COMBINING REGULARIZATION FRAMEWORKS FOR IMAGE DEBLURRING:

OPTIMIZATION OF COMBINED HYPER-PARAMETERS

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Outline

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- Direct Regularization Method
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- Iterative Regularization Methods
  - Conjugate Gradient Least-square [CGLS]
  - CGTik [CGLS+Tikhonov]
- Simulation Results

Problem Definition

- Consider the following problem:

$$d = Gm + n$$

$$\begin{aligned} \text{where } d &: measured\ data \\ m &: original\ image \\ G &: system\ matrix \\ n &: noise\ vector \end{aligned}$$

The goal is to solve for an estimate $\hat{m}$

Regularization Methods

- Deal with all difficulties related to ill-posed problems
  - Solution existence
  - Solution uniqueness
  - Solution Stability

- Inclusion of Prior knowledge to stabilize the solution in face of noise
- Smooth the data
- Constrain the solution in order to avoid noise amplification

Direct Tikhonov regularization

- Direct incorporation of prior information to the original least squares cost function

$$\hat{m}_{tik}(\alpha) = \arg\min_{m} \|d - Gm\|_2^2 + \alpha \|Lm\|_2^2$$

- Common choices for operator "L":
  - L=I
  - L=D

Direct Tikhonov regularization

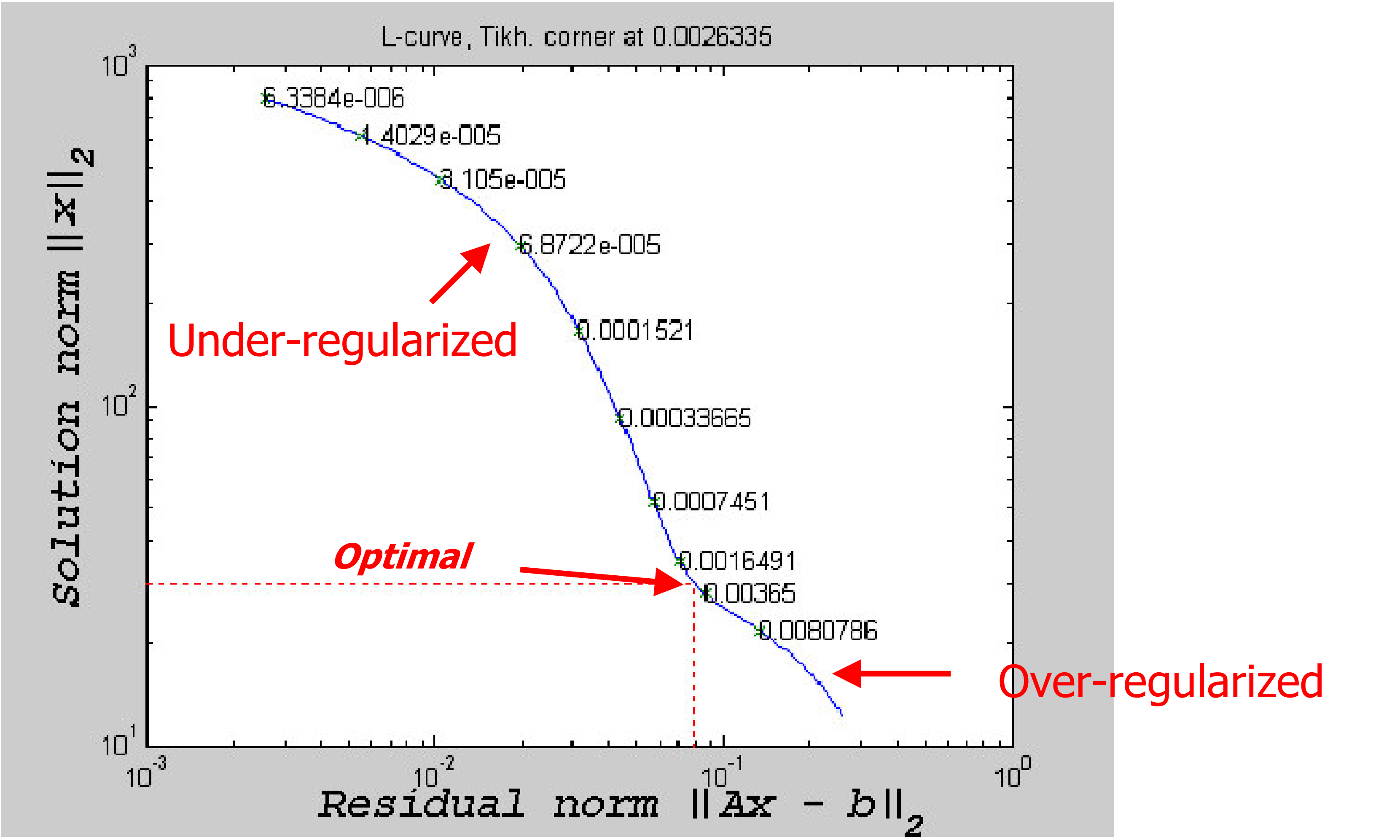
- The minimizer of the least-square formulation can be expressed as normal equations:

