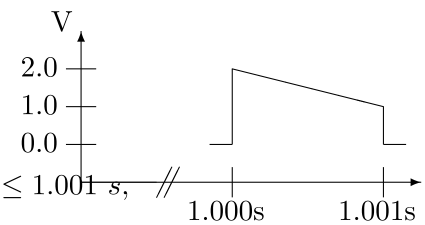
SYSC 4405: Midterm Exam #2. November 19, 2012
Carleton University, Systems and Computer Engineering

Instructions:
- This exam has **8** questions. Answer all questions. You have **80 minutes** to complete this exam.
- This is a closed book exam; however, you are permitted to bring one 8.5" × 11" sheet of notes.
- You are permitted to use a non-network connected calculator.
- Write your answers on an examination booklet. You may take this examination paper with you.
- An ideal low pass filter with cutoff frequency ω_c has impulse response: $h_{LP}[n] = \frac{\omega_c}{\pi} sinc\left(\frac{\omega_c}{\pi} n\right)$
- The FFT requires $N log_2(N)$ complex additions and $\frac{1}{2}N(log_2(N) - 2) + 1$ complex multiplications.
- Filter windows have the form: $w[n] = a_0 + a_1 cos(\pi \frac{n-L}{L}) + a_2 cos(2\pi \frac{n-L}{L}) + a_3 cos(3\pi \frac{n-L}{L})$, where

Window Name	Atten. (dB)	TBW (/L)	a_0	a_1	a_2	a_3
Rectangular	20.8	0.46	1	0	0	0
Hann	43.9	1.56	0.5	0.5	0	0
Hamming	53.9	1.90	0.53836	0.46164	0	0
Blackman	75.3	2.79	0.42	0.5	0.08	0
Blackman-Nutall	112.7	4.09	0.363582	0.489178	0.136510	0.010641

Consider a short segment of an input, $x(t)$, during the time $1.0\ s \leq t \leq 1.001\ s$ where the input is falling linearly from $x(1.000\text{ s}) = 2.0$ V to $x(1.001\text{ s}) = 1.0$ V. The signal is zero outside this range. A DSP system samples at $F_s = 3.0$ kSamples/s.



1. (5 points) **Plot the sampled signal** during the time $1.0\ s \leq t \leq 1.001\ s$, showing n and $x[n]$.

   ANSWER:
   n is 3000 . . . 3003 $x[n] = 2.0, 1.67, 1.33, 1$

2. (5 points) The input $x[n]$ is filtered with a high pass filter IIR filter such that

$$y[n] = x[n-1] - \tfrac{1}{2} y[n-1].$$

   A: Sketch the filter block diagram and **B: Calculate the output** $y[n]$ (to two significant figures) for time $1.0\ s \leq t \leq 1.001\ s$, assuming zero initial conditions.

   ANSWER:

n	$x[n]$	$x[n-1]$	$y[n-1]$	$y[n]$
3000	2.00	0.00	0.00	0.00
3001	1.67	2.00	0.00	2.00
3002	1.33	1.67	2.00	0.67
3003	1.00	1.33	0.67	1.00

3. (5 points) Calculate the impulse response $H(e^{j\omega})$ for the filter in question #2. **ANSWER:**

$$h[n] = (0.5)^{n-1} u[n-1] \tag{1}$$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (0.5)^{n-1} u[n-1] e^{-j\omega n} = \sum_{n=1}^{\infty} (0.5)^{n-1} e^{-j\omega n} \tag{2}$$

$$H(e^{j\omega}) = \sum_{m=0}^{\infty} (0.5)^m e^{-j\omega(m+1)} = e^{-j\omega} \sum_{m=0}^{\infty} (0.5)^m e^{-j\omega m} \tag{3}$$

$$H(e^{j\omega}) = e^{-j\omega} \sum_{m=0}^{\infty} \left(0.5 e^{-j\omega}\right)^m = e^{-j\omega} \frac{1}{1 - 0.5 e^{-j\omega}} \tag{4}$$

For the next questions, assume a signal $x[n] = \{10, 9, 8, 7, 6, 5, 4, 3, 2, 1, \ldots\}$, and a high-pass filter $h[n] = \{1, -1\}$. For convenience, the 4-point DFT of $h[n]$ is $H[k] = \{0, 1+j, 2, 1-j\}$.

4. (5 points) **Sketch the convolution operation** using the overlap-add method using $N = 4$, $M = 2$, and $B = 3$. Show the contents of the blocks of $x[n]$, the sizes of blocks $y[n]$. Also, show any zero-padding, and indicate the samples which need to be added together.

   ANSWER:
   Sketch here

5. (5 points) Which block of $x[n]$ is required to calculate $y[4]$? **Calculate high frequency term** $Y[2] = H[2]X[2]$ **for the block containing** $y[4]$.

   ANSWER:
   The block is the $x_2[n] = 7, 6, 5, 0$
   $X_2[2] = 1, -1, 1, -1 \times 7, 6, 5, 0 = 6$
   $Y[2] = H[2]X[2] = 2 \times 6 = 12$

For the next questions, assume a DSP system with $F_s = 3.0$ kSamples/s, we need to attenuate frequencies below 100 Hz by at least a factor of 10^3. The gain for frequencies above 200 Hz must be $1 \pm .01$.

6. (5 points) **Sketch the filter requirements**, and **choose an appropriate window** $w[n]$ for the requirements.

   ANSWER:
   This is a HighPass filter. $f_L = 100/3000 = .033$. $f_H = 200/3000 = .066$. $TBW = .033$

   Required attenuation is $20log_{10}(10^3) = 60$ dB. This is a Blackman window.

   $L = 2.79/.033 = 84.5 \approx 85$.

   $w[n] = 0.42 + 0.5cos(\pi\frac{n-L}{L} + 0.08cos(2\pi\frac{n-L}{L})$

7. (5 points) Calculate an ideal filter $h_{ideal}[n]$, and then **calculate the realizable FIR filter** $h[n]$ for these requirements.

   ANSWER:
   $h_{HP} = (-1)^n 2f_c sinc(2f_c n)$ where $f_c = \frac{1}{2} - f_a$. $f_a = 150/3000 = 0.05$, $f_c = 0.5 - 150/3000 = 0.4$
   $h[n] = (-1)^{n-L} 2f_c sinc(2f_c(n-L)) \times w[n]$

8. (5 points) What is the delay (in ms) of this filter if **A: it is implemented using convolution**, and **B: it is implemented using overlap-add block processing, with** $N = 2048$. Assume a very fast microprocessor for both cases.

   ANSWER:
   A: Delay $= L/f_s = 85/3000 = 0.028\ s = 28\ ms$.

   $B = N - M + 1 = 2048 - (2x85 + 1) + 1 = 1878$.

   B: Delay $= (L + B)/f_s = (1878 + 85)/3000 = 0.654\ s = 654\ ms$.