Multi-Antenna Aspects of Wireless Fixed Relays

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***Abstract*— Multiple antenna schemes can provide performance enhancement but the small size and cost constraints of wireless terminals preclude their use at these terminals. This paper investigates distributed fixed relays (infrastructure-based relaying) which are engaged in cooperation in a two-hop wireless network as a means of removing the burden of multiple antennas on wireless terminals.**

In contrast to mobile terminals, a small number of antennas can be deployed on infrastructure-based fixed relays. Hence, the paper investigates performance of the distributed cooperative multi-antenna fixed relays. Threshold maximal ratio combining and threshold selection combining of these multiple antenna signals are studied and analyzed. The following results are derived. For a given performance requirement, the multiple antennas at relays can significantly reduce the number of relays required in a network area, ultimately, reducing system deployment cost. In addition, threshold selection combining at the relays represents an excellent compromise between performance and system cost.

Finally, the analysis performed in this paper uses the versatile Nakagami fading channel model.

I. Introduction

The conventional cellular architecture appears incapable of delivering the ubiquitous high data-rate coverage expected of the future generation of wireless systems. This inadequacy suggests for a fundamental change in the way systems are designed and deployed as well as suggests for novel signal processing techniques [1], [2].

Recent literature attests to an explosive interest in multihop-augmented infrastructure-based networks, or simply, relay networks [3], which might be an underlying technology for deploying future generations of wireless systems. Most of the early works in literature on relaying are for coverage extension [4] or network capacity distribution [3]. These multi-hop relaying approaches are used usually because of their economic advantages over cell splitting methods which are used in conventional cellular networks to increase system capacity and spectral efficiency. This paper presents how these deployed relays can be used for providing spatial diversity gains for a wireless terminal which, otherwise, has limitations in the number of antennas it can bear.

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The exploitation of diversity was once considered as the "last frontier" in wireless communications. The recent experience with multiple input multiple output (MIMO) systems [5], has shown that joint time and space processing could push this frontier even further. Application of multi-antenna techniques to mobile systems is often plagued with implementation problems. The small size requirement for future wireless terminals limits the spatial separation that is needed to provide multi-antenna schemes with uncorrelated or at least weakly correlated channels to perform optimally.

As an add-on to network relays, we show that the difficulties of using multi-antenna on mobile terminals can be alleviated through the cooperative use of these relays. Infrastructure-based fixed relays may have the capability to carry multiple antennas. Therefore, this paper also investigates the impact of multi-antenna on the cooperative fixed relays in two-hop networks. Threshold maximal ratio combining and threshold selection combining of these multiple antenna signals are examined and analyzed. The following main results are derived. For a given performance requirement, the multiple antennas at relays can tremendously reduce the number of relays required in a network area, thereby, reducing system deployment cost. Furthermore, threshold selection combining at the relays represents an excellent compromise between performance and system cost in comparison to single antenna relaying and MRC-based relaying. Finally, cooperative diversity strategy employed as an add-on to infrastructure-based relays can harness the advantages of multiple antennas without the need for multiple antennas at wireless terminals.

In Section II the multi-antenna relay network is discussed. Section III presents the analysis of the end-to-end (E2E) error performance. This is followed by numerical examples in Section IV. Finally, conclusions are drawn in Section V.

II. Multi-Antenna Relay Networks

The conventional fixed protocol decode-and-forward relay network is shown in Fig. 1(a). The system model that is employed to form the two-hop network consists of N_R multiple adaptive (threshold decode-and-forward (TDF)) fixed relays carrying L micro-diversity antennas (Fig. 1(b)). The system architecture considered in this work is such that all link channels are independent but statistically identical. This scenario is referred to as a symmetric network as opposed

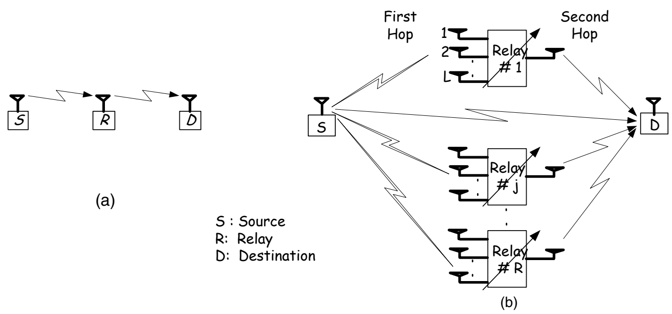

Fig. 1. Network relay deployment (a) DF fixed protocol (conventional relaying), (b) TDF protocol with multi-antenna cooperative relays.

to the asymmetric one where relays could experience non-identical channels.

