Electrical Impedance Tomography: *Image Reconstruction with Electrode Measurement Errors*

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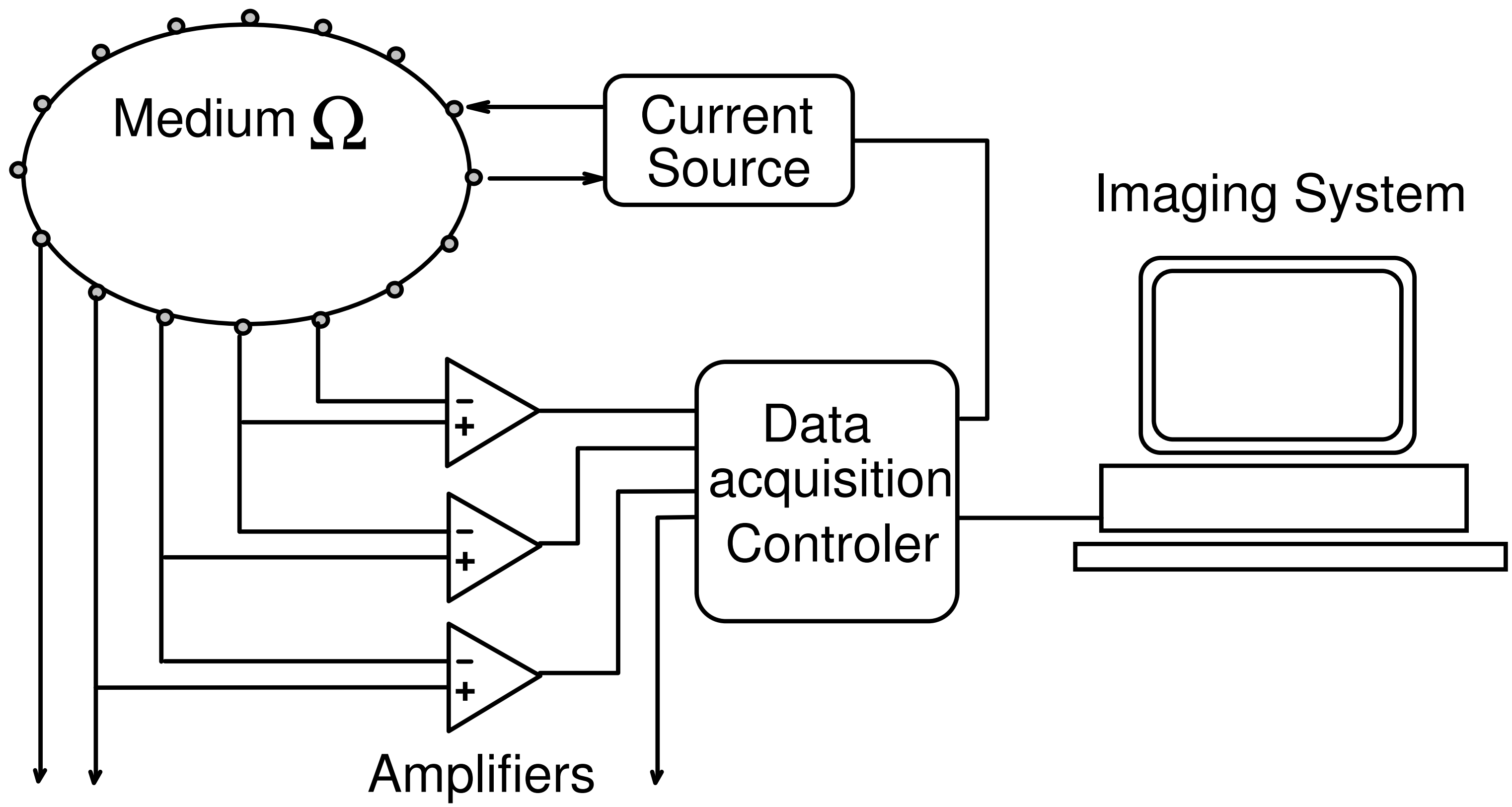
Outline

- Electrical Impedance Tomography
- Physics and Image Reconstruction
- Measurement Difficulties
  - Electrode Errors
- Work in Progress

Electrical Impedance Tomography

- Relatively new medical imaging technique (early 1990’s)
- Body Surface Electrodes apply current patterns and measure the resulting voltages
- Distribution of conductivity is calculated

EIT: Block Diagram



EIT: Applications

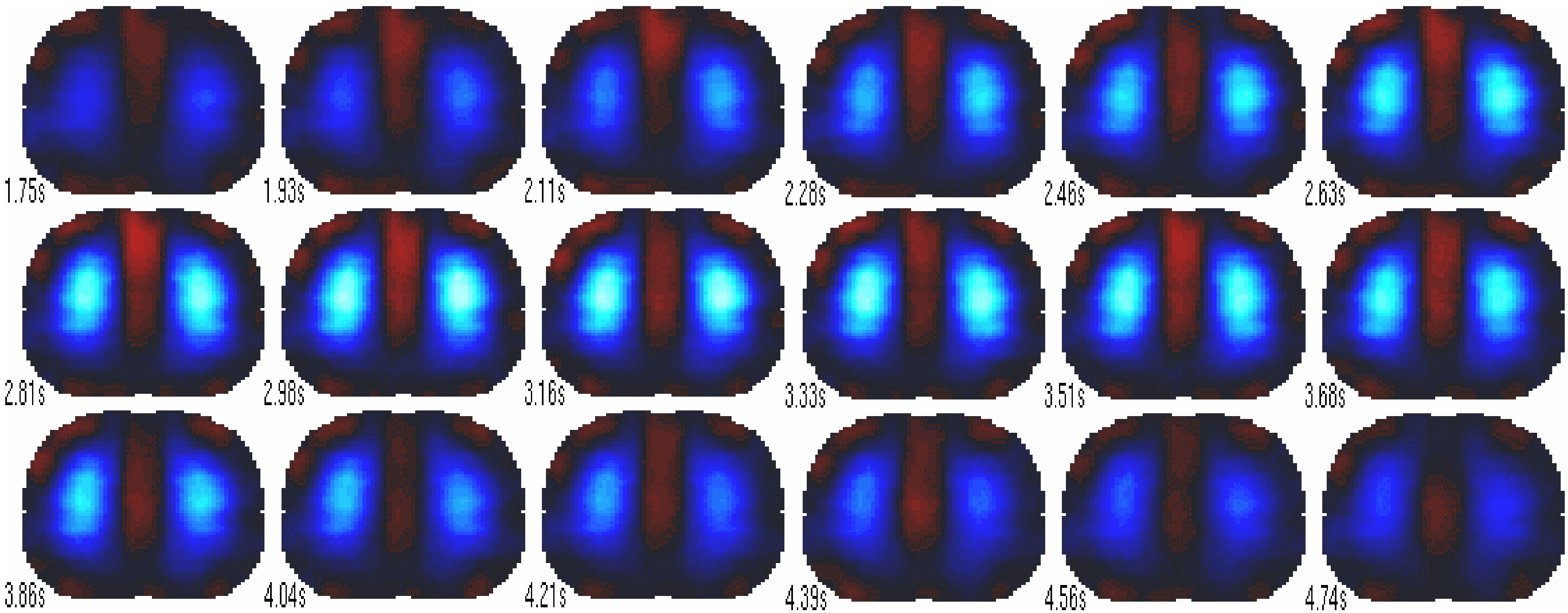
- EIT can image physiological processes involving movement of conductive fluids and gasses
- Lungs
- Heart / perfusion
- GI tract
- Brain
- Breast

