Variable Step-Size Affine Projection Algorithm with a Weighted and Regularized Projection Matrix

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Outline

- Introduction to Affine Projection Algorithm (APA)
- Optimal Variable Step-Size APA
- Optimal Variable Step Size APA with Forgetting Factor
- Regularization of the Ill-Conditioned Projection Matrix
- Conclusions

Introduction

- Evolution of Affine Projection Algorithm (APA)
  - Least Mean Square (LMS)
  - Normalized Least Mean Square (NLMS)
  - Affine Projection Algorithm (APA)
    - Variable Step-Size APA (VS-APA)
    - VS-APA with weighted input matrix processed by forgetting factor (VS-APA-FF)

Affine Projection Algorithm (APA) and Variable Step-Size APA(VS-APA)

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu U_i^* (U_i U_i^*)^{-1} \mathbf{e}_i$$

$$\mathbf{e}_i = \mathbf{d}_i - U_i \mathbf{w}_{i-1}$$

VS-APA:

$$\mu(i) = \mu_{\max} \frac{\|\hat{p}_i\|^2}{\|\hat{p}_i\|^2 + C}$$

$$p_i = U_i^* (U_i U_i^*)^{-1} \mathbf{e}_i$$

$$U_i = \begin{bmatrix} \mathbf{x}_i \\ \mathbf{x}_{i-1} \\ \vdots \\ \mathbf{x}_{i-K+1} \end{bmatrix} \qquad \mathbf{d}_i = \begin{bmatrix} d(i) \\ d(i-1) \\ \vdots \\ d(i-K+1) \end{bmatrix} \qquad \mathbf{w}_i = \begin{bmatrix} w_{0,i} \\ w_{1,i} \\ \vdots \\ w_{L-1,i} \end{bmatrix}$$

μ : step size
K : APA order
L : filter order

Variable Step-Size Affine Projection Algorithm with Forgetting Factor (VS-APA-FF)

- We proposed the optimal variable step-size APA with a forgetting factor(VS-APA-FF)

- Idea:
  - New data have more significance than old data during system convergence.

- Solution:
  - The project matrix is weighted by a forgetting factor

Variable Step-Size Affine Projection Algorithm with Forgetting Factor (VS-APA-FF)

- The projection matrix U is weighted by a forgetting factor λ. $(0 < \lambda \leq 1)$

$$
\begin{bmatrix}
x_i & x_{i-1} & \cdots & x_{i-L+1} \\
x_{i-1} & x_{i-2} & \cdots & x_{i-L} \\
\vdots & \vdots & \ddots & \vdots \\
x_{i-K+1} & x_{i-K} & \cdots & x_{i-K-L+2}
\end{bmatrix}
\begin{matrix}
\rightarrow \times \lambda^0 \\
\rightarrow \times \lambda^1 \\
\vdots \\
\rightarrow \times \lambda^{K-1}
\end{matrix}
$$

$$
\begin{matrix}
\uparrow & \uparrow & & \uparrow \\
\times \lambda^0 & \times \lambda^1 & & \times \lambda^{L-1}
\end{matrix}
$$

$$U_i' = \Lambda^{(K)} U_i \Lambda^{(L)}$$

$$\left[\Lambda^{(m)}\right]_{j,j} = \lambda^{j-1} \qquad j = 1, 2, \ldots, m$$

Variable Step-Size Affine Projection Algorithm with Forgetting Factor (VS-APA-FF)

- Algorithm proposed

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu(i) U_i^* (U_i U_i^*)^{-1} \mathbf{e}_i$$

Variable step size
$$\mu(i) = \mu_{\max} \frac{\|\hat{p}_i'\|^2}{\|\hat{p}_i'\|^2 + C}$$

Error estimation
$$p_i' = U'^*_i (U'_i U'^*_i)^{-1} \mathbf{e}_i$$

Smoothed error estimation
$$\hat{p}'_i = \alpha \hat{p}'_{i-1} + (1-\alpha) p'_i \quad 0 \le \alpha < 1$$