The protocol employed in this paper operates in the following mode. In the first hop, the source broadcasts a signal that is received by both destination and relays. The destination stores this signal for future processing. The received L signals at each relay are processed using either selection combining (SC) or maximal ratio combining (MRC) diversity techniques, depending on the processing complexity tolerable at the relays. Whether SC or MRC is used, the relay receiver checks its output SNR against a preset threshold. Each relay decodes and forwards only when the SNR is greater than this threshold. In this sense, the number of forwarding relays in the second hop is not a fixed number. For this reason, in addition to the fact that the power for fixed relays is available off the "wall plug" supply (posing no serious practical implications) explains why total power has not been constrained.

In the second hop, the relay either does or does not forward a new pre-processed signal to destination. If the relay forwards, the destination combines its delayed (buffered) signal with the new versions from the relays, assuming an MRC-type processor. In this phase, it is assumed that one antenna is utilized by each relay. A single transmitting antenna is adopted to keep a fair cost comparison with a conventional single antenna relay networks where one RF detection chain and one transmit antenna are required; the same requirement for the SC-based detection at the relays. For this reason, SC-based detection appears as a promising candidate, in terms of performance-cost trade-off, for the network discussed in this paper. The relays, however, could employ its multi-antenna in 'intelligent' transmit beamforming. Should this be the case the system performance could be improved.

III. Analysis of the Multi-Antenna Relay Networks

We define a general link in the network as a node t communicating with node j. A node could refer to a source, relay or destination. The received signal at node j can then be written as $r_{tj} = r_{t\longrightarrow j} = h_{tj}x_t + n_j$, where x_t is the signal emanating from node t and n_j is the noise at receiving node j. For a relay in the multi-antenna relay network, the input-output relation of the first hop, i.e., source (node S) to relays, can be expressed as

$$\mathbf{r}_j = \mathbf{h}_{Sj}x_S + \mathbf{n}_j, \quad j = 1, 2, \cdots, N_R, \tag{1}$$

where N_R is the total number of relays, $\mathbf{r}_j$ is the $L\times 1$ received vector at the jth relay, $\mathbf{h}_{Sj} = [h_{Sj}^{(1)}, \cdots, h_{Sj}^{(l)}, \cdots, h_{Sj}^{(L)}]^T$ denotes the random channel vector with independent components which are also independent of the components in the $L \times 1$ additive white Gaussian noise (AWGN) vector $\mathbf{n}_j$. In a coherent scheme, perfect phase and carrier recovery is possible. Therefore, each entry $h_{tj}^{(l)}$ represents the magnitude of the fade sample at an antenna l of a receiving node j. In the second hop, the relay j forwards the regenerated signal $(\hat{x}_j)$ to the destination (node D) provided that the received SNR is greater than the threshold γ_{th}. This signal is received as

$$r_{jD} = h_{jD}\hat{x}_j + n_{jD}, \quad j = 1, \cdots, N_R, \tag{2}$$

where h_{jD} is the fade sample in the link between relay node j and destination while r_{jD} and n_{jD} are the received signals and receiver noise during the reception, respectively. The relay transmissions are assumed to be decoupled at the destination through either suitable pre-coding, or time and frequency allocations (discussed in [6]). Therefore, interference is not considered in (2).

The analysis assumes that the fading sample $h_{tj}^{(l)}$ follows the Nakagami-m distribution which is a versatile statistical model that can be used to model a wide range of fading environments in mobile radio channels [7]. For instance, the Ricean model representing a line-of-sight (LOS) is related to the Nakagami distribution through the Ricean parameter K to Nakagami parameter m transformation $K = \frac{\sqrt{m^2-m}}{m-\sqrt{m^2-m}}$; moreover, $m = 1$ gives Rayleigh distribution. The SNR per symbol denoted as γ at an antenna of node k, is distributed according to the gamma distribution described by[1]

$$p_\gamma(\gamma, m) = \frac{m^m\gamma^{m-1}}{\bar{\gamma}^m\Gamma[m]}\exp\left(-\frac{m\gamma}{\bar{\gamma}}\right), \quad \gamma \geq 0,\, m \geq 1/2, \tag{3}$$

where $\gamma = (h_{tk}^{(l)})^2E_s/N_0$ and $\bar{\gamma} = E[(h_{tk}^{(l)})^2E_s/N_0] = \Omega_kE_s/N_0$ is the average SNR per symbol, E_s is the energy per symbol and $\Omega_k = E[(h_{tk}^{(l)})^2]$. The AWGN is characterized by one-sided power spectral density N_0 (W/Hz), and $\Gamma[\cdot]$ is the gamma function.