$$(G^T G + \alpha L^T L)\,\hat{m}_{tik} = G^T d$$

- Equation can be solved by:
  - Matrix inversion
  - Factorization methods (QR, SVD, Cholesky)
  - Iteration

Regularization parameter α

L-curve for direct Tikhonov method



Direct Tikhonov regularization

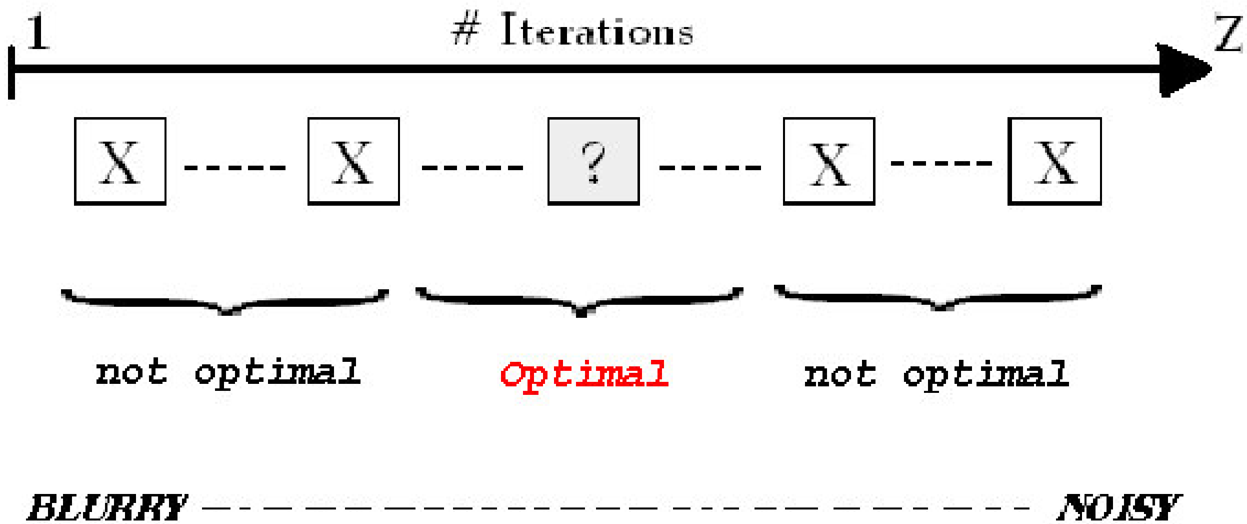
- Advantages:
  - Provide good solutions for small-scale problems
  - L-curve can be used to select the regularization parameter
- Disadvantages:
  - Inefficient for large problems → Large amount of storage
  - Image must be smooth → Blur edges

Iterative Methods

- Iterative techniques:
  - Very efficient for large size problems
  - Can be viewed as regularization methods
  - Restored images are monitored at each iteration

