Comparing D-bar and Regularization-based reconstruction

EIT 2018, Edinburgh, UK
11 June 2018

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Question:

How does $\boxed{\textit{D-bar}}$ *compare to the* $\boxed{\textit{“standard”}}$ *reconstructions we use in EIT?*

EIT reconstruction with Regularization

Calculate solution $\hat{x}$ where

$$\hat{x} = \underset{x}{\operatorname{argmin}} \left(\|y - F(x)\|^2 + \|x - \bar{x}\|^2_{\lambda P} \right)$$

- matrix λP is the regularization penalty
- used for difference and absolute EIT
- choice of parameters changes behaviour

Since $\|\cdot\|^2$ norms are used, solution is linear if $F(x) \approx Jx$

$$\hat{x} = \left(J^t J + \lambda P\right) J^t y$$

EIT reconstruction with D-bar

$$\begin{array}{c}\text{Current/Voltage Data}\\(\Lambda_\sigma,\Lambda_1)\end{array}\xrightarrow{1}\begin{array}{c}\text{Scattering Data}\\\mathbf{t}^{\mathbf{exp}}(k)\end{array}\xrightarrow{2}\begin{array}{c}\text{Conductivity}\\\sigma(x)\end{array}$$

Step 1: For each $k \in \mathbb{C} \setminus \{0\}$, evaluate the approximate scattering data

$$\mathbf{t}^{\mathbf{exp}}(k) = \begin{cases} \int_{\partial\Omega} e^{i\bar{k}\bar{x}} \left(\Lambda_\sigma - \Lambda_1\right) e^{ikx} dS(x), & 0 < |k| \le R \\ 0 & |k| > R. \end{cases}$$

Step 2: For each $z \in \Omega$, solve the D-bar equation via the integral equation

$$\mu^{\mathbf{exp}}(x,\kappa) = 1 + \frac{1}{4\pi^2}\int_{\mathbb{C}} \frac{\mathbf{t}^{\mathbf{exp}}(k)e^{-i(kx+\bar{k}\bar{x})}}{(\kappa - k)\bar{k}} \overline{\mu^{\mathbf{exp}}(x,k)}\, d\kappa_1 d\kappa_2,$$

and recover the approximate conductivity

$$\sigma^{\mathbf{exp}}(x) = \left[\mu^{\mathbf{exp}}(x,0)\right]^2.$$

EIT reconstruction with D-bar

$$\begin{array}{ccccc} \text{Current/Voltage Data} & \xrightarrow{1} & \text{Scattering Data} & \xrightarrow{2} & \text{Conductivity} \\ (\Lambda_\sigma, \Lambda_1) & & \mathbf{t}^{\mathbf{exp}}(k) & & \sigma(x) \end{array}$$

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Note: For *difference imaging*, replace Λ_1 with $\Lambda_{\mathbf{ref}}$ to recover $\sigma^{\mathbf{diff}}$.

Where to start comparing?

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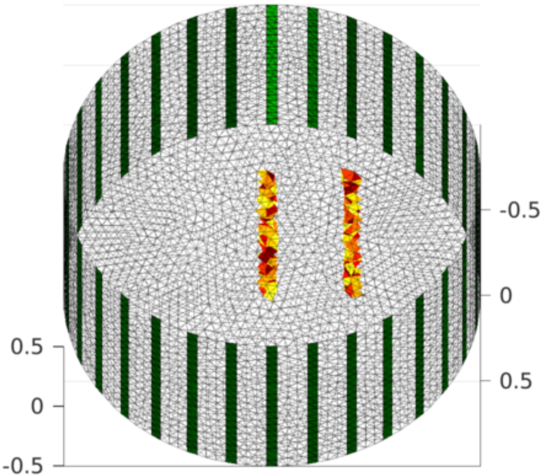
Simply!

