Filters Design

Ex.

★ Inspired by how to design a bridge

① Requirements - 5T

② Look up solutions - 2T x 3 = 6T

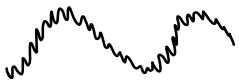
- 10T x 1 = 10T

③ Choose and Implement

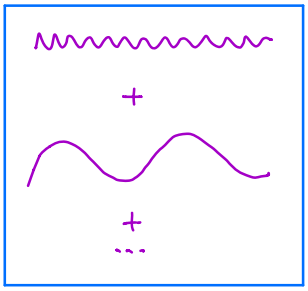
This is the same process for filters.

Filtering in Pics:

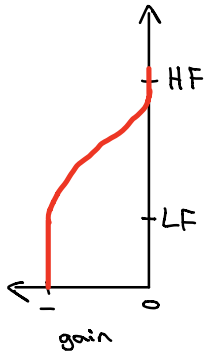
have

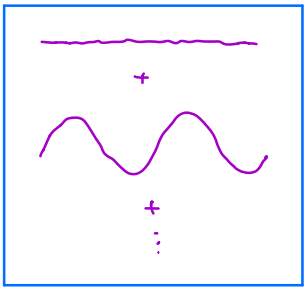


=



want





Low Pass Filter

N	Fp(0.8)	Fs(-40)	Fs(-60)	Fs(-80)	Fn	Zeta	Fn	Zeta	Fn	Zeta	Fn	Zeta
FILTER = Butterworth												
2	1.012	10.000	31.623	100.000	1.000	0.707						
4	1.007	3.162	5.623	10.000	1.000	0.924	1.000	0.383				
6	1.005	2.158	3.162	4.645	1.000	0.707	1.000	0.966	1.000	0.259		
8	1.005	1.778	2.371	3.162	1.000	0.981	1.000	0.831	1.000	0.556	1.000	0.195
FILTER = Chebychev 0.05dB												
2	2.291	21.577	68.234	215.774	2.162	0.668						
4	1.285	3.365	5.888	10.423	0.885	0.833	1.221	0.250				
6	1.125	1.897	2.673	3.846	0.569	0.860	0.870	0.412	1.091	0.120		
8	1.069	1.479	1.858	2.393	0.422	0.870	0.670	0.464	0.912	0.228	1.050	0.069
FILTER = Chebychev 0.1dB												
2	1.963	18.113	57.280	181.134	1.820	0.652						
4	1.219	3.097	5.408	9.550	0.789	0.808	1.153	0.229				
6	1.096	1.807	2.535	3.639	0.513	0.834	0.834	0.375	1.063	0.108		
8	1.054	1.432	1.791	2.296	0.382	0.843	0.645	0.423	0.894	0.204	1.034	0.062
FILTER = Chebychev 0.2dB												
2	1.690	15.205	48.084	152.055	1.535	0.628						
4	1.161	2.851	4.955	8.750	0.701	0.774	1.095	0.205				
6	1.072	1.722	2.399	3.436	0.460	0.799	0.803	0.335	1.038	0.095		
8	1.040	1.387	1.726	2.208	0.343	0.807	0.623	0.377	0.878	0.179	1.021	0.054
FILTER = Chebychev 0.5dB												
2	1.403	11.995	37.844	119.674	1.231	0.579						
4	1.096	2.553	4.416	7.780	0.597	0.709	1.031	0.170				
6	1.042	1.614	2.234	3.192	0.396	0.731	0.768	0.276	1.011	0.077		
8	1.026	1.334	1.644	2.094	0.297	0.739	0.599	0.310	0.861	0.144	1.006	0.043
FILTER = Chebychev 1.0dB												
2	1.227	9.954	31.405	99.312	1.050	0.523						
4	1.057	2.339	4.027	7.079	0.529	0.637	0.993	0.140				
6	1.026	1.538	2.113	3.006	0.353	0.657	0.747	0.227	0.995	0.062		
8	1.014	1.294	1.585	2.014	0.265	0.664	0.584	0.256	0.851	0.117	0.997	0.035