EIT: Advantages

EIT is a relatively low resolution imaging modality, *but*

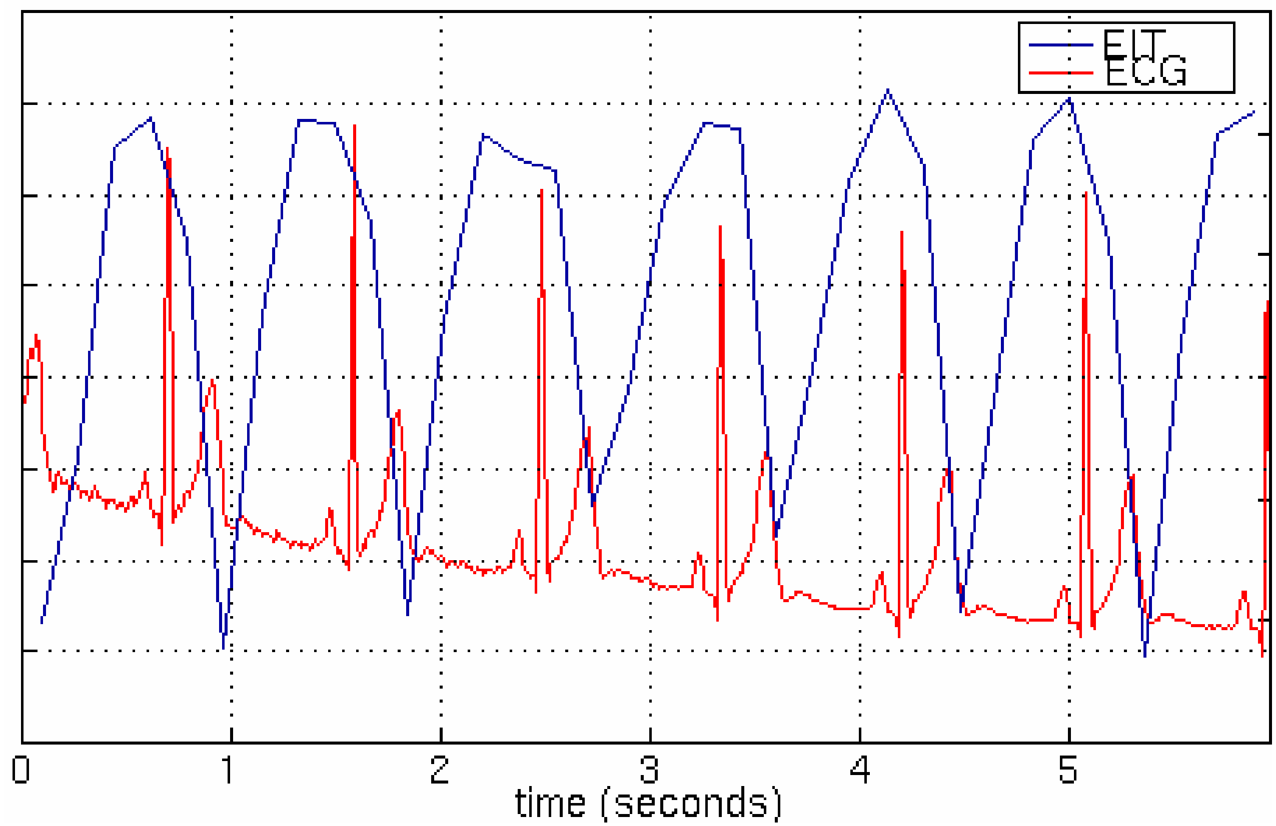
- Non-invasive
- Non-cumbersome
- Suitable for monitoring
- Underlying technology is low cost

