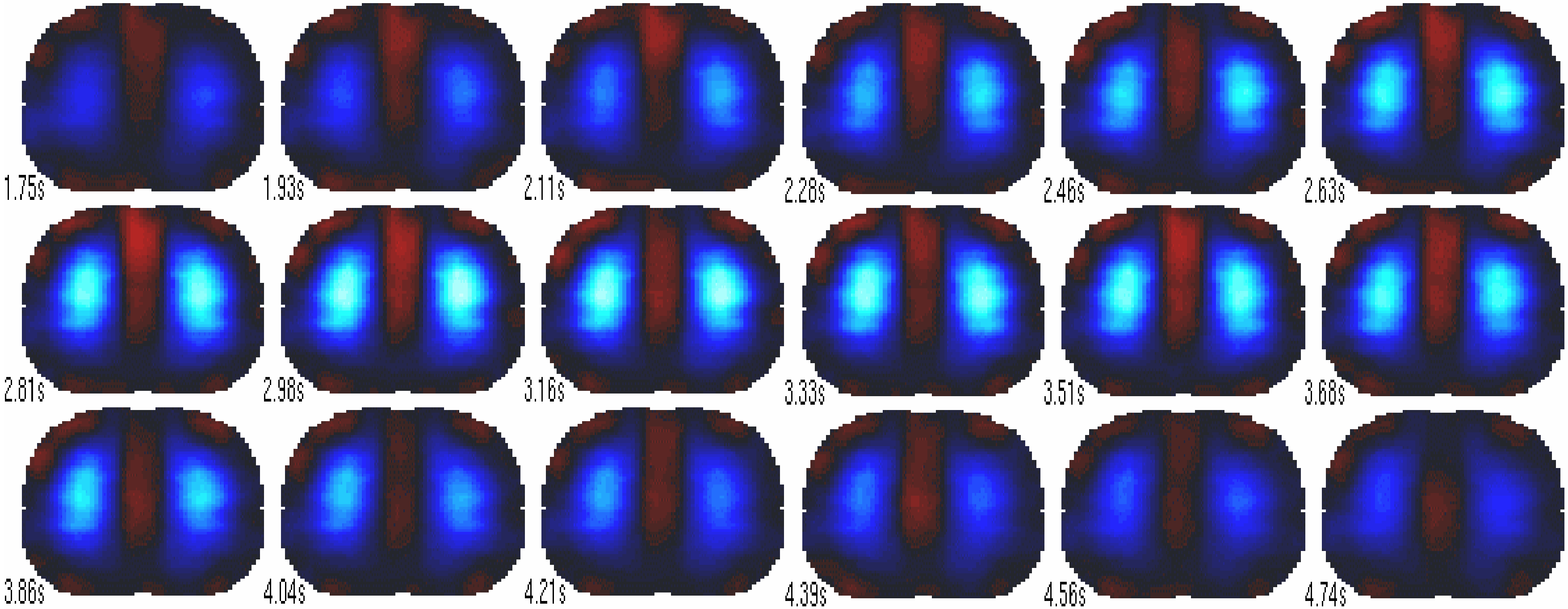
- Lungs
- Heart / perfusion
- GI tract
- Brain
- Breast

EIT: Advantages

EIT is a relatively low resolution imaging modality, *but*

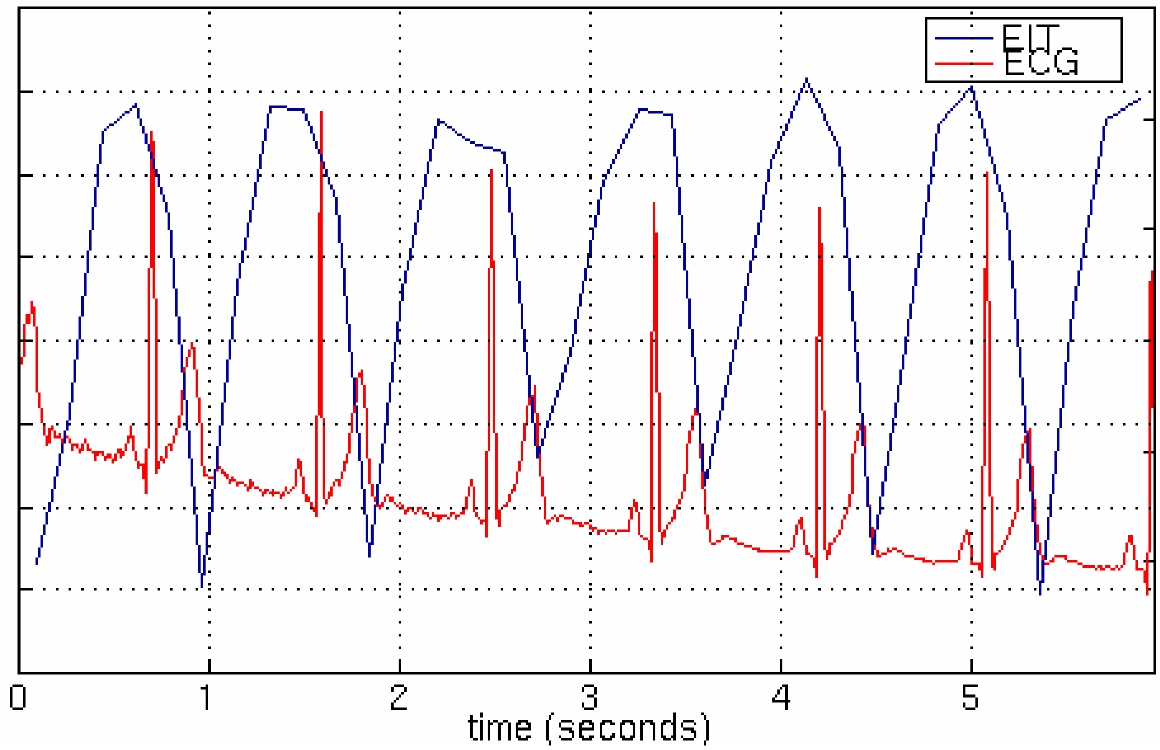
- Non-invasive
- Non-cumbersome
- Suitable for monitoring
- Underlying technology is low cost

Application: Breathing



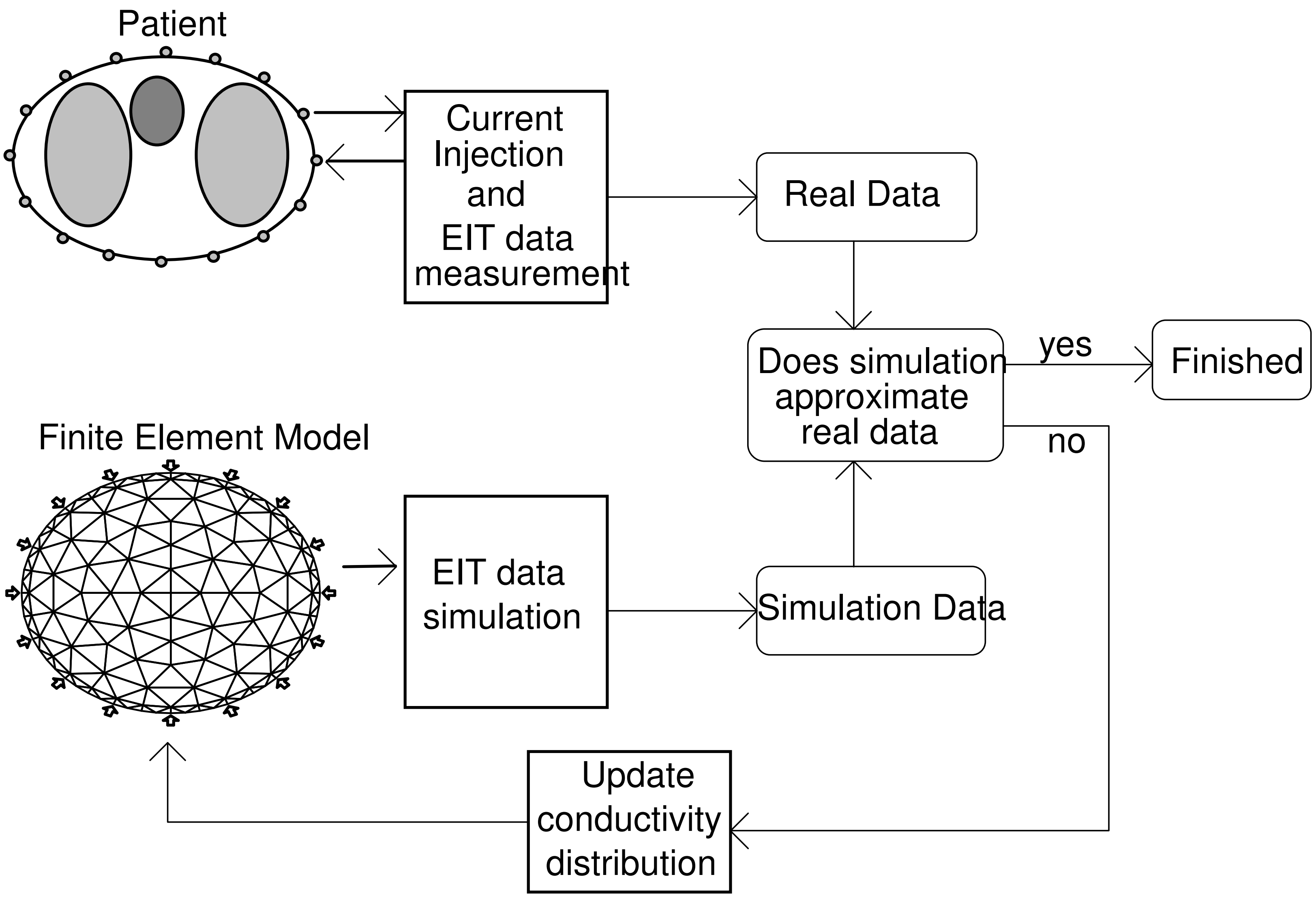
Chest images of tidal breathing in normal

Application: Heart Beat



EIT signal in ROI around heart and ECG

Iterative Image Reconstruction



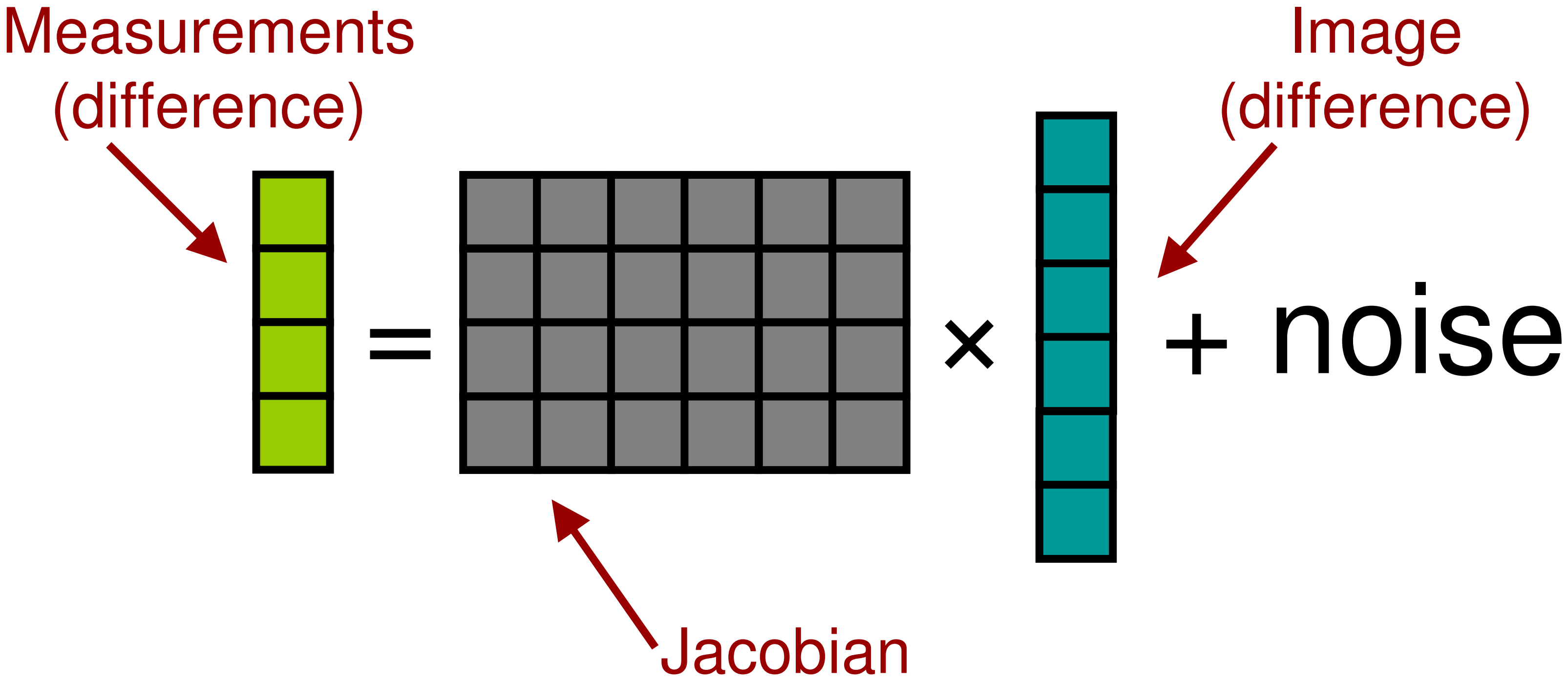
Dynamic Imaging

- Calculate Δ conductivity
  from Δ measurements

- Inverse problem *linearized*
- reduced sensitivity to electrode and hardware errors.
- Suitable for physiological imaging: lung, heart, GI