Iterative Methods

- # of iterations *N* → amount of regularization



- Low-frequency vs High-frequency components

Iterative Methods: CG, CGLS

- Conjugate Gradient (CG) techniques are able to solve positive definite equations of the form:

$$Ax = b$$

- CGLS solve the following least-square form

$$\min \| Gm - d \|$$

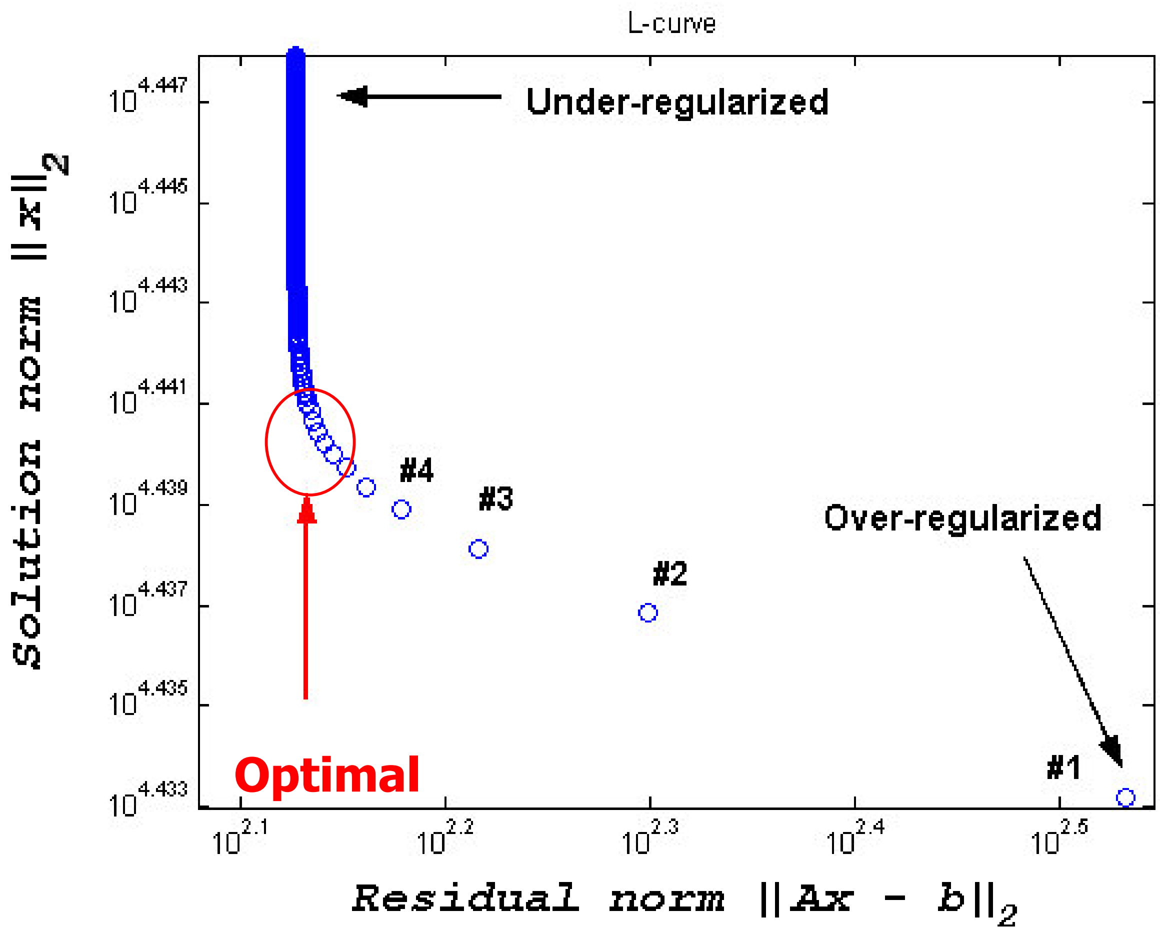
by applying CG to the normal equations :