Regularization of the Ill-Conditioned Projection Matrix

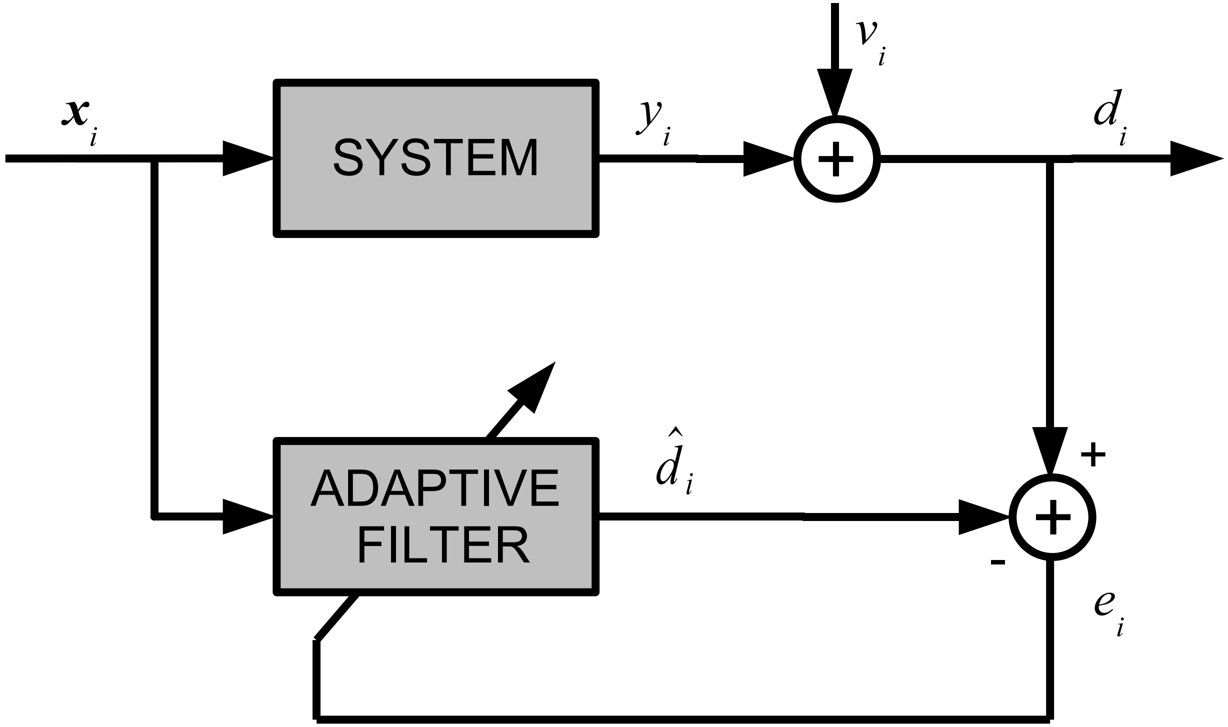
- Problem:
  - New data have more significance than old data during system convergence.

$$cond(U') = \sigma'_{\max} / \sigma'_{\min} = \sigma_1 / \left[\lambda^{2(K-1)} \sigma_K\right]$$
$$= \lambda^{2(1-K)} \cdot cond(U)$$