A. End-to-end Error Performance

The overall end-to-end (E2E) error rate $P_{e,\text{E2E}}^{(\text{TDF})}$ of the multi-antenna multiple relay schemes can be approximated, using an approach similar to that presented in [8] as

$$P_{e,\text{E2E}}^{(\text{TDF})} \approx P'_{\text{dec}}P_{\text{e,r}}P_{\text{e,p}} + P'_{\text{dec}}(1-P_{\text{e,r}})P_{e,\text{coop}} + (1-P'_{\text{dec}})P_{e,\text{dir}}. \tag{4}$$

[1]The indexes k and l are dropped because for a given node the statistics for all the antennas at k is the same.

The first and second terms in (4) represent the components due to cooperation while the third represents that due to no-cooperation case. The first term is needed to account for possible error propagation. $P_{\text{e,r}}$ is the error probability at the relay given that the received SNR, γ, is greater than γ_{th}. $P_{e,\text{coop}}$ is the destination error rate when cooperation is in effect (i.e., more than one branch is combined at the destination). $P^{'}_{\text{dec}}$ is the probability that at least one relay performs decoding and forwarding,[2] and $P_{e,\text{dir}}$ is the destination probability of error when no relay forwards. The impact of the number of relays is captured by averaging over the possible cooperative scenarios $P_{e,\text{coop}}$ and $P^{'}_{\text{dec}}$, and therefore, the error probability due to error propagation $P_{\text{e,p}}$ is bounded with the worst case bound: $P_{\text{e,p}} \leq 1/2$ [8].

$P^{'}_{\text{dec}}$ is obtained as follows. Denoting P_{DFP} as the decode-and-forward probability (DFP) at a relay, then P(relay r does not forward) $= 1 - P$(relay r does forward) $= 1 - P_{\text{DFP},r}$. Then, $P^{'}_{\text{dec}}$ can be expressed, after going through some steps, as

$$P^{'}_{\text{dec}} = \sum_{r=1}^{N_R} \binom{N_R}{r} (-1)^{r+1} (P_{\text{DFP}})^r, \tag{5}$$

where $P_{\text{DFP},r} = P_{\text{DFP}}$, for all r.

In symmetric network scenarios[3], the channels are considered to be balanced (same average SNR). Assuming that all the relays have the same error performance, $P_{e,\text{coop}}$ can be evaluated for equal-amplitude modulation as follows: The probability of error for T-branch MRC-receiver in Nakagami-m channel is derived as $\frac{q\,\Gamma[T\,m+1/2]}{\sqrt{\pi}\,\Gamma[T\,m]} B_\mu\,[T\,m, 1/2]$. The probability that i relays forward which give rise to $(i+1)$ diversity branches at the destination is given as ${}^{N_R}C_i (1-P_{\text{DFP}})^{N_R-i} P^i_{\text{DFP}}$ where ${}^{N_R}C_i = \frac{N_R!}{i!(N_R-i)!}$. Therefore, $P_{e,\text{coop}}$ can be expressed, by weighted average over the possible cooperative scenarios, as

$$\begin{aligned} P_{e,\text{coop}} &= \frac{q}{\sqrt{\pi}} \sum_{i=1}^{N_R} {}^{N_R}C_i\, (1 - P_{\text{DFP}})^{N_R-i} P^i_{\text{DFP}} \\ &\times \frac{\Gamma[(i+1)m + 1/2]}{\Gamma[(i+1)m]} B_\mu\,[(i+1)m, 1/2], \end{aligned} \tag{6}$$

where $\mu = m/(m + \lambda\bar{\gamma})$, and $B_\mu[\cdot,\cdot]$ and $\Gamma[\cdot]$ are the incomplete beta and gamma functions [9], respectively. The parameter $\lambda = g \sin^2(\pi/M)$, where M is the modulation constellation size. The g and q are defined according to the modulation and signal detection methods. For example, for BPSK $g = 1$, $q = 1/2$, and $M = 2$ [10].

It is obvious by examining (4) and (6) that how well the relay performs has a tremendous impact on the benefit of the cooperation. For instance, if $P_{\text{e,r}}$ is low and $P^{'}_{\text{dec}}$ is high, one obtains the most desirable benefit from the cooperation:

$$P^{(\text{TDF})}_{e,\text{E2E}} \approx P_{e,\text{coop}}. \tag{7}$$

[2]This implies there is diversity combining at destination.

[3]This assumption de-emphasizes the distance-dependent received power variation; this factor can be incorporated by generalizing our results to unequal branches analysis in the diversity combining.

The result given in (7) provides a number of insights into the cooperative scheme. First, (7) means that the relays always have good and reliable signals to transmit. Therefore, by combining these signals at the destination, the full benefit of diversity can be derived due to the distributed nature of the transmitting entities. This implies that the spatial constraint that usually hampers collocated multiple antenna schemes is, thus, removed. Furthermore, since the relays perfectly decode the source information, the source appears (to the destination) as if it were at the position of the relay, leading to path-loss reduction. This last point will be visible only in the asymmetric channel scenario which incorporates the distance-dependent received power loss.