Image Reconstruction

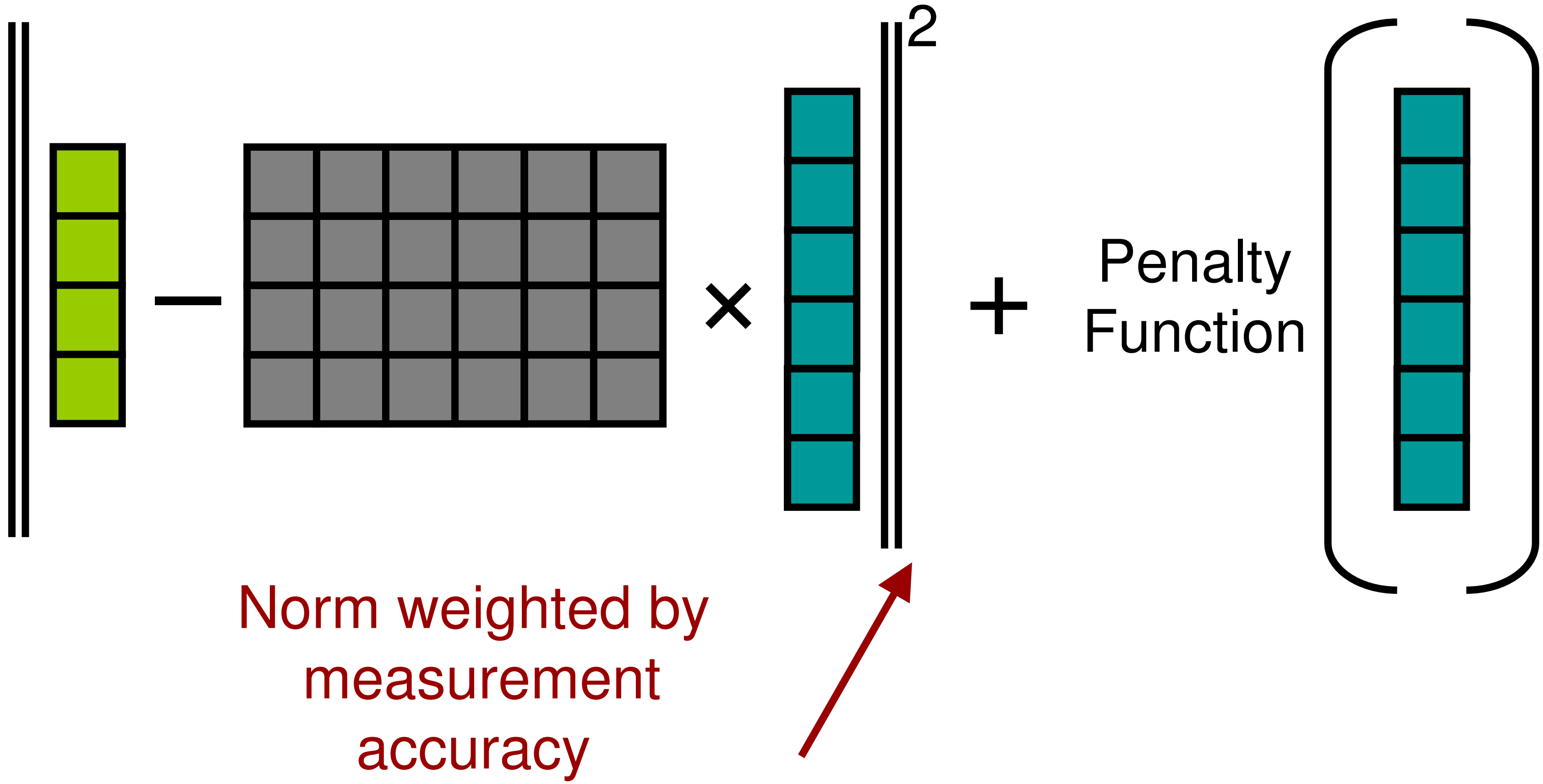
- Forward Model (linearized)



System is underdetermined

Image Reconstruction

Regularized linear Inverse Model



Measurement Norm

Penalize measurements by the SNR of each channel (ie 1/noise variance)

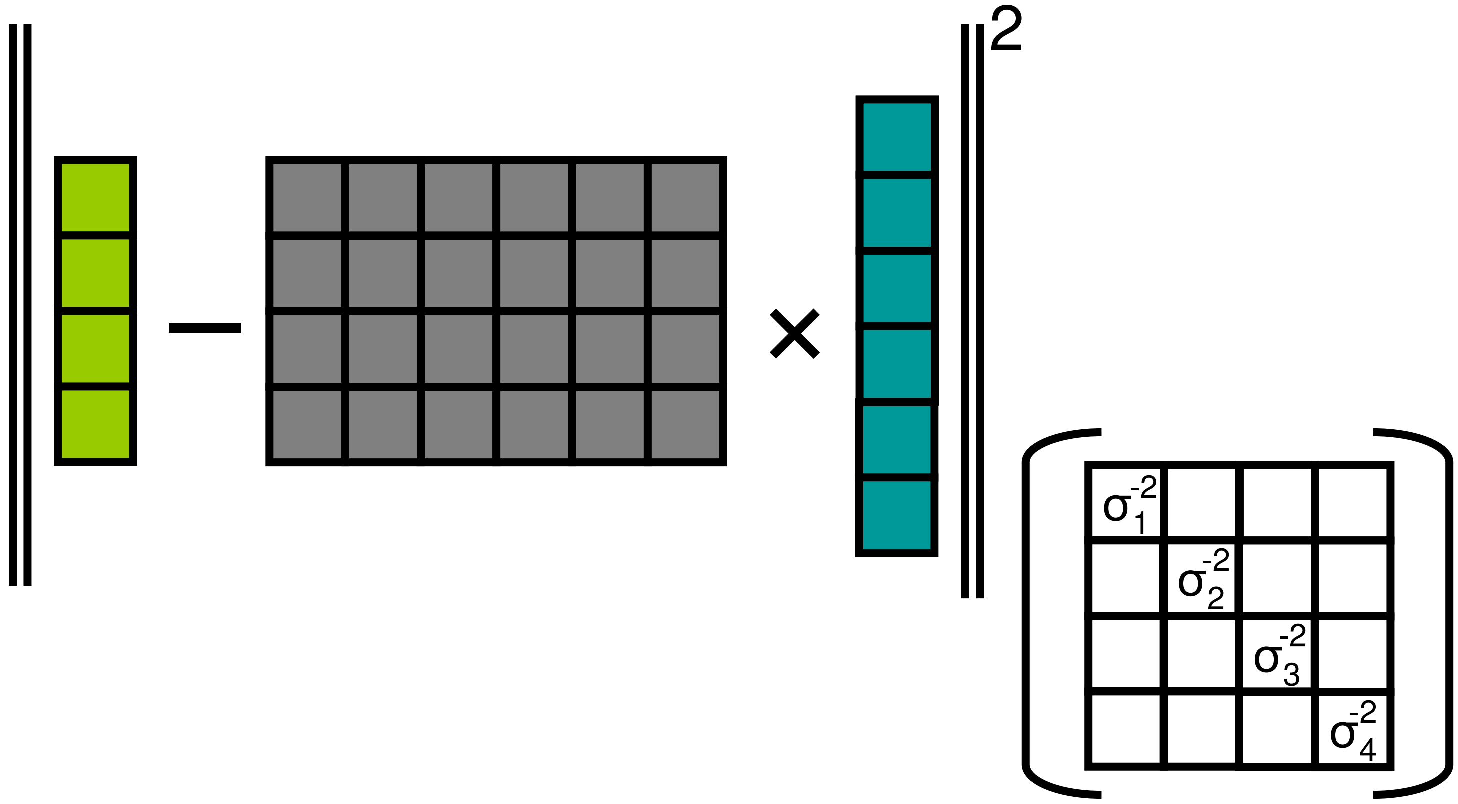


Image Reconstruction

Image Penalty Function

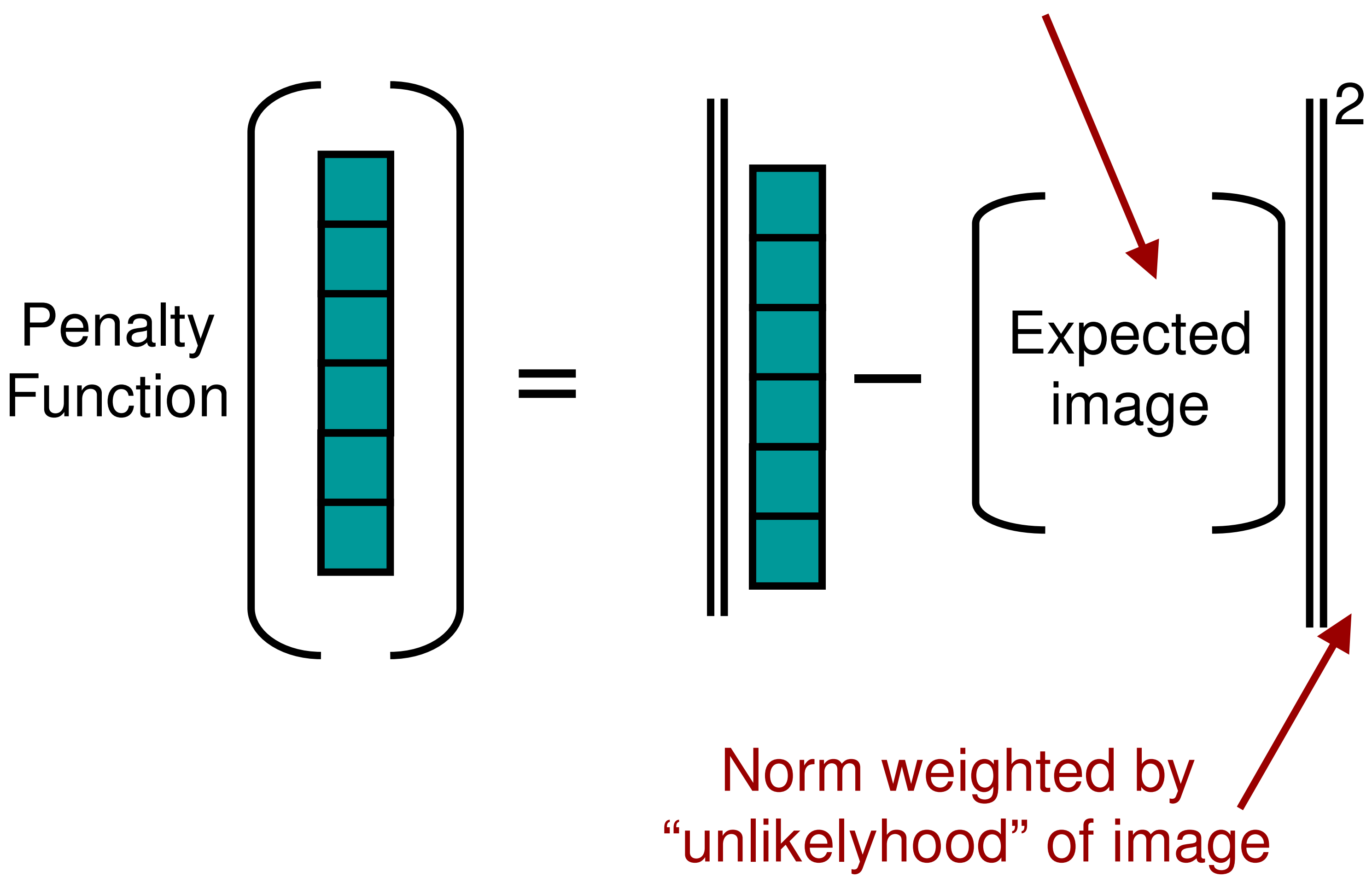


Image Reconstruction

- Penalty functions: Image Amplitude

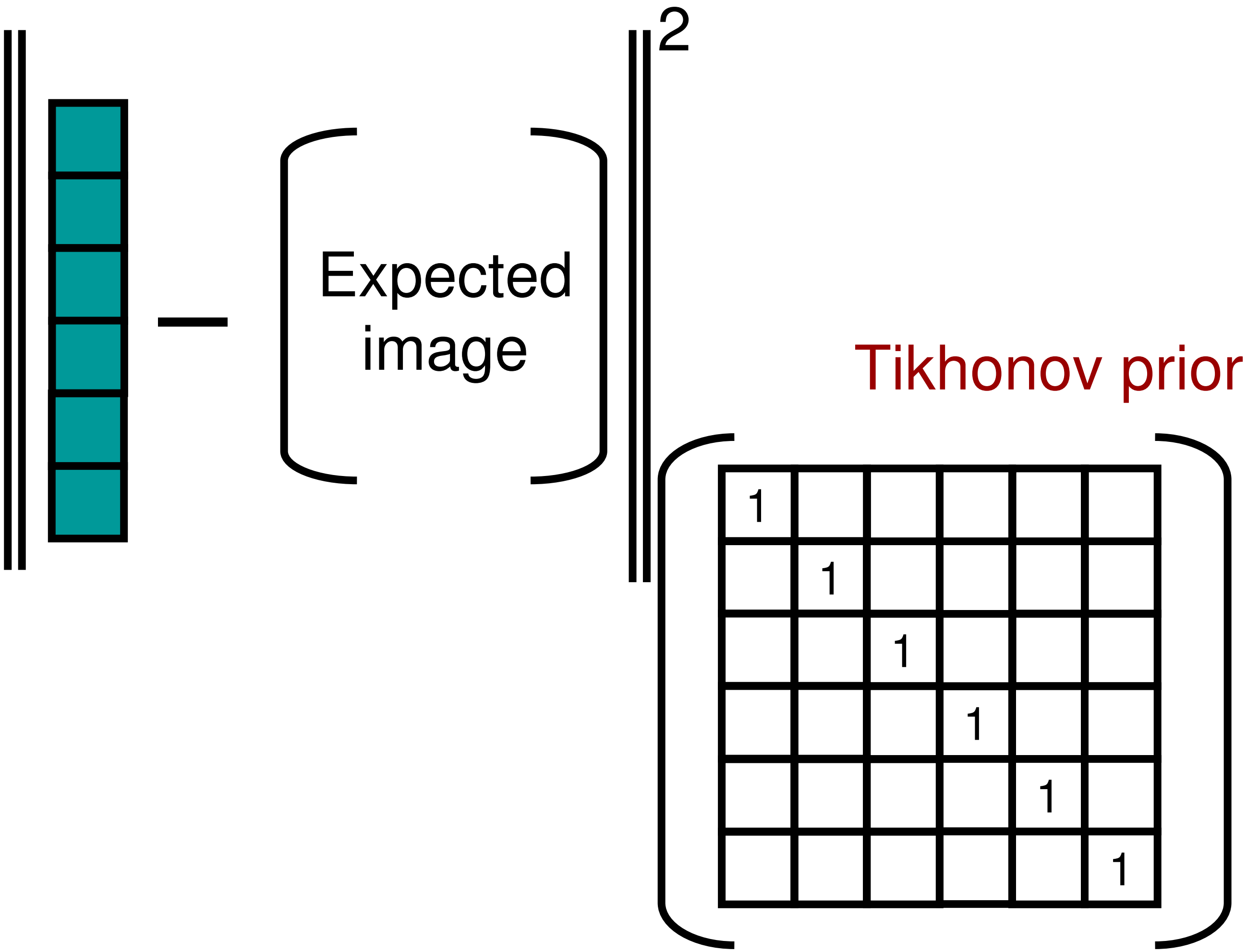
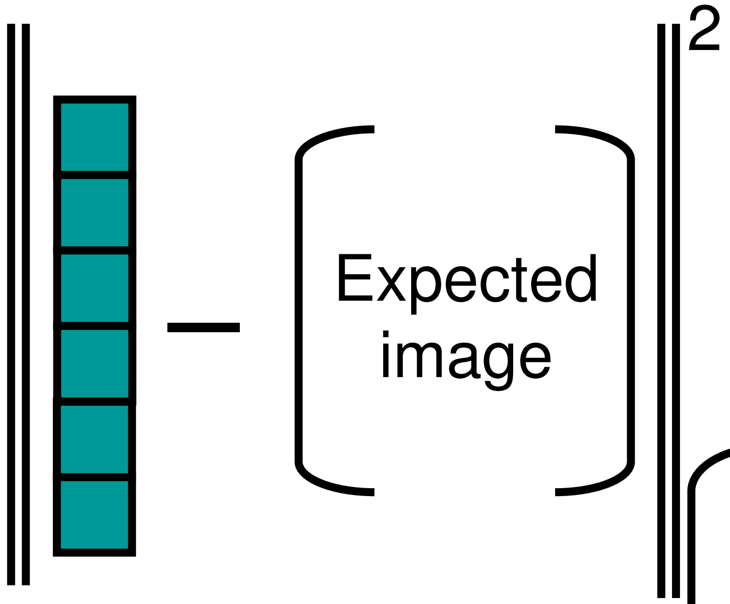


