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Search strategies for the acquisition of DS spread spectrum signals

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The performance, in terms of the mean acquisition time, of a direct sequence (DS) serial-search acquisition scheme that employs different search strategies is analysed. The mean value of the search length for the considered Z-search strategies is computed. This is then used to compute the mean acquisition time for each search strategy using the unified time domain method. In the presence of *a priori* information about the code phase, which is modelled as a truncated gaussian probability density function, various search strategies namely, Z-search, expanding window (EW) and alternate search strategies are analysed, evaluated and their performances compared. The performances are compared in the presence of narrowband interference, from which the DS spread spectrum suffers most, in addition to additive white gaussian noise (AWGN). Results have shown that the non-uniform expanding window (NUEA) search strategy outperforms other strategies, particularly at high interference-to-signal ratios (ISR). However, among the various Z-search strategies, the offset-Z-search strategy has been found to exhibit a mean acquisition time 3·3 times faster than that of the continuous centre (CC) Z-search strategy. An experimental serial-search DS acquisition scheme that employs different Z-strategies has been designed, implemented and tested. Experimental results compare very well with the theoretical results obtained.

1. Introduction

Three processes are performed by the spread spectrum receiver, namely code acquisition (i.e. initial synchronization), code tracking (i.e. fine synchronization) and data demodulation. Code acquisition is considered vital, because if it were to fail it would lead to unsuccessful data demodulation. Furthermore, rapid code acquisition in a spread spectrum receiver is essential to improve the interference immunity of the spread spectrum system. Thus, the faster the acquisition process, the more difficult it is for the would-be jammer to perform successful jamming (Ziemer and Peterson 1985).

Figure 1 illustrates a block diagram of a DS serial-search acquisition scheme. The search operation of the received pseudonoise (PN) code is performed by searching, serially, a phase uncertainty region cell-by-cell for the correct phase until the correct PN-code is found. Thus, the received PN-code is correlated with the locally generated PN-code. If both PN-codes are aligned in phase, a despreading process occurs and the spread spectrum bandwidth collapses to the information signal bandwidth. The output signal energy of the correlator is integrated over a dwell-time (T_{d1}) and compared to a threshold level. If the output of the integrator is above the threshold, acquisition is declared; otherwise the PN-code generator is shifted by, normally, half a chip and the acquisition process is repeated.

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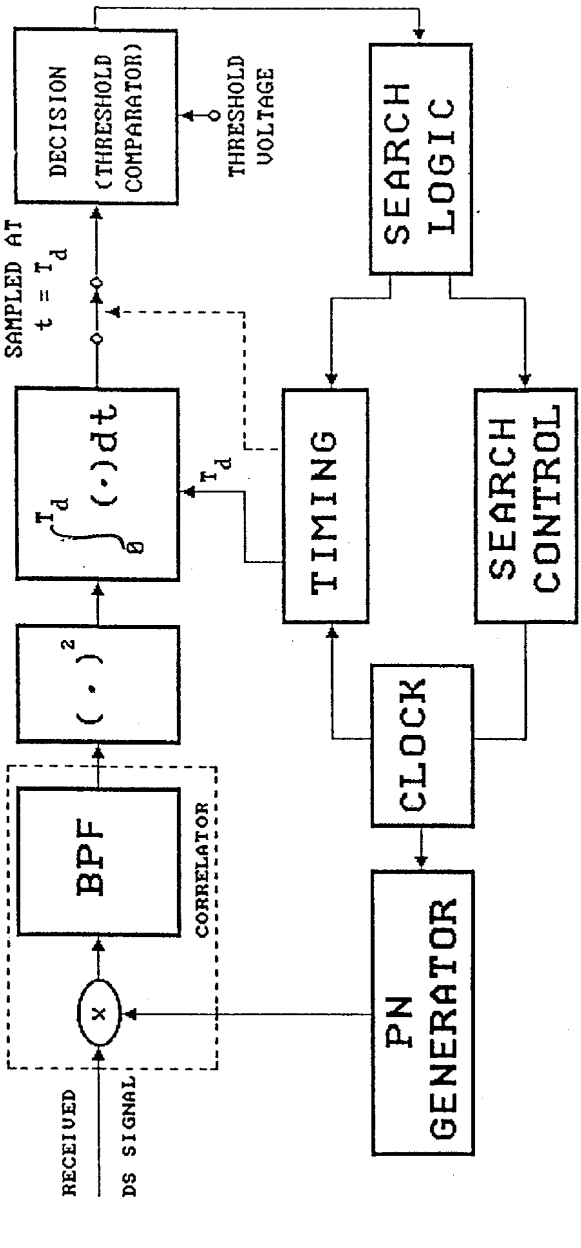
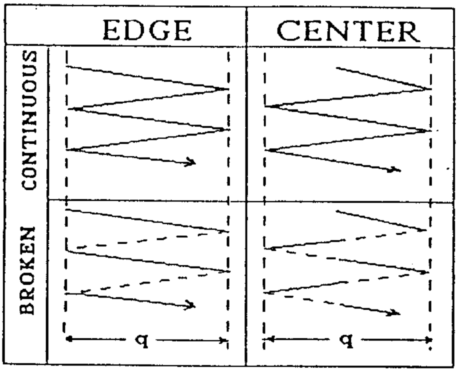


Figure 1. Serial-search PN code acquisition system.

The initial synchronization event is usually verified using a search/verification logic/lock operation. If the correlator output fails to exceed the threshold, this cell is rejected and the next cell is tested. However, if threshold is exceeded, the search process enters a lock mode in a single-dwell time scheme, and it enters a verification state in a multiple-dwell time scheme (Pan *et al.* 1991 b).

Generally, serial-search acquisition schemes require longer times to acquire synchronization, in particular when long PN-codes are employed by the spread spectrum system. However, the performance of the code acquisition process, in terms of the mean acquisition time, may be improved by employing the appropriate search strategy (Jovanovic 1988, Ziemer and Peterson 1985).

Various forms of Z-search and expanding window (EW) search strategies have been proposed for more rapid code acquisition in the presence of *a priori* information about the code phase (Jovanic 1988, Pan *et al.* 1991 b). Fig. 2(*a*) illustrates different Z-search strategies, and Fig. 2(*b*) illustrates the offset Z-search strategies that were introduced by Pan *et al.* 1991 a). The offset Z-search strategies gives an improvement, in terms of the mean acquisition time, when compared with the broken centre Z-search strategy. This improvement in the mean acquisition time has been gained for a better utilization of *a priori* information of the code phase.



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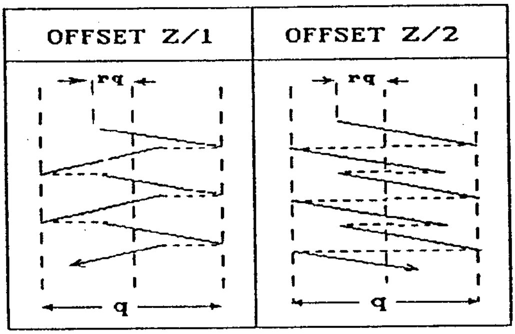

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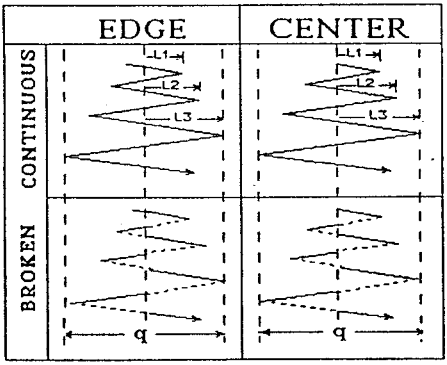
Figure 2. (*a*) Different Z serial-search strategies; (*b*) offset-Z search strategies.