B. Relay Performance (DFP and Error rate)

Let us now examine the decode-and-forward (DF) probability of the multi-antenna relay with threshold strategy. We begin with SC where the relay first selects the branch with the largest SNR, i.e., $\gamma = \max[\gamma_1, \cdots, \gamma_L]$, and then compares it with the set decoding threshold γ_{th}. The joint probability density function (PDF) of selecting n largest from L independently and identically distributed random variables is given in a general form in [11] as $p_{\gamma_1,\ldots,\gamma_n}(\gamma_1,\cdots,\gamma_n) = n! C^L_n [F(\gamma)]^{L-n} \prod_{l=1}^n p(\gamma_l)$. In our case, the required PDF reduces to $p(\gamma, m) = L[F(\gamma, m)]^{L-1} p_\gamma(\gamma, m)$. Further, employing the functional series representation [9], the decoding probability of the selection-based DF relay can be expressed as [6]

$$\begin{aligned} P^{(\text{r, sc})}_{\text{DFP}} &= \frac{L}{\Gamma[m]^L} \sum_{p=0}^{\infty} c_p \exp\left(-\frac{m\gamma_{th}}{\Omega_{\text{sr}}\bar{\gamma}}\right) \\ &\times \sum_{k=0}^{p+mL-1} \frac{(p+mL-1)!}{k!} \left(\frac{m\gamma_{th}}{\Omega_{\text{sr}}\bar{\gamma}}\right)^k, \end{aligned} \tag{8}$$

where, $c_0 = a_0^{L-1}$, $c_p = \frac{1}{pa_0}\sum_{w=1}^p (w(L-1)-p+w) a_w c_{p-w}$, and $a_w = \frac{(-1)^w}{w!(m+w)}$. The parameters γ_{th} and Ω_{sr} are the relay decoding threshold and channel power of the source-relay link, respectively. In the same manner, the decoding probability of MRC-based DF relay can be expressed as

$$P^{(\text{r, mrc})}_{\text{DFP}} = \exp\left(-\frac{m\gamma_{th}}{\bar{\gamma}\Omega_{\text{sr}}}\right) \sum_{k=0}^{Lm-1} \frac{1}{k!} \left(\frac{m\gamma_{th}}{\Omega_{\text{sr}}\bar{\gamma}}\right)^k. \tag{9}$$

The error rate of a multi-antenna relay that performs threshold decoding can be expressed as [6]

$$\begin{aligned} P^{(\text{r, mrc})}_{e} &= \frac{q\,\Gamma[Lm+1/2]}{\sqrt{\pi}\,\Gamma[Lm, m\gamma_{th}/\bar{\gamma}]} B_\mu\,[Lm, 1/2] \\ &- \frac{q\,\Gamma[Lm] - \Gamma[Lm, \frac{m\gamma_{th}}{\bar{\gamma}}]}{\Gamma[Lm, m\gamma_{th}/\bar{\gamma}]} \\ &+ \frac{4\,q}{\sqrt{\pi}} \sum_{p=0}^{\infty} \frac{(-1)^p}{p!\,(2p+1)} \left(\frac{g}{2}\right)^{p+1/2} \\ &\times \frac{\Gamma[Lm+p+1/2] - \Gamma[Lm+p+1/2, \frac{m\gamma_{th}}{\bar{\gamma}}]}{\Gamma[Lm, m\gamma_{th}/\bar{\gamma}] \left(\frac{m}{\bar{\gamma}}\right)^{(p+1/2)}}, \end{aligned} \tag{10}$$

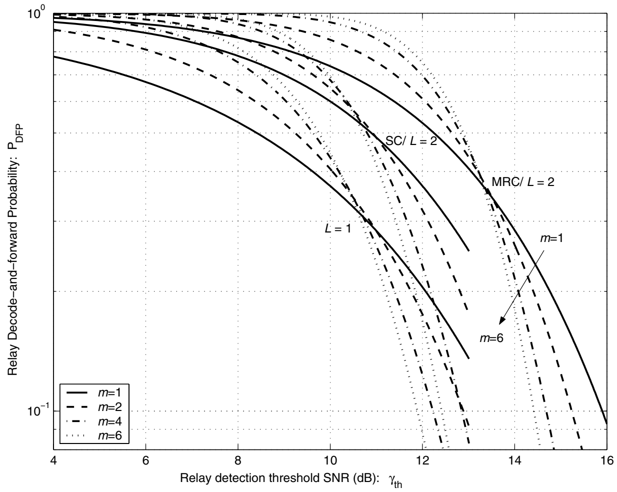

Fig. 2. Probability of decode and forward at a relay ($L = 2$).