Image Reconstruction

- Penalty functions: Image Smoothness



Laplacian prior

1	-½				
-½	1	-½			
	-½	1	-½		
		-½	1	-½	
			-½	1	-½
				-½	1

Compare Penalty Functions

Images

(A)	(B)
	1
1	
1	1
1	
	1

Priors

1					
	1				
		1			
			1		
				1	
					1

1	-½				
-½	1	-½			
	-½	1	-½		
		-½	1	-½	
			-½	1	-½
				-½	1

Penalties

A) 3
B) 3

More reasonable
Image A is more likely

A) 1
B) 3

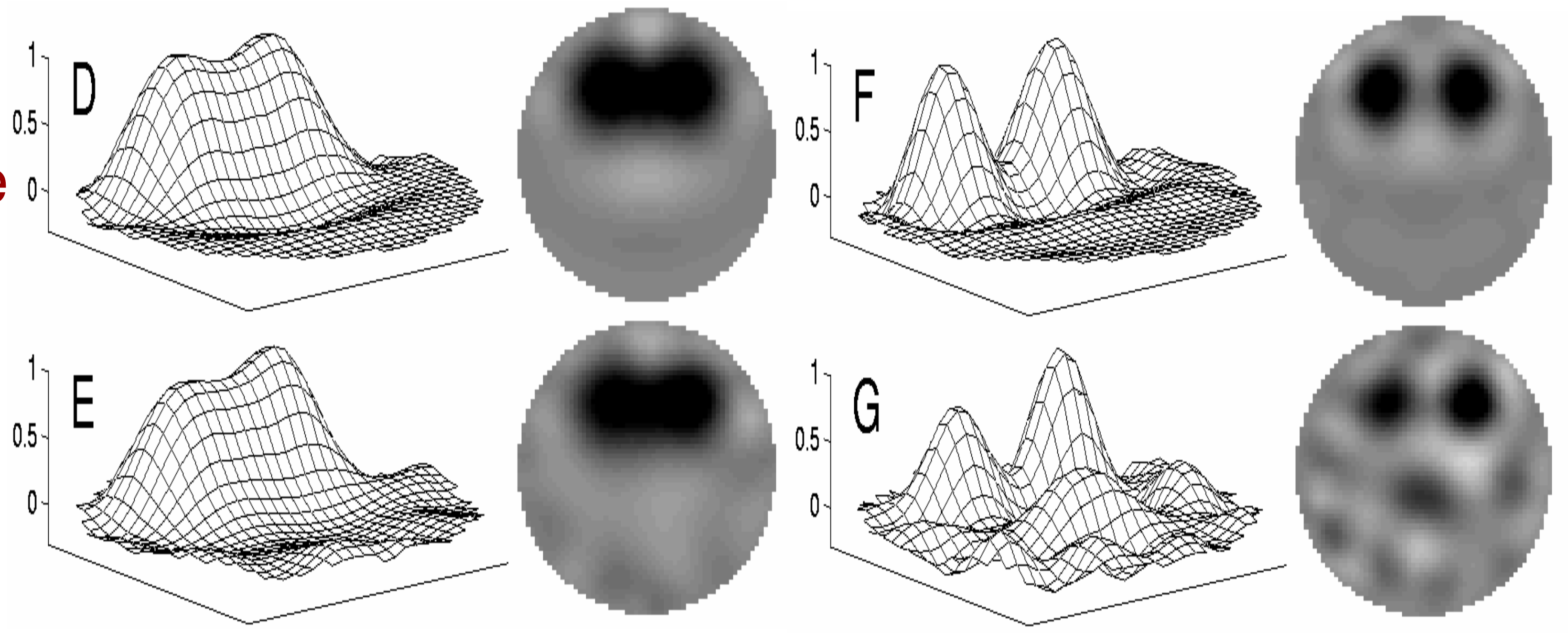
Noise – Resolution Tradeoff

Lots of Regularization (large penalty)

Little Regularization (small penalty)

No Noise

-3dB SNR



Applications …

- Electrode Errors
- Electrode Movement
- Temporal Filtering

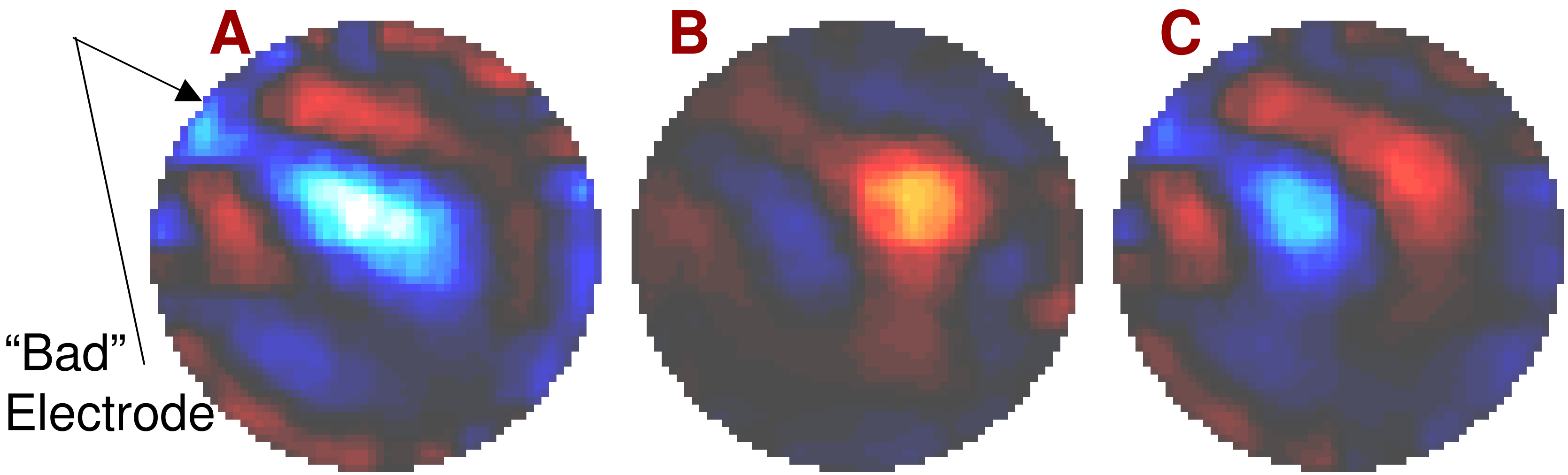
Electrode Measurement Errors

Experimental measurements with EIT quite often show large errors from one electrode

Causes aren't always clear

- Electrode Detaching
- Skin movement
- Sweat changes contact impedance
- Electronics Drift?

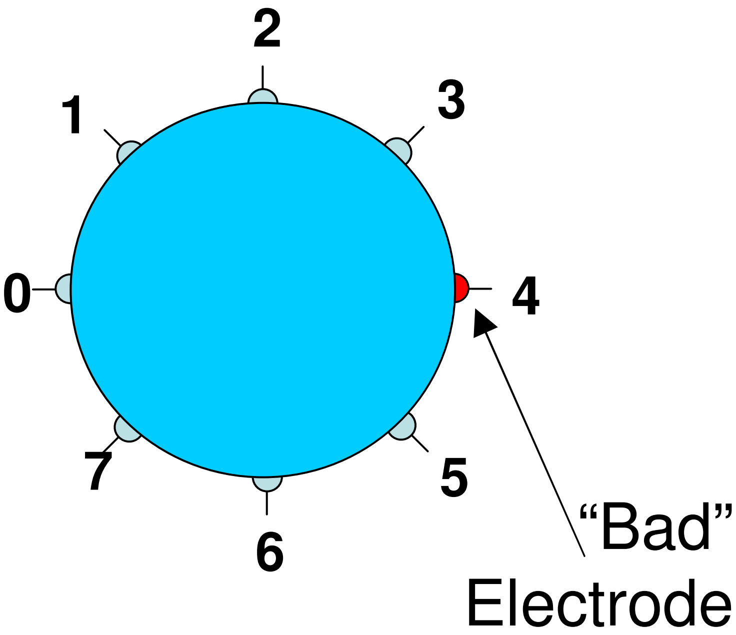
Example of electrode errors



Images measured in anaesthetised, ventilated dog

A. Image of 700 ml ventilation
B. Image of 100 ml saline instillation in right lung
C. Image of 700 ml ventilation and 100 ml saline

Measurements with “bad” electrode



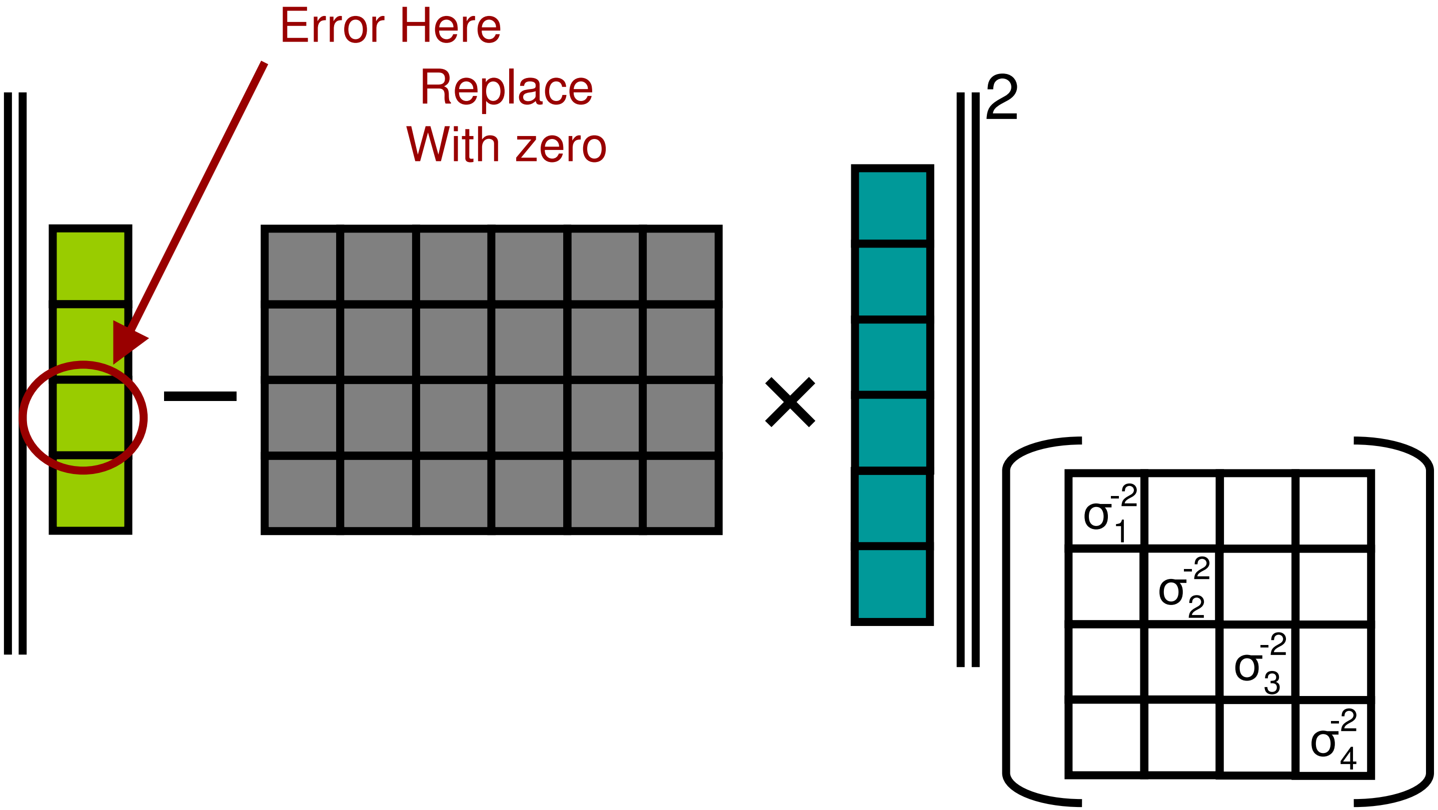
	01	12	23	34	45	56	67	70
01	X	X		*	*			X
12	X	X	X	*	*			
23		X	X	X	*			
34	*	*	X	X	X	*	*	*
45	*	*	*	X	X	X	*	*
56				*	X	X	X	
67				*	*	X	X	X
70	X			*	*		X	X

* “bad” measurement

X measurement at current injection

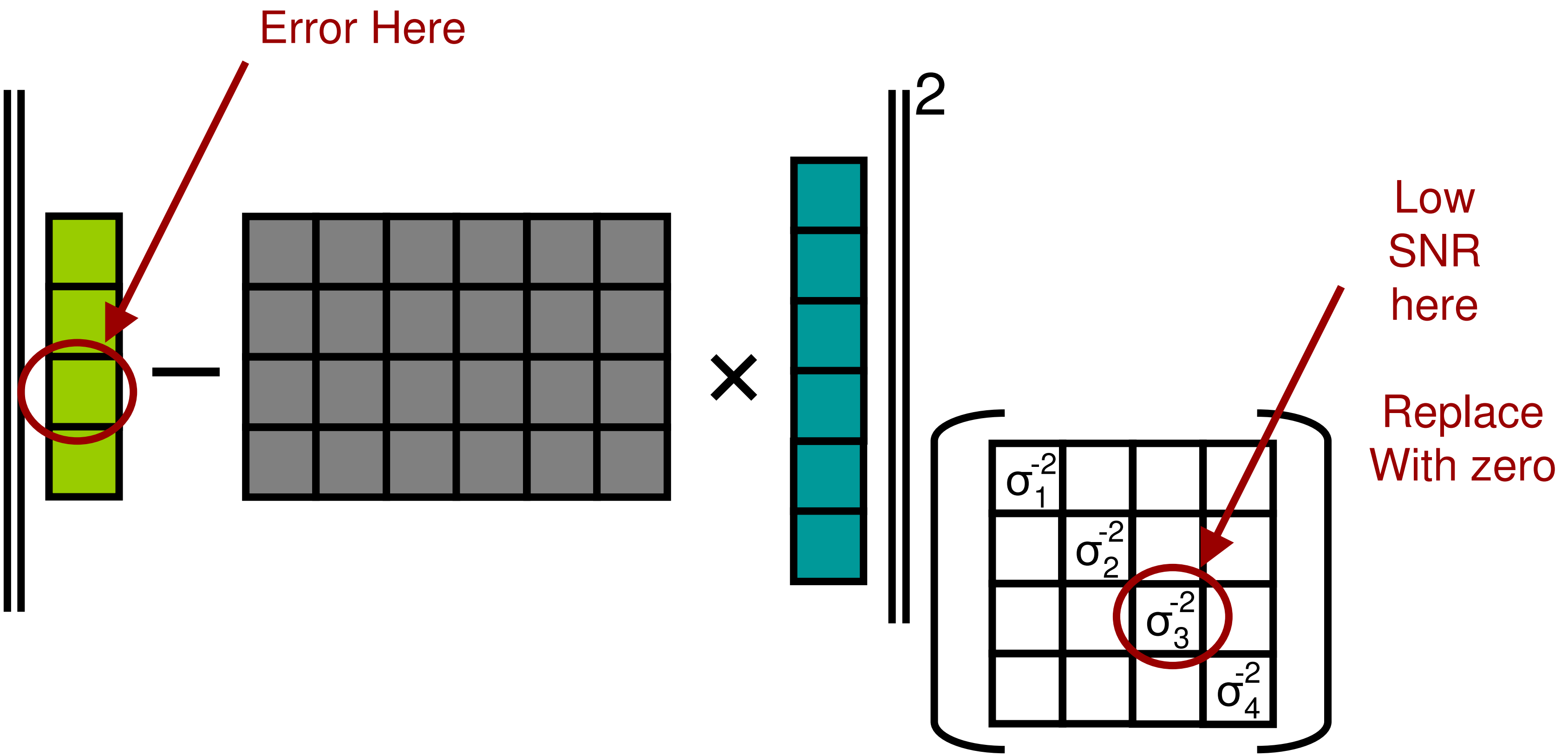
"Zero bad data" solution

"Traditional solution" (in the sense that I've done this)