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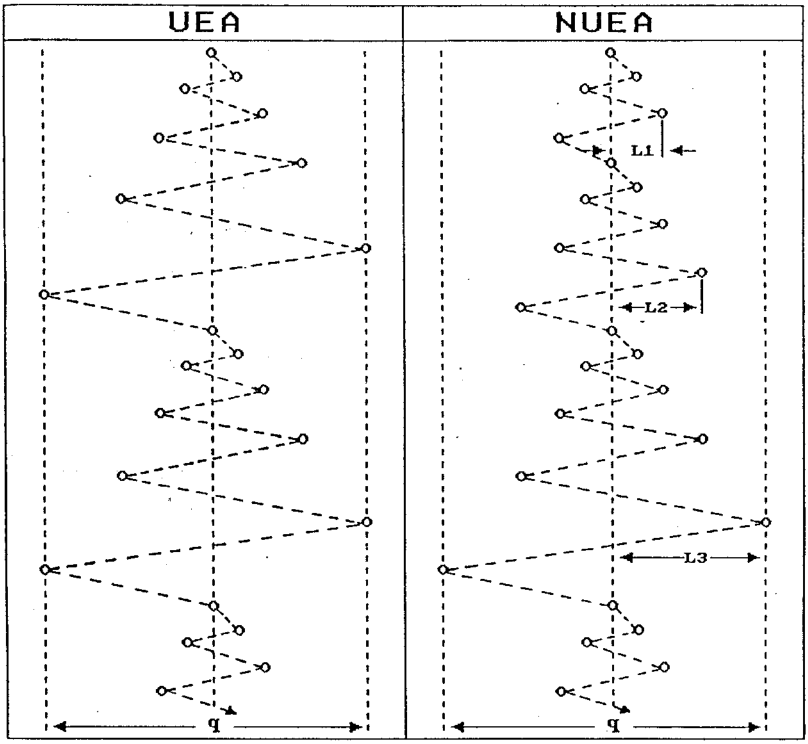

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Figure 3. (*a*) Expanding window serial-search strategies; (*b*) alternative serial-search strategies.

In most systems *a priori* information about the code phase is modelled as a triangular or truncated gaussian density function (Jovanovic 1988, and Pan *et al.* 1991 a). Moreover, Figs 3(*a*) and (*b*) illustrate, respectively, EW and alternate serial-search strategies. In EW search strategies (Weingberg 1983, Braun 1982) the most likely cells are swept more frequently than others. This would save more acquisition time, especially at low probability of detection P_{d} (e.g. $P_{\mathrm{d}} < 0{\cdot}75$) (Jovanovic 1988).

However, in alternative search strategies, the search is performed by jumping sequentially on cells, following the order of decreasing *a priori* probability, until all the uncertainty region has been tested. A uniform expanding alternate (UEA) search strategy outperforms all known search strategies for $P_d = 1$; however, for small probability of detection UEA fails to perform the best search strategy, because it tests all cells with the same frequency. Thus, the non-uniform expanding window alternate (NUEA) search strategy has been introduced to combine the good features of both alternate and EW search strategies (Jovanovic 1988, Jibrail *et al.* 1994), as shown in Fig. 3(*b*).

In this paper the mean acquisition time as a function of the search strategy for a DS serial-search acquisition scheme is computed in an explicit form. This made possible a direct performance comparison of various search strategies, when operating in the same communication environment. Performances of these search strategies have been obtained then compared in the presence of narrowband interference, from which the DS spread spectrum suffers most, in addition white gaussian noise (AWGN).

2. Computation of the mean acquisition time

A commonly used performance measure of a spread spectrum acquisition system is the mean acquisition time (Dixon 1975, Ziemer and Peterson 1985). This is the average time that it takes the acquisition scheme to search for, locate and verify the correct cell in an uncertainty region of q cells. Different approaches have been suggested to compute the mean acquisition time for a serial-search code acquisition process, taking into account the search strategy employed in the process.

A generalized analysis for the evaluation of the search strategy effect on PN-code acquisition has been proposed by Weinberg (1983), and a unified approach to analyse serial-search acquisition techniques via transform domain flow graphs have been proposed by Polydoros and Weber (1984). Moreover, the direct approach method, which was proposed by Jovanovic (1988), permits the measurement of performance for serial-search acquisition schemes for different search strategies, namely Z-search, EW and alternate search strategies. Finally, a unified time-domain approach to analyse the code acquisition techniques, that employ, various Z-search and EW search strategies, has been utilized by Pan *et al.* 1991 b. In this approach three random variables are defined to describe the acquisition process, namely the number of cells searched before reaching the correct cell position x, the number of times that a correct phase is missed during the acquisition process n_q and the total number of T_{d2} periods spent in verification/lock time slips. The number of cells searched x may take any value in a limited set of 0, 1,..., v, whereas n or n_q may take any value in the infinite set of integers. Thus, the acquisition time T_a as a function of these three random variables (Pan *et al.* 1991 b) may be expressed as

$$T_a = nT_{d2} + WT_{d1} + T_{d1} + vT_{d2} \qquad (1)$$

where v is the number of verification states; W is the number of cells searched before accepting the true cell position (i.e. the search length); T_{d1} is the dwell-time in the search mode; and T_{d2} is the dwell-time in the verification/lock mode.

Thus, the mean acquisition time $E(T_a)$ which depends on $E(n_q)$, $E(W)$, P_{d1}, P_{d2}, P_{f1}, P_{f2} and other system parameters may be expressed as (Pan *et al.* 1991 a):

$$E(T_a) = (P_{f1}\bar{m}T_{d2} + T_{d1})E(W) + \left[\frac{(1 - P_{d2})P_{d1}(P_{d2})^v}{P_{d2}(1 - P_{d1}(P_{d2})^v)} E(n_q) + v - P_{f1}mE(n_q)\right] T_{d2} + T_{d1} \qquad (2)$$

where P_{d1} and P_{d2} are, respectively, the probabilities that the correct cell position is accepted in search and verification states; P_{f1} and P_{f2} are, respectively, the false alarm probabilities in the search and verification states; and m is the number of verification and lock states before returning to the search mode at the next cell position.

Moreover, $E(nq)$ and $\bar{m}$ are given as

$$E(n_q) = \frac{1 - P_{d1}(P_{d2})^v}{P_{d1}(P_{d2})^v} \qquad (3a)$$

$$\bar{m} = \frac{1}{(1 - P_{f2})^2} \qquad (3b)$$

However, the mean value of the search length $E(W)$ is now to be developed for each Z-search strategy.

2.1. *Mean acquisition time for Z-search strategies*

In this section the mean value of the search length for a broken edge-Z (BE-Z) search, a broken centre-Z (BC-Z) search, a continuous edge-Z (CE-Z) search, a continuous centre-Z (CC-Z) search and an offset-Z (O-Z) search is to be developed.

Figure 4 illustrates the BE-Z search path, together with the probability density function (p.d.f.) $P(x)$ of the phase. Thus, W can be expressed as

$$W = x + qn_q \qquad (4a)$$

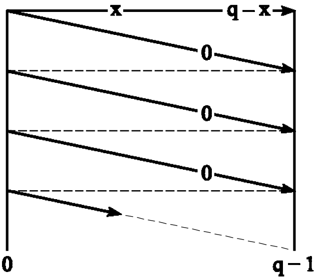
$$E(W) = \sum_{n_q=0}^{\infty}\sum_{x=0}^{\infty} WP(n_q)P(x) \qquad (4b)$$

$$E(W)_{(\text{BE-Z})} = E(x) + qE(n_q) \qquad (4c)$$

$E(x)$ may be obtained from the p.d.f. of x. This is developed from the assumed truncated gaussian *a priori* model as follows.

$$E(x) = \frac{1}{\sqrt{2\pi}\sigma}\sum_{x=0}^{\sigma-1} x \exp\left\{-\frac{(x - q/2)^2}{2\sigma^2}\right\} \qquad (5)$$

Thus, substituting for $E(W)$ and $E(n_q)$ in (2), $E(T_a)$ for the BE-Z search strategy may be computed. Similarly, $E(T_a)$ for other Z-search strategies may be computed.



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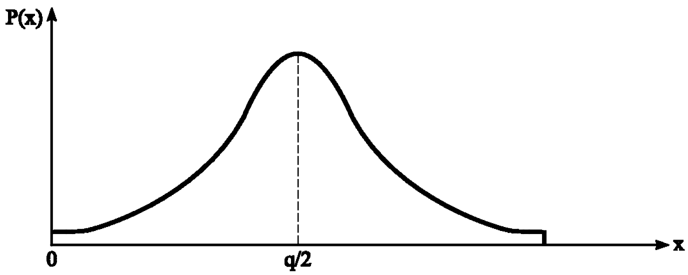

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Figure 4. (*a*) Broken edge Z search path; (*b*) probability of first meeting true code phase versus sweep length.