$$\left(G^T G m = G^T d\right)$$

- Stop the algorithm when $\left\| G m_k - d \right\|_2 < \delta$

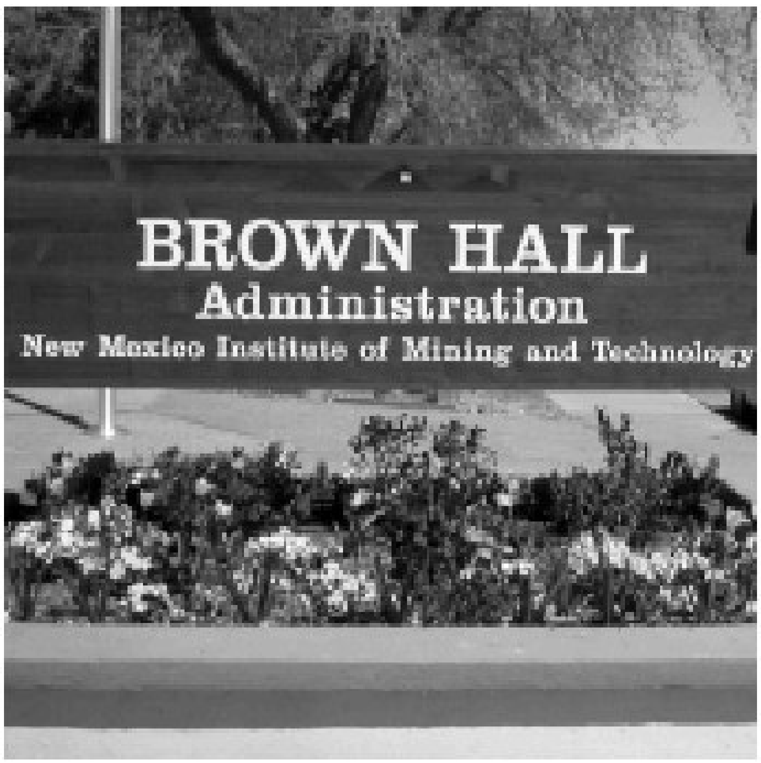
CGLS Iterative Technique

- CGLS L-curve for hyper-parameter selection *(N)*:

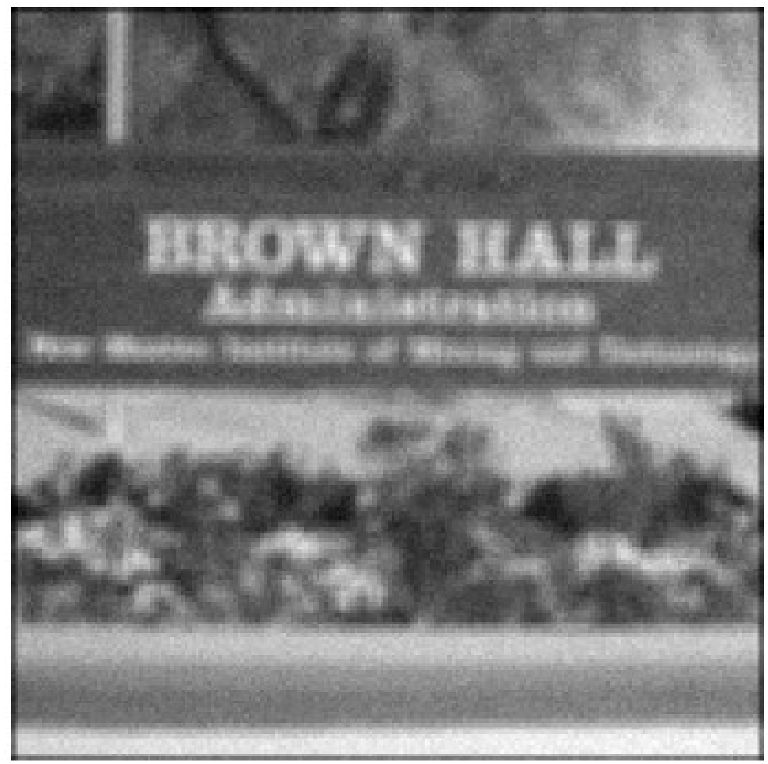


Test data

Original Image



Blurred Noisy Image



CGLS Iterative Technique

- Results:

→ *Video:*

CGTik [CGLS+Tikhonov]

- Tikhonov combined with CGLS

$$\hat{m}_{tik}(\alpha) = \arg\min_{m} \|d - Gm\|_2^2 + \alpha\|Lm\|_2^2$$

and

$$\hat{m}_{CGTik}(\alpha) = \left\| \begin{bmatrix} G \\ \alpha L \end{bmatrix} m - \begin{bmatrix} d \\ 0 \end{bmatrix} \right\|_2^2 \qquad (3)$$