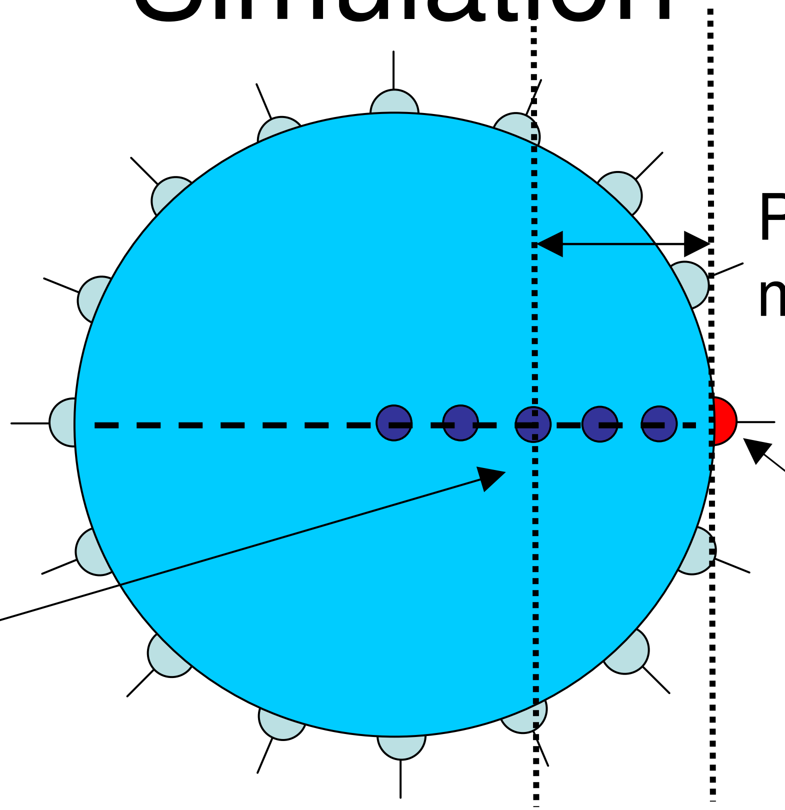


Regularized imaging solution

Electrode errors are large measurement noise on affected electrode



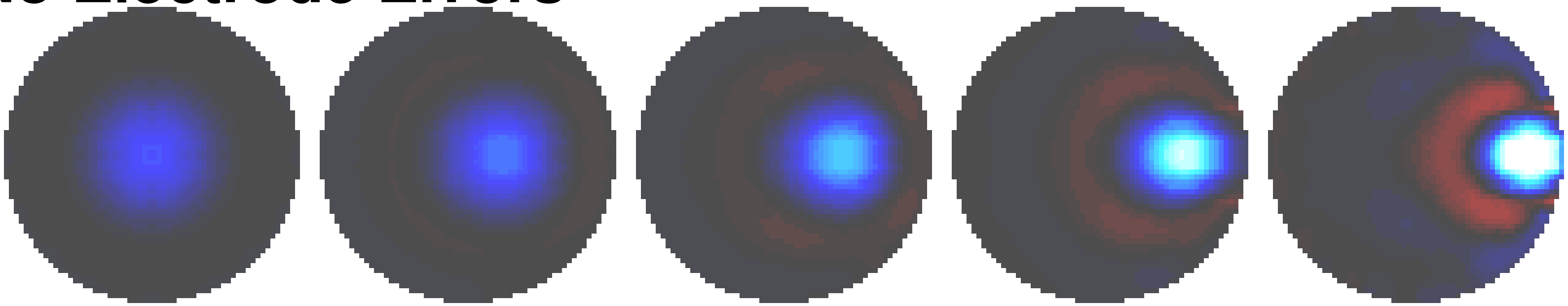
Simulation



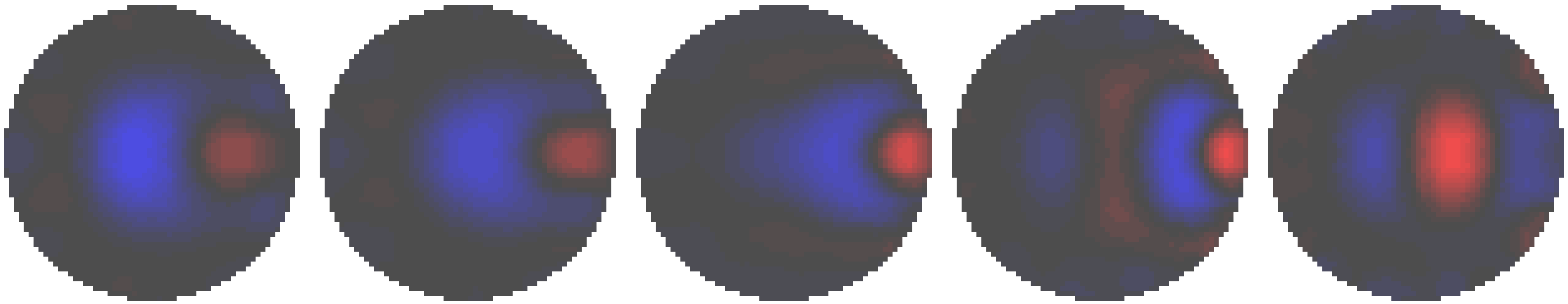
Data simulated with 2D FEM with 1024 elements
– not same as inverse model

Simulation results for opposite drive

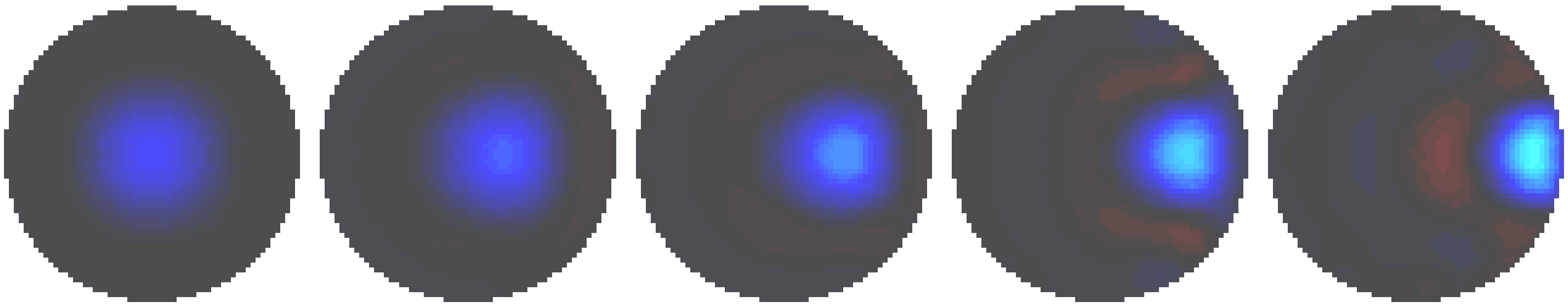
No Electrode Errors



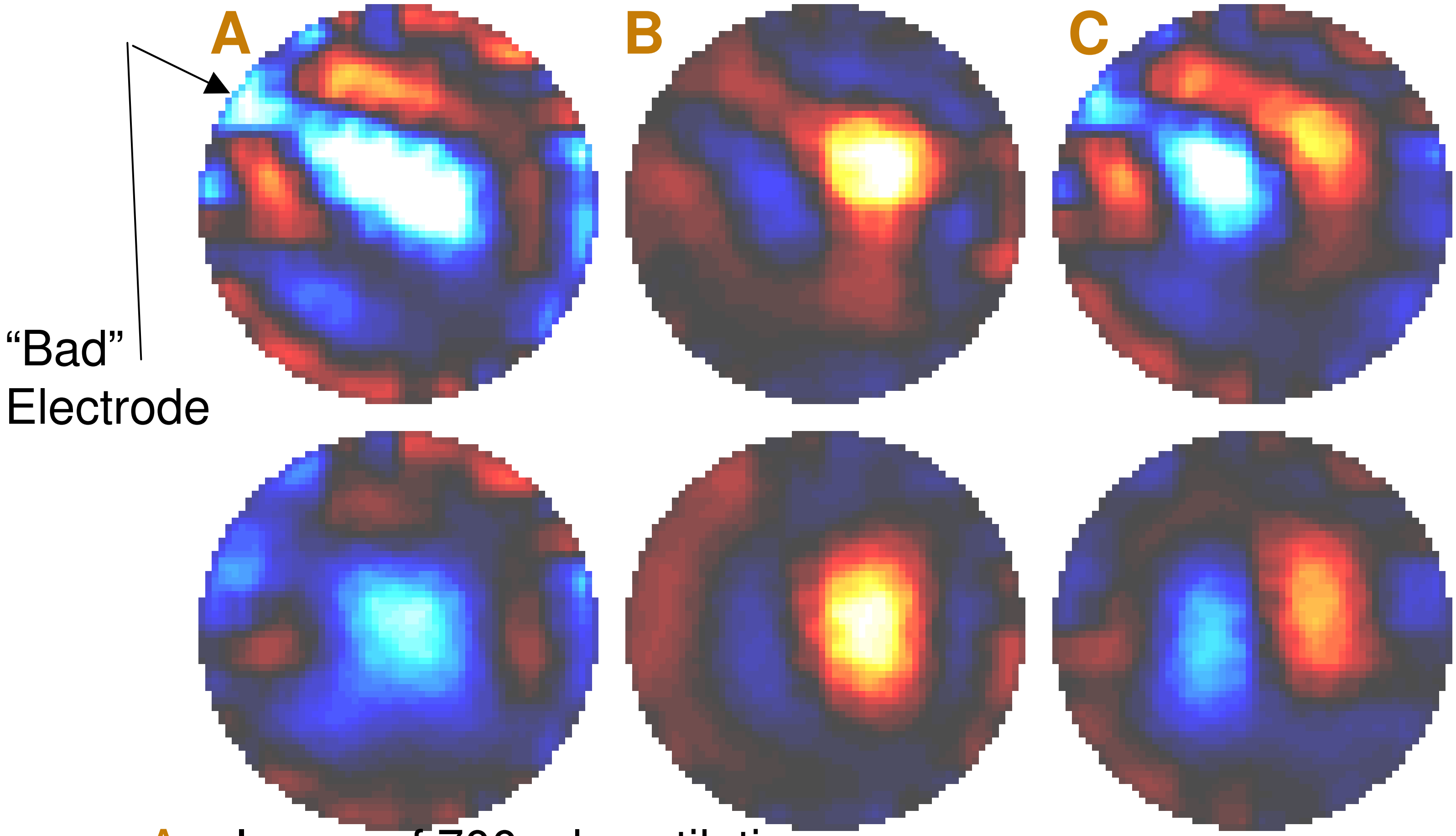
Zero Affected Measurements



Bayesian Inverse

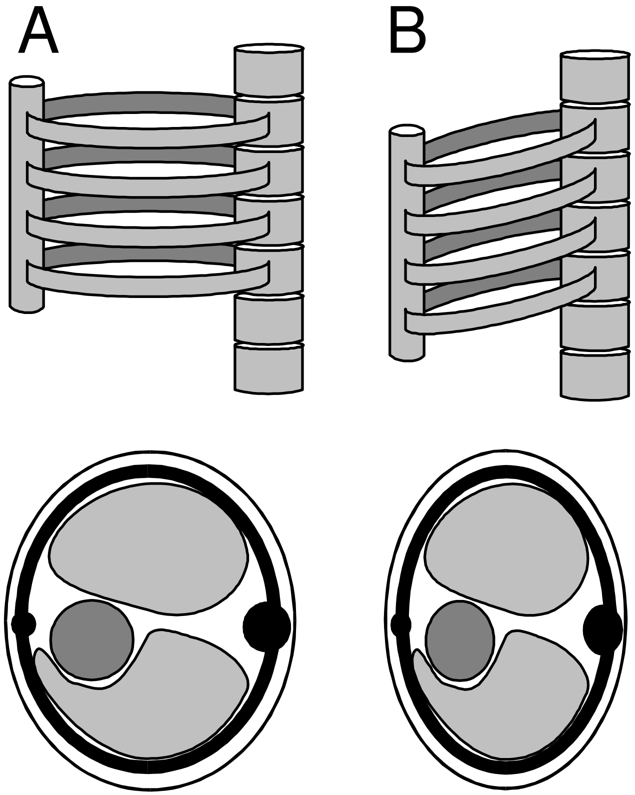


How does this work with real data?



