COMBINING REGULARIZATION FRAMEWORKS FOR IMAGE DEBLURRING: OPTIMIZATION OF COMBINED HYPER-PARAMETERS

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Abstract **Regularization is an important tool for restoration of images from noisy and blurred data. In this paper, we present a novel regularization technique (CGTik) that augments the conjugate gradient least-square (CGLS) algorithm with Tikhonov-like prior information term. This technique requires the appropriate selection of two hyper-parameters, the number of iterations (*N*) and amount of regularization (α). A method to select good values for these parameters is developed based on the L-curve technique. Tests were performed by calculating reconstructed images for each algorithm for heavily blurred images. CGTik showed improved restored images compared to the separate algorithms Tikhonov and CGLS.**

Keywords: Regularization, Tikhonov, inverse problems, iterative methods

1. INTRODUCTION

Image restoration and reconstruction are widely used in applications where an unknown and desired image $f(x,y)$ must be recovered from a set of distorted data $g(x,y)$. If the system is linear and shift invariant, the relationship between the unknown and observed image can be described by a distortion model $h(x,y)$ representing the known point-spread-function (PSF) of the system. Unfortunately, blurring makes the restoration problem ill-conditioned, and therefore not amenable to direct matrix inverse techniques, leading to noise amplification [1]. Regularization methods solve this problem by using prior information about the image to calculate the estimate. For example, the Tikhonov [2,3,10] technique provides an approximate solution by augmenting matrix inversion or factorization solutions. It requires the selection of a regularization parameter α that controls the tradeoff between fidelity to measurements and to prior information [4]. The Conjugate Gradient Least-square (CGLS) [5,6,10] is an iterative algorithm that can be viewed as a restoration method because the low frequency image components converge much faster than the high-frequency ones. The CGLS hyper parameter is thus the number of iterations (*N*).

This paper proposes a new algorithm (CGTik) formulated as an iterative CGLS inverse augmented with a Tikhonov-like image prior information term. The CGLS algorithm is used to efficiently solve the least-square minimization representation of the Tikhonov formulation. While techniques exist to choose an optimum value for single hyper-parameters, these techniques are not adequate for this algorithm, in which two hyper-parameters, α and *N,* are required. We develop an approach based on the L-curve technique to choose parameter values.

2. METHODS

We consider the following problem

$$g = H\,f + n \tag{1}$$

where g and f are vectors of length m representing the measured and original images respectively, and H is an $m \times m$ linear operator that characterizes the image degradation. n is a vector of length m that representing additive white noise contaminating the measurements The goal is to solve for an acceptable estimate $\hat{f}$ of the original image without excessive noise.

2.1 Tikhonov Regularization

Tikhonov Regularization requires finding the image estimate $\hat{f}_{tik}$ that minimizes:

$$\hat{f}_{tik}(\alpha) = \arg\min_{f} \|g - Hf\|_2^2 + \alpha\|Lf\|_2^2 \tag{2}$$

where $\|.\|_2^2$ denotes the L-2 norm. The first term measures the fidelity of the solution to the data while the second term measures the fidelity to prior knowledge expressed in operator L. α controls the tradeoff between these terms and represents the amount of regularization. L is often chosen to be a smoothing function such as the identity

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matrix or the Laplace operator D .The minimizer of (2) expressed as normal equations is:

$$(H^T H + \alpha L^T L)\hat{f}_{tik} = H^T g \qquad (3)$$

where H and L are block Toeplitz matrices. Equation (3) can be solved by singular value decomposition (SVD), factorization or by iteration [7,9].

2.2 The conjugate Gradient Least-Square Method (CGLS)

The conjugate gradient least-square method [6,7,8,10] solves for an expression of the form

$$\arg\min_f \|Hf - g\|_2^2 \qquad (4)$$

or expressed in normal equations:

$$H^T H f = H^T g \qquad (5)$$

This expression is the Tikhonov equation with no regularization term (i.e. α=0). When CGLS is applied on unregularized normal equations as in (5), the low-frequency components of the image tend to converge faster than the high-frequency components [5,6]. In order to control high-frequency noise in the solution, the number of iterations plays the role of regularization parameter.

2. 3 The Proposed Method

The proposed method (CGTik) uses the basis of the CGLS algorithm, and introduces a Tikhonov-like parameter to control the smoothing in the regularized image. The CGTik image estimate is:

$$\hat{f}_{CGTik}(\alpha) = \arg\min_f \quad \|H' f - g'\|_2^2$$

where

$$H' = \begin{bmatrix} H \\ \alpha L \end{bmatrix} \; and \;\; g' = \begin{bmatrix} g \\ 0 \end{bmatrix}$$

This equation is solved by CGLS methods [7,10] in terms of augmented terms *H'* and *g'*. However, for this solution, two parameters, *N* and α must be appropriately selected.