Application: Breathing



Chest images of tidal breathing in normal

Application: Heart Beat



EIT signal in ROI around heart and ECG

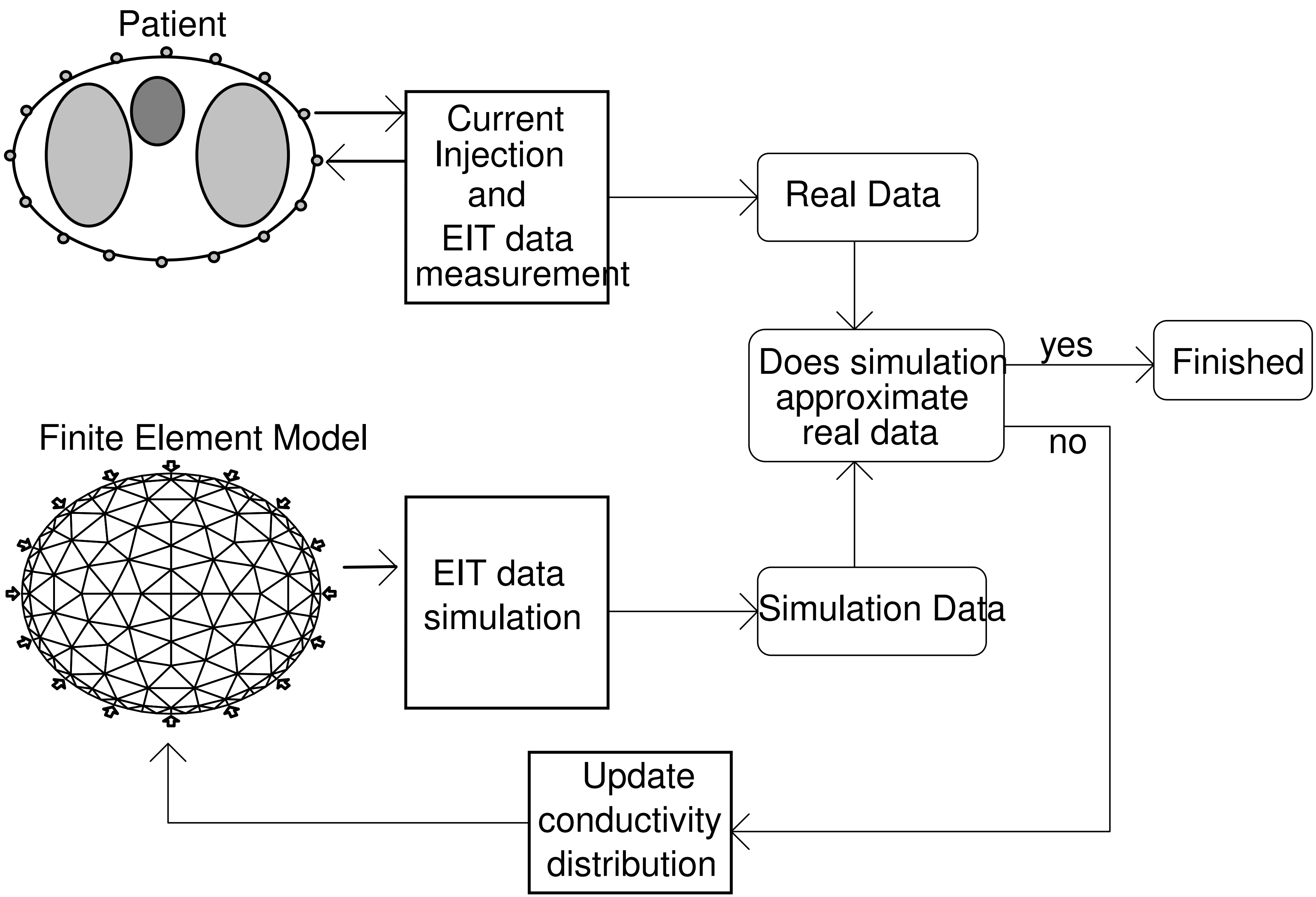
Image Reconstruction: Static Imaging

Static imaging reconstructs the absolute conductivity from measurements.

Algorithms:

- Iterative (Newton-Raphson)
- Layer Stripping

Block Diagram of Iterative Algorithm



Static Imaging Difficulties

- Extremely sensitive to uncertainties in electrode position
- Ill-conditioned problem
- Numerical instability

Dynamic Imaging

- Calculate change in conductivity distribution from change in measurements
- Inverse problem *linearized*
- Much reduced sensitivity to electrode and hardware errors.
- Very suitable for physiological imaging: lung, heart, GI

Inverse Techniques

- We can pose dynamic imaging as linear inverse, using a *sensitivity matrix*

$$\mathbf{z}_j = \frac{\mathbf{z}(\sigma_h) - \mathbf{z}(\sigma_h + \delta_j)}{\delta_j}$$

$$\mathbf{z} = \mathbf{H}\Delta\sigma$$

Parametrize Conductivity

- We want to parameterize conductivity
  - So that all reconstructed valued are physically valid
  - To reflect physical importance of low and high values
- Most common parameterization is r = *log*(conductivity)

Inverse Techniques

- Classic least-squares inverse

$$\mathbf{z} = \mathbf{H}\mathbf{x}$$

$$\hat{\mathbf{x}} = \left(\mathbf{H}^t\mathbf{H}\right)^{-1}\mathbf{H}^t\mathbf{z}$$

Least squares inverse

However, problem is:

- ill-conditioned: measurements depend much more on data near electrodes than in centre
- ill-formed: more unknowns than measurements

Regularized Imaging

Handwaving argument for regularization:
used for ill-posed and ill-formed problems to find a solution with:

- Low error: small (**z – Hx**)
- Stable: small change in **x** for small **Δz**
- Good looking:
  - Somewhat hard to define, but includes smoothness, clean edges, etc.

MAP estimates

- MAP approach says choose **x** such that *f*(**x**|**z**) is maximized
  - In other words, choose the image that is most likely, considering the measured data
- Bayes Rule

$$f\left(\mathbf{x}\middle|\mathbf{z}\right)=\frac{f\left(\mathbf{z}\middle|\mathbf{x}\right)f\left(\mathbf{x}\right)}{f\left(\mathbf{z}\right)}$$

MAP estimates

$f(\mathbf{z}|\mathbf{x})$ the distribution of measurements given an image

- Based on forward model and noise properties

$f(\mathbf{z})$ distribution of measurements

- Not a parameter of MAP estimate

$f(\mathbf{x})$ distribution of image

- Based on *a priori* knowledge of physically possible and likely images distributions

Regularized Imaging

Given Linear Model:

$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

Maximum A Posteriori (MAP) estimate is:

$$\hat{\mathbf{x}} = \left(\mathbf{H}^t \mathbf{R}_n^{-1} \mathbf{H} + \mathbf{R}_x^{-1}\right)^{-1} \left(\mathbf{H}^t \mathbf{R}_n^{-1} z + \mathbf{R}_x^{-1} \mathbf{x}_\infty\right)$$

Regularized Imaging

- Parameters $\mathbf{R_x}$, $\mathbf{R_n}$, x_∞ , represent *a priori* statistical knowledge of problem

$$\mathbf{x}_\infty = E[\mathbf{x}]$$

$$\mathbf{R_x} = E\left[(\mathbf{x} - \mathbf{x}_\infty)^t (\mathbf{x} - \mathbf{x}_\infty)\right] = E\left[\mathbf{x}^t \mathbf{x}\right] - \mathbf{x}_\infty{}^t \mathbf{x}_\infty$$

$$\mathbf{R_n} = E\left[\mathbf{n}^t \mathbf{n}\right] = \begin{bmatrix} \sigma_1^2 & 0 & \cdots \\ 0 & \sigma_2^2 & \\ \vdots & & \ddots \end{bmatrix}$$

Choice of parameter $\mathbf{R_x}$

- Parameter is a "penalty function"
- Many regularization approaches use a diagonal matrix
  - Tikhonov regularization uses the scaled identity matrix
  - This will penalize large amplitude pixels in image
- We choose a dense matrix
  - Penalize image frequency content above maximum possible with measurements

Choice of parameter $\mathbf{R_x}$

- In order to avoid problems inverting $\mathbf{R_x}$, we directly calculate the inverse
  - Since $\mathbf{R_x}$ represents spatial low pass filter, $\mathbf{R_x}^{-1}$ represents a high pass
- Choose a Gaussian high pass of form

$$F(u,v) = 1 - e^{-\omega_0\left(u^2+v^2\right)}$$

Regularization: Hyperparameters

Regularizations techniques must finally introduce a "hyperparameter" (μ)

$$\hat{\mathbf{x}} = \left(\mathbf{H}^t\mathbf{W}\mathbf{H} + \mu\mathbf{Q}\right)^{-1}\mathbf{H}^t\mathbf{W}\mathbf{z}$$

where

$$\mathbf{W} = \frac{1}{\sigma_n^2}\mathbf{R}_\mathbf{n}^{-1}$$, ie. the relative noise amplitudes

$$\mathbf{Q} = \frac{1}{\sigma_x^2}\mathbf{R}_\mathbf{x}^{-1}$$, ie. the relative image correlations