New Least-square problem is expressed as follows:

$$\hat{m}_{CGTik}(\alpha) = \arg\min \|C m - data\|_2^2 \qquad (4)$$

$$where \quad C = \begin{bmatrix} G \\ \alpha L \end{bmatrix} \quad and \quad data = \begin{bmatrix} d \\ 0 \end{bmatrix}$$

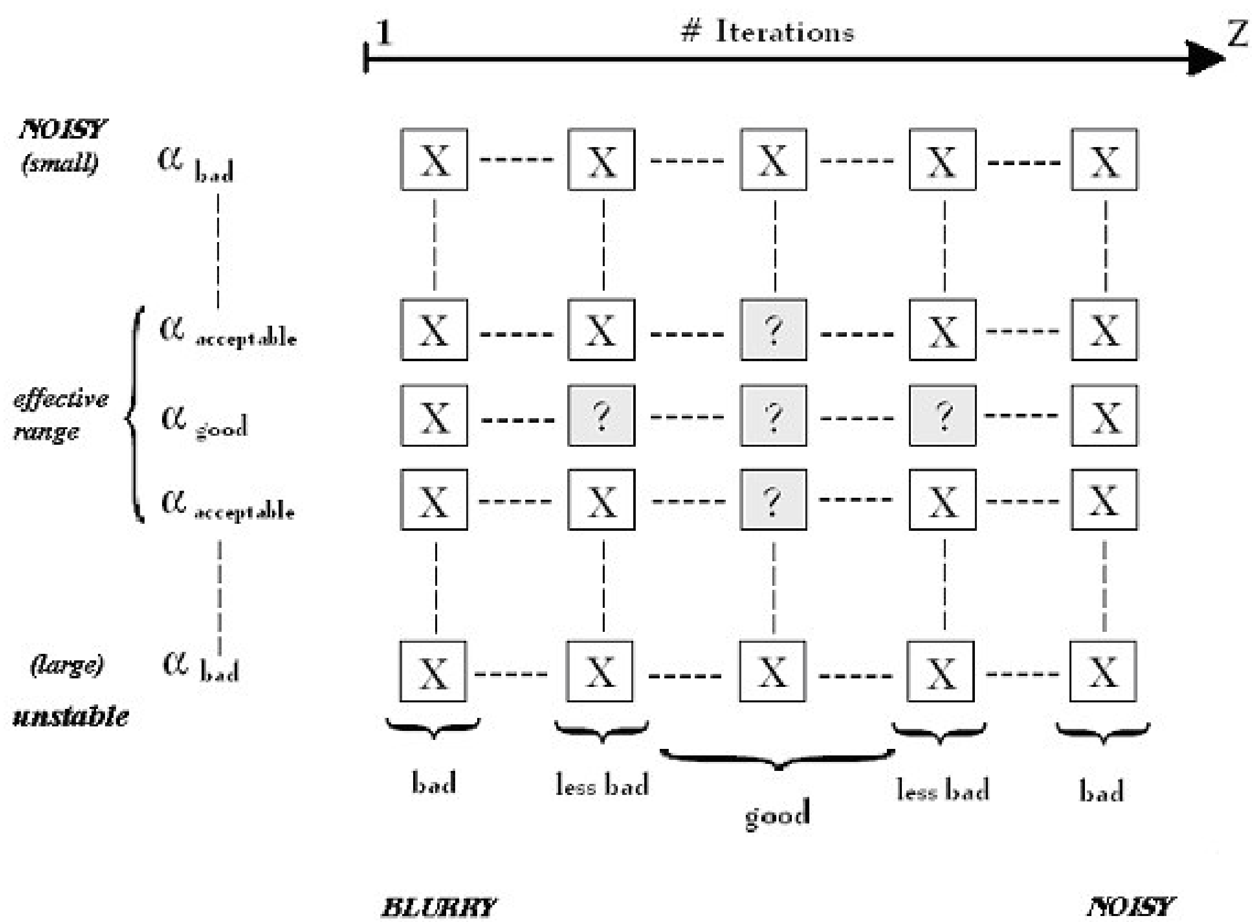
CGLS + Tikhonov (L=D)

- $\|Dm\|$ becomes a measure of the variability or roughness of the solution
- Forces image estimates with limited high-frequency energy
- Captures prior belief that solution images should be smooth

