On the Asymptotic Analysis of Average Interference Power Generated by a Wireless Sensor Network

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Abstract—**Massive deployments of Wireless Sensor Networks (WSNs) are expected in near future. In one of the most likely scenarios, these WSNs would share a licensed frequency band with a primary user. So, it is essential to understand the behavior of the interference generated by a WSN towards the primary user. This paper provides an asymptotic analysis of the average interference power generated by a WSN. The analysis is extended to a special but important shape of a sensor field. This shape can be used to provide an upper bound of the average interference power generated by any sensor field with an arbitrary shape. The paper shows that the expansion of the sensor field does not necessarily cause an increase in the average interference power. For most practical values of path loss exponent, the average interference power asymptotically approaches constant levels with the increase in the field size provided that the minimum distance from the field to the primary user is fixed. The paper provides expressions for these constants. Moreover, results indicate that a key parameter in determining the average interference power is the ratio of the radial depth of the field to the minimum distance from the field to the primary user. Also, this paper illustrates how a WSN can be equivalently represented by a single virtual node producing the same level of average interference power.**

Keywords- wireless sensor network; interference; spectrum sharing

I. INTRODUCTION

Massive deployments of wireless sensor networks (WSNs) are expected in near future [1]. These networks require a frequency band to operate in. Since it is not a viable solution to acquire a spectrum license for a WSN due to the high cost associated with it, a WSN is likely to share a frequency band with other systems. Current WSN implementations share unlicensed frequency bands with other unlicensed systems, e.g., WiFi and Bluetooth [2], [3]. FCC Spectrum Policy Task Force proposed that licensed frequency bands could be shared between licensed users (primary users) and unlicensed users (secondary users) [4]. This spectrum sharing will be under the condition that the performance of primary users' communication is not degraded.

The spectrum sharing proposal would create many opportunities for unlicensed systems like WSNs and lead to an efficient use of this invaluable resource, i.e., RF spectrum. Some measurements were conducted for the bands below 3 GHz at six locations including the New York City [5]. The measurements' results show that the maximum frequency occupancy was 13.1% which was measured in New York City. The average of the frequency occupancies for the six locations was 5.2%.

Other advances in wireless communications would empower secondary users to dynamically share a frequency band with other primary users. The secondary users with a technology such as Cognitive Radio would be able to sense the environment around them and identify spectrum holes that can be used without affecting the performance of primary users [6]. A spectrum hole is defined in [6] as a frequency band that is underutilized by the primary user. A Cognitive Radio could be implemented for WSNs, however on the expense of node complexity [7]. We envision that these complexity issues could be resolved by future advancement in hardware technologies.

If a WSN shares a licensed frequency band with a primary user, the requirement is that the WSN transmission does not create harmful interference towards the primary user. The total interference power that a WSN generates towards a primary user is the sum of all the power coming from each transmitting node in that WSN. This problem is similar in some aspects to co-channel interference in cellular networks [8]. The power that a primary user receives from a single interfering node has three components: distance-dependent attenuation, shadowing, and multipath fading. In the context of interference, the multipath fading is of less significance compared to the shadowing component [9]. In our analysis, we model the interference power generated by a single node towards the primary user with a distance-dependent attenuation and shadowing components. Therefore, this received power can be modeled by a lognormal random variable [9]. Based on this, the total power that a primary user receives from all transmitting nodes in WSN is a sum of lognormal random variables. Finding the distribution of a sum of lognormal variables is a well-studied problem, however, there is not a closed-form solution for it. Instead, in many sources, the sum of lognormal random variables is approximated by another lognormal random variable in many works (see [10] and references therein.)

Applying these results to the context of the interference generated by a WSN is considered in [11] and [12]. The authors in [11] focus on finding a more accurate model for the sum of a large number of correlated lognormal random variables. They highlight the applicability of their model to the case of WSNs. The focus in [12] is on finding the

This work was supported by Saudi Aramco, Dhahran, Saudi Arabia.

distribution of interference power coming from simultaneously transmitting nodes in a WSN towards another node in the same WSN. The authors in [12] do not propose a new model but rather apply some of the previous models to the context of WSN.