Regularization: Hyperparameters

μ is thus the ratio of image and noise amplitudes,

$$\mu = \frac{\sigma_x^2}{\sigma_y^2}$$

it can be interpreted as a the *noise figure* of a signal receiver

Regularized Inverse

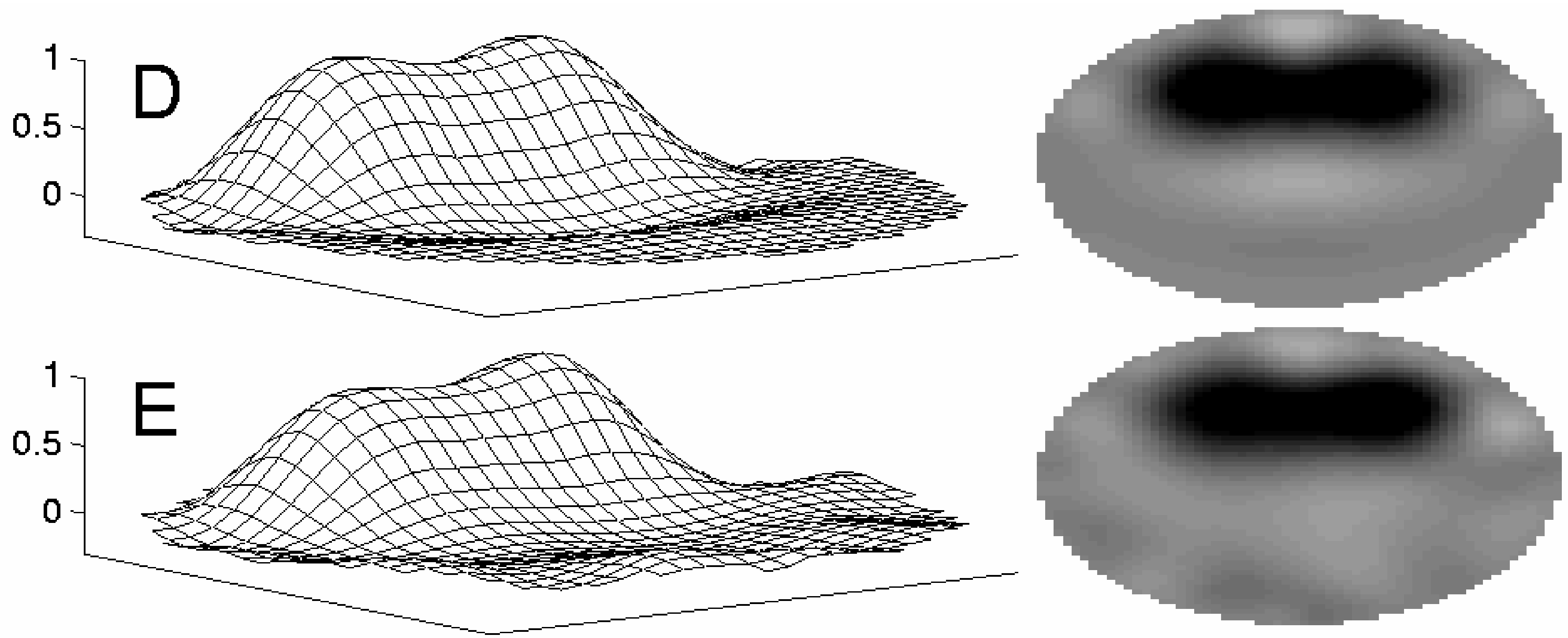
Parameters:

- **W**: models measurement noise
- **Q**: penalizes image features which are greater than data supports
- $\mathbf{x}_\infty$: represents the background conductivity distribution (heart,lungs,etc)
- **μ**: "hyper-parameter" amount of regularization

Advantages of Regularization

- Stabilizes ill-conditioned inverse
- Introduction of *a priori* information
- Control of *resolution-noise* performance trade-off
- MAP inverse justifies the formulation in terms of Bayesian statistics