A. Image of 700 ml ventilation
B. Image of 100 ml saline instillation in right lung
C. Image of 700 ml ventilation and 100 ml saline

Electrode Movement



Electrodes move

- with breathing
- with posture change

Simulations show broad central artefact in images

Imaging Electrode Movement

- Forward model *image* includes movement

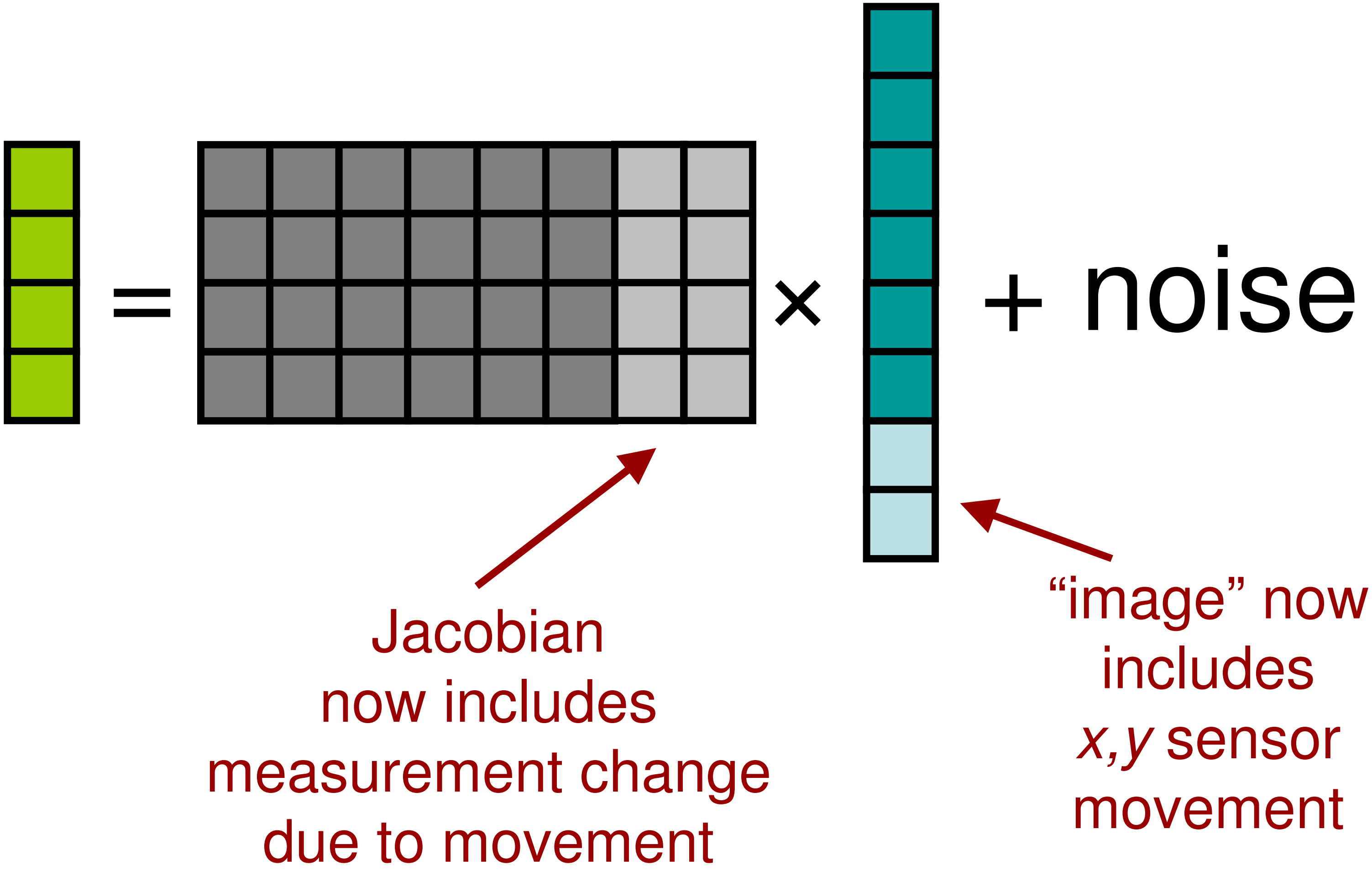
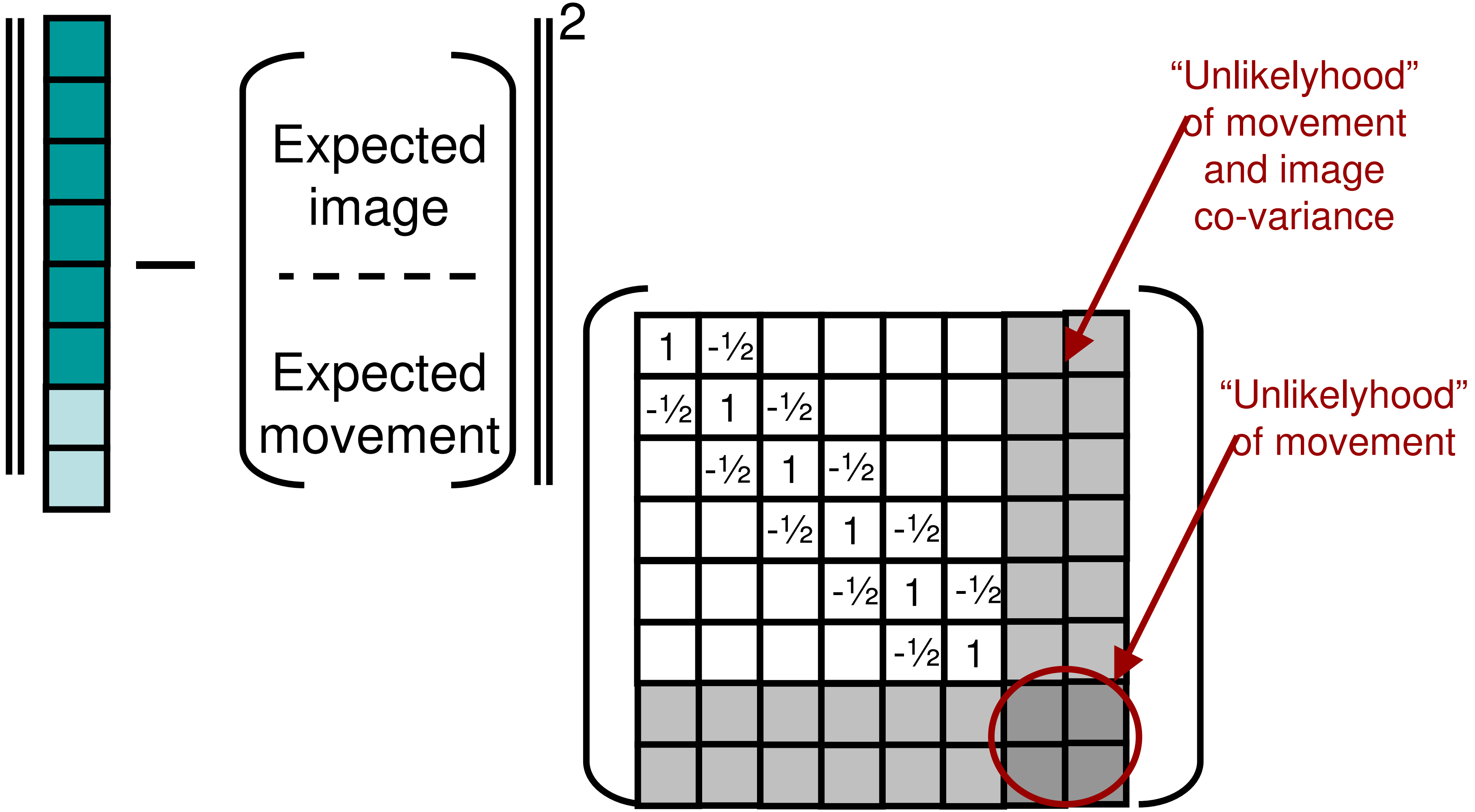