To the best of our knowledge, no work has been devoted to study the effect of the sensor field dimensions on the total interference power generated by a WSN towards another system. The focus of the present paper is on the behavior of the average interference power of WSN towards a primary user with respect to the changes in the field size. The mathematical tractability of average interference power allows us to obtain some important insights. Moreover, in some applications or engineering decisions the probability density function of interference may not be needed, instead the average interference power may simply be sufficient. The average interference power could also be used to make a conservative decision that might eliminate the need for finding the distribution function of the interference.

The rest of the paper is organized as follows: Section II describes the system model. Section III provides the initial formulation for the average interference power generated by a WSN towards a primary user. Section IV extends the formulation developed in Section III to a sensor field with the shape of an annular sector. This section, IV, also discusses how the average interference power would change with the expansion of the sensor field. Finally, Section V summarizes the main points about the behavior of the average interference power generated by a WSN towards a primary user.

II. SYSTEM MODEL

The system model used in this paper is shown in Fig. 1. This model assumes that there is a WSN deployed over an area of A_n. The area that is covered by the WSN might be called the WSN field [2] or simply the sensor field. The WSN is assumed to be operating in an outdoor environment. Point X represents the location of a primary user operating in the same frequency band. The distance between the closest point of the sensor field to point X is denoted by r_o. This distance is assumed to be fixed. The radial depth of the sensor field is denoted by L. The sensor field is spread over an angle of θ_n as seen in point X. The primary user at point X has an antenna gain of G_X.

The number of nodes transmitting simultaneously in a unit area represents the density of the interfering (active) nodes, denoted by D_n. The interfering nodes are assumed to be uniformly distributed over the sensor field. Each node has a transmitting power of P_n and an antenna gain of G_n.

The formulation of interference power in this paper is built on a propagation model that incorporates the effect of distance-dependent attenuation and shadowing effect ([12] and [13]). The shadowing effect is modeled as a lognormal random variable in the form of $10^{Y/10}$ where Y is a Gaussian random variable with zero mean and a standard deviation σ_Y. For the sake of convenience, the shadowing model would be written as $e^Z = 10^{Y/10}$ where $Z = (\ln 10/10)Y$ and $\sigma_Z = (\ln 10/10)\sigma_Y$. The shadowing parameters are included in our analysis to make the analysis results ready for future work. For example, the average interference power expressions provided here could be used to find the distribution function of the total interference power through the use of moment matching or other related techniques.

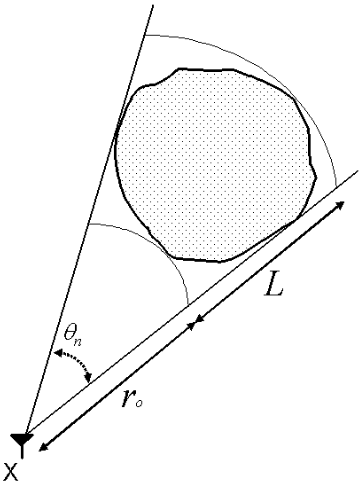

Fig. 1. System Model.

III. INTERFERENCE GENERATED BY A WSN

The objective of this section is to develop a mathematical formulation of the average interference power generated by a sensor field towards a primary user at point X. Let us assume that there is a wireless sensor node at a distance r_i from point X. The average interference power that would reach point X from this node would be

$$P_X = P_n G_n G_X \left(\frac{\lambda}{4\pi d_o}\right)^2 d_o^m \left(\frac{1}{r_i}\right)^m e^{Z_i}, \quad (1)$$

where d_0 is a reference close-in distance, and m is the path loss exponent [14]. We use the propagation model shown in (1), however, the use of any exponential path loss model would lead to similar results. Then, the total interference power can be written as

$$P_X = \sum_i P_n G_n G_X \left(\frac{\lambda}{4\pi d_o}\right)^2 d_o^m \left(\frac{1}{r_i}\right)^m e^{Z_i}. \quad (2)$$

So, the mean value of P_X is

$$E[P_X] = E\left[\sum_i P_n G_n G_X \left(\frac{\lambda}{4\pi d_o}\right)^2 d_o^m \left(\frac{1}{r_i}\right)^m e^{Z_i}\right]. \quad (3)$$