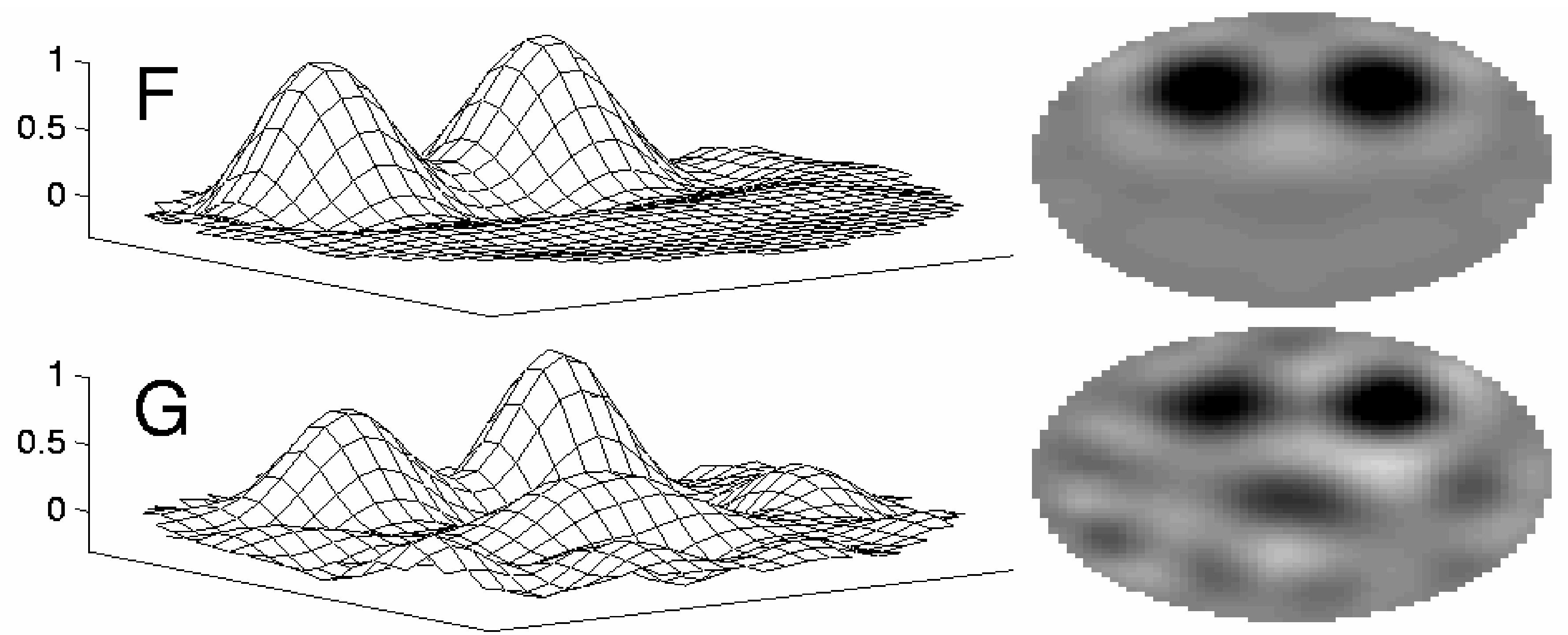
Noise – Resolution Tradeoff



D: *Meas:* No Noise *Reconst:* NF= 0.4

E: *Meas:* -3dB SNR *Reconst:* NF= 0.4

Noise – Resolution Tradeoff



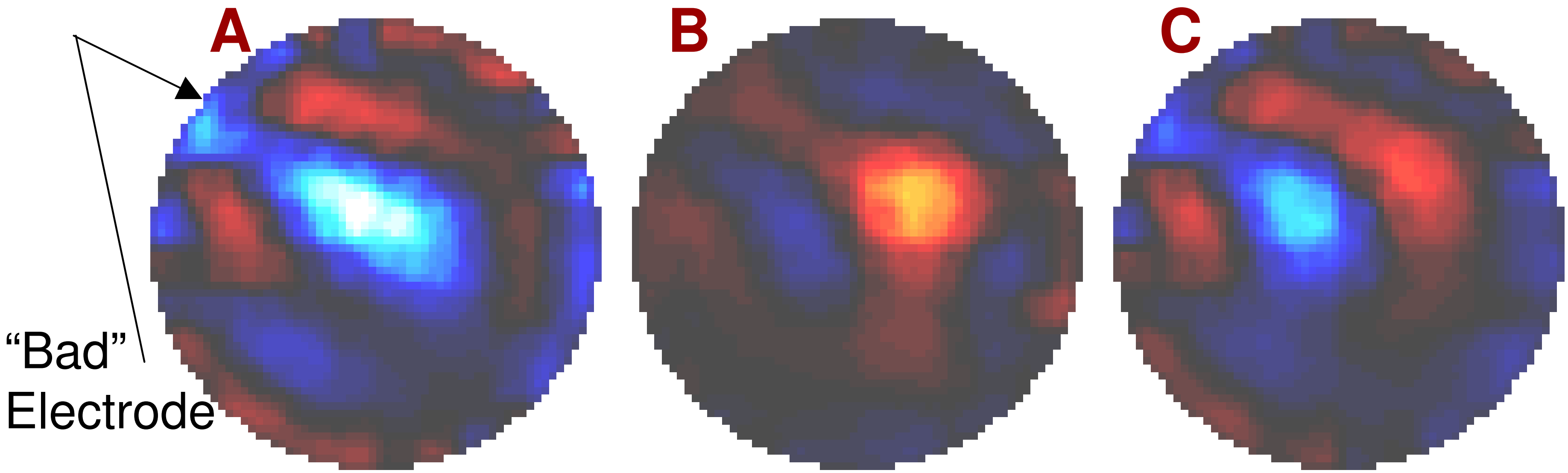
F: *Meas:* No Noise *Reconst:* NF= 2.0

G: *Meas:* -3dB SNR *Reconst:* NF= 2.0

Electrode Measurement Errors

- Experimental measurements with EIT quite often show large errors from one electrode
- Causes aren't always clear
  - Electrode Detaching
  - Skin movement
  - Sweat changes contact impedance
  - Electronics Drift?

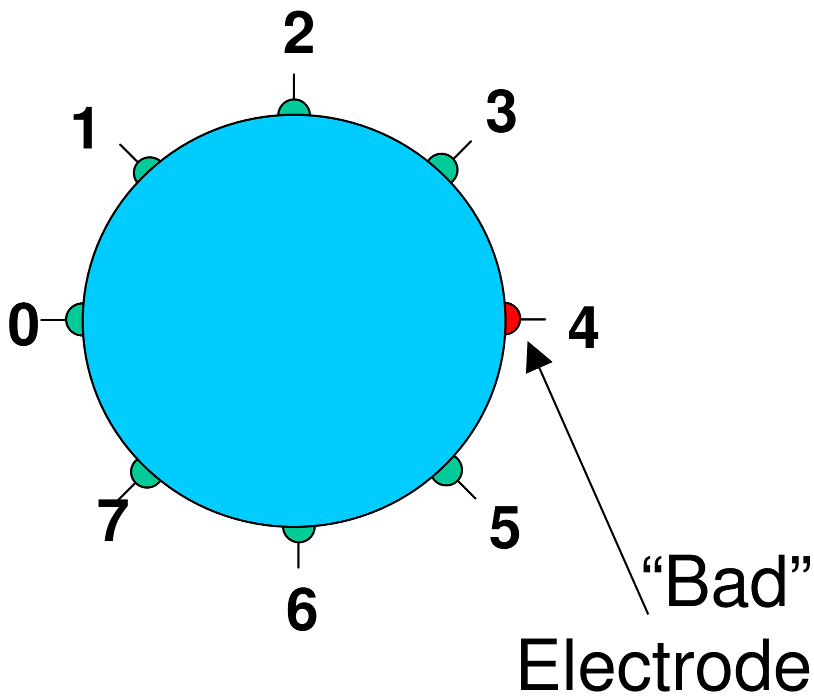
Example of electrode errors



Images measured in anaesthetised, ventilated dog

A. Image of 700 ml ventilation
B. Image of 100 ml saline instillation in right lung
C. Image of 700 ml ventilation and 100 ml saline

Measurements with “bad” electrode



	01	12	23	34	45	56	67	70
01	X	X		*	*			X
12	X	X	X	*	*			
23		X	X	X	*			
34	*	*	X	X	X	*	*	*
45	*	*	*	X	X	X	*	*
56				*	X	X	X	
67				*	*	X	X	X
70	X			*	*		X	X

* “bad” measurement

X measurement at current injection

Possible solution: zero erroneous data

- Set all measurement and injection data on “bad” electrodes to zero
- “Traditional solution” in the sense I’ve used it. I’m not aware of any formal description

Solution: zero erroneous data

Issues

- Reduces amplitude of contrasts
- Error in reconstructed contrast Position
- Decreases image resolution