LPF ex

① Accept all $f < 100$ Hz

(within 10%)

1.1
1.0
0.9

$20\log(1.1) = 0.83$ dB

$20\log(1.0) = 0$ dB

$20\log(0.9) = -0.92$ dB

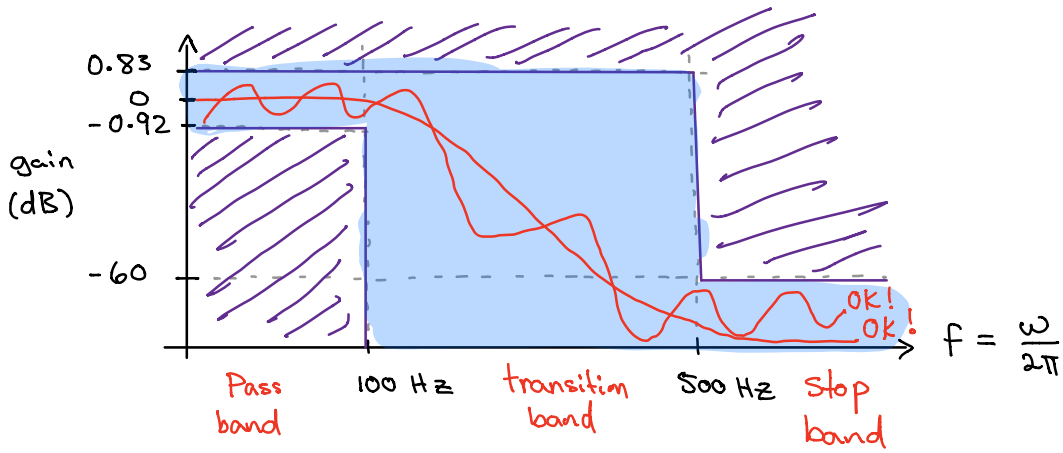
② Reject all $f > 500$ Hz

(< 0.1 %)