- Solution:
  - The projection matrix needs to be regularized

$$p_i' = U_i'^* (U_i' U_i'^* + \alpha^2 I)^{-1} \mathbf{e}_i$$

Simulations



system identification model is used for algorithm verification

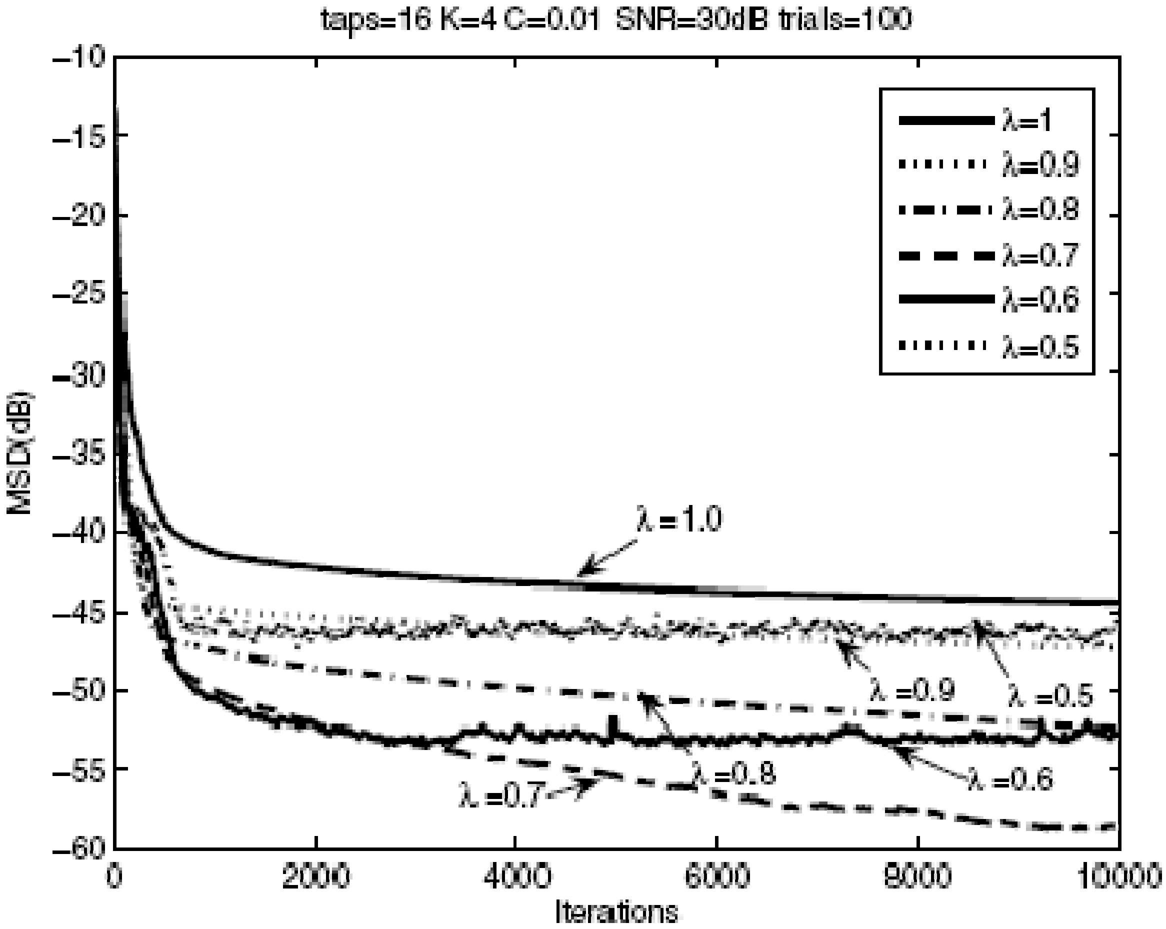
Simulations

Two input colorizations

$$G_1(z) = 1/1 - 0.9z^{-1}$$

$$G_2(z) = \frac{1 + 0.9z^{-1} + 0.6z^{-2} + 0.81z^{-3} - 0.329z^{-4}}{1 + z^{-1} + 0.21z^{-2}}$$