Proposed solution: Bayesian Imaging model

- Maximum *a posteriori* (MAP) models allow incorporation of known constraints into regularized image calculation
- Model electrode errors as *a priori* large measurement noise on all measurements using affected electrode

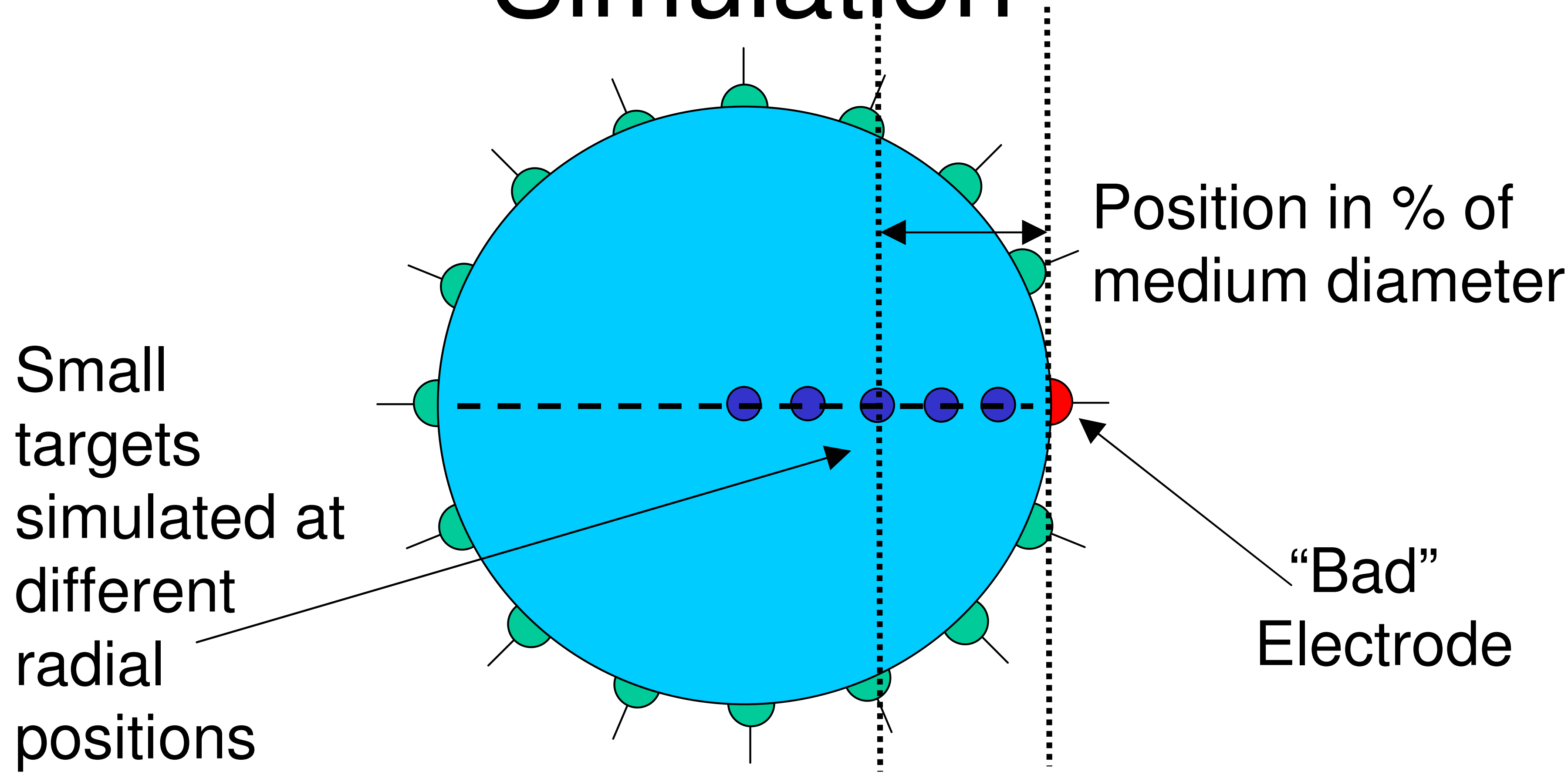
Regularized Imaging Model

- Parameters $\mathbf{R_x}$, $\mathbf{R_n}$, represent *a priori* statistical knowledge of problem

$$\mathbf{R_n} = E\left[\mathbf{n}^t\mathbf{n}\right] = \begin{bmatrix} \sigma_1^2 & 0 & \cdots \\ 0 & \sigma_2^2 & \\ \vdots & & \ddots \end{bmatrix}$$

- If a $\sigma^2 = \infty$, then inverse matrix will have 0 in corresponding position