Fig. 5 illustrates the search path for a BC-Z search, together with the p.d.f. of (x). The search length for this strategy may be expressed as (Al-Zubiady 1994):

$$W = \begin{cases} x + 0\cdot 5qn_q; & 0 < x < q/2 \\ x + 0\cdot 5q(n_q - 1); & q/2 < x < q \end{cases} \tag{6a}$$

$$E(W) = \sum_{n_q=0}^{\infty} \left\{ \sum_{x=0}^{0\cdot 5q} (x + 0\cdot 5qn_q)P(x) + \sum_{x=0\cdot 5q}^{q} (x + 0\cdot 5qn_q - 0\cdot 5q)P(x) \right\} P(n_q) \tag{6b}$$

$$E(W)_{(\text{BC-Z})} = E(x) + 0\cdot 5qE(n_q) - 0\cdot 25q \tag{6c}$$

Moreover, for the CE-Z search strategy, the search path is as shown in Fig. 6(*a*), and it is given by

$$W = \begin{cases} x + qn_q, & \text{for } n_q \text{ even} \\ x + qn_q + q - 2x, & \text{for } n_q \text{ odd} \end{cases} \tag{7a}$$

Therefore

$$E(W) = \sum_{n_q=0,2}^{\infty} \sum_{x=0}^{q} (x + qn_q)P(n_q)P(x) + \sum_{n_q=1,3}^{\infty} \sum_{x=0}^{q} (x + qn_q + q - 2x)P(n_q)P(x) \tag{7b}$$

$$E(W) = \sum_{n_q=0,2}^{\infty} \sum_{x=0}^{q} (x + qn_q)P(n_q)P(x) + \sum_{n_q=1,3}^{\infty} \sum_{x=0}^{q} (q - 2x)P(n_q)P(x) \tag{7c}$$

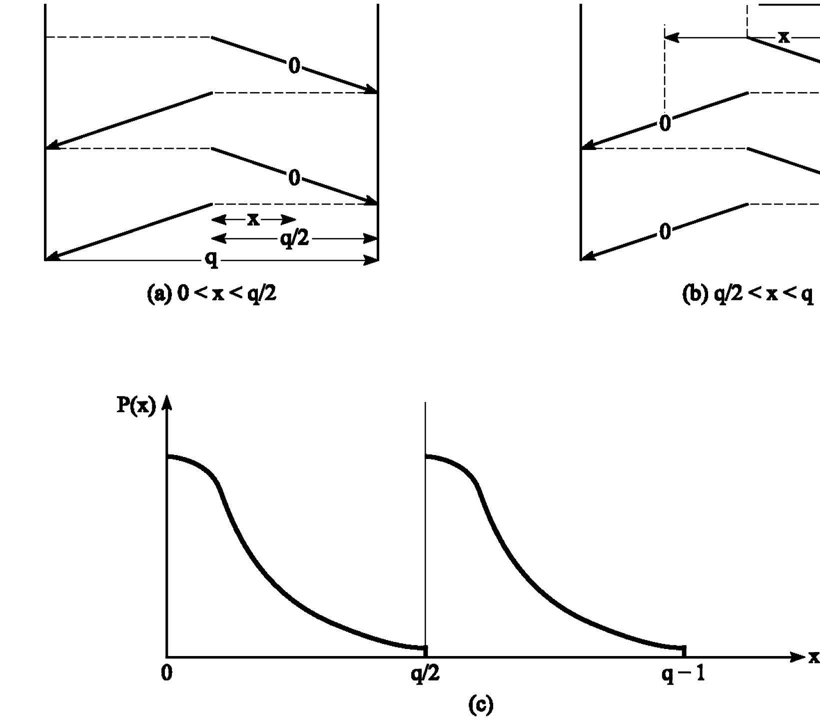


Figure 5. (a) Sweep path in BC-Z search strategy, $0 < x < q/2$; (b) sweep path in BC-Z search strategy, $q/2 < x < q$; (c) p.d.f. of x for BC-Z search strategy.

For a verification/lock strategy with v verification states, $P(n_q)$ is given as (Pan *et al.* 1991 b):

$$P(n_q) = P_{d1}(P_{d2})^v \{1 - P_{d1}(P_{d2})^v\}^{n_q} \tag{8a}$$

Thus

$$\sum_{n_q=1,3}^{\infty} P(n_q) = \frac{1 - P_{d1}(p_{d2})^v}{2 - p_{d1}(P_{d2})^v} \tag{8b}$$

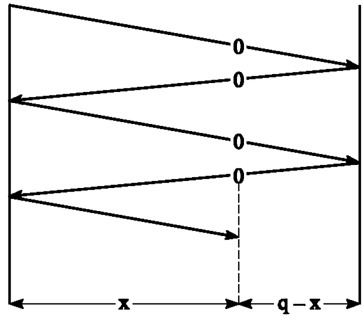
This yields

$$E(W)_{(\text{BC-Z})} = E(x) + qE(n_q) + \frac{1 - P_{d1}(P_{d2})^v}{2 - P_{d1}(P_{d2})^v}\{q - 2E(x)\} \tag{9}$$

Therefore, substitute into (9) for $E(n_q)$ from (3a) and for $E(x)$ from (5) to obtain $E(W)$ for BE-Z search strategy.

The search path (W) for the continuous centre-Z (CC-Z) search shown in Fig. 6(b) may be expressed as follows (Al-Zubiady 1994):

$$W = \begin{cases} x + qn_q, & 0 < x < 0{\cdot}5q \\ x + qn_q, & \text{for } n_q \text{ even; } q < x < 1{\cdot}5q \\ x + qn_q + 2(q - x), & \text{for } n_q \text{ odd; } q < x < 1{\cdot}5q \end{cases} \tag{10}$$



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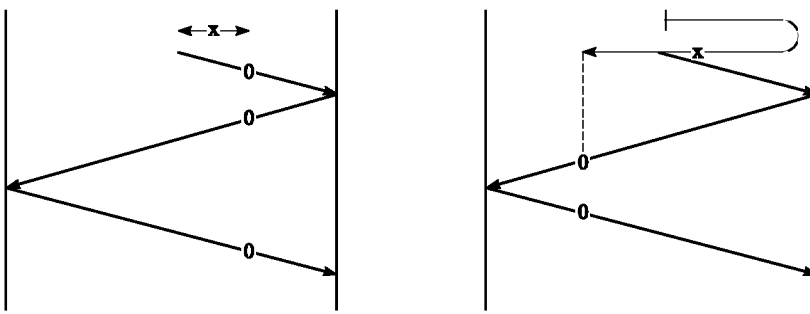

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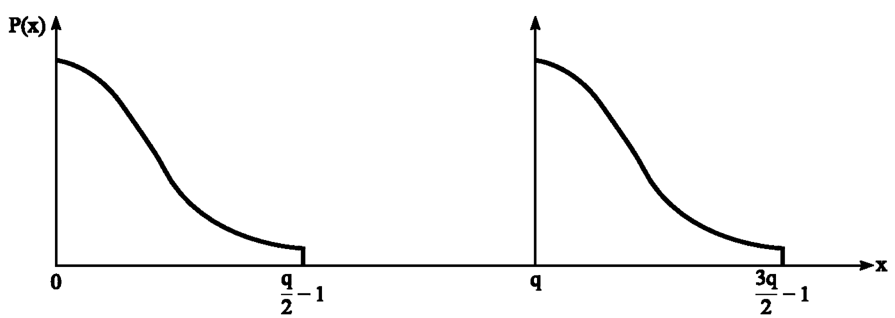


(*c*)

Figure 6. (*a*) Continuous edge Z-search strategy; (*b*) continuous centre Z-search strategy; (*c*) p.d.f. of x for continuous centre Z-search strategy.

