Accelerating Space-Time Reconstructions

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Introduction

Regularized image reconstruction uses a penalty to impose spatial smoothness. Several approaches exist to also impose temporal smoothness. We formulate spatio-temporal reconstruction to helps clarify the impact of parameter choices.

Spatio-temporal (S-T) regularized time difference EIT reconstruction can be formulated in (a) two stages (spatial then temporal) [3], (b) via an augmented S-T matrix [1], or (c) as a Kalman smoother [2]. Here we extend (b) to provide a simplified and efficient calculation.

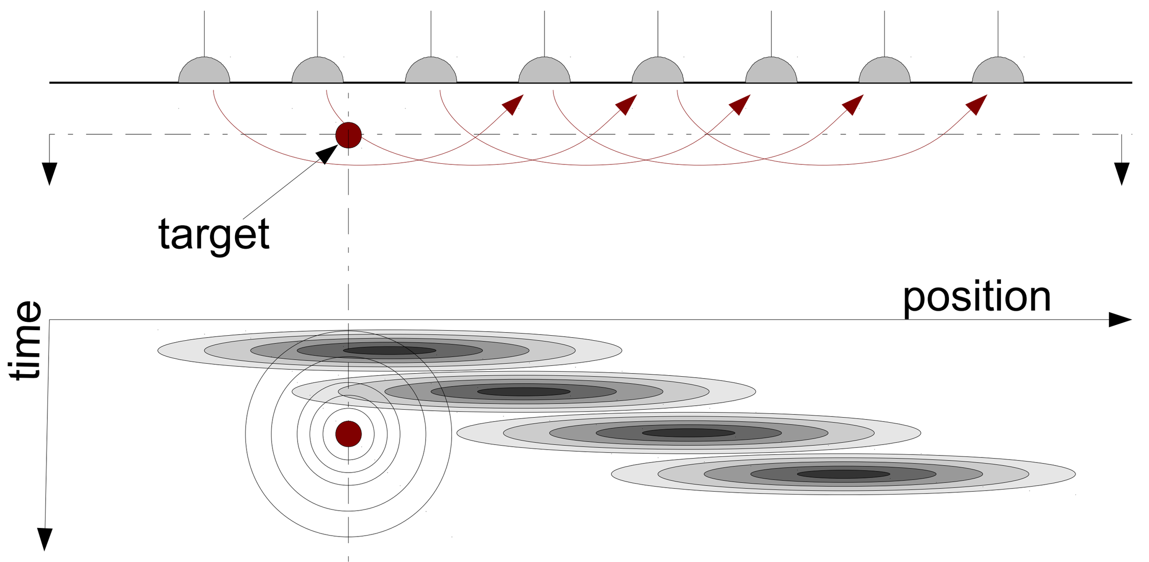


Fig. 1: Block diagram of a geophysical EIT system with a temporal effect. *Top:* horizontal plane beneath surface electrodes *Bottom: Space* and *Time* Interpolation

Space then Time

For a frame of data, y, image x

$$\|y - Jx\|_P^2 + \|x\|^2$$

with solution, $\hat{x} = Ry$

$$R = \left(J^tJ + P\right)^{-1} J^t$$

A S-T formulation

$$\begin{bmatrix} y_+ \\ y_0 \\ y_- \end{bmatrix} = \begin{bmatrix} J & & \\ & J & \\ & & J \end{bmatrix} \begin{bmatrix} x_+ \\ x_0 \\ x_- \end{bmatrix}$$

Model time-correlation, Γ

$$\begin{bmatrix} \hat{x}_+ \\ \hat{x}_0 \\ \hat{x}_- \end{bmatrix} = f \underbrace{\begin{bmatrix} 1 & \gamma & \gamma^2 \\ \gamma & 1 & \gamma \\ \gamma^2 & \gamma & 1 \end{bmatrix}}_{\Gamma} \begin{bmatrix} R & & \\ & R & \\ & & R \end{bmatrix} \begin{bmatrix} y_+ \\ y_0 \\ y_- \end{bmatrix}$$

Choose f so gain=1; $f \to \frac{1-\gamma}{1+\gamma}$.