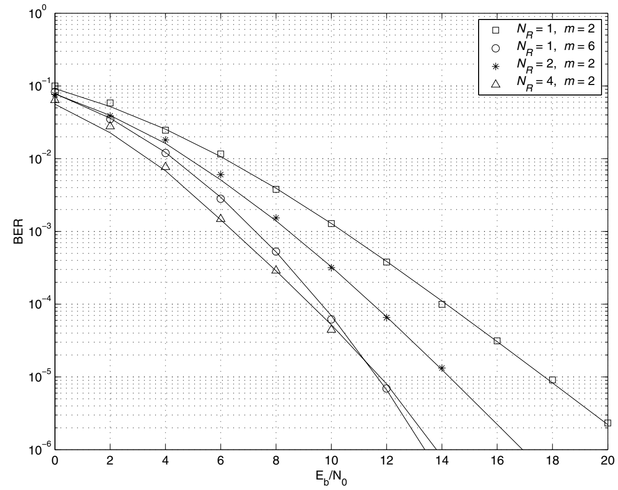

Fig. 3. End-to-end BER performance of single antenna TDF cooperative relay networks in Nakagami fading, $N_R = 1$, 2, 4.

where $\Gamma[\cdot, \cdot]$ is the incomplete gamma function.

Similarly, the expression for evaluating the performance of SC-based detection at the relay has also been derived in [6], a sketch is shown in the Appendix.

IV. Numerical Results

Fig. 2 shows the improvement in the probability of decoding at the relay when two antennas are deployed for SC- and MRC-based detections. The figure also shows the performance curves for the single antenna at the relay for comparison purposes. Different Nakagami parameters $m = 1, 2, 4, 6$ are shown. A significant increase in the number of times the relay decodes and forwards is observed for the dual-antenna cases. This indicates the increase in the number of times the destination relies on diversity combining. This, however, has to be complemented with improved error performance at the relay.

Strong agreement between the simulated and analytical results for the end-to-end performance of multiple (parallel) relays with a single antenna for different values of the Nakagami parameter, m is presented in Fig. 3. The solid curves represent the analytical results while the symbols denote the simulated performance. The relay decoding threshold is set at $\gamma_{th} \approx \sqrt{2\bar{\gamma}}$ and in all the numerical examples.

Fig. 4 depicts the performance of the multi-antenna relay in symmetrical networks for different numbers of antennas (L) at the relay station. BPSK modulation is used in all the links. The relays utilize MRC to combine the branch signals. The results for the Nakagami parameter m = 1, 2, 6 and L = 1, 2, 4 is indicated. In these curves, it has been assumed that the fading distributions between the tripartite (source to relay, source to destination and relay to source) are the same. The performance of conventional relaying (Fig. 1 (a)) is shown as well.

To evaluate the system performance for non-identical fading distributions scenarios is straightforward. For instance, a fixed relay can be positioned in such a way that it experiences NLOS from the source and LOS to the destination. Since the source would likely have NLOS to destination, the relay-destination channel can be modelled with a suitable m using the Nakagami-m to Ricean K-factor transformation while the source-relay and source-destination channels are both modelled with the Rayleigh distribution ($m = 1$).

Furthermore, Fig. 4 shows that the multi-antenna relay systems yield tremendous gains over the conventional form of relaying. Let us compare the SNR requirements at BER = 10^{-2}. The following gains are obtained over the conventional relaying, 10.5 dB ($L = 1$), 13 dB ($L = 2$), and 14.5 dB ($L = 4$) for $m = 1$ channel condition. In addition, at BER $= 10^{-3}$, the dual antenna case exhibits about 4 dB superiority over the single antenna case. When four antennas are deployed, this gain is about 5 dB. For $m = 2$, however, the gain of dual antennas over the single antenna, is about 1.5 dB while that of four antennas is 2.5 dB. From these cases, it can be deduced that more diversity gain is obtained for multi-antenna system in Rayleigh ($m = 1$) than in less scattering channels ($m > 1$). With dual antennas, it seems that the necessary diversity is already acquired, and that only marginal gains are observed for increasing the number of antenna elements. In this sense, a dual antenna at the relay might be strongly recommended, as the overall diversity order is improved with this number of antenna deployment.

The trends observed in Fig. 4 are generally observed for the multi-antenna relay networks ($N_R = 4$ is shown in Fig. 5).