Since Z_i and r_i are independent random variables, (3) can be rewritten as

$$E[P_X] = P_n G_n G_X \left(\frac{\lambda}{4\pi d_o}\right)^2 d_o^m \times \sum_i E\left[\left(\frac{1}{r_i}\right)^m\right] E\left[e^{Z_i}\right]. \quad (4)$$

By assuming that all Z_i have the same distribution with $\sigma_{Zi=}\ \sigma_Z$ (similar assumption in [7]), (4) can be rewritten as

$$E[P_X] = P_n G_n G_X \left(\frac{\lambda}{4\pi d_o} \right)^2 d_o^m \times e^{\sigma_Z^2/2} \sum_i E\left[\left(\frac{1}{r_i} \right)^m \right]. \tag{5}$$

Writing E[P_X] in terms of the average interference power coming from a node at a distance r_o results into

$$E[P_X] = I(r_o)\, r_o^m \sum_i E\left[\left(\frac{1}{r_i} \right)^m \right] = KI(r_o)\,, \tag{6}$$

where $I(r_o)$ is the average interference power experienced at X from a node r_o meters away, and K is a scaling factor which absorbs the spatial distribution of interfering nodes and distance-dependent attenuation.

If we take an infinitesimal area dA centered at a distance of r from point X, the interference contributed by this small area would be $I(r_o)$ scaled by

$$dK = r_o^m E\left[\left(\frac{1}{r_i} \right)^m \right] D_n dA\,, \tag{7}$$

where $D_n dA$ is the number of interfering nodes in dA. In polar coordinates, dA is equal to $rdrd\theta$. Assuming that r_i is uniformly distributed over dr,

$$E\left[\left(\frac{1}{r_i} \right)^m \right] = \left(\frac{1}{r} \right)^m. \tag{8}$$

So, dK is

$$dK = \left(\frac{r_o}{r} \right)^m D_n rdrd\theta. \tag{9}$$

Integrating dK over A_n gives

$$K = \iint_{A_n} \left(\frac{r_o}{r} \right)^m D_n rdrd\theta. \tag{10}$$

By using (6) and (10), the whole field can be represented by a single virtual node that would generate an equivalent level of average interference power at point X. Assuming that the virtual node is at a distance r_o away from point X, its power is

$$P_V = KP_n\,, \tag{11}$$

where P_V is the average power of the virtual node and P_n is the average power of a single sensor node (see Fig. 2).

In order to examine the behavior of the average interference power with respect to the changes in the field size, a simple but important shape for the sensor field is considered. This shape is an annular sector shape as shown in Fig. 3. This shape can be used to provide a conservative approximation (upper bound) of the interference power generated by any sensor field with an arbitrary shape.

In the next section, a formulation of K will be derived for a field of an annular sector shape.

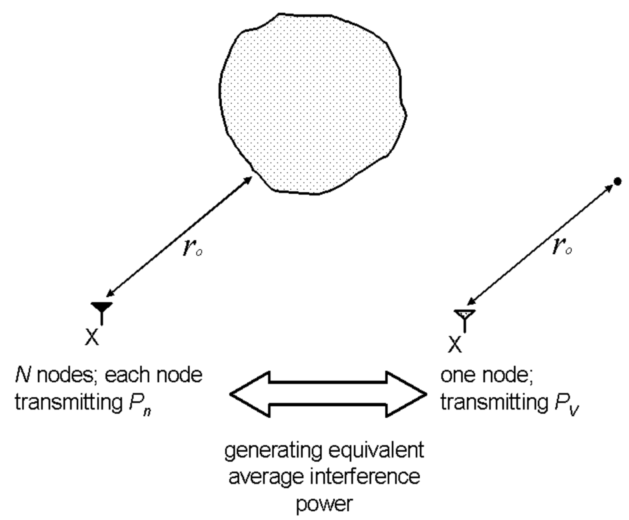

Fig. 2. Representing a sensor field by a single virtual node generating equivalent level of average interference power.

IV. Sensor Field of Annular Sector Shape

In this section, we work with a sensor field with the shape of an annular sector as shown in Fig. 3.