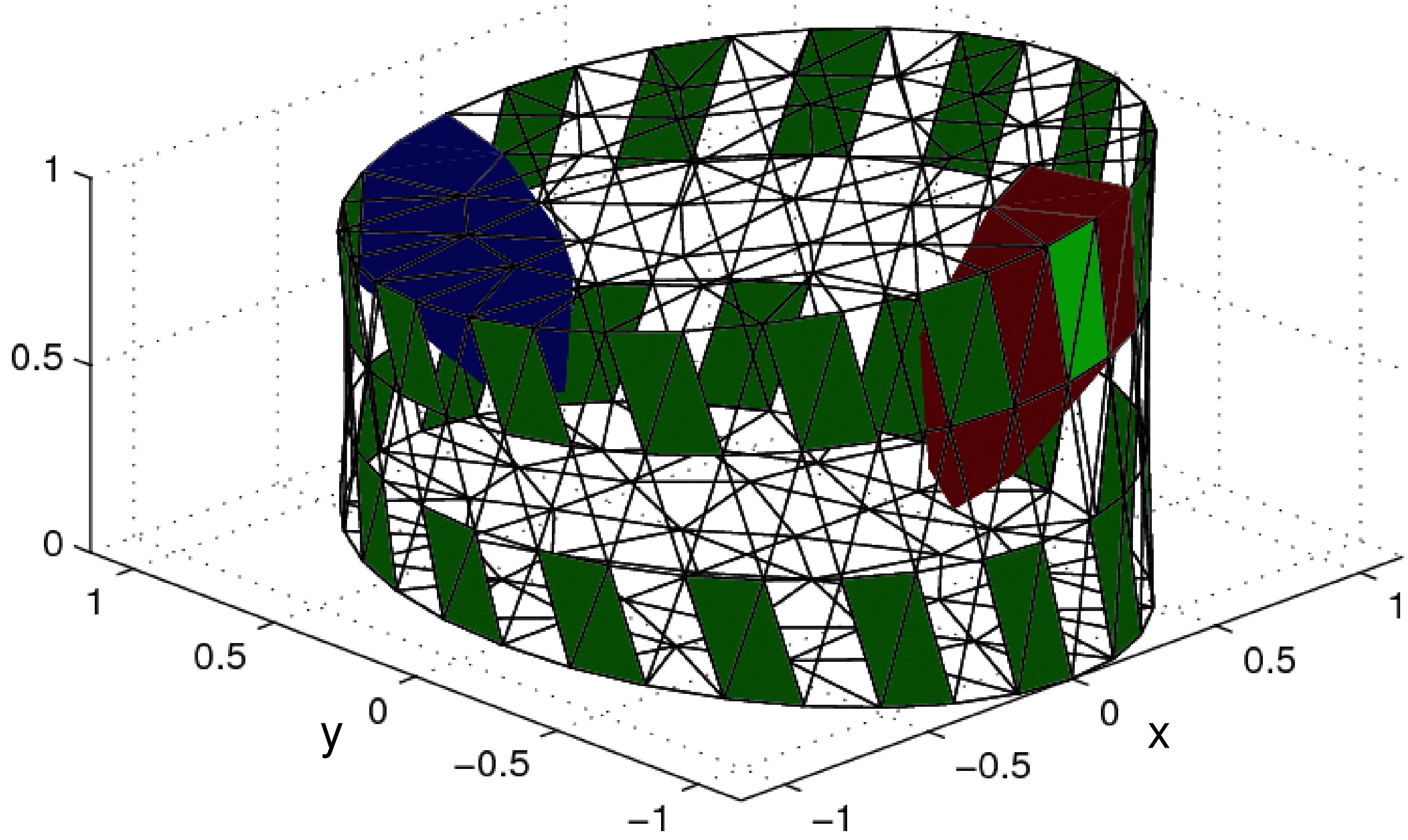
Image and movement

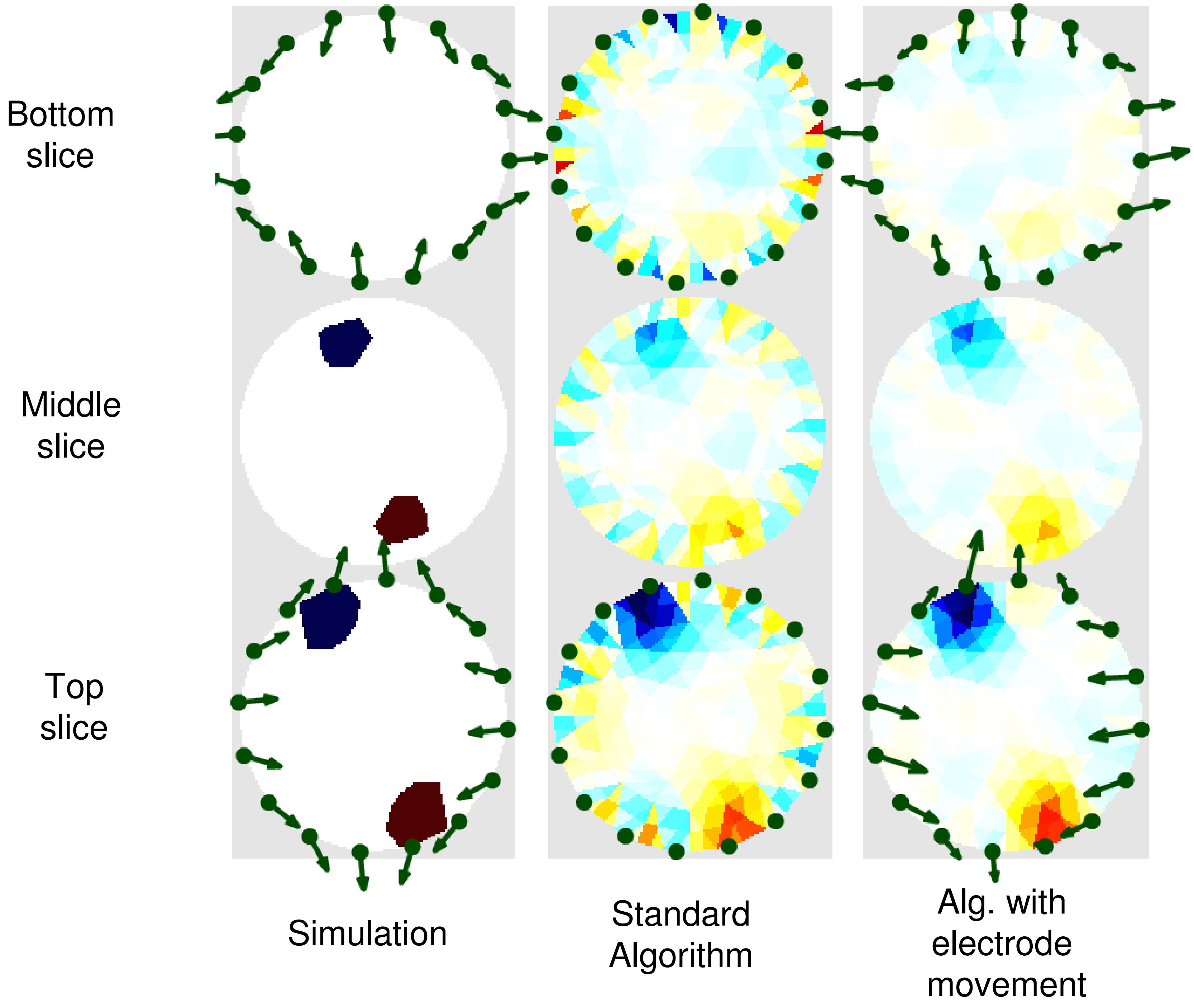
Penalty: Image and movement Smoothness



Images of electrode movement

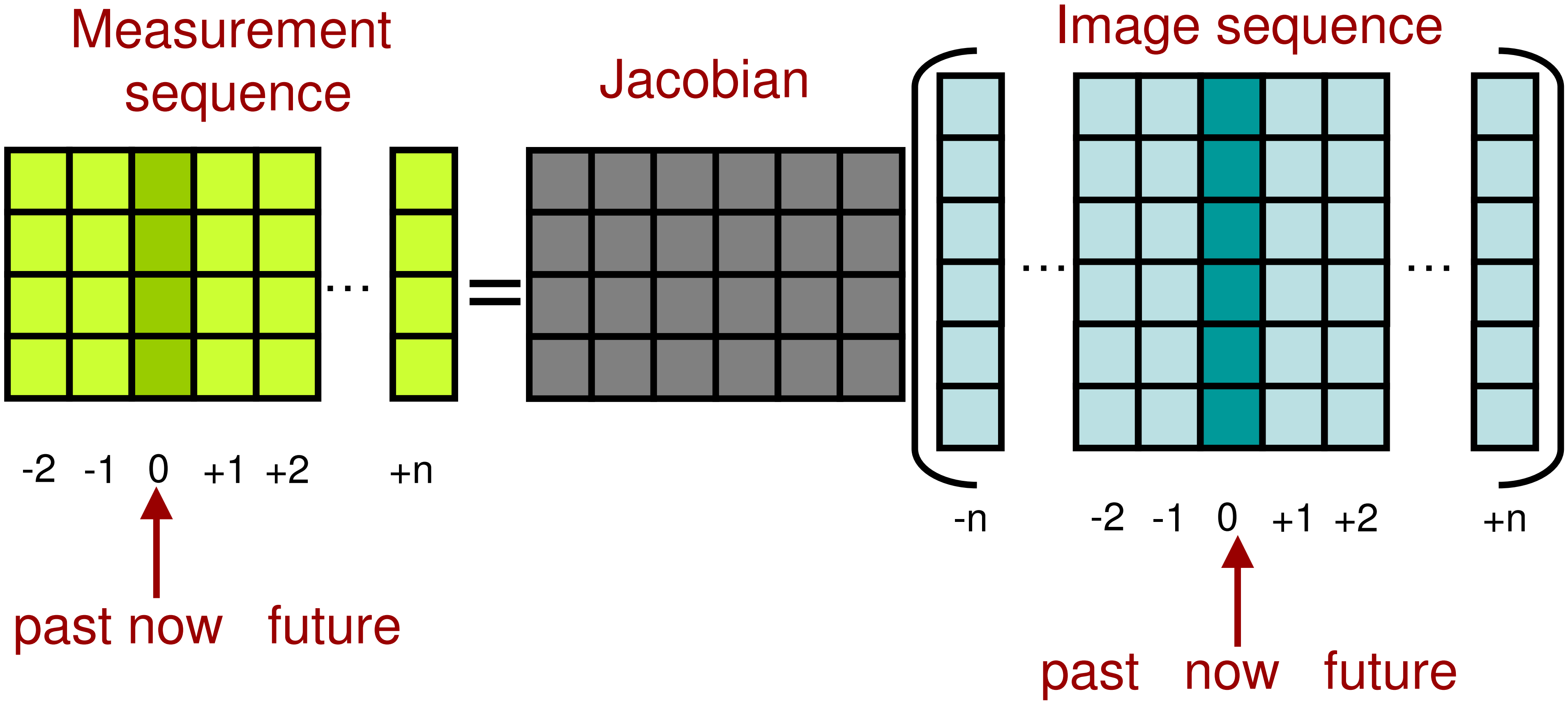
Simulation: tank twisted in 3D





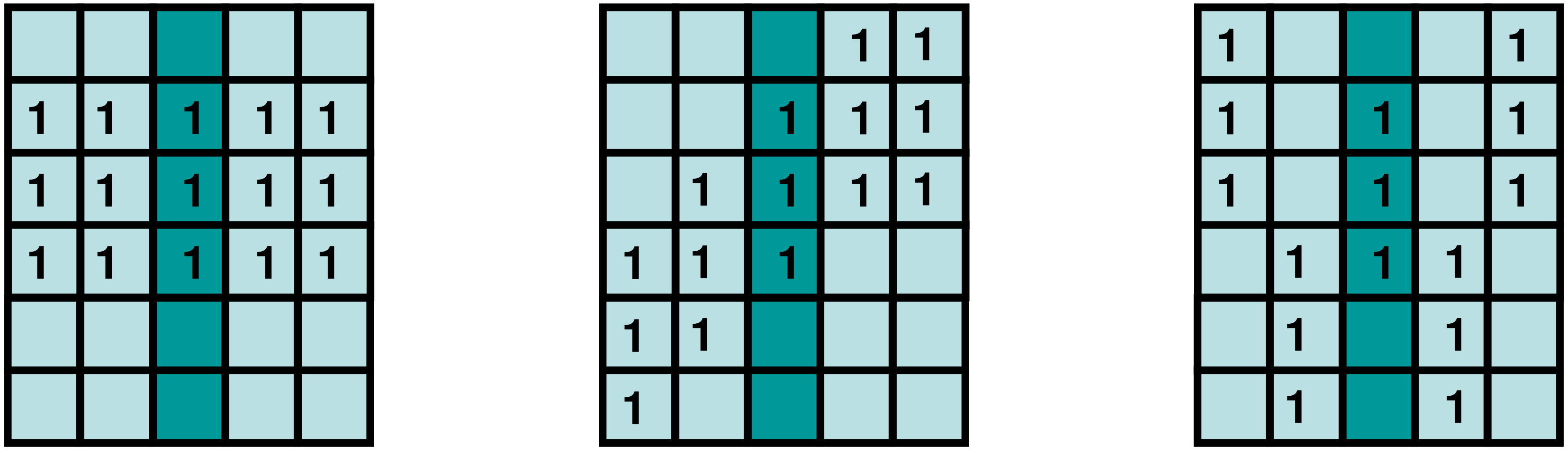
Bottom slice
Middle slice
Top slice
Simulation
Standard Algorithm
Alg. with electrode movement

EIT makes fast measurements. Can we use this fact?



Temporal Reconstruction

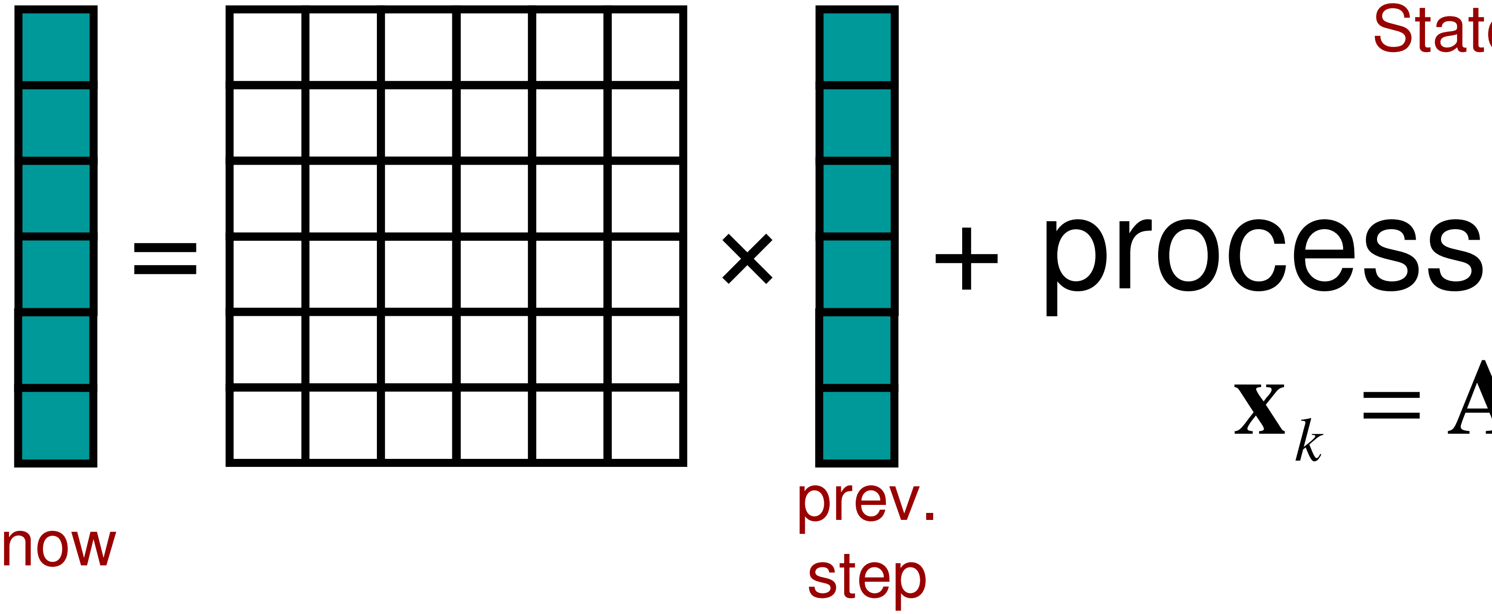
Temporal Penalty Functions



likely quite likely unlikely

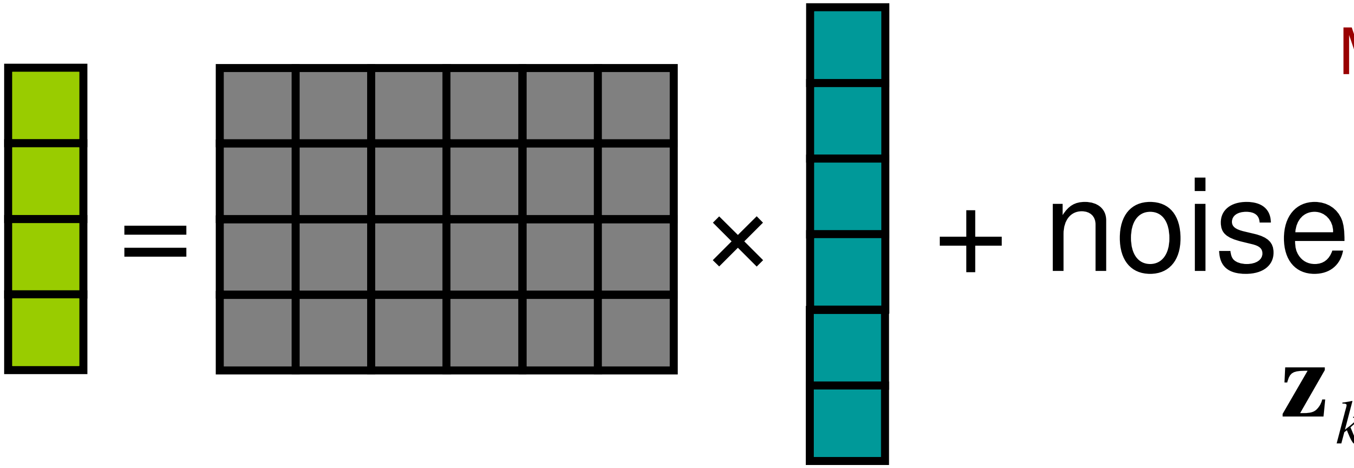
Standard EIT approaches to not take this into account

Kalman Filtering



State transition
Matrix

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{w}$$



Observation
Matrix (Jacobian)

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{n}$$

Kalman Filtering

Two stage process

- Prediction:

Estimate of now based on old data only

$$\mathbf{x}_k^- = \mathbf{A}\mathbf{x}_{k-1}$$

- Update:

$$\hat{\mathbf{x}}_k = \mathbf{x}_k^- + \mathbf{K}_k\left(\mathbf{z}_k - \mathbf{H}\mathbf{x}_k^-\right)$$

- K is *Kalman gain*:
  - Need to update at each step
  - Depends on $\mathbf{P}_k = \mathrm{cov}\left(\hat{\mathbf{x}}_k - \mathbf{x}_k\right)$

Reconstructed Movies

- Algorithm is regularized one-step Gauss-Newton using Laplace prior

Netgen simulation of
moving ball,
Using 100,000 elements
per frame

Total simulation time =
3 days

Measurements of
moving plexiglas rod
in saline tank
(thanks to IIRC)

Total model time =
60 seconds

Gauss-Newton vs. Kalman

Data with added 0dB SNR noise

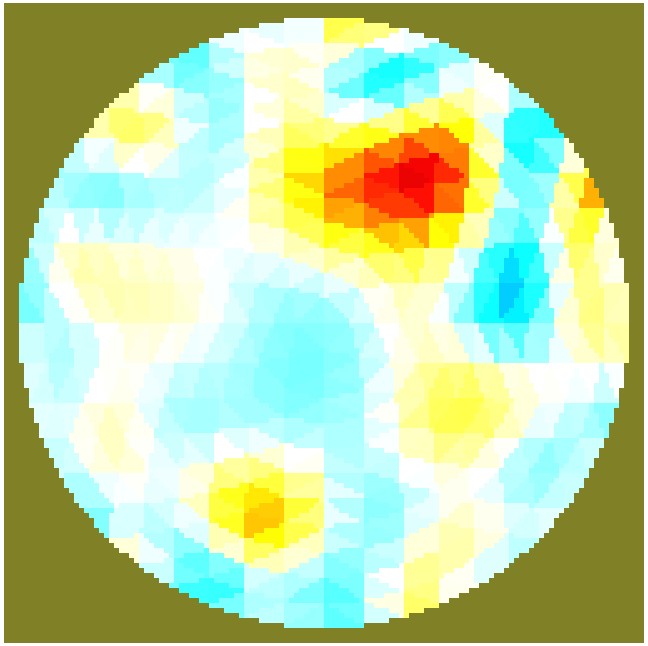
Gauss-Newton solver

Solve time = 5.33 s
(with caching) = 0.22 s

Kalman solver

Solve time = 29.6 min

Gauss-Newton vs. Kalman (0dB SNR)



Gauss-Newton solver

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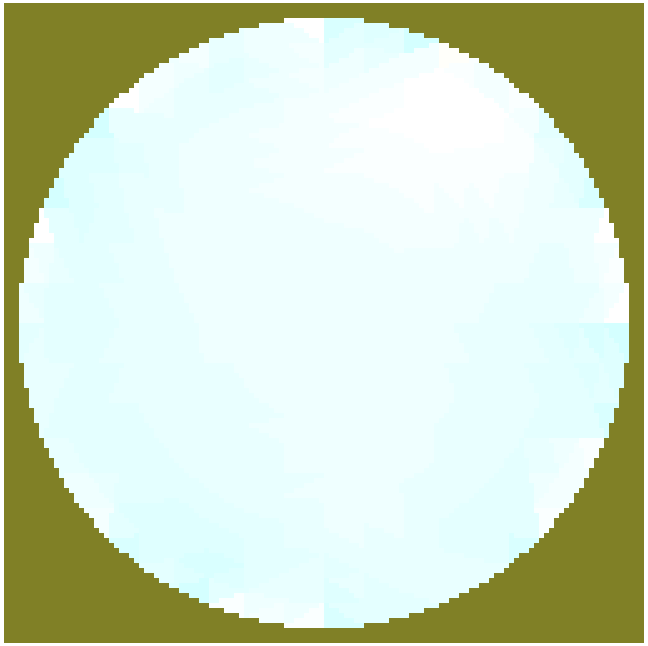