We will now compare the SC-based detection for different system configurations (Table I). We consider the SNR required for an error rate 10^{-4} for different parameters, L and m. We start off with $\{N_R = 4, L = 1\}$, $\{N_R = 4, L = 2\}$, and $\{N_R = 2, L = 4\}$. In the case of $\{N_R = 4, L = 1\}$ and $\{N_R = 4, L = 2\}$ four RF detection chains are required but $\{N_R = 4, L = 2\}$ requires extra four inexpensive antennas and switching mechanisms. It is observed that in Rayleigh fading environments, a gain as high 3.8 dB is obtained with $\{N_R =$

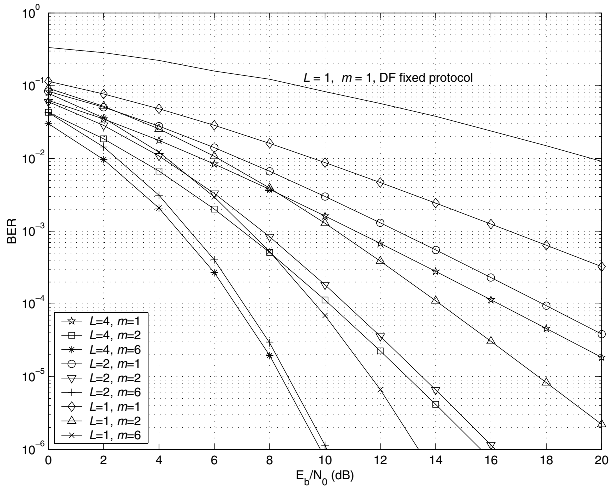

Fig. 4. End-to-end BER performance of MRC-based multi-antenna TDF cooperative relay networks in Nakagami fading for $N_R = 1$.

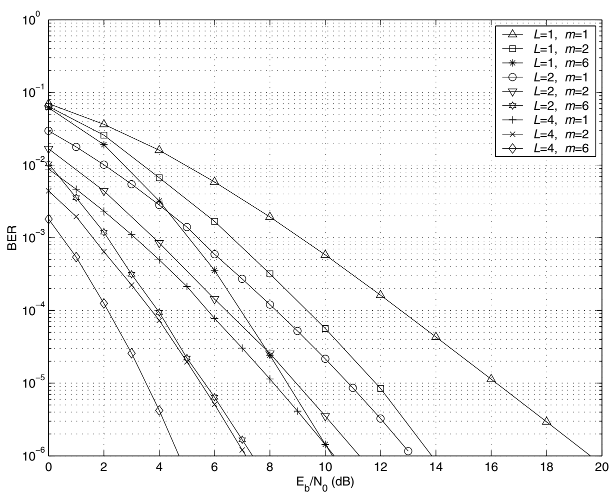

Fig. 5. End-to-end BER performance of MRC-based multi-antenna TDF cooperative relay networks in Nakagami fading for $N_R = 4$.

$4, L = 2\}$ over $\{N_R = 4, L = 1\}$ configuration and this gain drops to 1.6 dB for $m = 2$. When the $\{N_R = 2, L = 4\}$ system configuration is employed in place of $\{N_R = 4, L = 2\}$, we observe that degradation of 1.6 and 0.2 dB, for $m = 1$ and $m = 2$, respectively, are incurred. However, two detection chains are required for $\{N_R = 2, L = 4\}$ as compared to four that are required for $\{N_R = 4, L = 2\}$. This is an indication that deployment of multiple antennas at relays could help to reduce the number of relays required in service area.

Furthermore, Table I reveals that deploying two relays each with two antennas $\{N_R = 2, L = 2\}$ and the relay selects one antenna at a time yields almost the same performance as deploying four relays each with one antenna $\{N_R = 4, L = 1\}$. We reiterate that two detection resources are required in the $\{N_R = 2, L = 2\}$ case as compared to four in $\{N_R = 4, L = 1\}$. Besides, the resources required for deploying extra relays

TABLE I

SC-based Relay Detection: System SNR Comparison at BER = 10^{-4}

	$N_R = 2$		$N_R = 4$	
# of antennas L	$m = 1$	$m = 2$	$m = 1$	$m = 2$
1	18.0 dB	11.5 dB	13.0 dB	9.2 dB
2	13.2 dB	9.2 dB	9.2 dB	7.6 dB
4	10.8 dB	7.8 dB	7.6 dB	6.1 dB

is much higher than that for installing microdiversity antenna elements on relays. Therefore, deploying microdiversity at relays result in considerable savings in the number of relays to be deployed in a given area.

V. CONCLUSIONS

Since space constraints prohibit the use of a large number of antennas at mobile terminals, the future wireless communication systems can exploit the promises associated with multi-antenna techniques by employing multiple relays which are engaged in cooperation, to mimic a large array of antennas. We have demonstrated that, with modest signal processing, multiple relays can be used to enhance the performance of wireless communication networks.

In infrastructure-based fixed relaying, multiple antennas can be deployed. Threshold maximal ratio combining and threshold selection combining techniques to diversity process relay signals are studied and analyzed. From the end-to-end performance it is derived that for a given performance requirement, the multiple antennas at relays can significantly reduce the number of relays required in a network area, which is associated with system deployment cost. Furthermore, threshold selection combining at the relays represents an excellent performance-cost tradeoff.