Based on (10), the formulation of K for this shape can be written as

$$K = D_N \int_0^{\theta_n} \int_{r_o}^{r_o+L} \left(\frac{r_o}{r} \right)^m rdrd\theta, \tag{12}$$

which leads to

$$K = \begin{cases} D_n \theta_n r_o^2 \ln\left(1 + \frac{L}{r_o}\right); & m = 2 \\ D_n \theta_n r_o^2 \frac{1}{m-2} \left[1 - \frac{1}{\left(1 + \frac{L}{r_o}\right)^{m-2}} \right]; & m > 2. \end{cases} \tag{13}$$

Plots of K for different values of m are shown in Fig. 4. This figure highlights the behavior of K and, hence, the average interference power with respect to the changes in L/r_o. In this paper, r_o is assumed to be fixed, thus, the changes in L/r_o are due to the changes in L. Equation (14) shows that the asymptotic values of K when $L \ll r_o$, and when $L \gg r_o$.

$$K \approx \begin{cases} D_n \theta_n r_o L; & m \geq 2 \text{ and } L \ll r_o \\ \left.\begin{array}{ll} D_n \theta_n r_o^2 \ln\left(\frac{L}{r_o}\right); & m = 2 \\ D_n \theta_n r_o^2 \frac{1}{m-2}; & m > 2 \end{array}\right\} & \text{for } L \gg r_o. \end{cases} \tag{14}$$

From (13), (14) and Fig. 4, the following observations can be made:

- The maximum value of K for an unbounded sensor field is limited by

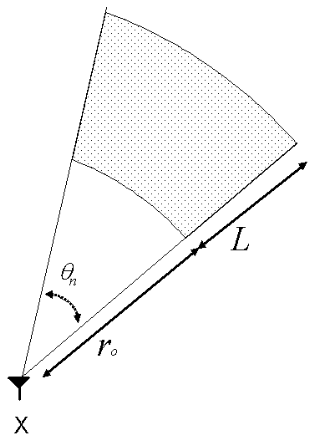

Fig. 3. A sensor field with the shape of an annular sector.

$$K = \begin{cases} D_n \theta_n r_o^2 \ln\left(\dfrac{L}{r_o}\right); & m = 2 \text{ and } L >> r_o \\ D_n \theta_n r_o^2 \dfrac{1}{m-2}; & m > 2 \text{ and } L >> r_o, \end{cases} \qquad (15)$$

- For m=2, the value of K and, hence, the average interference power increases logarithmically with the increase in L.
- For m>2, the value of K and, hence, the average interference power asymptotically approaches a constant with the increase in L. The asymptotic constant decreases with a factor of *(1/m-2)* as m increases. In [12], a relevant observation is made for m=3 and m=6 based on simulation and numerical results. It is highlighted in the paper that an increase in the network size of a WSN does not necessarily lead to an increase in the total interference power. However, the paper does not provide expressions for the average interference power or the asymptotic constants. In the present paper, we provide exact expressions describing the behavior of the average interference generated by a sensor field towards a primary user. Expressions for the asymptotic constants for the case of m>2 are also provided. Moreover, we show that the behavior of the average interference power with the changes in L is different when m=2.
- For $L<<r_o$, the value of K and, hence, the average interference power depends linearly on L.

The value of K can be interpreted as the number of active nodes in an effective area (A_{eff}) within the sensor field. From (13) and $K=D_n A_{eff}$, we can write A_{eff} as

$$A_{eff} = \begin{cases} \theta_n r_o^2 \ln\left(1+\dfrac{L}{r_o}\right); & m = 2 \\ \theta_n r_o^2 \dfrac{1}{m-2}\left[1-\dfrac{1}{\left(1+\dfrac{L}{r_o}\right)^{m-2}}\right]; & m > 2. \end{cases} \qquad (16)$$

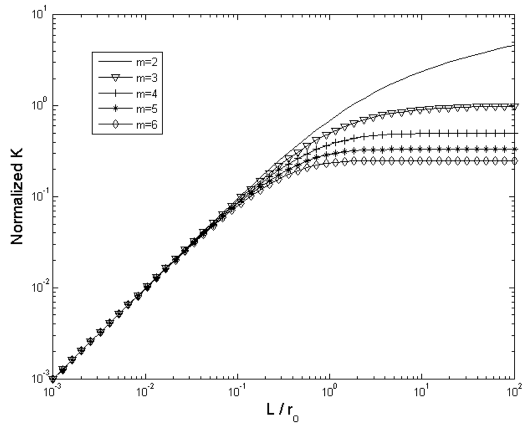

Fig. 4. Normalized K ($K/D_n\theta_n r_o^2$) vs. L/r_o for different values of m.