Thus, the mean value of W is given as

$$E(W) = \sum_{n_q=0}^{\infty}\sum_{x=0}^{0\cdot 5q}(x - qn_q)P(n_q)P(x) + \sum_{n_q=0,2}^{\infty}\sum_{x=q}^{1\cdot 5q}(x + qn_q)P(n_q)P(x)$$

$$+ \sum_{n_q=1,3}^{\infty}\sum_{x=q}^{1\cdot 5q}\{(x + qn_q) + 2(q - x)\}P(n_q)P(x) \tag{11}$$

By rearranging (11) and substituting for $P(n_q)$ from (8*b*), $E(W)$ becomes

$$E(W)_{(\text{CC-Z})} = E(x) + qE(n_q) + \frac{1 - P_{\text{d}1}(P_{\text{d}2})^{v}}{2 - P_{\text{d}1}(P_{\text{d}2})^{v}}\sum x \tag{12}$$

where

$$\sum x = \sum_{x=q}^{1\cdot 5q} 2(q - x)P(x)$$

$E(x)$ and (x) are found using the p.d.f. of (x) from Fig. 6. Finally, the mean value of the search path for offset-Z1 (O-Z1) and offset-Z2 (O-Z2) search strategies (Pan *et al.* 1991 a) may be developed for a truncated gaussian p.d.f. $P(x)$ as in the Appendix, and $E(W)$ for each search strategy is given as follows:

$$E(W)_{(\text{O-Z1})} = E(x) + (r + 0\cdot 5)q\left\{\frac{1 - P_{\text{d1}}(P_{\text{d2}})^v}{P_{\text{d1}}(P_{\text{d2}})^v}\right\}(2 - \alpha_r) + \left\{\frac{1 - P_{\text{d1}}(P_{\text{d2}})^v}{P_{\text{d1}}(P_{\text{d2}})^v}\right\}(2rq\alpha_r - 2\beta_r) \tag{14}$$

where

$$\alpha_r = \sum_{x=0}^{2rq} P(x), \quad \beta_r = \sum_{x=0}^{2rq} xP(x)$$

Similarly, the search path (given in the Appendix) for (O-Z2), search strategy and $E(W)$ as derived in the Appendix may be expressed as follows:

$$E(W)_{(\text{O-Z2})} = E(x) + (r + 0\cdot 5)q\left\{\frac{1 - P_{\text{d1}}(P_{\text{d2}})^v}{P_{\text{d1}}(P_{\text{d2}})^v}\right\}(2 - \alpha_r) + \left\{\frac{1 - P_{\text{d1}}(P_{\text{d2}})^v}{P_{\text{d1}}(P_{\text{d2}})^v}\right\}(0\cdot 5 - r)q\alpha_r \tag{15}$$

2.2. *Mean acquisition time for EW search strategies*

The normalized mean acquisition time for EW search strategies (shown in Fig. 2) was developed by Jovanovic (1988) using the direct approach method. This may be expressed as

$$E(T_n)_{(EW)} = \frac{E(T_a)}{qE(T_r)} = \frac{1}{q}\sum_{i=1}^{\infty} P_d(1 - P_d)^{i-1}\sum_{j=1}^{0\cdot 5q} P(j)\{m_+(i,j) + m_-(i,j)\} \tag{16}$$

where T_r is the time required for the rejection of an incorrect cell (including the time required by the verification logic in the case of a false alarm), and $P(j)$ is the *a priori* distribution of the phase.

The functions $m(i,j) = m_+(i,j)$ for $j > 0$, and $m(i,j) = m_-(i,j)$, for $j < 0$, denotes the total number of cells tested during acquisition, assuming that the correct location is in the cell marked with j and that the correct detection happened during the ith test of the jth cell.

Expressions for $m(i,j)$ for different EW search strategies are as shown in Table 1 (Jovanovic 1988). Moreover, the sum SL_k may be expressed as

$$SL_k = \sum_{n=0}^{k} L_n, \quad SL_o = 0 \tag{17}$$

where k denotes the largest integer such that, $L_k < j$, L_n is the sweep radii.

Strategy	$m_+(i,j)$	$m_-(i,j)$
BE-EW	$2SL_{k+i-1} + L_{k+1} + j$	$2SL_{k+i-1} + L_{k+1} + j - 1$
CC-EW	$4SL_{k+(i-1)/2} + 2L_{k+(i-1)/2} + j,$ i odd $4SL_{k+i/2} - j + 1,$ i even	$4SL_{k+(i-1)/2} - j,$ i odd $4SL_{k+(i-2)/2} + 2L_{k+i/2} + j + 1,$ i even
BC-EW	$2SL_{k+i-1} + j$	$2SL_{k+i-1} + L_{k+1} - j$
CE-EW	$4SL_{k+(i-1)/2} - L_{k+(i-1)/2} + j,$ i, odd $4SL_{k+i/2} - L_1 - j + 1,$ i even	$-L_1 < j < -1$ $4SL_{(i+1)/2} + 2L_{(i+1)/2} + j + 1 - L_1,$ i odd $4SL_{i/2} - j - L_1,$ i even $-q < j < -L_1$ $4SL_{(i-1)/2} - L_1 - j,$ i odd $4SL_{k+(i-2)/2} + L_1 + 2L_{k+1/2} + j + 1,$ i even

Table 1. $m(i,j)$ for the EW search strategies.

In the case of an equi-expanding window search, the radii L_n are given as

$$L_n = \frac{nq}{2N} \tag{18}$$

where N is the number of sweeps through the uncertainty region.

Thus, substituting for $m(i,j)$ from Table 1 into (16) for a given *a priori* distribution, the normalized mean acquisition time may be computed for any type of EW search strategy.

2.3. *Mean acquisition time for alternate search strategies*

This mean acquisition time of the uniform expanding alternate (UEA) search strategy may be obtained using the direct approach method (Jovanovic 1988). First, the function $m(i,j)$ may be evaluated with the aid of Fig. 3(b), as follows:

$$\left.\begin{aligned} m_+(i,j) &= (i-1)q + 2j - 1, \quad \text{for } j > 0 \\ m_-(i,j) &= (i-1)q + 2j, \qquad \text{for } j < 0 \end{aligned}\right\} \tag{19}$$

By substituting for $m_+(i,j)$ and $m_-(i,j)$ in (16), the normalized mean acquisition time for UEA search strategy may be expressed as

$$E(T_{\mathrm{n}})_{(\mathrm{UEA})} = \frac{1}{P_{\mathrm{d}}} - 1 + \frac{2}{q}\sum_{j=1}^{0 \cdot 5q}\{2jP(j)\} \tag{20}$$

Similarly, the function $m(i,j)$ must be calculated for the non-uniform expanding window alternate (NUEA) search strategy with the aid of Fig. 3(b). $m(i,j)$ may be expressed as

$$\left.\begin{aligned} m_+(i,j) &= SL_{k+i-1} + 2j - 1, \quad L_k < j < L_{k+1} \\ m_-(i,j) &= SL_{k+i-1} - 2j, \qquad -L_k < j < -L_{k+1} \end{aligned}\right\} \tag{21}$$

As SL_k is given by (17), the normalized mean acquisition time for an NUEA search strategy may be expressed as

$$E(T_n)_{(\mathrm{NUEA})} = \frac{2}{q}\sum_{j=1}^{0\cdot5q} 2jP(j) + \frac{2}{q}\sum_{k=0}^{N-t}\sum_{t=1}^{N-k}\{P_d(1-P_d)^{t-1}SP_kSL_{k+t-1}\}$$
$$+\sum_{k=0}^{N-t}(1-P_d)^{N-k}SP_k\left\{\frac{2}{q}SL_{N-t}+\frac{1}{P_d}\right\} \tag{22}$$

where

$$SP_k = \sum_{j=L_k+1}^{L_{k+1}} 2P(j)$$

2.4. *Threshold selection*

The importance of threshold setting in the acquisition process stems from the fact that appropriate threshold settings may significantly improve (i.e. reduce) the probability of a false alarm (P_f). Threshold setting at the spread spectrum receiver is a function of several system parameters. Hence, the selection of best threshold setting must be based on as many parameters concerned as possible. These parameters are the probability of detection (P_d), the probability of a false alarm (P_f), the single-to-noise ratio (SNR), the interference-to-signal ratio (ISR), etc. Moreover, because the maximum of ($P_d - P_f$) may lead to a rapid acquisition time, thus P_d and P_f (computed using the formulae given in the Appendix) may be used to determine the best threshold setting for various ISRs, as shown in Fig. 7. It is evident from this figure that only certain threshold values may maximize ($P_d - P_f$) for a serial-search DS acquisition system. This fact suggests that an adaptive threshold level system based on the estimation of ISR is required to estimate the best threshold values for each ISR.