Kalman solver

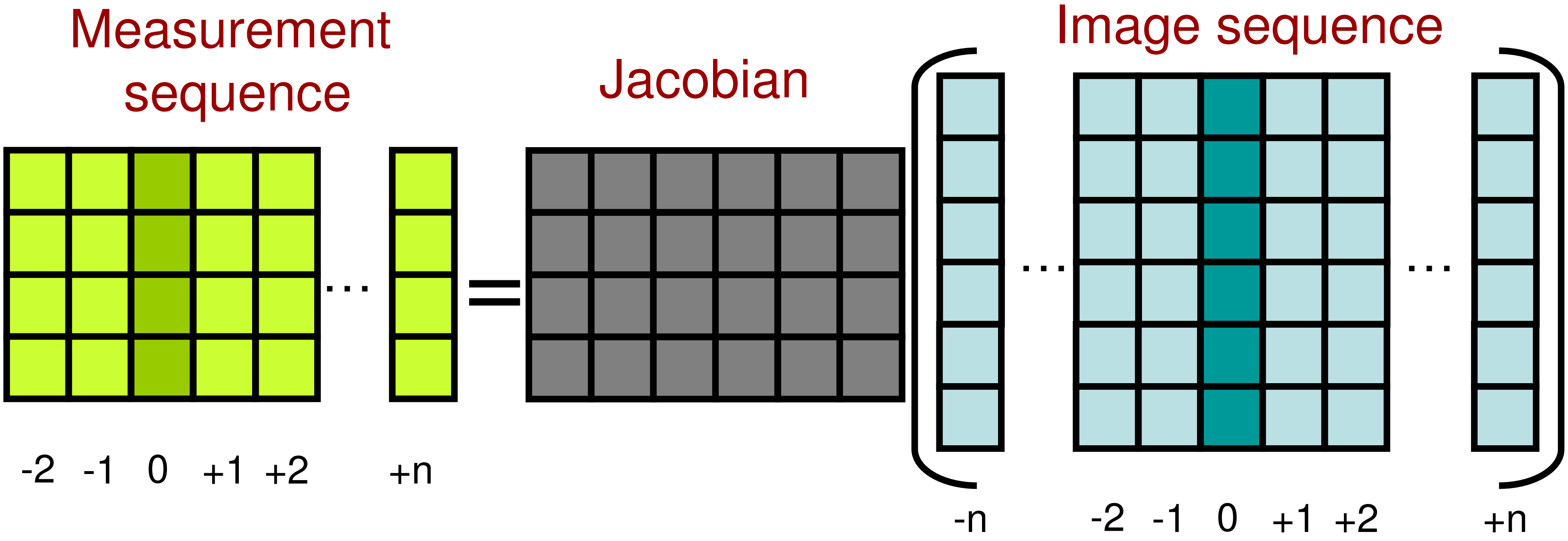
Solve time = 29.6 min

We need a faster solver

We can improve on Kalman in two ways

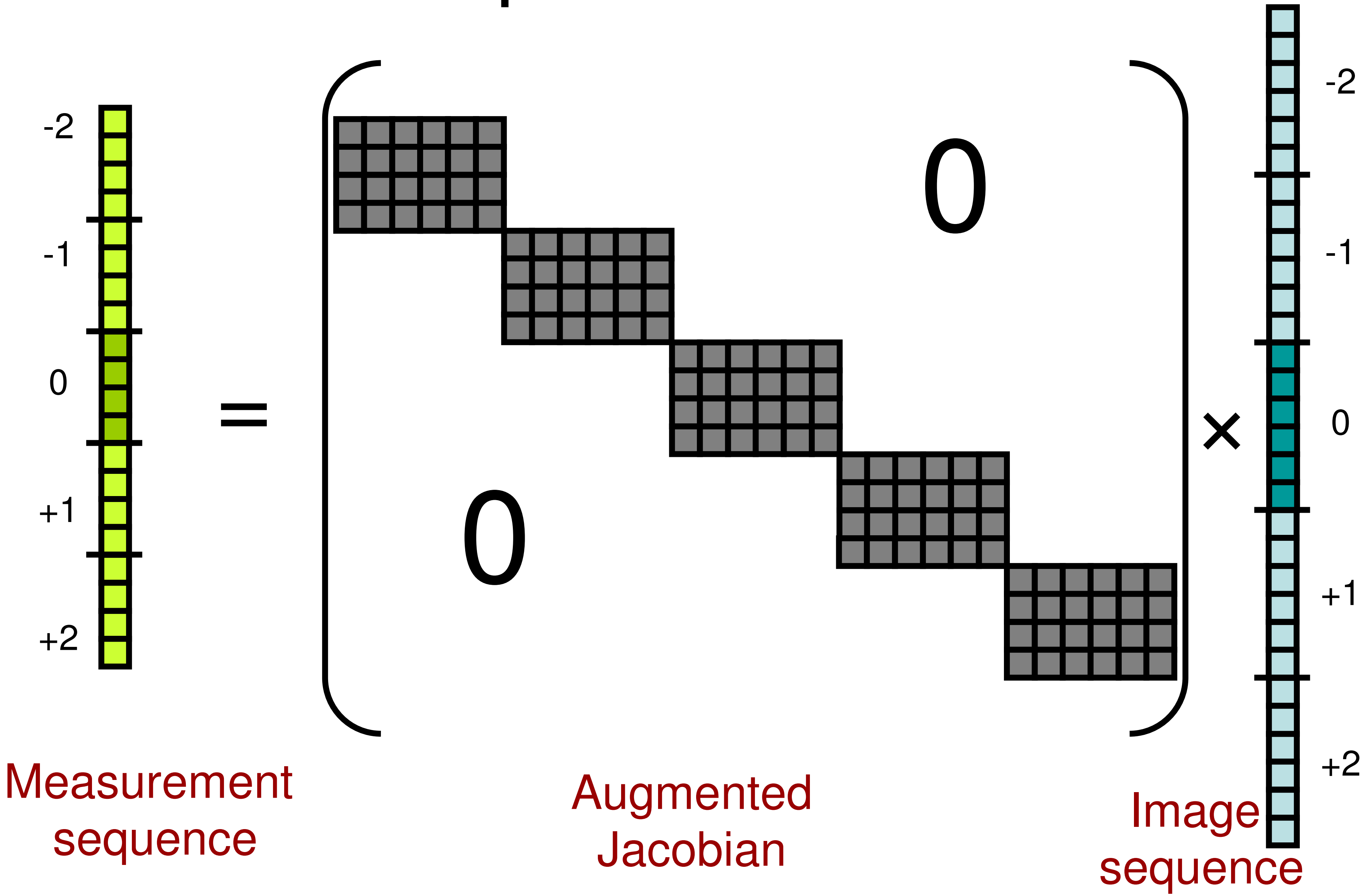
- We can go faster.
  - Kalman calculates the temporal prior. We can directly tell the algorithm
- Use *future* and *past* data
  - Most EIT reconstruction is post-processing
  - For online images, we can delay by a few frames (≈ 100ms)

Direct temporal solver

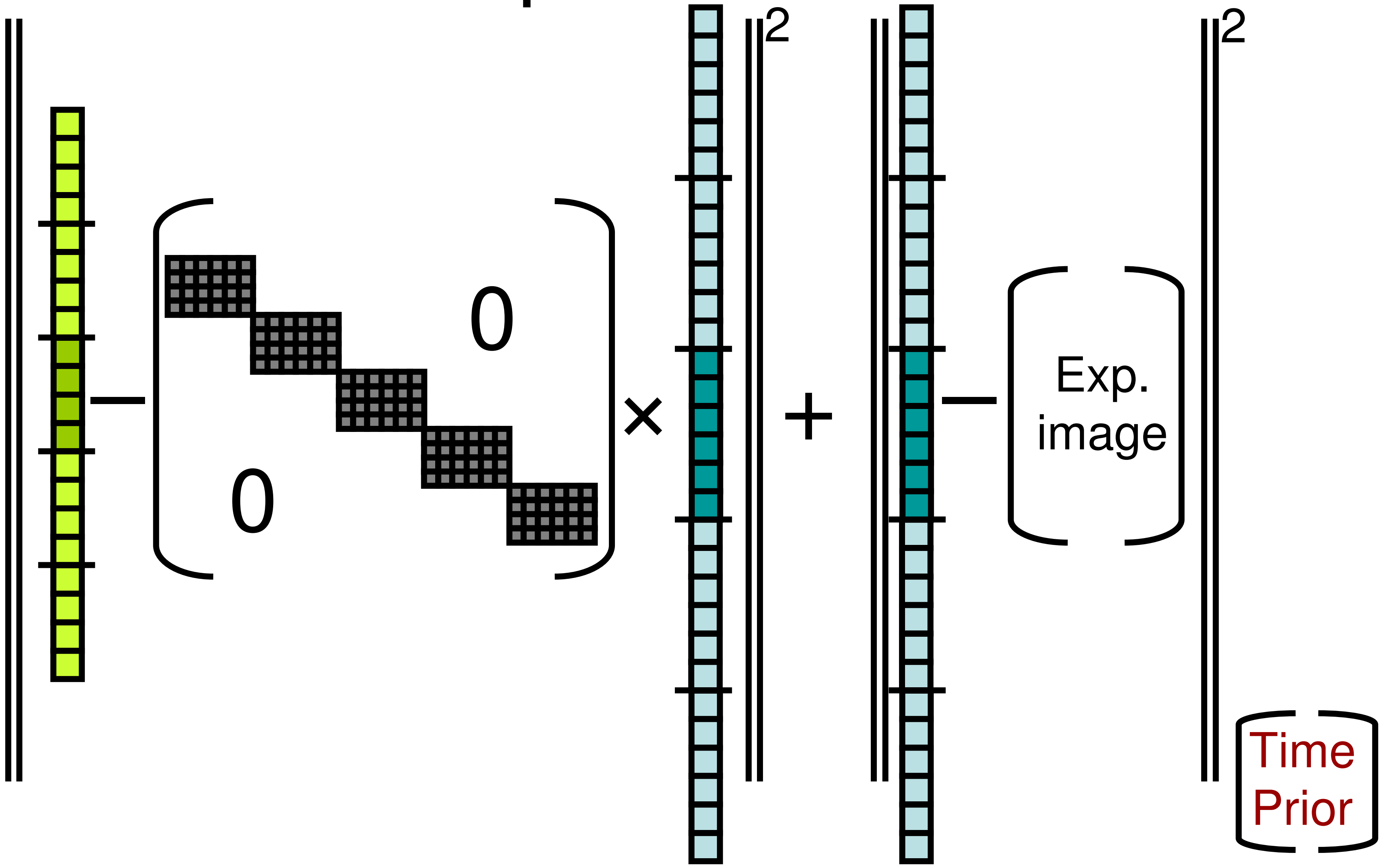


Rewrite as …

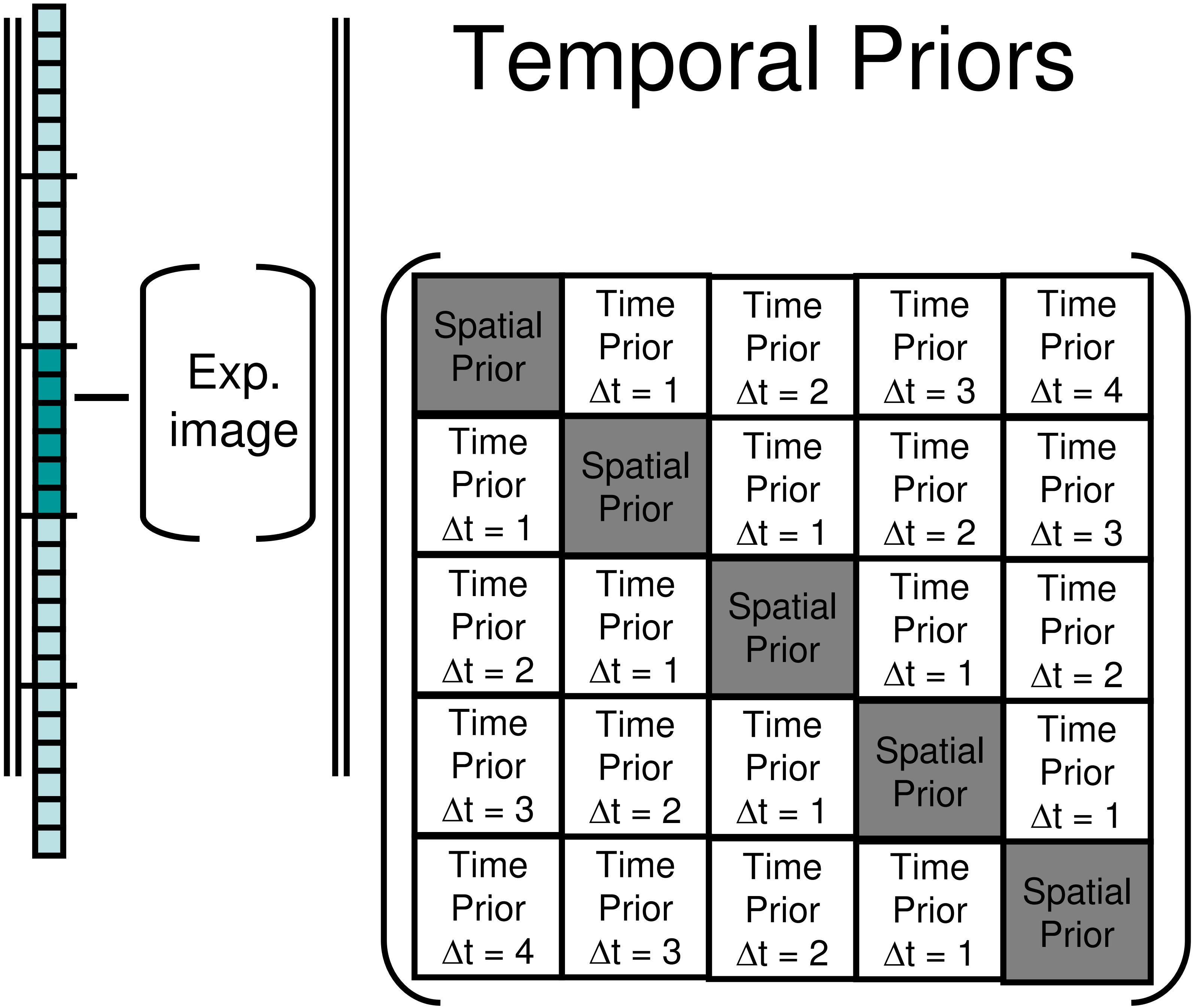
Direct temporal forward model



Direct temporal inverse model



Temporal Priors



One-step inverse

We formulate the one step inverse as:

$$\|\mathbf{z}-\mathbf{H}\mathbf{x}\|_{\mathbf{W}}^{2}+\lambda^{2}\|\mathbf{x}\|_{\mathbf{R}}^{2}$$

$$\hat{\mathbf{x}}=\left(\mathbf{H}^{t}\mathbf{W}\mathbf{H}+\lambda^{2}\mathbf{R}\right)^{-1}\mathbf{H}^{t}\mathbf{W}\mathbf{z}$$

Need to cut matrix afterward, we only want to estimate current image from data

Problem is size of matrix inverse:

For 2 time steps, we have 5 x num_elems square

Underdetermined formulation

We formulate the one step inverse as:

$$\hat{\mathbf{x}} = \left(\mathbf{H}^t\mathbf{W}\mathbf{H} + \lambda^2\mathbf{R}\right)^{-1}\mathbf{H}^t\mathbf{W}\mathbf{z}$$

$$\hat{\mathbf{x}} = \mathbf{R}^{-1}\mathbf{H}^t\left(\mathbf{H}\mathbf{R}^{-1}\mathbf{H}^t + \lambda^2\mathbf{W}^{-1}\right)^{-1}\mathbf{z}$$

Now matrix inverse is smaller:

For 2 time steps, we have 5 x num_meas square

$\mathbf{R}^{-1}$ and $\mathbf{W}^{-1}$ are modelled directly. No need to take the inverse