We look only at

- 2D
- circular domains
- difference EIT with small contrasts (i.e. linear)

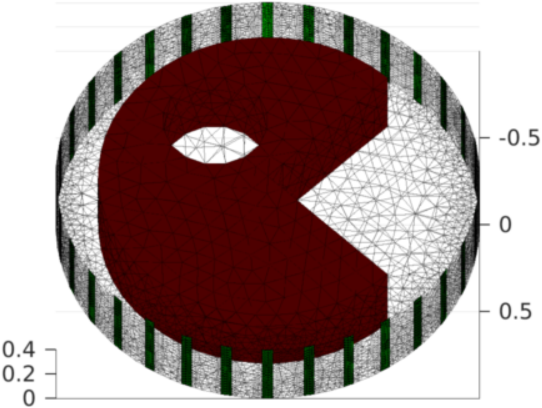
Clearly, the plan is to move on from here.

Simulation phantoms #1



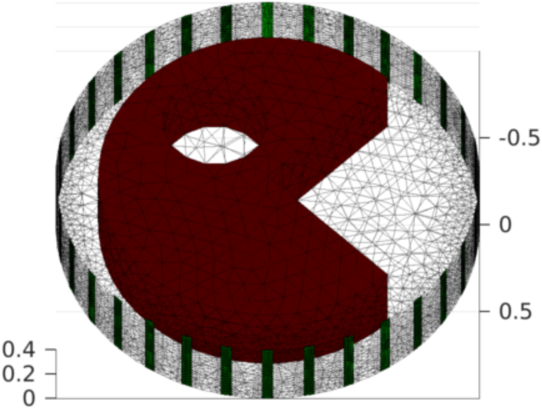
Using 32 equally spaced electrodes of the indicated width. Stimulation patterns were “skip 4” with monopolar voltage measurements on all electrodes (including driven ones).

Simulation phantom #2



designed to give edges and holes → difficult to reconstruct

Simulation phantom #2



designed to give edges and holes → difficult to reconstruct
Any similarity to “pac-man” is coincidence

Parameter selection

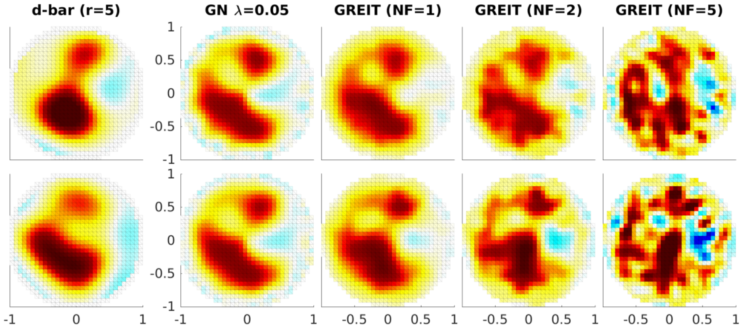
Control of the trade-off: resolution $\Longleftrightarrow$ noise performance.

Algorithm	*Parameter*
D-bar	truncation radius (r) for the scattering data
GN	hyperparameter (λ)
GREIT	noise figure (NF)

We first select parameters which for which the noise performance is equal, and then subsequently evaluate other characteristics.

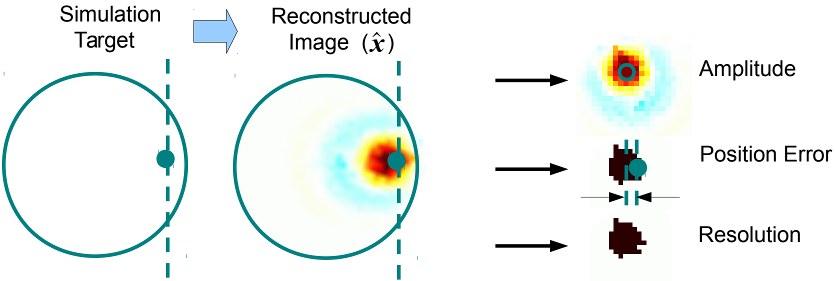
Reconstuctions of noisy data

Reconstructions of data with added Gaussian noise (noise sample per row) for algorithms and parameter settings.

