IMAGE RECONSTRUCTION IN ELECTRICAL IMPEDANCE TOMOGRAPHY: A NEURAL NETWORK APPROACH

Andy ADLER, Robert GUARDO, Greg SHAW
Institut de Génie Biomédical
Ecole Polytechnique et Université de Montréal
Montréal, Québec, CANADA, H3C 3A7

ABSTRACT - **Reconstruction of images in electrical impedance tomography requires the solution of an inverse problem which is typically ill-conditioned due to the effects of noise and therefore requires regularisation based on a priori knowledge. This paper presents a linear reconstruction technique using neural networks which adapts the solution to the noise level used during the training phase. Results show a significantly improved resolution compared to the weighted equipotential backprojection method.**

INTRODUCTION

Electrical Impedance Tomography estimates the conductivity distribution of a medium from potential measurements produced by injected currents on the medium boundary. We are interested in dynamic imaging, which estimates conductivity changes in the medium from changes in these measurements, because these changes in measurements are much more stable than the measurements themselves to variations in electrode position, resistance, and amplifier gain.

The best image reconstruction techniques are based on fitting the measured voltages to finite element models (FEMs) of tissue conductivity. This inverse problem, however, is ill-posed for any reasonable number of elements (>50) in noisy data. Regularization, based on a priori knowledge of the problem is typically necessary. The neural network approach, however, can be used to calculate a linear inverse to the problem without any need of a priori knowledge.

FORWARD PROBLEM

Using the FEM, we simulate the voltage measurement vector by

$$v = F(r_h + r) = \frac{1}{\exp(r_h)} F(r) \tag{1}$$

where r_h is the uniform homogenous log conductivity, r is a vector of the M element log conductivity changes, and F is a linear function of the injected current and a non-linear function of r. The voltage measurements are obtained from a 16 electrode system, using 16 current injection patterns, resulting in $N=256$ differential voltage readings. From (1) we calculate our dynamic measurement vector f by

$$f_i = \frac{[v_{inhom}]_i - [v_{hom}]_i}{[v_{hom}]_i} = \frac{F(r)_i}{F(0)_i} - 1 \text{ , for } 1 \le i \le N \tag{2}$$

where $[v_{hom}]_i$ and $[v_{inhom}]_i$ represent the ith element of the voltage measurement vector before and after, respectively, a conductivity change.

We look for a linear approximation to this problem, in order to simplify the design and reduce the training time of the neural network. Linearizing about r = 0 results in:

$$f = Yr \quad \text{where,} \quad Y_{ij} = \frac{1}{F(0)} \frac{\partial F_i}{\partial r_j}. \tag{3}$$

INVERSE PROBLEM

This inverse problem may be stated as finding the matrix Z which, in the presence of noise n, best approximates,

$$r \approx Z(f + n) \tag{4}$$

in the least squared error sense. The neural network model considered here is the "adaptive linear element", or ADALINE[2]. One ADALINE corresponds to each value of r and sums each value of f by the corresponding row in Z. The values of Z are calculated or "trained" by the Widrow-Hoff learning rule, using a set of input vectors f_k and their (known) desired responses from the network d_k. Training aims to reduce the error ϕ for all training sets k.

$$\phi = \sum_k (d_k - Zf_k)^t (d_k - Zf_k) \tag{5}$$

We choose the desired responses to be individual objects in each element, i.e. the column vectors of $I_{n \times n}$, and we obtain the input vectors from the direct problem, $f = YI = Y$. In order to train the network to deal with noise, we must include the expected noise in the input. Using this training set we carry out the following algorithm:

- Initially, all weights are set to zero.

- The training vectors are presented to the current network weights, outputting: $O = Z_k(Y + n)$

- The error $E = O - D$ is defined as the difference between the output, O, and the desired response, $D = I_{n \times n}$

- Network weights are updated by the learning rule:
$$Z_{k+1} = Z_k - \alpha E O^t \tag{6}$$

- Iteration is continued until the error is below an acceptable limit.

The parameter α controls the learning rate; for stability, it's value must be less than the reciprocal of the maximum eigenvector of YY^t.