Simulations

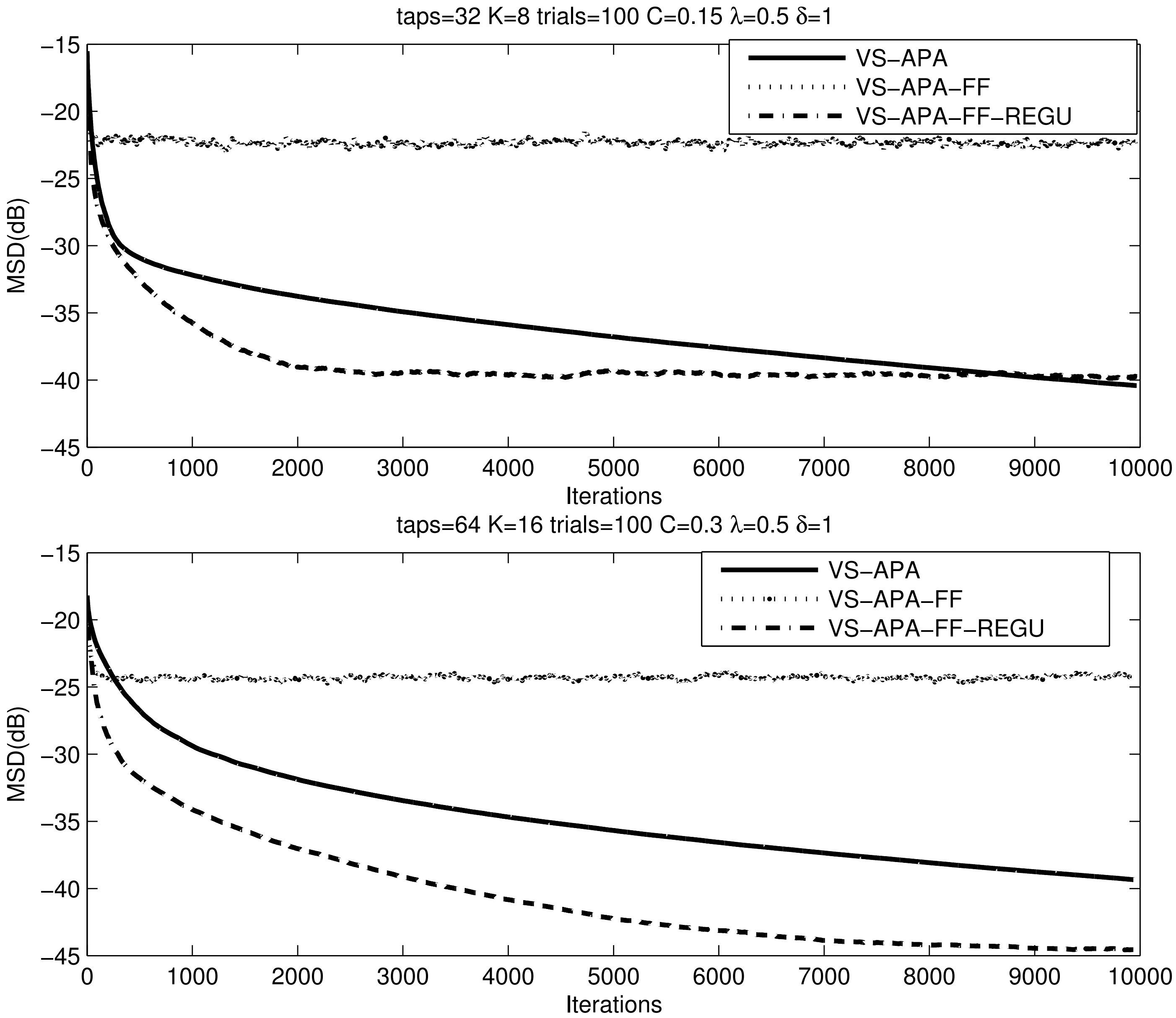


MSD vs. iterations for *VS-APA-FF* for effect of different forgetting factors *λ*. (L=16, K=4, SNR=30dB, G2 colorization)

Simulations

- *Recommended values of forgetting factor λ for* VS-APA-FF. *(L=16)*

K	C	λ			
		G1		G2	
		SNR =30dB	SNR =40dB	SNR =30dB	SNR =40dB
1	0.0001	**0.8**	**0.4**	**0.5**	**0.1**
2	0.001	**0.9**	**0.8**	**0.5**	**0.3**
4	0.01	**1**	**0.9**	**0.7**	**0.6**
8	0.15	**1**	**0.9**	**0.8**	**0.8**



Comparisons among VS-APA, VS-APA-FF, and VS-APA-FF-REGU, G1 colorization. $\lambda = 0.5$. (a) K=8, taps=32, C=0.15; (b)K=16, taps=64, C=0.3

Conclusions

- Weighted by a forgetting factor, VS-APA-FF is an upgrade of VS-APA
- VS-APA-FF suffers from ill-conditionness for some cases (large K, small λ)
- The regularized VS-APA-FF greatly fixed ill-conditionness
- How to correctly choose a regularization parameter is still remain for further research