$$\Gamma^{-1} = f' \begin{bmatrix} 1 & -\gamma & 0 \\ -\gamma & 1+\gamma^2 & -\gamma \\ 0 & -\gamma & 1 \end{bmatrix}$$

where $f' = (1-\gamma)^{-2}$

Space with Time

The augmented reconstruction matrix, $\tilde{R}$

$$\begin{bmatrix} \tilde{R} \end{bmatrix} = \underbrace{\left(\begin{bmatrix} J^TJ & & \\ & J^TJ & \\ & & J^TJ \end{bmatrix} + P \otimes \Gamma^{-1} \right)^{-1}} \begin{bmatrix} \tilde{J}^T \end{bmatrix}$$

$$\left(\begin{bmatrix} J^TJ{+}P & & \\ & J^TJ{+}P & \\ & & J^TJ{+}P \end{bmatrix} + P \otimes (\Gamma^{-1} - I) \right)^{-1}$$

$$= \begin{bmatrix} R & & \\ & R & \\ & & R \end{bmatrix} \underbrace{\left(\begin{bmatrix} I & & \\ & I & \\ & & I \end{bmatrix} + RP \otimes (\Gamma^{-1} - I) \right)^{-1}}_{(\quad I \quad + \quad \delta \quad)^{-1}}$$

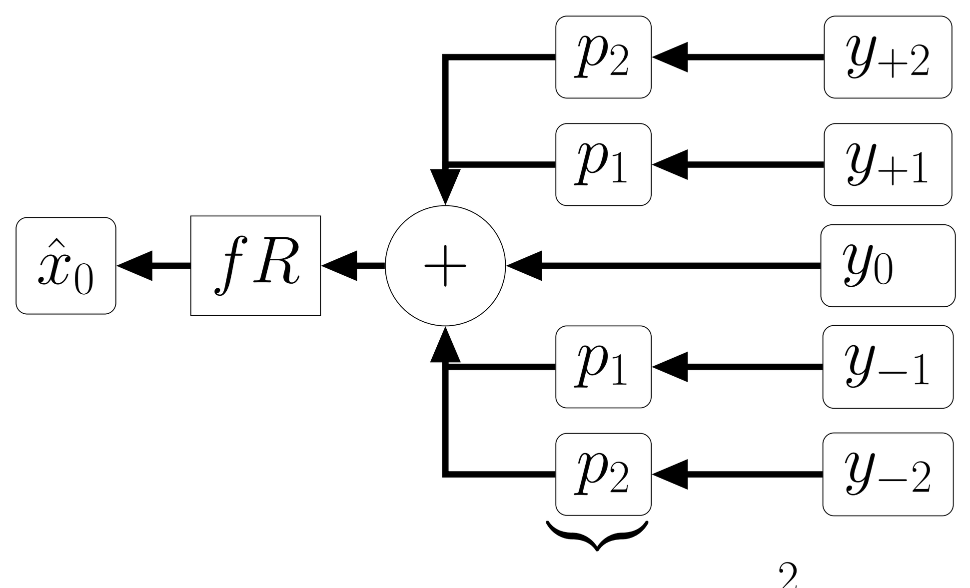
where $(I+\delta)^{-1} \approx 1 - \delta + \delta^2 - \delta^3 \cdots$

Define factors:

$$p_{i,j} = P^j \left[(\Gamma^{-1} - I)^j \right]_i$$

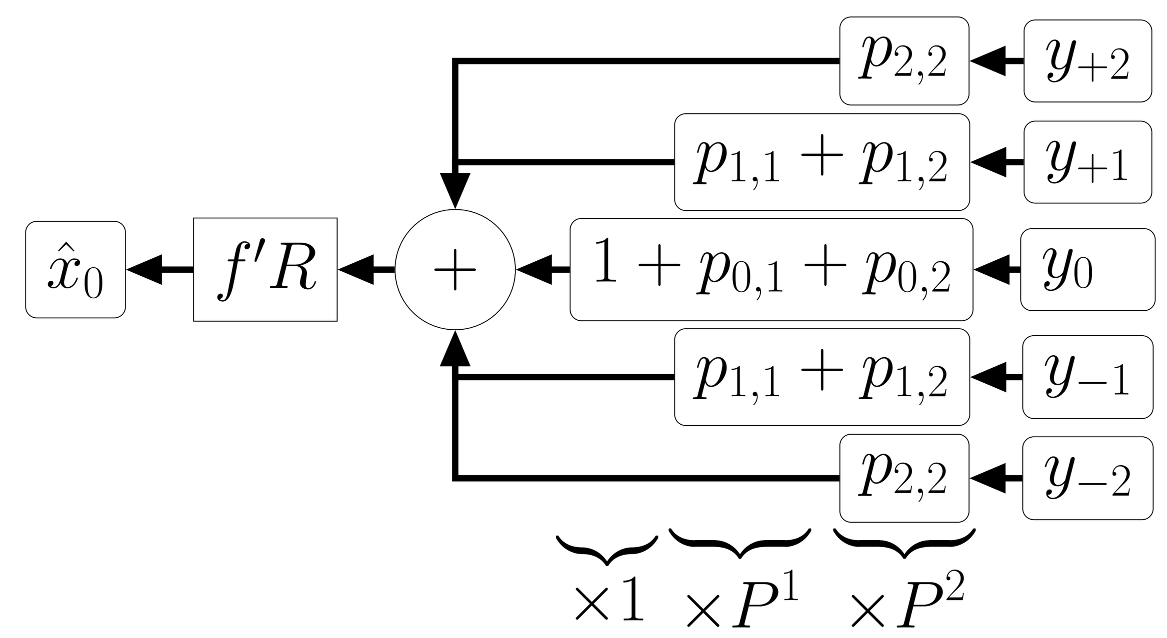
where $[\cdot]_i$ is the i^{th} offset from the matrix diagonal.