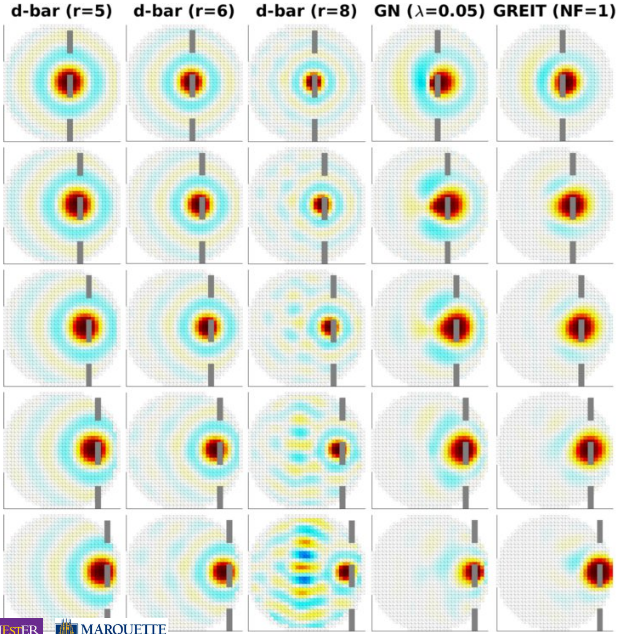


D-bar shows a different pattern (lower spatial frequency) for the reconstructed noise compared GN and GREIT.

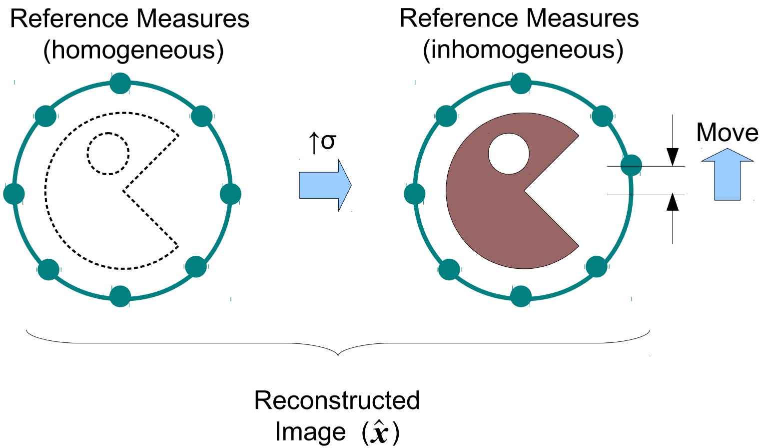
Point spread function



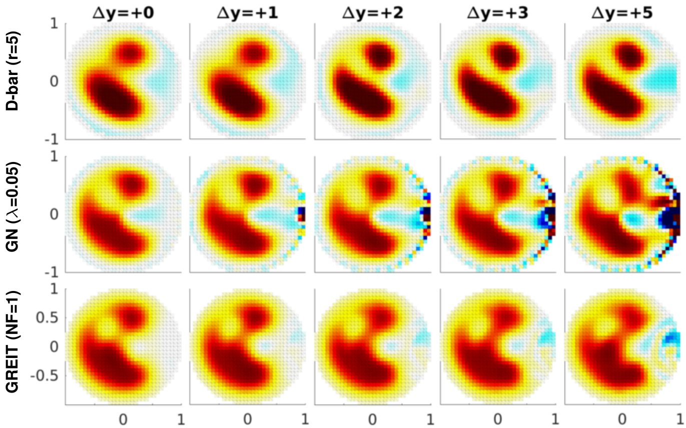
Point spread function



Sensitivity to movement



Sensitivity to movement



the right centre electrode moves by the indicated amount (in degrees). Results show D-bar is least affected.

Observations

D-bar (v.s. the others) has

- has position invariant point-spread function
- projects noise into images very differently
- much less sensitive to electrode position errors

There is lots of work to understand these effects

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