$20\log(0.001) = -60$ dB $=$ gain

↑

≡ attenuation of 60 dB



HPF Ex

① Accept ±1% all $f > 100$ Hz

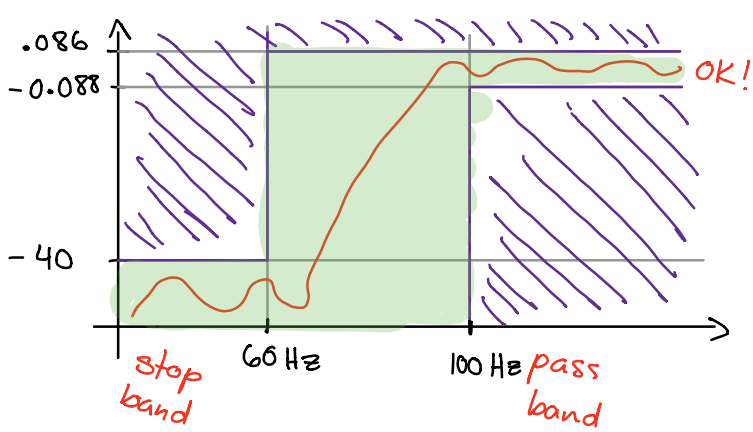
② Reject (<1%) of $f < 60$ Hz

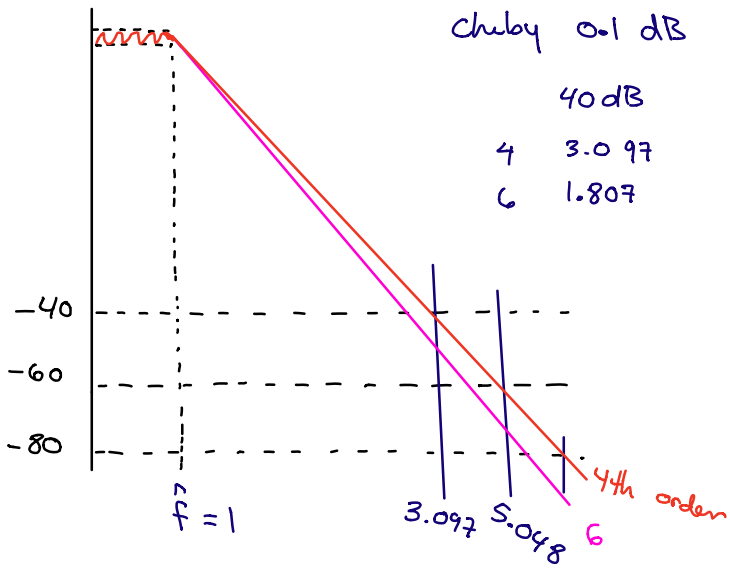
$20 \log(0.01) = -40$ dB

$20 \log(1.01) = +0.086$ dB

$20 \log(0.99) = -0.088$

} amount of deviation allowed, but can pick 0.05 to do better.





Chebyv 0.1 dB

	40 dB	60 dB	80 dB
4	3.097	5.048	9.55
6	1.807	2.535	

N	Fp(.8)	Fs(-40)	Fs(-60)	Fs(-80)	Fn	ζ	Fn	ζ	Fn	ζ	Fn	ζ
FILTER = Chebychev 0.5dB												
2	1.403	11.995	37.844	119.674	1.231	0.579						
4	1.096	2.553	4.416	7.780	0.597	0.709	1.031	0.170				
6	1.042	1.614	2.234	3.192	0.396	0.731	0.768	0.276	1.011	0.077		
8	1.026	1.334	1.644	2.094	0.297	0.739	0.599	0.310	0.861	0.144	1.006	0.043

LowPass

Use cheby 0.5 dB ✓

(0.5 < 0.83)

req. $F_s = \frac{f_s}{f_p} = \frac{500\ Hz}{100\ Hz} = 5$

N	-60 dB	
2	37	✗
4	4.4	✓ (already attenuates by $4.4 \times f_p = 440\ Hz$, we need 500 Hz, so OK!)
6	2.2	These also meet the requirements, but need more components than necessary.
8	1.6	

Choose lowest N which meets req.

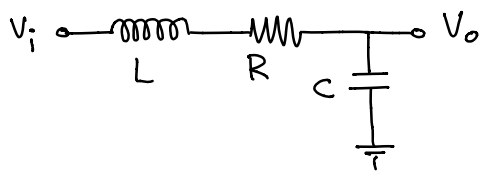
$F_n = .597$, $\zeta = .709$ → $F_n = 1.031$, $\zeta = .170$

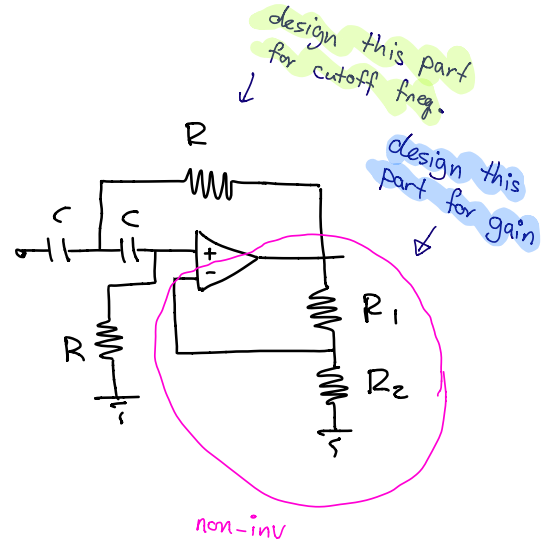
← cutoff freq.

← damping const.