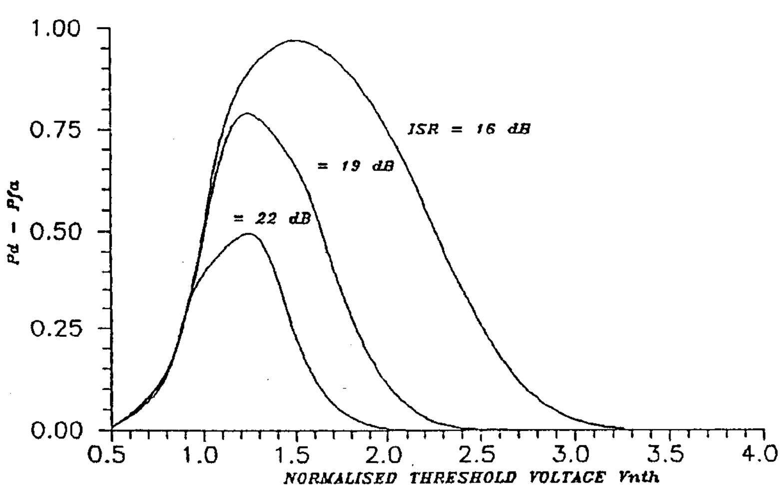


Figure 7. $P_d - P_{fa}$ against normalized threshold for different ISRs.

Parameter	Description
Code chip rate	50 k chip/s
Code length	100 chip (*m*-sequence)
Searched cells q	200 cell
Process gain	26 dB
Information bandwidth	500 bit/s
Signal power	1
First dwell-time T_{d1}	0·5 ms
Second dwell-time T_{d2}	10 ms
Verification/lock strategy	One verification and one lock state
A priori information $P(x)$	Truncated gaussian ($\sigma = q/24$)

Table 2. System parameters.

3. Performances of different search strategies

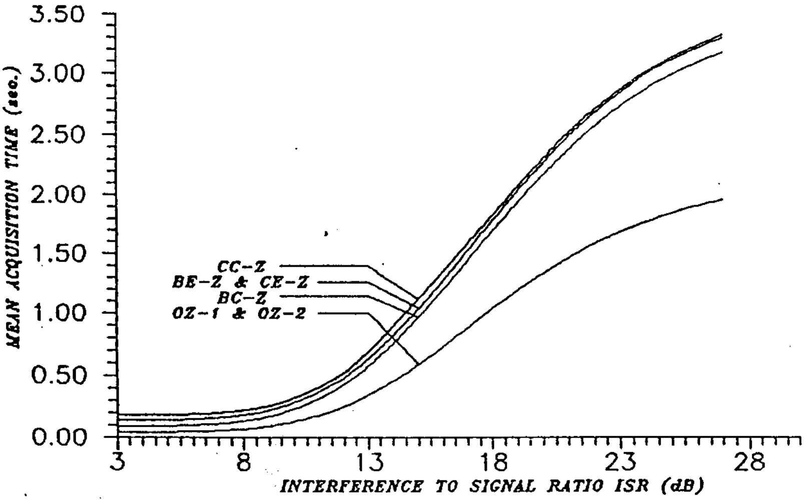
The performances of various search strategies, in terms of the mean acquisition time, for different ISRs are presented in this section. The acquisition system parameters used in computing the mean acquisition time are as given in Table 2.

Narrowband interference is selected because it is the most effective type of interference on DS spread spectrum systems (Dixon 1975, Ziemer and Peterson 1985). Thus, the mean acquisition time for the Z-search strategies has been computed using the expressions obtained in § 2.1, and then plotted against ISR, as shown in Fig. 8(*a*). It is evident that the offset Z-search strategies (i.e. O-Z1 and O-Z2) outperform all the other Z-search strategies, particularly at high ISRs. This is due to the more frequent tests performed by offset Z-search strategies on the centre part of the uncertainty region, as compared with other Z-search strategies. However, Fig. 8(*b*) illustrates the gain in the mean acquisition time over the CC-Z search strategy for various Z-search strategies considered. At ISR $\leqslant$ 6 dB, O-Z search strategies exhibit a mean acquisition time that is 3·3 times faster than that of the CC-Z strategy, and the BC-Z strategy exhibits a mean acquisition time 1·6 times faster than the CC-Z strategy. However, at higher ISRs, namely ISR $\geqslant$ 18 dB, O-Z search strategies feature a mean acquisition time 1·6 times faster than the CC-Z search strategy, and the mean acquisition time for the BC-Z strategy is 1·2 times faster.

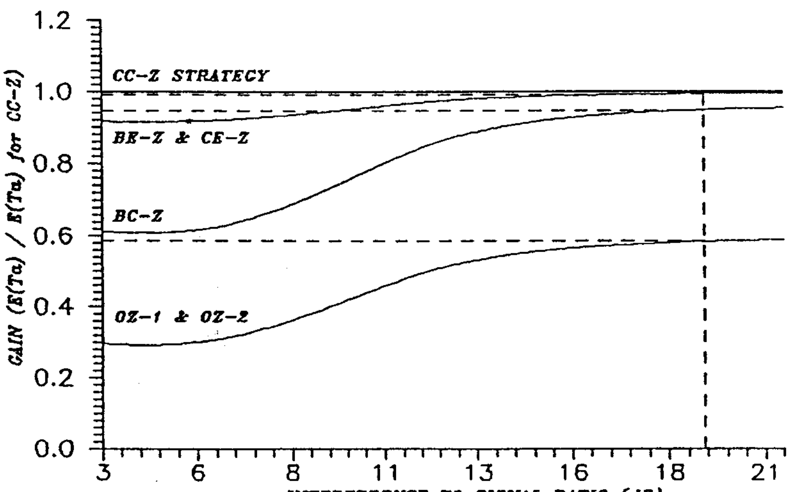
Figure 9(*a*) illustrates the normalized mean acquisition time $E(T_n)$ versus the probability of detection P_d for different EW search strategies with various *a priori* gaussian distributions of the phase. Better probability of detection may be obtained and hence faster mean acquisition times, as the *a priori* distribution is more peaked. Moreover, Fig. 9(*b*) illustrates the normalized mean acquisition time for the BC-EW search strategy versus ISR for different *a priori* gaussian p.d.f.s. It is evident that the BC-EW search strategies perform very well at high ISR environments (e.g. ISR $\geqslant$ 30 dB), particularly for more peaked *a priori* p.d.f.s (e.g. $\sigma = q/24$).

Figure 10 illustrates the normalized mean acquisition time $E(T_n)$ versus ISR for the two types of alternate search strategies, namely UEA and NUEA. It is clear that NUEA performs better than UEA, particularly at high ISRs and for more peaked *a priori* p.d.f.s ($\sigma = q/24$).

Moreover, Fig. 11 illustrates the performances of various search strategies considered, in terms of the mean acquisition time, versus ISR, for a normalized threshold voltage $V_{nth} = 1{\cdot}5$ and *a priori* p.d.f. of $\sigma = q/24$. At relatively low ISRs, EW and alternate search strategies enjoy better performances. Thus, at



(*a*)

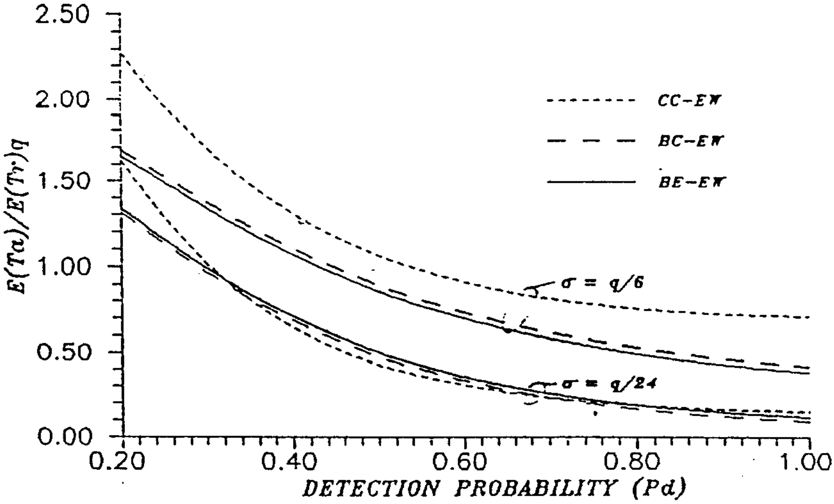


(*b*)