Simulation

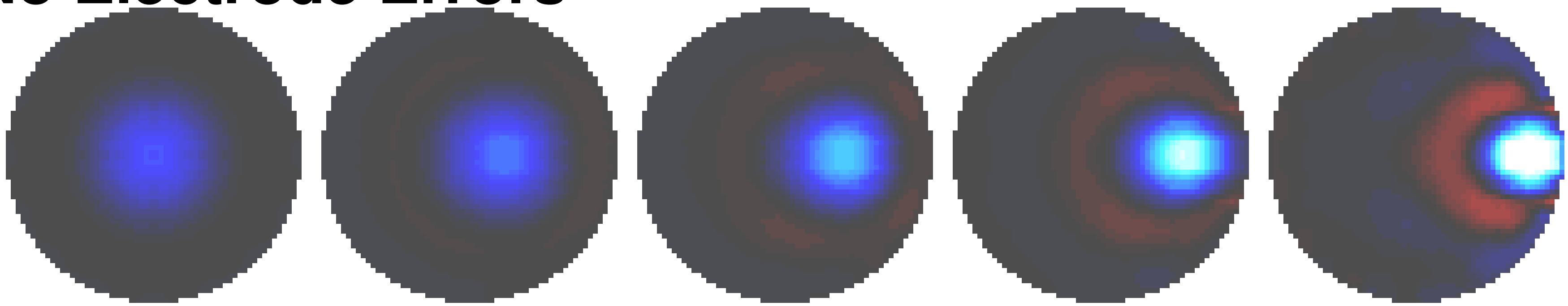


Data simulated with 2D FEM with 1024 elements

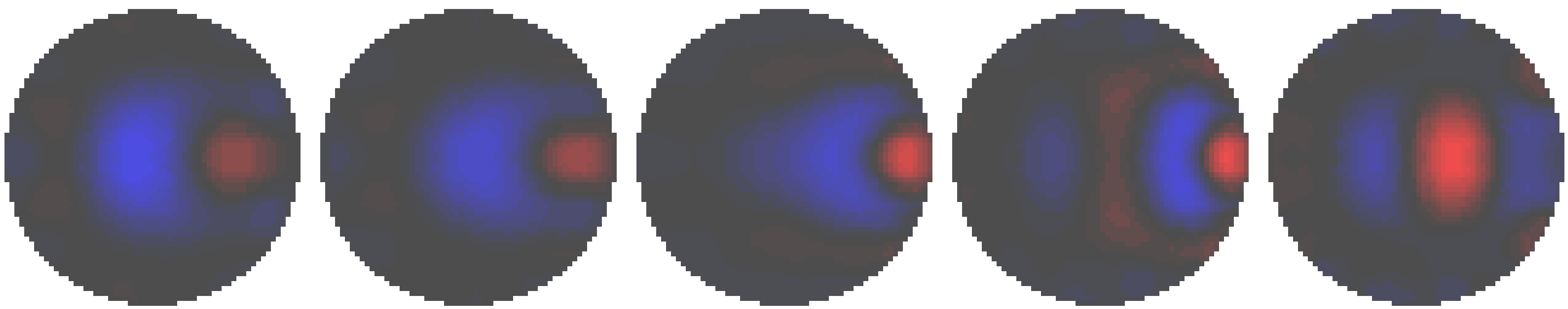
– not same as inverse model

Simulation results for opposite drive

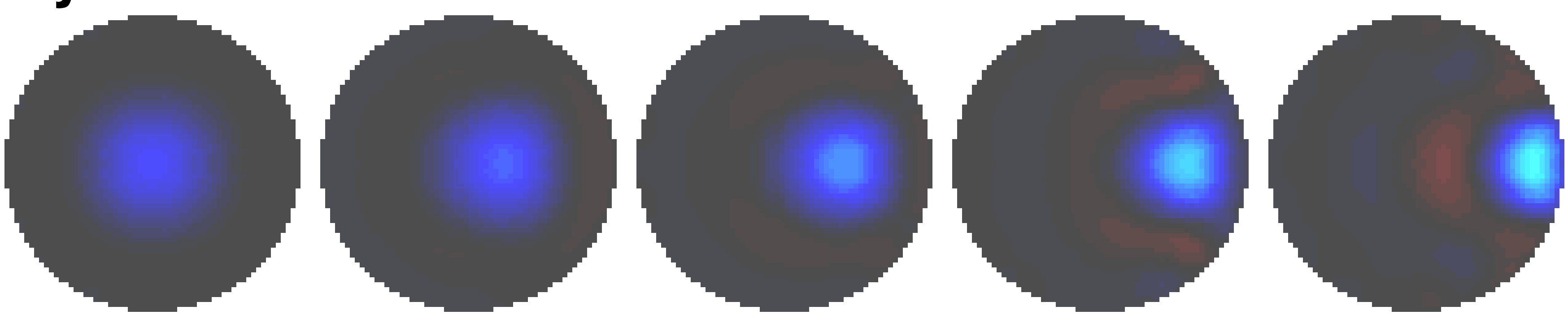
No Electrode Errors

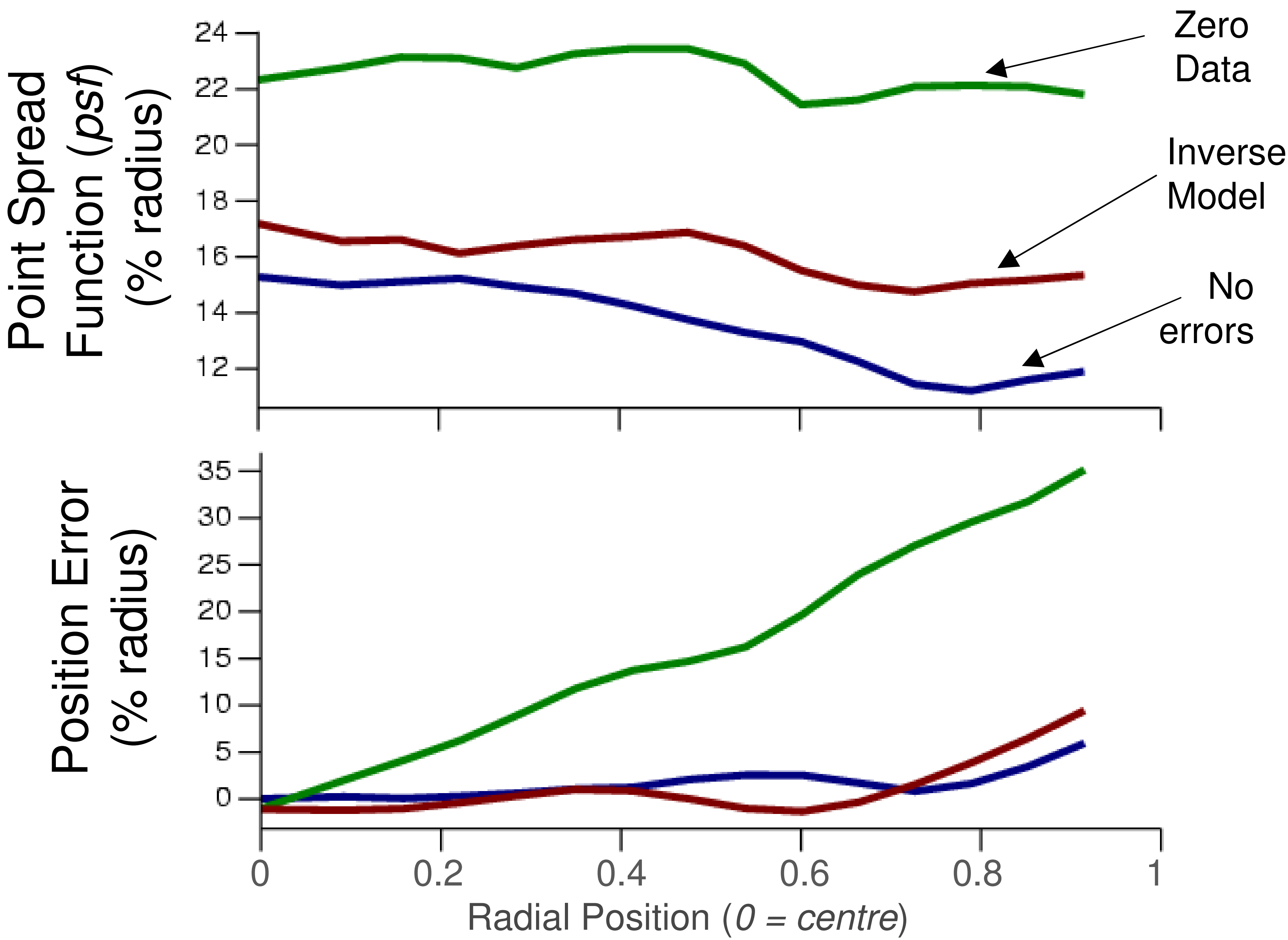


Zero Affected Measurements

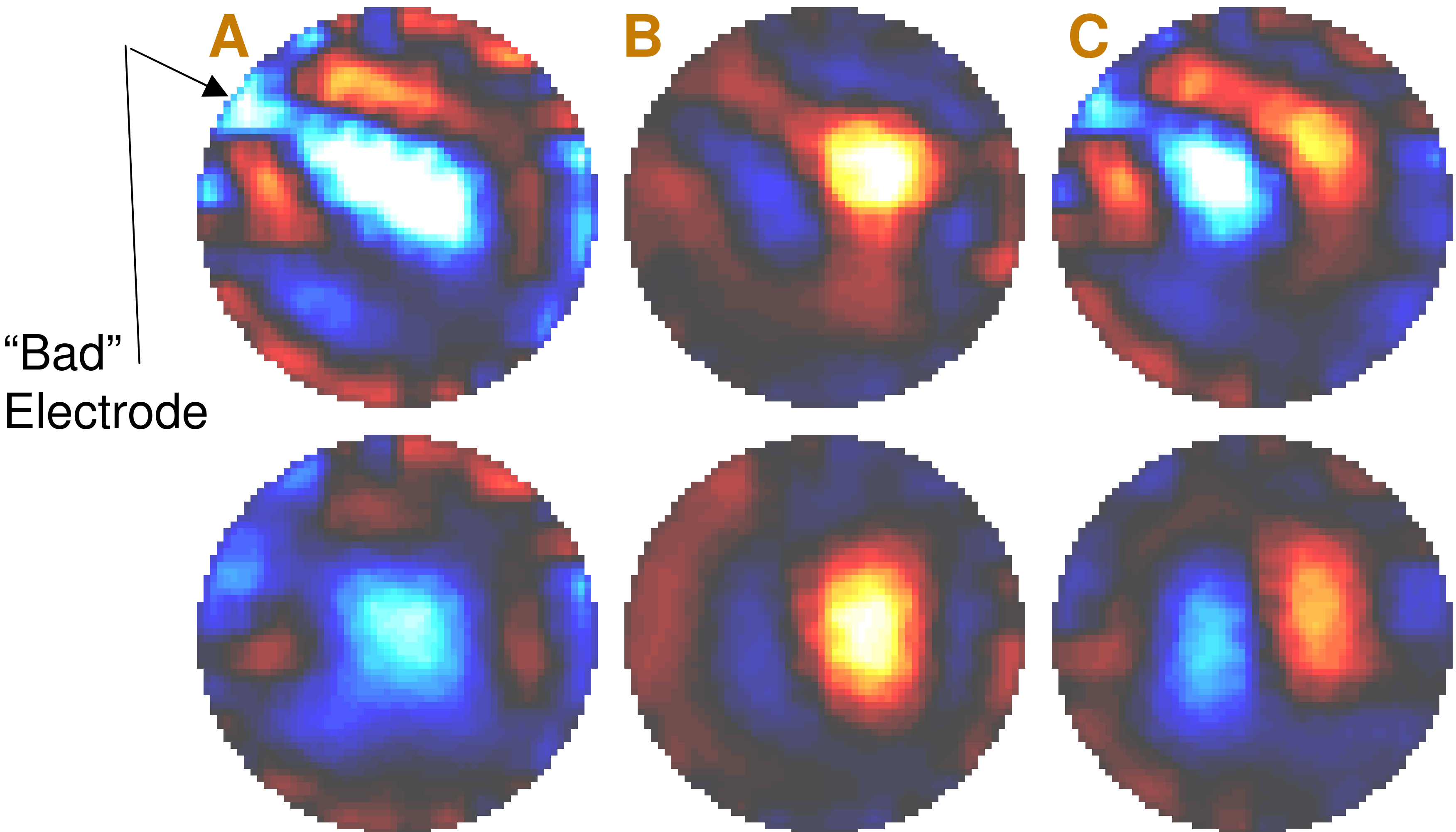


Bayesian Inverse




Point Spread Function (psf) (% radius)
24
22
20
18
16
14
12
Zero Data
Inverse Model
No errors
Position Error (% radius)
35
30
25
20
15
10
5
0
0
0.2
0.4
0.6
0.8
1
Radial Position (0 = centre)

How does this work with real data?



A. Image of 700 ml ventilation
B. Image of 100 ml saline instillation in right lung
C. Image of 700 ml ventilation and 100 ml saline

Discussion

Image Approaches to Electrode Errors

- Set “bad data” to zero
  – Works reasonably well > 25% diameter from electrode
- MAP model
  – Works well up to 15% diameter from electrode
  – Close to no errors for opposite drive
  – Natural extension of Bayesian prior info

However, should also try to better understand the causes of electrode errors …

Work in progress

Model Electrode Errors

- Physical modelling
  - Electronics Drift
  - Electrode movement
  - Change in skin impedance due to sweat, irritation, etc.
- Numerical Modelling
  - Finite element modelling of both mechanical and electrical properties of thorax
- We hope to be able to build better image reconstruction algs., using detailed prior knowledge

Work in progress

- Automatic identification of erroneous electrode data
- Approach: *for all electrodes* (e_i)
  - Using all electrodes, except e_i, reconstruct image
  - Using image, estimate measurements on e_i