GN vs. Temporal Inverse

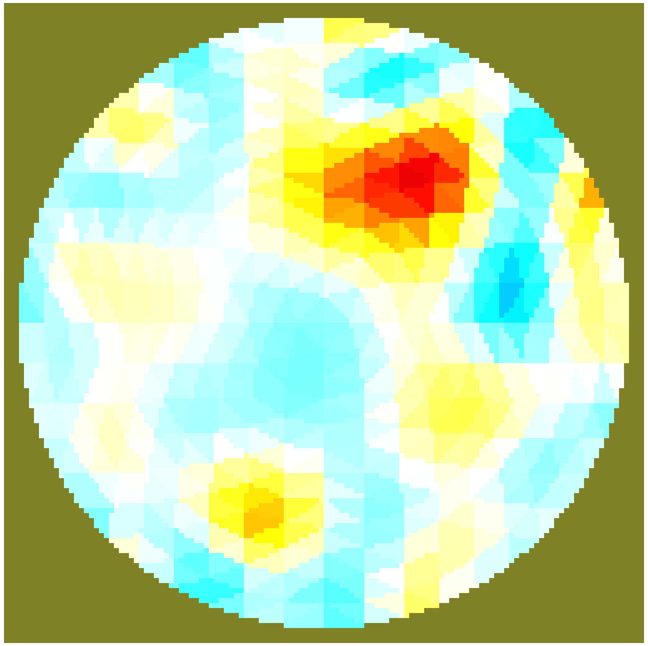
1. Noise free data (IIRC tank)
2. Data with added 6dB SNR noise

Gauss-Newton solver

Solve time = 5.33 s
(with caching) = 0.22 s

Temporal solver
(4 time steps)
Solve time = 34.81 s
(with caching) = 0.60 s

Gauss Newton vs. Temporal Inverse (6db SNR)



Gauss-Newton solver

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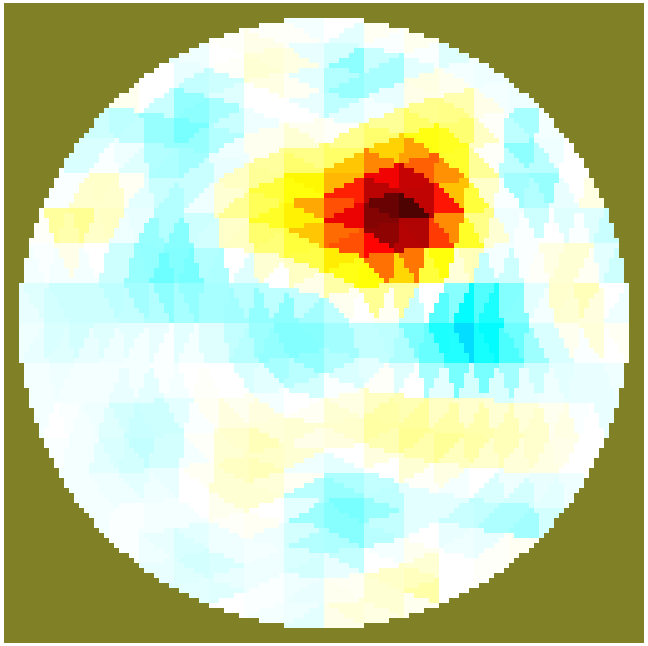

Temporal solver
(4 time steps)
Solve time = 34.81 s
(with caching) = 0.60 s

EIDORS: community-based extensible software for EIT

Andy Adler[1], William R.B. Lionheart[2]

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[2]School of Mathematics, University of Manchester, U.K.

EIDORS Tutorial

- Introduction to EIDORS
  - Goal
  - Features
- Examples (worked together)
  - Forward solutions
  - Inverse solutions
- Examples (worked alone)
  - Based on EIDORS tutorial (with V3.1)

Goal: software community

Project: **E**lectrical
Impedance and
Diffuse
Optical
Tomography
Reconstruction
Software

Blobby the Walrus?

1. EIT images blobby objects in aqueous media; Blobby the Walrus is a fat animal that lives in water.
2. Walrus is EIDORS logo
3. Walruses are much funnier than a talk about software architecture

Images credit: www.biosbcc.net
© Genny Anderson

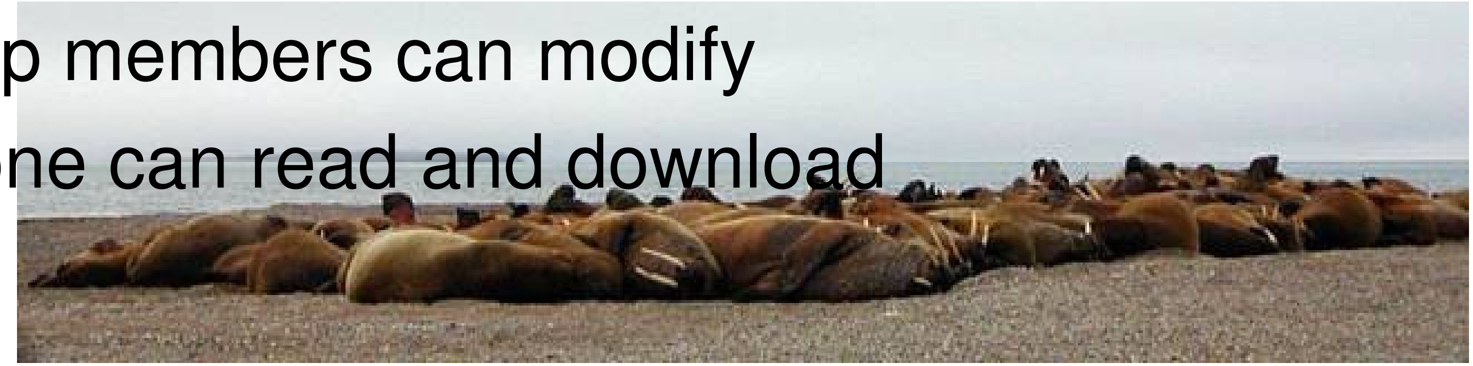
EIDORS Features

Open-source:

- License: GNU General Public License.
- Free to use, modify, and distribute modifications.
- May be used in a commercial product

Hosted on Sourceforge.net

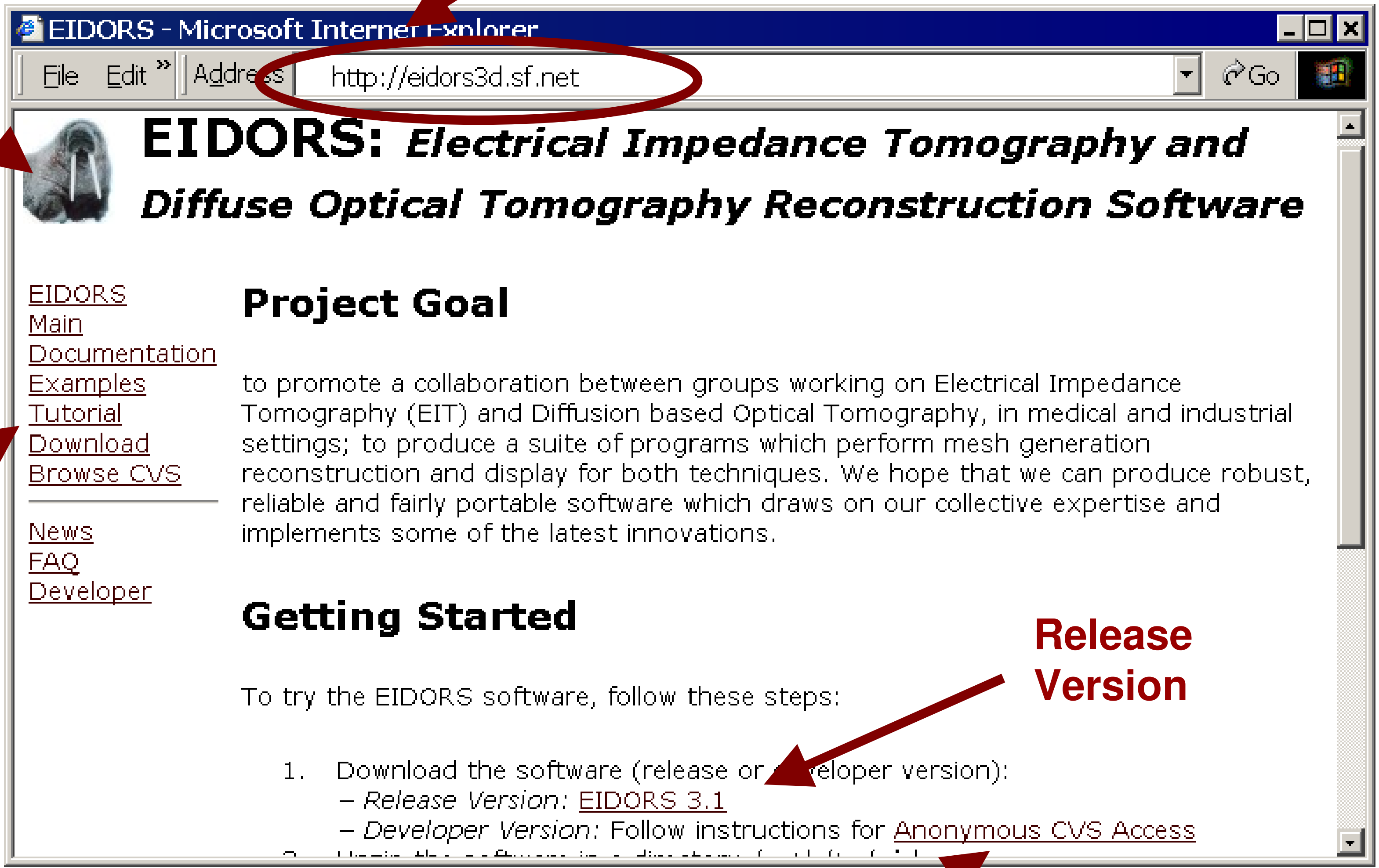
- Software is available for download (version 2.0)
- CVS access to latest developer versions
- Group members can modify
- Anyone can read and download



Web Site

Walrus

This Tutorial



Release Version

Developer Version

Features

Language independence:

- Octave (octave.org, ver≥ 2.9)
- Matlab (version ≥ 6.0).

Usage examples:

- new software is based on demos.
- simple and more complex usage examples.

Tests:

- Software is intrinsically difficult to test.
- Numerical software is probably more difficult
- Implement of regression test scripts

Features

Pluggable code base:

- Object-oriented: *Packaging* and *Abstraction*.
- Don’t use the Matlab OO framework
- Instead, EIDORS designed as "Pluggable" software using function pointers.

Features

Automatic matrix caching:

- Save computationally expensive variables
  - ie Jacobian , Image priors.
- Caching complicates software
- Caching managed in `eidors_obj`

Features

Generalized data formats:

- EIT has a wide variety of stimulation, measurements
- general EIT data format : *fwd_model*
  - electrode positions
  - contact impedances
  - stimulation and measurement patterns.

Interface software for common EIT systems:

- Load data from some EIT systems
- Please contribute

getting started

- Download
  - Run tutorial examples

- Join Mailing list

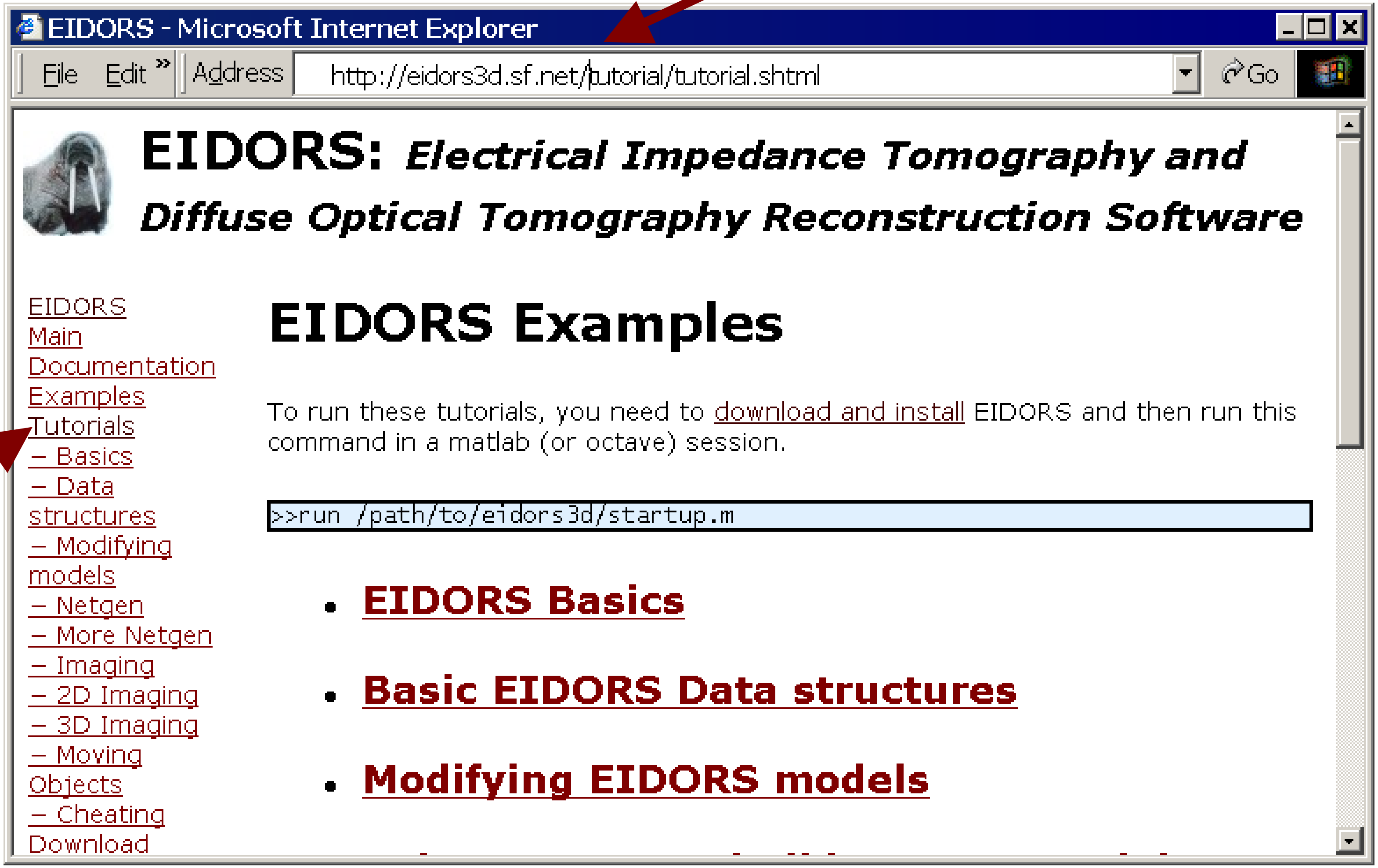
  `eidors3d@listserv.umist.ac.uk`

- Sign up as developer at:
  `sourceforge.net`

- Contribute your code

Tutorials

Also `tutorial.shtml`
In eidors-v3.1 distribution



Tutorials

Discussion

- EIT and Image Reconstruction
  - Electrode Errors
  - Electrode Movement
  - Temporal Filtering
  - EIDORS Project
- Significant recent developments in EIT image algorithms will improve EIT's clinical applicability