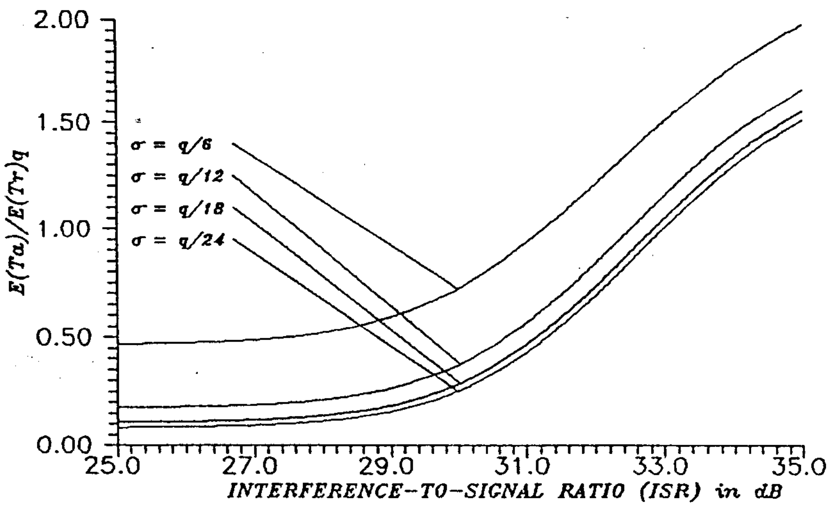
Figure 8. (*a*) Mean acquisition time versus ISR for different Z-search strategies; (*b*) gain in acquisition time when using different Z strategies with respect to CC-Z strategy versus ISR.

10 dB < ISR < 15 dB the NUEA search strategy is four times faster than that of the O-Z1 search strategy in terms of the mean acquisition time. However, at high ISRs the NUEA search strategy outperforms all the other search strategies. Hence, at ISR = 20 dB NUEA features a mean acquisition time that is 20 times faster than that of the O-Z1 search strategy. Nevertheless, the implementation complexity of the NUEA search strategy could be a prohibitive factor for using it in practical spread spectrum acquisition systems.

Finally, the equations given in §§ 2.1–2·3 may easily be used to establish the performance of various search strategies for frequency-hopped (FH) acquisition



(*a*)



(*b*)

Figure 9. Normalized mean acquisition time: (*a*) versus P_d for various EW search strategies with gaussian *a priori* p.d.f.; (*b*) versus ISR for BC-EW search strategies with gaussian *a priori* p.d.f.

schemes. However, the type of interference from which the FH spread spectrum suffers most are partial band noise and partial band tone interfering signals (Ziemer and Peterson 1985, Jibrail *et al.* 1994).

4. Experimental results

A block diagram of the experimental system is as shown in Fig. 12. The transmitter part of the system comprises a PN-code generator which is an *m*-sequence,

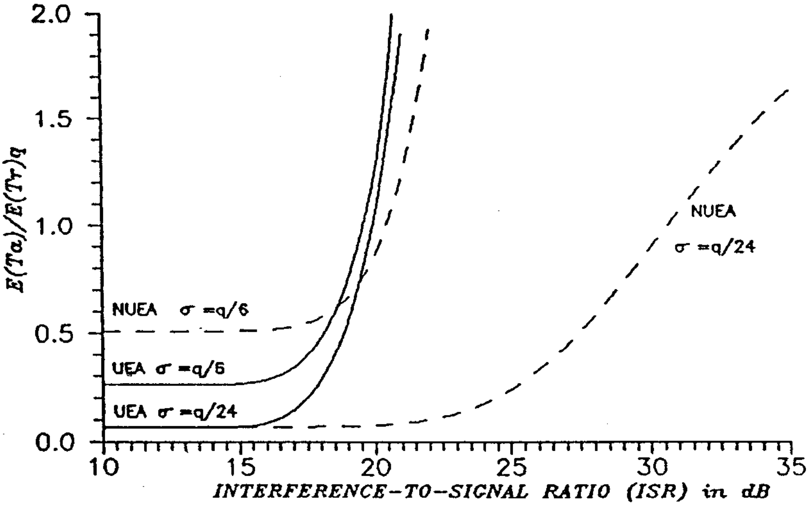


Figure 10. Normalized mean acquisition time versus ISR, for the UEA and NUEA search strategies, with different gaussian *a priori* distributions.

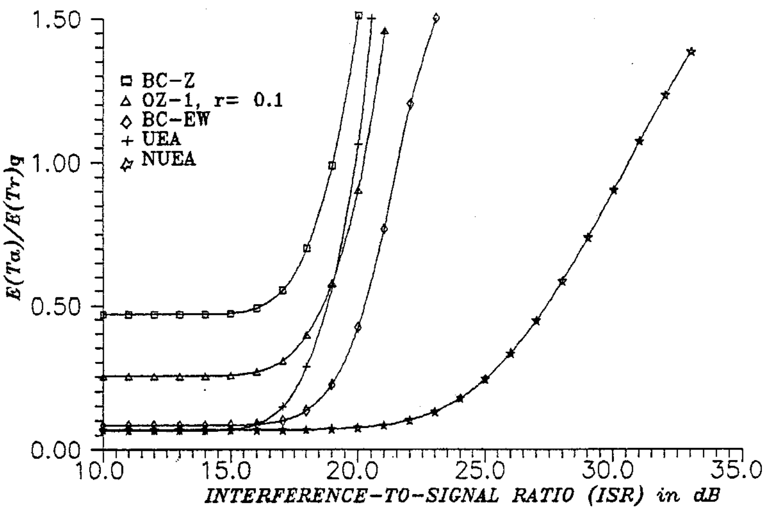


Figure 11. Normalized mean acquisition time versus interference-to-signal ratio for various search strategies with $\sigma = q/24$, $V_{nth} = 1{\cdot}5$.

and the channel is linear and it combines AWGN and narrowband interference. At the receiver, the acquisition scheme consists of: a lowpass filter that limits the noise bandwidth to $(1/T_c)$, where T_c is the chip duration; a digital sliding correlator (Jibrail and Houmadi 1991); a local PN-code generator; a threshold comparator; and timing and control circuits. These circuits are designed and implemented to perform all the Z-search strategies under consideration (Al-Zubiady 1994). The parameters of the experimental system are as given in Table 3.

A large number of acquisition time readings are recorded for each value of ISR and a given SNR, then an average value is computed. Fig. 13 illustrates the