Order 2 + 2 = 4

Two High Pass Filters





$$\omega_c = \frac{1}{\sqrt{LC}}$$

$$\zeta = \frac{R}{2L}$$

$$\boxed{\begin{aligned} G &= 1 + \frac{R_1}{R_2} \\ \zeta &= \tfrac{1}{2}(3 - G) \\ \omega_c &= \frac{1}{RC} \end{aligned}}$$

N	Fp(.8)	Fs(-40)	Fs(-60)	Fs(-80)	Fn	ζ	Fn	ζ	Fn	ζ	Fn	ζ
FILTER = Chebychev 0.05dB												
2	2.291	21.577	68.234	215.774	2.162	0.668						
4	1.285	3.365	5.888	10.423	0.885	0.833	1.221	0.250				
6	1.125	1.897	2.673	3.846	0.569	0.860	0.870	0.412	1.091	0.120		
8	1.069	1.479	1.858	2.393	0.422	0.870	0.670	0.464	0.912	0.228	1.050	0.069

High Pass

Choose .05 dB Cheby

(0.05 dB < .086 dB)

arrange the num. & denom. so that F_s is always >1

N	-40 dB
2	21
4	3.3
6	1.89
8	1.4 ✓

Req. $F_s = \frac{f_p \; 100\,Hz}{60\,Hz} = 1.66$

Continue choosing values from the table.

$F_N = .422$, $\zeta = .870$ → $F_N = .670$, $\zeta = .464$ → $F_N =$, $\zeta =$ → $F_N =$, $\zeta =$

$f_c = \frac{100}{0.422} = 237\ Hz$

$f_c = \frac{f_p}{F_N} = \frac{100}{.67} = 149\ Hz$

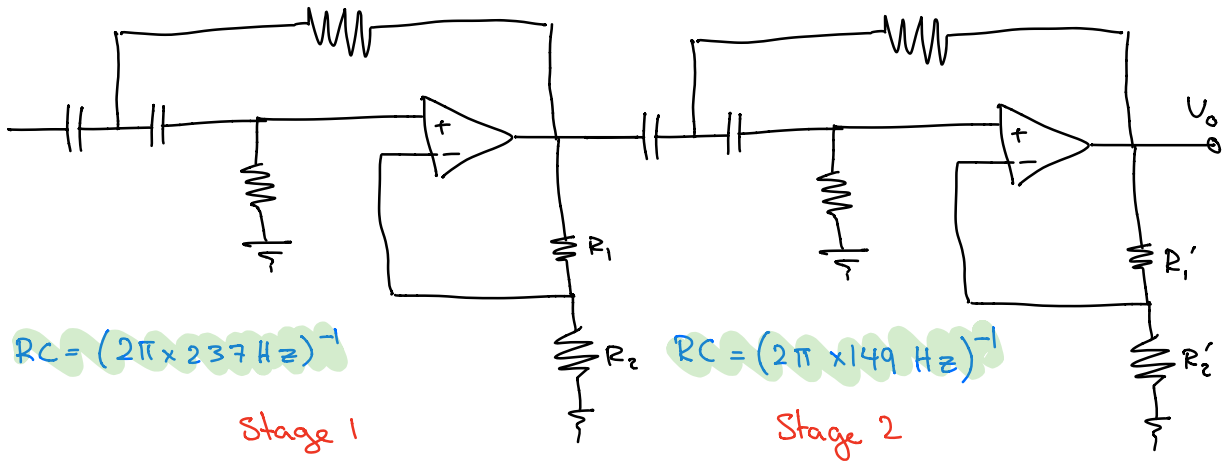
Low Pass

$(f_c)_{stage} = f_p \times F_N$

High Pass

$(f_c)_{stage} = f_p / F_N$

Now, design the HPF:



$$\zeta = \frac{1}{2}(3-G) = \frac{1}{2}\left[3 - \left(1 + \frac{R_1}{R_2}\right)\right]$$

$$= \boxed{1 - \frac{R_1}{2R_2}}$$

$$\hookrightarrow \frac{R_1}{R_2} = 2(1-\zeta)$$

$$\therefore \frac{R_1}{R_2} = 2 - 2\zeta = 2 - 2(.87)$$

$$\frac{R_1}{R_2} = 2 - 2(.464)$$