CGTik (L=D)

→ Regularization parameter (α)

→ Stopping time of the algorithm (*N*)



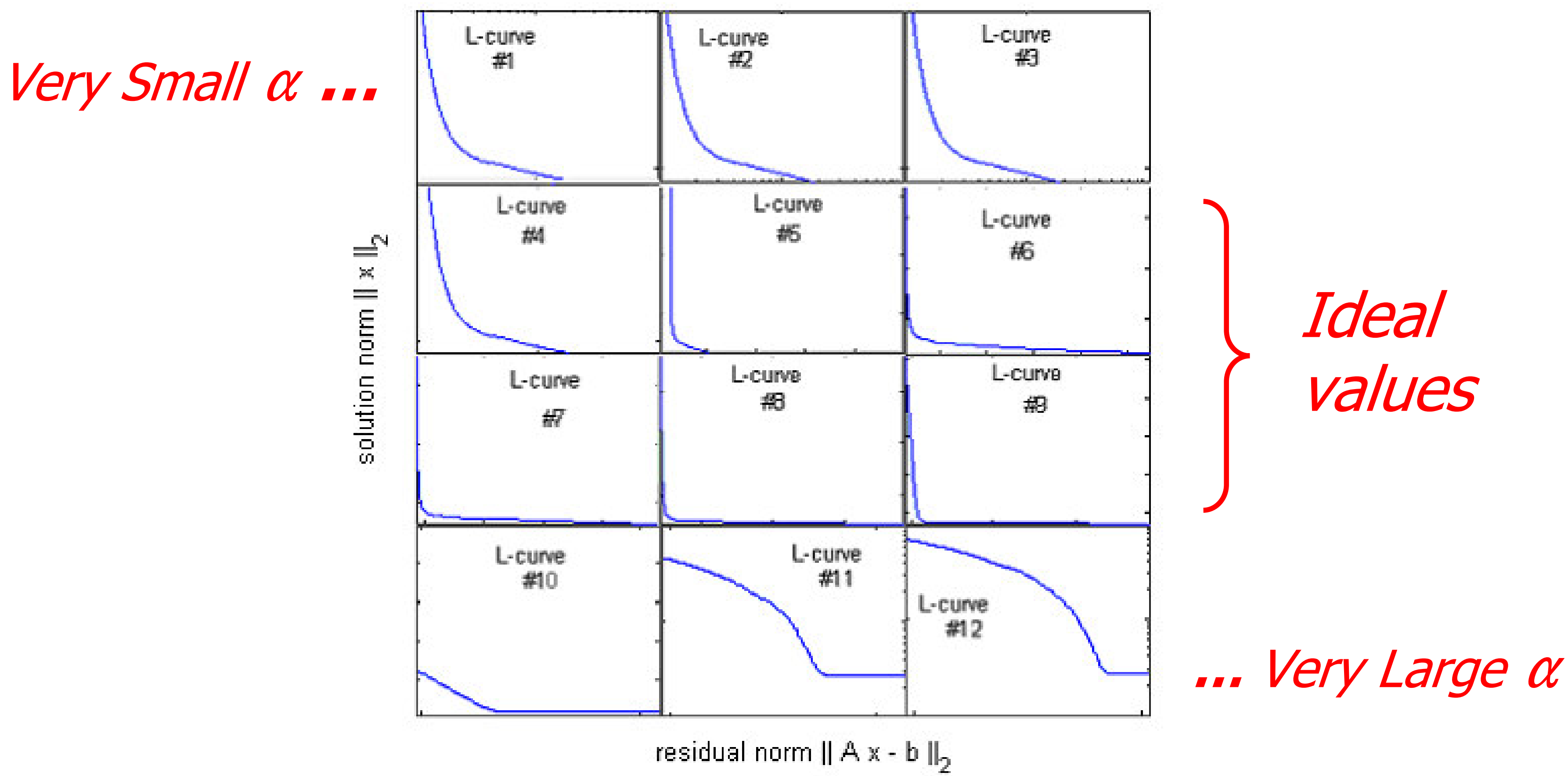
Back to the initial problem…

- PRE-SELECTED range of α 's for the initial problem:

$$\alpha = \begin{bmatrix} 1x10^{-6} \\ 1x10^{-5} \\ 1x10^{-4} \\ 1x10^{-3} \\ 10x10^{-3} \\ 20x10^{-3} \end{bmatrix} \quad \cdots\cdots \quad \begin{bmatrix} 30x10^{-3} \\ 40x10^{-3} \\ 50x10^{-3} \\ 100x10^{-3} \\ 300x10^{-3} \\ 500x10^{-3} \end{bmatrix}$$