Appendix

A. SC-based Multi-antenna Relay and Threshold Decode-and-Forward Strategy

We consider SC-based diversity at a relay (to illustrate our derivations) since the basic steps are the same for MRC-based. For brevity, the error probability of the SC-based threshold relay is represented as $P_e^{(\mathrm{r,\,sc})} = (I_1^{(\mathrm{sc})} - I_2^{(\mathrm{sc})})/\upsilon_{\mathrm{sc}}$. Using the PDF [12], $I_1^{(\mathrm{sc})}$ is given as

$$\begin{aligned} I_1^{(\mathrm{sc})} &= q\int_0^\infty \frac{L}{(m-1)!}\sum_{l=0}^{L-1}(-1)^l \left(\begin{array}{c} L-1 \\ l \end{array}\right)\sum_{k=0}^{l(m-1)} b_k^l \\ &\times \left(\frac{m}{\bar{\gamma}}\right)^{m+k}\gamma^{m+k-1}\exp\left(-(l+1)\frac{m\gamma}{\bar{\gamma}}\right)\mathrm{erfc}\left(\sqrt{\gamma\lambda}\right)d\gamma, \\ &= \frac{q\,L}{(m-1)!}\sum_{l=0}^{L-1}(-1)^l\left(\begin{array}{c} L-1 \\ l \end{array}\right)\sum_{k=0}^{l(m-1)} b_k^l\left(\frac{m}{\bar{\gamma}}\right)^{m+k} \\ &\times \int_0^\infty \gamma^{m+k-1}\exp\left(-(l+1)\frac{m\gamma}{\bar{\gamma}}\right)\mathrm{erfc}\left(\sqrt{\gamma\lambda}\right)d\gamma. \end{aligned}$$

After some steps, $I_1^{(\mathrm{sc})}$ can be expressed as

$$I_1^{(\mathrm{sc})} = \frac{q\,L}{(m-1)!}\sum_{l=0}^{L-1}(-1)^l\left(\begin{array}{c}L-1\\ l\end{array}\right)\sum_{k=0}^{l(m-1)} b_k^l\left(\frac{m}{\bar{\gamma}\lambda}\right)^{m+k} \times \frac{\Gamma(1/2+k+m)}{\sqrt{\pi}(k+m)} \times {}_2F_1[k+m,k+m+1/2;k+m+1;\frac{-m(1+l)}{\lambda\bar{\gamma}}] \quad (11)$$

With the help of [9] we can express the hypergeometric function as ${}_2F_1[k+m,k+m+1/2;k+m+1;\frac{-m(1+l)}{\lambda\bar{\gamma}}] = \left(\frac{\lambda\bar{\gamma}+m(1+l)}{\lambda\bar{\gamma}}\right)^{-(k+m)} {}_2F_1[k+m,1/2,k+m+1,\frac{m(1+l)}{m(1+l)+\lambda\bar{\gamma}}]$, thereafter employing [9, pp. 960, (8.391)] to arrive at a simplified final expression

$$I_1^{(\mathrm{sc})} = \frac{q\,L}{\sqrt{\pi}(m-1)!}\sum_{l=0}^{L-1}(-1)^l\left(\begin{array}{c}L-1\\ l\end{array}\right) \times \sum_{k=0}^{l(m-1)} b_k^l\frac{\Gamma(k+m+1/2)}{(1+l)^{k+m}}B_y\left[k+m,1/2\right], \quad (12)$$

where $y=\frac{m\,(1+l)}{\lambda\,\bar{\gamma}+m\,(1+l)}$, $b_0^l=1$, $b_1^l=l$, $b_{l(m-1)}^l=\frac{1}{((m-1)!)^l}$, and $b_k^l=\frac{1}{k}\sum_{j=1}^{\min[k,m-1]}\frac{j(l+1)-k}{j!}b_{k-j}^l$ are recursively computed with $k=2,3,\cdots,l(m-1)-1$.

Finally, we pursue the derivation of $I_2^{(sc)}$ and it has been obtained as

$$\begin{aligned} I_2^{(\mathrm{sc})} &= q\int_0^{\gamma_{th}}\frac{L}{(m-1)!}\sum_{l=0}^{L-1}(-1)^l\left(\begin{array}{c}L-1\\ l\end{array}\right)\sum_{k=0}^{l(m-1)} b_k^l \\ &\times \left(\frac{m}{\bar{\gamma}}\right)^{m+k}\gamma^{m+k-1}\exp\left(-(l+1)\frac{m\gamma}{\bar{\gamma}}\right)\mathrm{erfc}\left(\sqrt{\gamma\lambda}\right)d\gamma, \\ &\approx \frac{q\,L}{(m-1)!}\sum_{l=0}^{L-1}(-1)^l\left(\begin{array}{c}L-1\\ l\end{array}\right)\sum_{k=0}^{l(m-1)} b_k^l\left(\frac{m}{\bar{\gamma}}\right)^{m+k} \\ &\times \frac{\Gamma(m+k-1/2)-\Gamma[m+k-1/2,\lambda\gamma_{th}+\frac{m(l+1)\gamma_{th}}{\bar{\gamma}}]}{\sqrt{\pi}\lambda\,(\lambda+\frac{m(l+1)}{\bar{\gamma}})^{m+k-1/2}}. \end{aligned} \quad (13)$$

