Estimating Electrode Movement in Two Dimensions

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Abstract: Four methods for estimating the electrode movement Jacobian were compared under a range of simulation conditions using the Finite Element Method (FEM). Mesh density, electrode diameter and contact impedance were varied over orders of magnitude, and the results were plotted to demonstrate the points of agreement and illustrate numerical instabilities between methods.

1 Introduction

A Fréchet derivative (1) has been constructed [1] for calculating an electrode movement Jacobian $\mathbf{J}_m$ based on the contact impedance z_c, nodal voltages u, v and measurements U_m, V_m from the FEM forward solutions in a similar manner to the adjoint method for the conductivity Jacobian.

$$\mathbf{J}_m = \frac{1}{z_c} \int_{\partial E} (h \cdot \upsilon_{\partial E})(U_m - u)(V_m - v) ds \qquad (1)$$

The formulation takes the change in voltage around the surface of the electrode denoted ∂_E tangential to the surface $h \cdot \upsilon_{\partial E}$ to estimate a Jacobian. We implement the Fréchet derivative and compare it to three perturbation methods: direct perturbations, selection of adjacent nodes, and the matrix rank-one update [2].

2 Methods

A rectangular two-dimensional model with four electrodes (CEM) on its upper surface was constructed in EIDORS using NetGen (fig. 1). Many models were generated that conformed to the same geometry but with variations in the electrode diameter, mesh density and contact impedance.

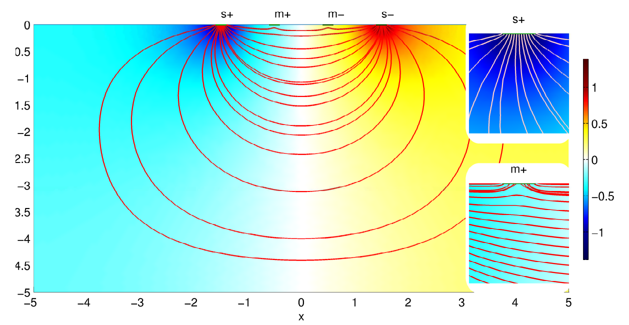
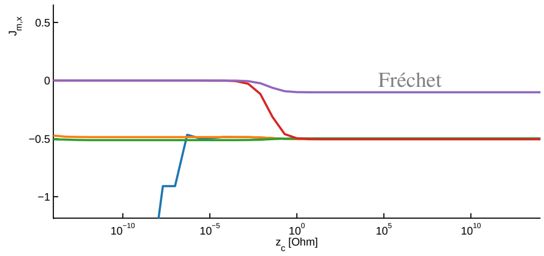


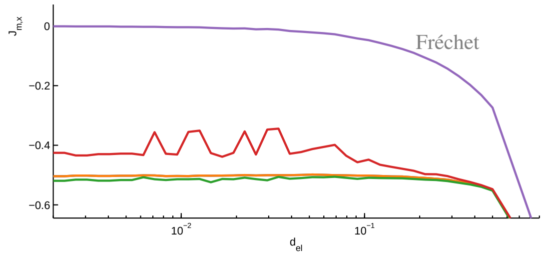
Figure 1: A four electrode half-space model; outer electrodes $s+$, $s-$ were used for stimulus and inner electrodes $m+$, $m-$ were used for measurements. Streamlines (red) in the inset figures show current density near the electrodes. Image background colour shows the voltage distribution (voltage scale to the right of the figure)

3 Simulations

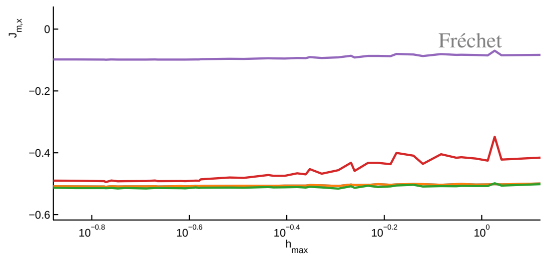
Plots illustrating variations in the Jacobian with respect to contact impedance (fig. 2a), mesh density (fig. 2b) and electrode diameter (fig. 2c) follow. Plots show the direct perturbation (blue), nodal selection (green), matrix rank-one update (orange), Fréchet derivative (purple) and Fréchet derivative divided by electrode diameter (red).



(a) Contact impedance z_c



(b) Electrode diameter d



(c) Mesh density h_{max}

Figure 2: Four Jacobian estimation methods; (blue) perturbation, (orange) matrix rank-one update [2], (green) select new electrode nodes, (purple) Fréchet derivative [1], (red) Fréchet derivative divided by electrode diameter d; showing variations in the Jacobian of the $m+$ measurement electrode for tangential surface movements $J_{m,x}$

4 Discussion

An initial evaluation of the technique for constructing a Fréchet derivative for electrode movement was confirmed to give the correct direction. Magnitudes differed between methods.

Movement estimates tangential to the surface were faithfully estimated for many scenarios. Estimating movement normal to the surface, though technically feasible with current perturbation implementations, appears to suffer from severe numerical instabilities that render the resulting Jacobian unusable in its current form.

A Fréchet derivative for the normal component has recently been developed and offers the possibility of a more stable solution for normal boundary movement. The methods in this paper can be extended to three dimensions; preliminary results for perturbations show similar outcomes.

References

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[2] Gómez-Laberge C, Adler A. *Physiol Meas* **29**(6):S89–S99, 2008