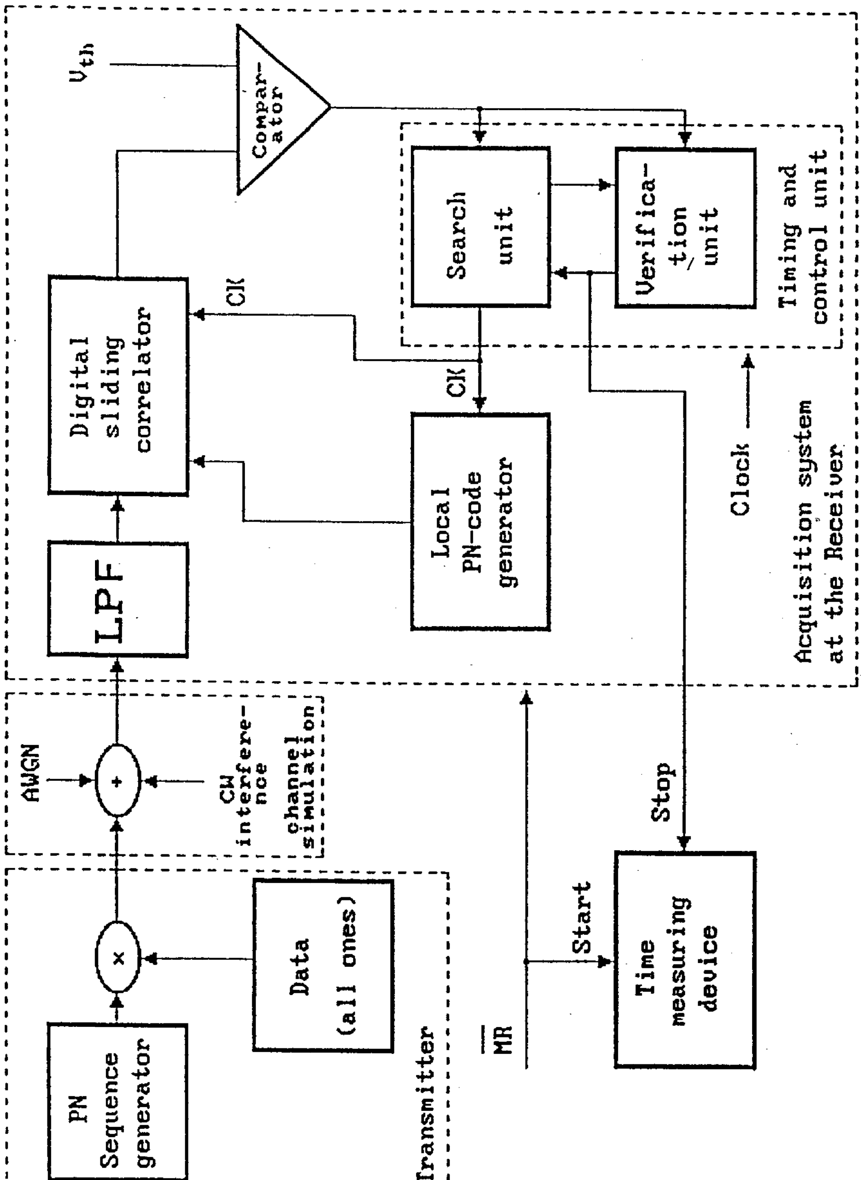


Figure 12. Experimental system block diagram.

Parameter	Description
PN-code	m-sequence, $L = 100$
Chip rate	50 k chip/s
First dwell-time T_{d1}	0·5 ms
Second dwell-time T_{d2}	10 ms
Phase updating	half a chip
Uncertainty region	200 cells
Noise	Additive white gaussian
Interference	CW with $f_0 = 50$ kHz

Table 3. Experimental system parameters.

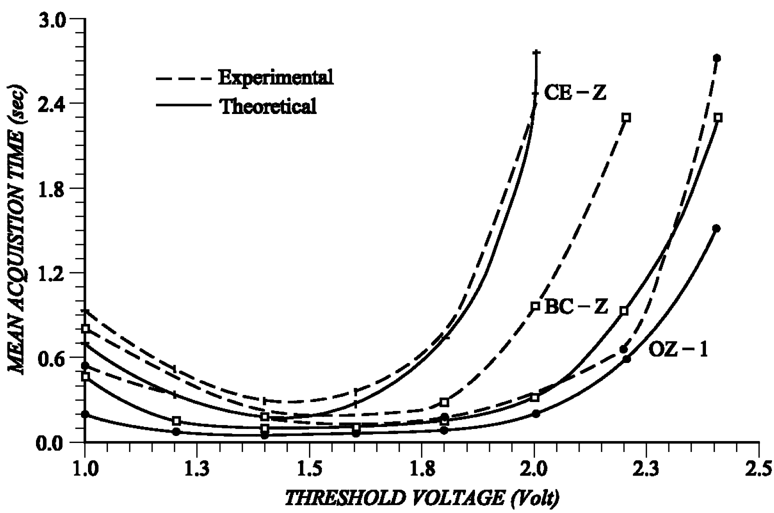


Figure 13. Mean acquisition time versus normalized threshold for different Z search strategies, $T_{d1} = 0{\cdot}5$ ms.

mean acquisition time (both experimental and theoretical) against ISR for several Z-search strategies. Results confirm the superiority in performance of the offset Z-search strategies, as compared with other Z-search strategies, especially at high ISRs.

5. Conclusions

Performances, in terms of the mean acquisition times, of various search strategies, namely the Z-search, EW-search, UEA-search and NUEA-search strategies, have been computed and compared for a DS spread-spectrum acquisition scheme.

Among the Z-search strategies, it has been shown that at ISR = 6 dB, the O-Z search strategy exhibits a mean acquisition time that is 3·3 times faster than that of the CC-Z search strategy. Yet at higher ISRs namely, ISR $\geqslant$ 18 dB, O-Z search strategies give a mean acquisition time that is 1·6 times faster than that of the CC-Z search strategy. Moreover, experimental results have shown that O-Z search strategies outperform other Z-search strategies (see Fig. 13).

It has been also shown that a significant reduction may be obtained in the mean acquisition time of a DS spread-spectrum acquisition scheme by using the NUEA search strategy. However, the implementation complexity of the NUEA search strategy could be the prohibitive factor to employ this strategy in practical DS acquisition systems. Furthermore, more peaked gaussian *a priori* p.d.f.s for the phase information of the PN-code have been shown to give faster mean acquisition times for EW and alternate search (i.e. UEA and NUEA) strategies, particularly at higher ISRs (see Figs 9(*a*) and (*b*)).

Appendix A: Derivation of $E(W)$ for O-Z search strategies

Offset-Z1 search (Pan *et al.* 1991a)

Figure 14 illustrates the p.d.f. of (x) in the case of the O-Z1 search strategy. The path length W for an O-Z1 search may be obtained with the aid of Fig. 15 as follows:

$$\begin{aligned} W &= n_q(r+0{\cdot}5)q + x, && \text{for } n_q \text{ even, } 0 < x < 2rq \\ W &= n_q(r+0{\cdot}5)q + x + 2(rq - x), && \text{for } n_q \text{ odd, } 0 < x < 2rq \\ W &= 2n_q(r+0{\cdot}5)q + x, && 2rq < x < (2r+1)q \end{aligned} \tag{A 1}$$

The mean value of the search length W is given as

$$E(W) = \sum_{n_q=0}^{\infty} \sum_{x=0}^{\infty} WP(n_q)P(x) \tag{A 2}$$

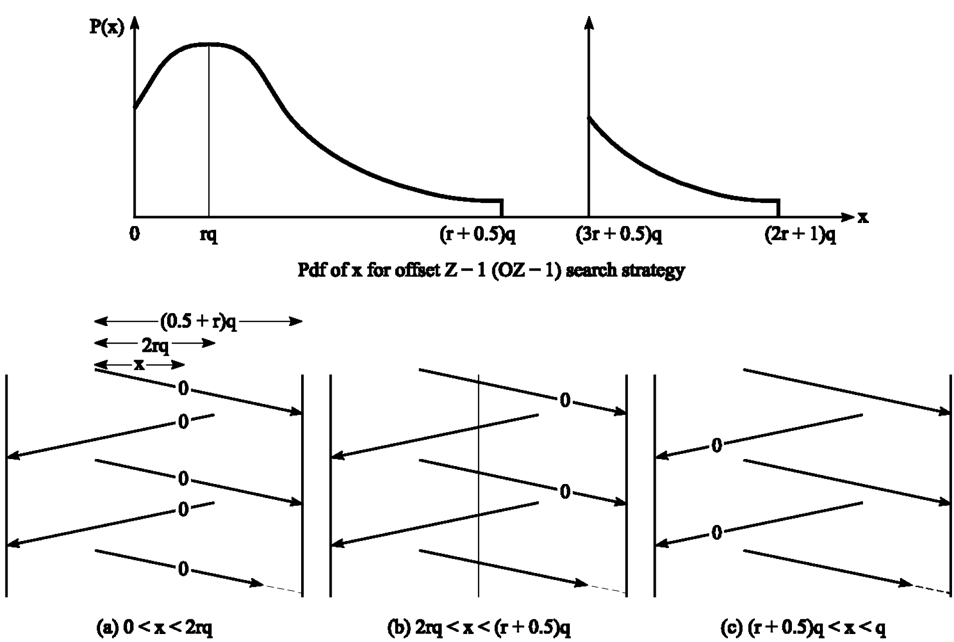
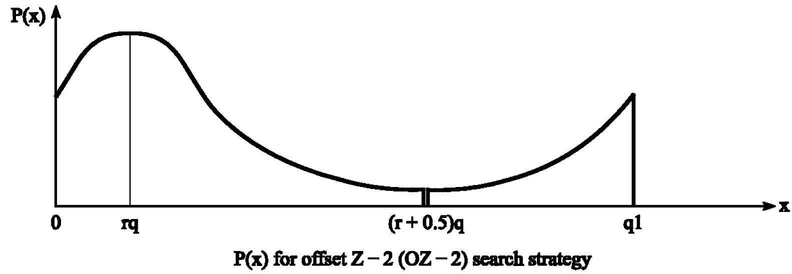


Figure 14. (*a*) Search path in OZ-1 search strategy: (*a*) $0 < x < 2rq$; (*b*) $2rq < x < (r+0{\cdot}5)q$; (*c*) $(3r+0{\cdot}5)q < x < (2r+1)q$.



