For a decreasing exponential with time constant τ, which is offset by $V(\infty)$, we have $\Delta V(t) = V(t) - V(\infty)$,

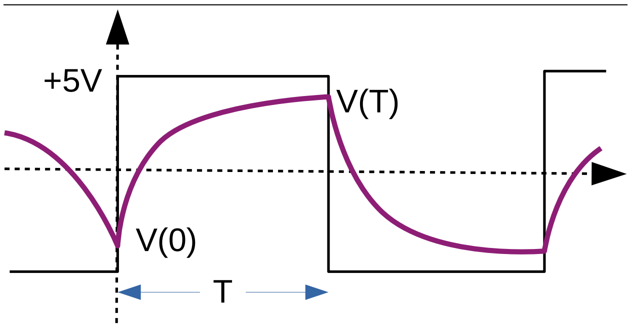
$$\Delta V(t) = \Delta V(0)e^{-t/\tau} \longrightarrow V(t) - V(\infty) = (V(0) - V(\infty))e^{-t/\tau}$$

and

$$\frac{V(t) - V(\infty)}{V(0) - V(\infty)} = e^{-t/\tau}$$

For a square-wave of frequency f, period is $1/f = 2T$, as below

Low-pass RC filter



Here, $V(\infty) = +5\,V$.
When stable $V(0) = -V(T)$.

$$V(T) - V(\infty) = (V(0) - V(\infty))e^{-T/\tau}$$

$$-V(0) - 5\,V = (V(0) - 5\,V)e^{-T/\tau}$$

$$-V(0) - V(0)e^{-T/\tau} = -5\,Ve^{-T/\tau} + 5\,V$$

$$-V(0)(1 + e^{-T/\tau}) = +5\,V(1 - e^{-T/\tau})$$

$$V(0) = -5\,V\left(\frac{1 - e^{-T/\tau}}{1 + e^{-T/\tau}}\right)$$

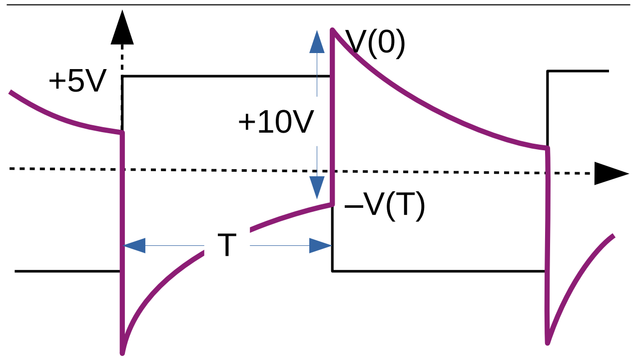
As $-T/\tau \to 0$,

$$V(0) = -5\,V\left(\frac{1-1}{1+1}\right) = 0$$

As $-T/\tau \to \infty$,

$$V(0) = -5\,V\left(\frac{1-0}{1+0}\right) = -5\,V$$

High-pass RC filter



Here, $V(\infty) = 0\,V$.
When stable $V(T) = 10\,V - V(0)$.

$$V(T) - V(\infty) = (V(0) - V(\infty))e^{-T/\tau}$$

$$10 - V(0) - 0\,V = (V(0) - 0\,V)e^{-T/\tau}$$

$$10 - V(0) = V(0)e^{-T/\tau}$$

$$10\,V = V(0)e^{-T/\tau} + V(0) = V(0)(1 + e^{-T/\tau})$$

$$V(0) = 10\,V\left(\frac{1}{1 + e^{-T/\tau}}\right)$$

As $-T/\tau \to 0$,

$$V(0) = 10\,V\left(\frac{1}{1+1}\right) = 5\,V$$

As $-T/\tau \to \infty$,

$$V(0) = 10\,V\left(\frac{1}{1+0}\right) = 10\,V$$