2.3.1 Regularization Parameter selection. Appropriate selection regularization parameter is important to achieve good restoration. If α is too small, noise will dominate the solution; if it is too large, the resulting image will be over blurred. For Tikhonov-like regularization, α may be selected using the L-curve technique ([2]). The norm of the solution ($\| \hat{f} \|$) is plotted against the norm of the residual ($\|g - Hf\|$) in order to visualize the tradeoff between data misfit and the energy in the solution. α is selected corresponding to the corner of the curve, found as the point that has the maximum curvature.

In order to select appropriate regularization parameters for CGTik, the L-curve technique is initially applied for classic Tikhonov regularization, and α_{tik} calculated. Subsequently, a range of α values (12 values corresponding to 12 different L-curves) is selected by visual inspection and CGTik is run for 200 iterations for each value of α, and he corresponding L-curve (data fidelity versus number of iterations) is calculated. These 12 α values presented the best perceived quality range for this specific restoration problem. α and *N* are chosen at the point with the most well defined "knee-shape" edge.

2.3.2 Test methodology. Test images (200x200 pixels) were chosen from [10] where a Gaussian blur (PSF width 3x3 pixels) was applied and Gaussian white noise added (SNR=25 dB). The original and blurred images are shown in Fig. 2. This image was then restored using the Tikhonov, CGLS, and the proposed CGTik algorithms. The constraint L=D was used for the Tikhonov and CGTik algorithms. This constraint penalizes non-smooth content in the regularized image, and produces image estimates with limited high-frequency energy. The regularization results obtained are compared using the noise power spectrum density (PSD). The PSD illustrates the level of noise power in the processed image over the entire frequency range. Therefore, the restored images tend to have a reduced high frequency noise compared to the original blurred noisy data. The PSD is used to show the evaluate the spectral properties of algorithm performance. Another parameter to define the improvement brought to a degraded image is the improvement signal to noise ratio (ISNR) [11], defined as:

$$ISNR = 10\log\left[\|g - f\|_2^2 \Big/ \|\hat{f} - f\|_2^2\right] \qquad (6)$$

3. RESULTS

The simulation results obtained for the proposed algorithm show a significant improvement in perceived quality and a reduction of noise in the restored images. Fig. 1 illustrates the L-curves obtained by CGTik. As mentioned previously, in order to select an appropriate hyper parameter using the L-curve method, the curve must have a well defined "knee-shaped" edge. This is illustrated by L-curves #[6,7,8,9] in fig. 1. They correspond to the optimal parameters for this case.

The CGTik regularized image (Fig. 2D) with α=0.03,contains less high-frequency noise than the CGLS restoration (Fig. 2C). In fact, the CGTik image (Fig. 2D) does not show the edge artefacts or ringing of the CGLS image (Fig. 2C). ISNR values achived are 16 dB for CGTik, 12.5 dB for CGLS and 12.4 dB for Tikhonov. Iterative techniques provide an advantage over direct

Tikhonov by controlling the stopping time of the algorithm and selecting the estimates with the highest perceived quality. Also, the solution power spectrum density shows that CGTik reduces high-frequency noise with no impact on low-frequency image components. If the too large value is chosen for α, CGTik will produce unusable (over-regularized) solutions (Fig. 2E, α=0.9).

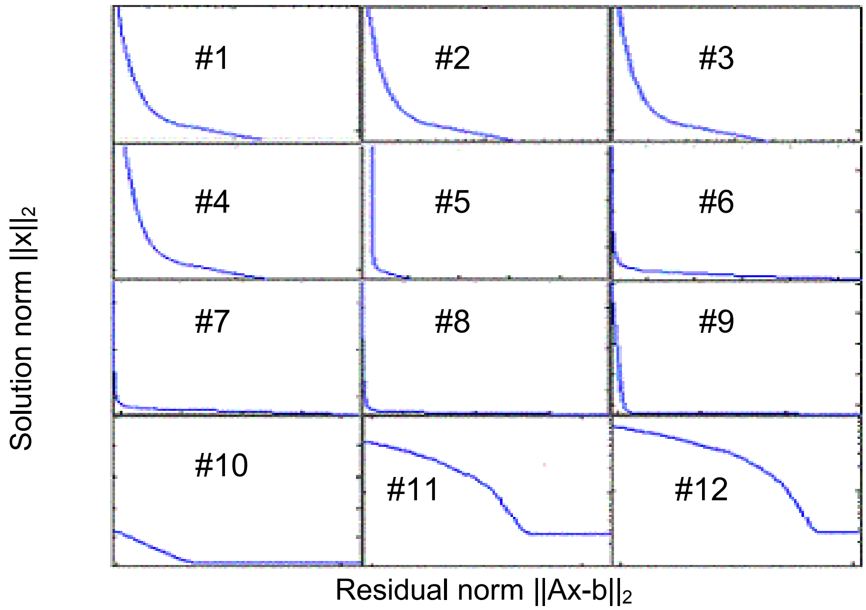

Figure 1: L-curve values for twelve different selections of α. Each sub-figure is a log-log plot of residual norm vs. solution norm. Axes are arbitrary but identical for each sub-figure.