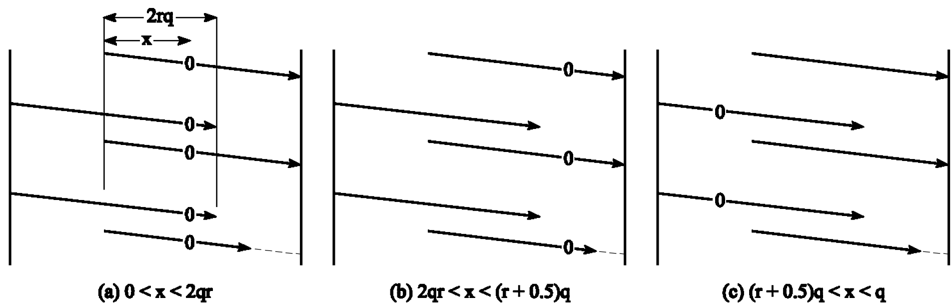


Figure 15. OZ-2 search path: $(a)\ 0 < x < 2rq$; $(b)\ 2rq < x < (r + 0{\cdot}5)q$; $(c)\ (r + 0{\cdot}5)q < x < q$.

For the O-Z1 search strategy, substitute W from (A 1) into (A 2), and $E(W)$ becomes

$$
\begin{aligned}
E(W) = & \sum_{n_q=0,2}^{\infty} \sum_{x=0}^{2rq} \{n_q(r + 0{\cdot}5)q + x\} P(n_q) P(x) \\
& + \sum_{n_q=1,3}^{\infty} \sum_{x=0}^{2rq} \{n_q(r + 0{\cdot}5)q + x + 2(rq - x)\} P(n_q) P(x) \\
& + \sum_{n_q=0}^{\infty} \sum_{x=0}^{2rq} \{2n_q(r + 0{\cdot}5)q + x\} P(n_q) P(x) \qquad \text{(A 3)}
\end{aligned}
$$

As the term $\left[nq(r + 0{\cdot}5)q + x\right]$ is common in all three terms in (A 3), $E(W)$ becomes

$$
\begin{aligned}
E(W) = & \sum_{n_q=0}^{\infty} \sum_{x=0}^{\infty} \{n_q(r + 0{\cdot}5)q + x\} P(n_q) P(x) \\
& + \sum_{n_q=1,3}^{\infty} \sum_{x=0}^{2rq} 2(rq - x) P(n_q) P(x) + \sum_{n_q=0}^{\infty} \sum_{x=2rq}^{\infty} n_q(r + 0{\cdot}5)q P(n_q) P(x) \qquad \text{(A 4)}
\end{aligned}
$$

The summation in the second term may be expressed as follows:

$$
\sum_{n_q=1,3}^{\infty} P(n_q) = \frac{1 - P_{\mathrm{d1}}(P_{\mathrm{d2}})^{v}}{2 - P_{\mathrm{d1}}(P_{\mathrm{d2}})^{v}} \qquad \text{(A 5)}
$$

By substituting (A 5) into (A 4), $E(W)$ becomes

$$E(W) = E(x) + (r+0\cdot5)qE(n_q) + \left\{\frac{1-P_{d1}(P_{d2})^v}{2-P_{d1}(P_{d2})^v}\right\} \sum_{x=0}^{2rq} 2(rq-x)P(x)$$

$$+ (r+0\cdot5)qE(n) \sum_{x=2rq}^{\infty} P(x) \qquad \text{(A 6)}$$

where $E(n_q)$ is given by (3 a). By denoting

$$\sum_{x=0}^{2rq} P(x) = \alpha_r$$

$$\sum_{x=0}^{2rq} xP(x) = \beta_r$$

Hence

$$\sum_{x=2rq}^{\infty} P(x) = 1-\alpha_r$$

By substituting the above into (A 6), $E(W)$ for the O-Z1 search becomes

$$E(W)_{(\text{O-Z1})} = E(x) + (r+0\cdot5)q\left\{\frac{1-P_{d1}(P_{d2})^v}{P_{d1}(P_{d2})^v}\right\}(2-\alpha_r)$$

$$+ \left\{\frac{1-P_{d1}(P_{d2})^v}{2-P_{d1}(P_{d2})^v}\right\}(2rq\alpha_r - 2\beta_r) \qquad \text{(A 7)}$$

Offset-Z2 search strategy

The p.d.f. of (x) for the O-Z2 search strategy, the search path and the length W may be expressed as (Pan *et al.* 1991 a)

$$\begin{aligned} W &= n_q(r+0\cdot5)q + x, && \text{for } n_q \text{ even}, \quad 0 < x < 2rq \\ W &= n_q(r+0\cdot5)q + x + (0\cdot5-r)q, && \text{for } n_q \text{ odd}, \quad 0 < x < 2rq \\ W &= 2n_q(r+0\cdot5)q + x, && \text{otherwise} \end{aligned} \qquad \text{(A 8)}$$

By substituting (A 8) into (A 2), $E(W)$ becomes

$$E(W) = \sum_{n_q=0}^{\infty}\sum_{x=0}^{\infty}\{n_q(r+0\cdot5)q + x\}P(n_q)P(x)$$

$$+ \sum_{n_q=1,3}^{\infty}\sum_{x=0}^{2rq}(0\cdot5-r)qP(n_q)P(x) + \sum_{n_q=0}^{\infty}\sum_{x=2rq}^{\infty} n_q(r+0\cdot5)qP(n_q)P(x) \qquad \text{(A 9)}$$

Again, by substituting (A 5) into (A 9), $E(W)$ becomes

$$E(W) = E(x) + (r+0\cdot5)qE(n_q) + \left\{\frac{1-P_{d1}(P_{d2})^v}{2-P_{d1}(P_{d2})^v}\right\}(0\cdot5-r)\sum_{x=0}^{2rq} P(x)$$

$$+ (r+0\cdot5)qE(n_q) \sum_{x=2rq}^{\infty} P(x) \qquad \text{(A 10)}$$

Thus, by substituting for $E(nq)$ in (A 10), $E(W)$ for O-Z2 becomes

$$E(W)_{(\text{O-Z2})} = E(x) + (r + 0{\cdot}5)q\left\{\frac{1 - P_{d1}(P_{d2})^v}{P_{d1}(P_{d2})^v}\right\}(2 - \alpha_r) + \left\{\frac{1 - P_{d1}(P_{d2})^v}{2 - P_{d1}(P_{d2})^v}\right\}(0{\cdot}5 - r)q\alpha_r, \tag{A 11}$$

Appendix B: Transition probabilities

Two of the most important acquisition process parameters are the probability of detection P_d and the probability of a false alarm P_f. These probabilities describe the detector performance, and they are functions of some detector parameters such as dwell-time T_d and threshold voltage V_{th}. However, for given system parameters, these probabilities may be expressed as (Simon *et al.* 1985, Holmes and Chen 1977)

$$P_f = Q\left[(V_{nth} - 1)(BT_d)^{1/2}\right] \tag{B 1}$$

$$P_d = Q\left[(V_{nth} - 1 - \text{SNR})\left[BT_d/(1 + 2\text{SNR})\right]^{1/2}\right] \tag{B 2}$$

where

$$Q = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du$$

$V_{nth} = V_{th}/N_0 B$, is the normalized threshold voltage, N_0 is the noise spectral density, B is the predetection bandpass filter bandwidth, and SNR is the predetection signal-to-noise ratio.

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