CGLS+Tikhonov (L=D)

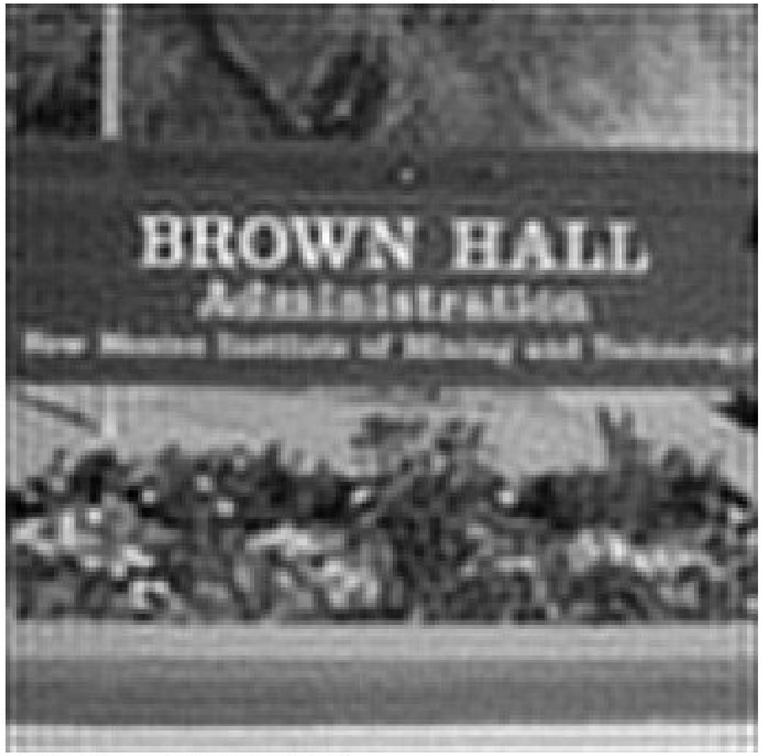
- L-curves for different hyper-parameters:



SIMULATION RESULTS

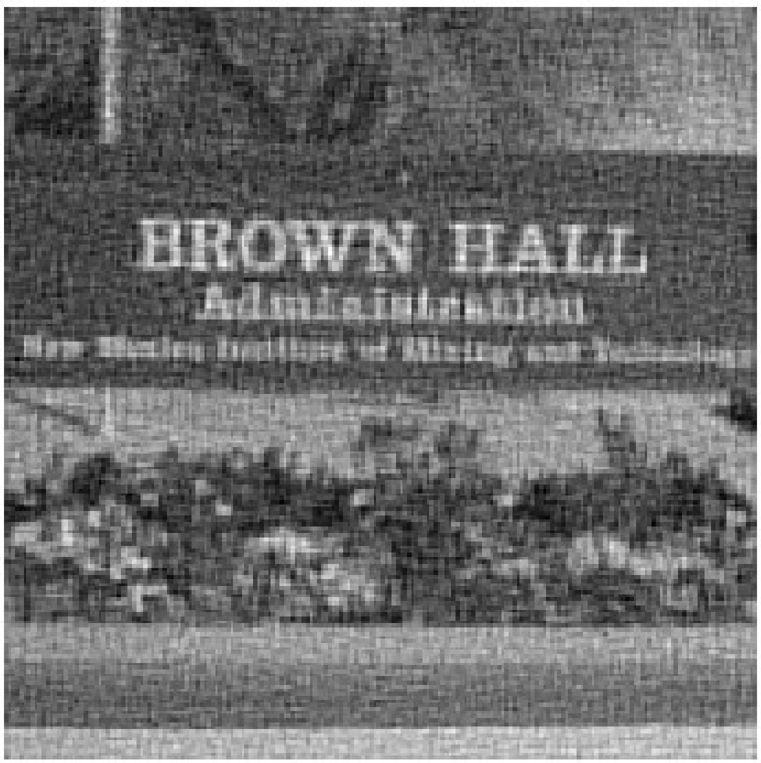
AND

COMPARISON



Tikhonov α=0.02

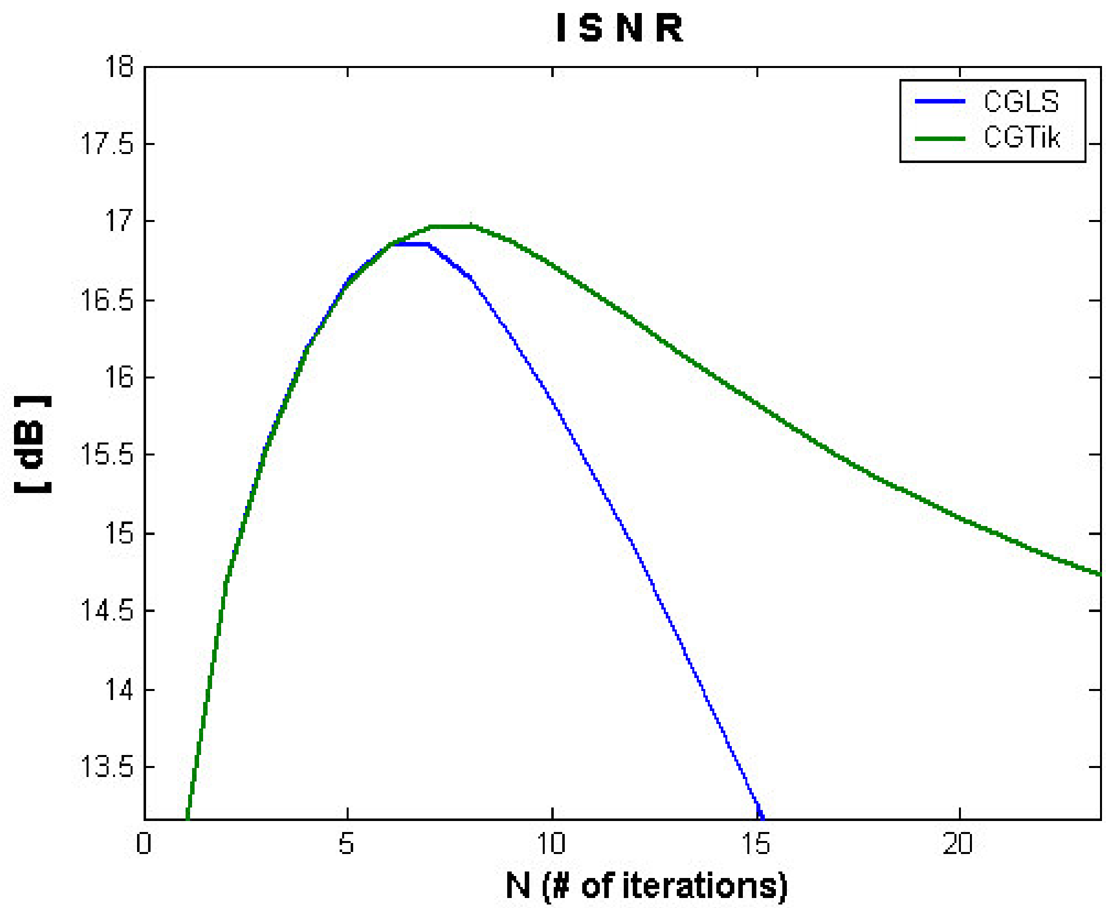
CGLS
N=7



CGTik
N=11
α=0.02

ISNR Curve for 25 iterations

α=20x10^{-3}



TiKhonov ISNR=10.6 dB

Results of CGTik (L=D) restoration

- Advantages:
  - Quality of image is enhanced
  - Noise is reduced
  - Details in image are well recovered

- Disadvantages:
  - α and stopping time N must be re-selected for different problem
  - Image must be smooth in order to have an efficient noise removal without edge blurring
  - Tradeoff between noise and amount of blur