  - Compare measured vs simulated data

Electrical Impedance Tomography: Image Reconstruction with Electrode Measurement Errors.

A. Adler, VIVA Lab Seminar, U. Ottawa, 29 May 2003

Abstract: Electrical Impedance Tomography (EIT) is a relatively new medical imaging technique which allows imaging of the change in conductivity distribution within a body using body surface current applications and voltage measurements. We are particularly interested in its use as a monitoring technique of lung and heart activity in anesthetized and critical care patients. EIT's advantages - non-invasive, non-cumbersome, and relatively low cost - make it ideal for this kind of monitoring application. Reconstruction of the conductivity change image involves the solution of a non-linear, ill-posed problem from noisy data. Stable solutions are typically achieved by the use of regularization. The talk will present an approach using Maximum a Posteriori (MAP) based regularization, in which the data and image priors are based on detailed modelling of image and noise priors. One of the most challenging problems in EIT – especially for long term monitoring applications - is dealing with errors in electrode measurements. Electronics drift, electrode movement and changing electrode impedance due to sweat and irritation introduce difficult-to-model errors into the data. Our work in progress to deal with some of these effects will be presented. The regularized image reconstruction model is modified to account for known data errors in terms of Bayesian prior information, allowing for the calculation of remarkably good images in the presence of severe single electrode data errors.