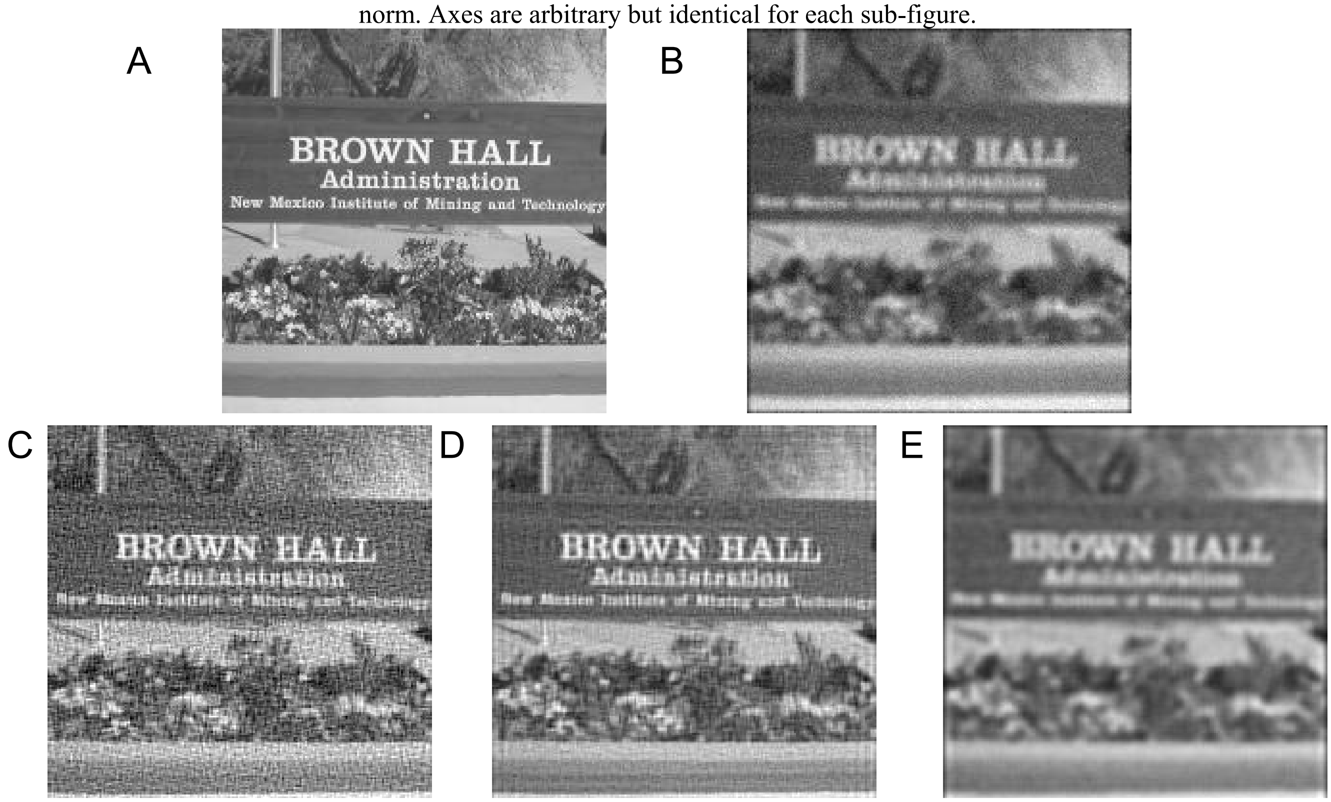

Figure 2: Original and reconstructed images: A) Original Image (200x200), B) Blurred noisy image (SNR=25 dB), C) CGLS restored image, D) CGTik restored image with best parameter choice, E) Over-regularized CGTik restored image

4. DISCUSSION

This paper develops a novel regularization approach, CGTik, to restore unknown images from distorted data. Reconstructed image with CGTik showed better perceived quality, elimination of ringing artefacts, and a significant noise reduction in the PSD plot compared to CGLS. One additional complication of CGTik is the requirement to select appropriate values of two hyper-parameters, α and N. An L-curve based technique is developed to select α. N is chosen by visual inspection. One disadvantage of CGTik is that its parameter choice is computationally expensive, as these operations must be repeated for several candidate solutions. On the other hand, it provides the possibility of observing several estimates at different number of iterations within a good perceived quality range. This was not possible using direct Tikhonov method.

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