Block Diagram



$p_1 = \gamma, p_2 = \gamma^2$

Block Diagram



References

[1] A Adler, T Dai, WRB Lionheart, *Physiol Meas* 28:S1-S11, 2007.

[2] M Vauhkonen, PA Karjalainen, JP Kaipio, *IEEE T Biomed Eng* 45:486–493, 1998.

[3] RJ Yerworth, I Frerichs, R Bayford, *J Clin Monit Comput* 31:1093–1011, 2017

Representative Images

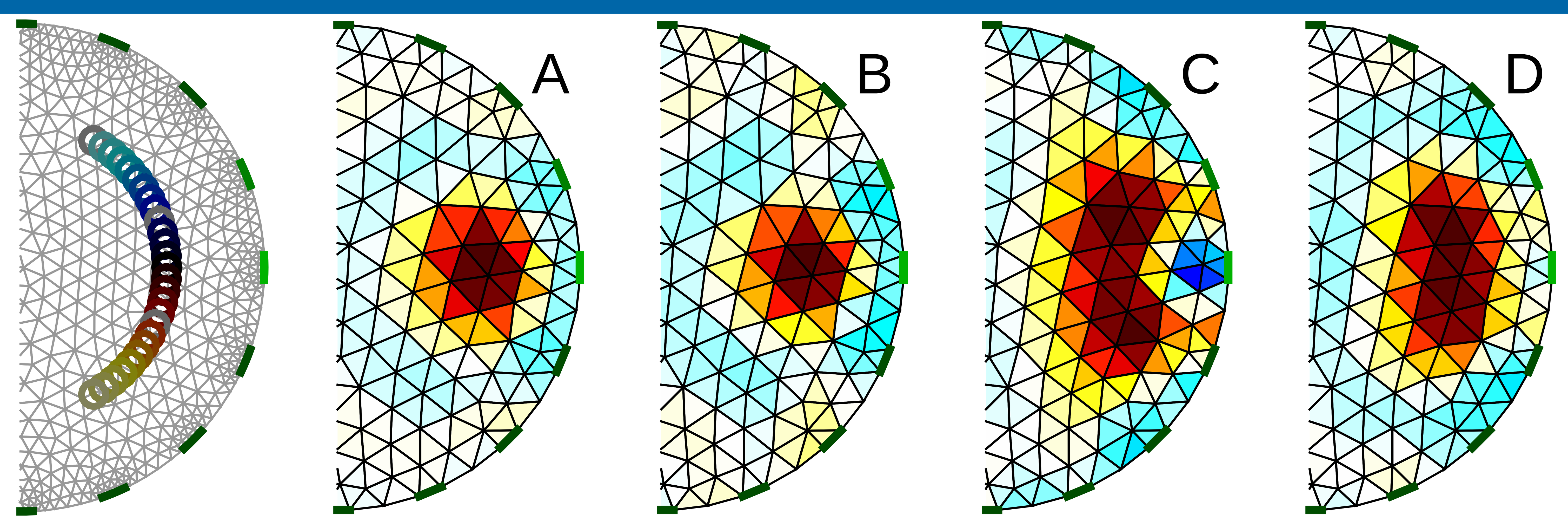


Fig 2: Simulation and Reconstruction images (on circular domain, half shown). *Left:* Simulation matrix, with an object moving from top (blue) to bottom (red) during three acquisition frames; the first acquisition of each frame is marked white; *A:* Reconstruction of a frame of data with the object still at 90° (reference image); *B:* Temporal ignorance (average all measures); *C:* Linear temporal interpolation; *D:* Temporal reconstruction [1].