Equation (13) represents an approximation which is only tight at high SNR. Therefore, we need to find expression for the low SNR regime. This expression is obtained by going through the following steps:

$$\begin{aligned} I_2^{(\mathrm{sc})} &= q\int_0^{\gamma_{th}}\frac{L}{(m-1)!}\sum_{l=0}^{L-1}(-1)^l\left(\begin{array}{c}L-1\\ l\end{array}\right) \\ &\times \sum_{k=0}^{l(m-1)} b_k^l\left(\frac{m}{\bar{\gamma}}\right)^{m+k}\gamma^{m+k-1} \\ &\times \exp\left(-(l+1)\frac{m\gamma}{\bar{\gamma}}\right)\mathrm{erfc}\left(\sqrt{\gamma\lambda}\right)d\gamma, \\ &= \frac{q\,L}{(m-1)!}\sum_{l=0}^{L-1}(-1)^l\left(\begin{array}{c}L-1\\ l\end{array}\right) \\ &\times \sum_{k=0}^{l(m-1)} b_k^l\left(\frac{1}{1+l}\right)^{(k+m)}(\Gamma[k+m] \\ &- \Gamma[k+m,\frac{m(1+l)\gamma_{th}}{\bar{\gamma}}]-\frac{4h}{\sqrt{\pi}}\sum_{p=0}^{\infty}\frac{(-1)^p}{p!\,(2p+1)}\left(\frac{m}{\lambda\bar{\gamma}}\right)^{-(p+1/2)} \\ &\times \frac{\Gamma[k+m+p+1/2]-\Gamma[k+m+p+1/2,m(1+l)\gamma_{th}/\bar{\gamma}]}{(1+l)^{p+1/2}}), \end{aligned} \quad (14)$$

which should be utilized in the low SNR regime. The analysis for SC-based TDF is concluded with the evaluation of υ_{sc} as, $\upsilon_{\mathrm{sc}}=\frac{L}{(m-1)!}\sum_{l=0}^{L-1}(-1)^l C_l^{L-1}\sum_{k=0}^{l(m-1)} b_k^l\left(\frac{1}{1+l}\right)^{k+m}\Gamma[k+m,(1+l)\frac{m\gamma_{th}}{\bar{\gamma}}]$.

REFERENCES

[1] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior", *IEEE Trans. on Inform. Theory*, 50(11), pp. 3062-3080, December 2004.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part I: System description", *IEEE Trans. on Commun.*, 51(11), pp. 1927-1938, November 2003.

[3] R. Pabst, B. Walke, D. Schultz, P. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. Falconer, and G. Fettweis, "Relay-based deployment concepts for wireless and mobile broadband radio", *IEEE Commun. Magazine*, 42(9), pp. 80-89, September 2004.

[4] H. Hu, H. Yanikomeroglu, D. Falconer, and S. Periyalwar, "Range extension without capacity penalty in cellular networks with digital fixed relays", *IEEE Globecom*, 2004.

[5] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas", *Wireless Personal Commun.*, pp. 311-335, 1998.

[6] A. Adinoyi and H. Yanikomeroglu, "Cooperative relaying in multi-antenna fixed relay networks", submitted to *IEEE Trans. on Wireless Commun.*, March 2005.

[7] R. Braun and U. Dersch, "A physical mobile radio channel model", *IEEE Trans. on Vehic. Technol.*, 40(2), pp. 472-482, May 1991.

[8] P. Herhold, E. Zimmermann, and G. Fettweis, "A simple cooperation extension to wireless relaying", *Int'l Zurich Seminar on Commun.*, February 2004.

[9] I. S. Gradshteyn and I. M Ryzhik, Table of Integrals, Series, and Products, San Diego, CA: Academic Press, 1994.

[10] M. Simon, S. Hinedi, and W. Lindsey, Digital Communication Techniques- Signal Design and Detection, Englewood Cliffs, NJ: PTR Prentice Hall, 1995.

[11] A. Papoulis, Probability, Random Variables, and Stochastic Processes, New York: McGraw-Hill, 1991.

[12] G. Fedele, "Error probability for diversity detection of binary signals over Nakagami fading channels," *IEEE Int'l Conf. on Personal Indoor Radio and Mobile Commun. (PIMRC'94),* pp. 609-611, 1994.