Once training is completed, Z can be used as a reconstruction operator which calculates the log element conductivities from the voltages measured by:

$$r_{reconst} = Z\left(\frac{v_{inh} - v_{hom}}{v_{hom}}\right) \tag{7}$$

EXPERIMENTAL RESULTS

The above procedure was used to train neural networks *N1* and *N2* on a two dimensional circular geometry for no noise and 15dB signal to noise ratio (SNR) respectively. Training encompassed 5000 iterations, during which time the least mean squared error (the reconstructed output minus the desired output) decreased by 50 percent.

While this technique works well on real data obtained from our tomographic system, the performance compromises involved are most clearly seen on simulated data. Figure 1A shows the pattern to be imaged: two small non-conductive objects in a circular milieu separated by one third of the diameter. This pattern was simulated on a much finer finite element mesh than was used for the training of the neural networks. Figures 1B to 1D were reconstructed using the voltage measurements from 1A with no noise added, using reconstruction based on weighted backprojection (1B), network *N1* (1C), and network *N2* (1D). Figures 1E to 1G were reconstructed using measurements with a SNR of 15dB, again using backprojection (1E), network *N1* (1F) and network *N2* (1G).

We define the SNR as $(f^t f) / (n^t n)$.

DISCUSSION AND CONCLUSIONS

The results show that the neural network produces significantly better resolution images than weighted equipotential backprojection, and also offers the advantage of being adaptable to the noise level present in the measurements. The network trained for no noise displays the best resolution, but has little ability to reject noise. The network trained with noise, while having slightly degraded resolution, has better ability to reject noise. Although the training is a long process; image reconstruction times with a trained neural network and with the backprojection method are similar, as both processes require only one matrix multiplication.

This neural network approach seems to show promise as a reconstruction technique which can control the compromise between the noise performance and the resolution of the image.
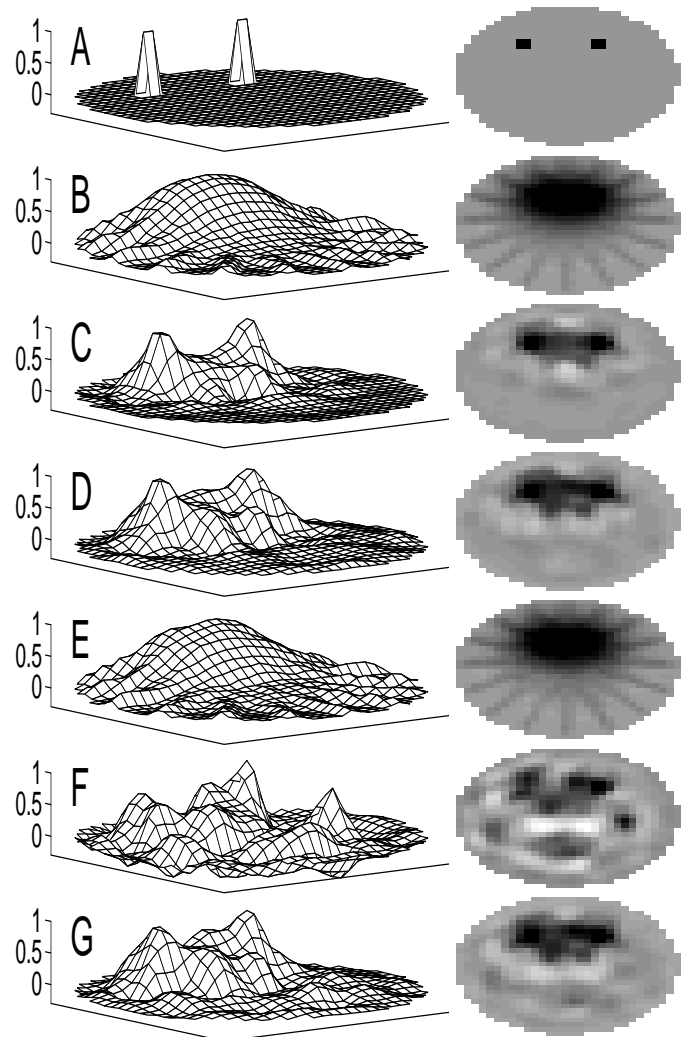

Figure 1: Images Produced by Neural Network
A: Theoretical Object
B: Backprojection: *no noise*
C: Image: *no noise*   Training: *no noise*
D: Image: *no noise*   Training *15dB SNR*
E: Backprojection: *15dB SNR*
F: Image: *15dB SNR*   Training *no noise*
G: Image: *15dB SNR*   Training *15dB SNR*

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