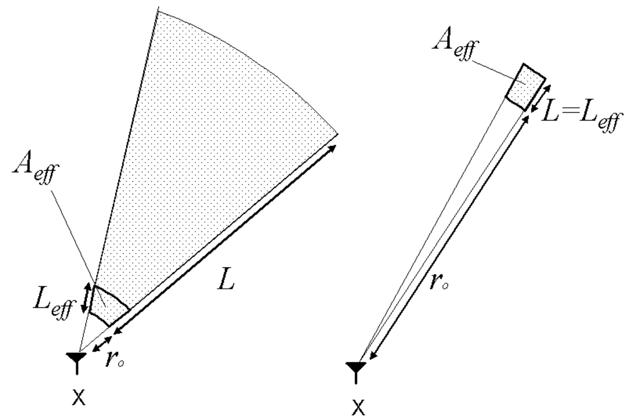

Fig.5. Sensor fields with $L>>r_o$ and $L<<r_o$.

When $L<<r_o$, the A_{eff} is equivalent to the total area of the sensor field. On the other hand, when $L>> r_o$, the value of A_{eff} is much less than the total area of the sensor field. For m>2 and $L>> r_o$, A_{eff} is limited by a maximum value regardless of how big the sensor field is. These remarks are deduced from (17) and represented graphically in Fig. 5.

$$A_{eff} \approx \begin{cases} \left\{\theta_n r_o L; \quad m \geq 2\right\} \text{ for } L << r_o \\ \left.\begin{matrix} \theta_n r_o^2 \ln\left(\dfrac{L}{r_o}\right); & m = 2 \\ \theta_n r_o^2 \dfrac{1}{m-2}; & m > 2 \end{matrix}\right\} \text{ for } L >> r_o. \end{cases} \qquad (17)$$

The effective depth (L_{eff}) of the sensor field corresponding to A_{eff} can be approximated for the asymptotic cases as

$$L_{eff} \approx \begin{Bmatrix} \left\{L; \quad m \geq 2\right\} \text{ for } L << r_o \\ \left.\begin{matrix} r_o\left[\sqrt{1+2\ln\left(\dfrac{L}{r_o}\right)}-1\right]; & m = 2 \\ r_o\left[\sqrt{1+\dfrac{2}{m-2}}-1\right]; & m > 2 \end{matrix}\right\} \text{ for } L >> r_o \end{Bmatrix}. \qquad (18)$$

For example, if there is a sensor field with an L=10r_o and m=3. In this case, A_{eff} would be 1/60 of the total area of the sensor field and the value of L_{eff} would be around 0.7r_o.

In summary, to find the average interference power of a sensor field towards a primary user at point X, the following needs to be done: 1) find the average interference power generated by a single node at a distance r_o from point X, and 2) multiply that average by D_nA_{eff} which represents the number of interfering nodes in the effective area, A_{eff}. The value of A_{eff} can be calculated from (16).

V. CONCLUSIONS

In this paper, we provide an asymptotic analysis of the average interference power generated by a wireless sensor field towards a primary user sharing the same frequency band. Applying this analysis to a special but important shape of the wireless sensor field produces closed-form formulas. These formulas provide further insights into the behavior of the average interference power of a sensor field towards a primary user. These insights are general for any propagation model that is based on an exponential decay of power with distance. Moreover, the analysis and results presented in this paper can also be applied to the case when a WSN shares an unlicensed frequency band with another unlicensed user.

The following conclusions are reached on the basis of the present study:

- An expansion of the sensor field does not necessarily cause an increase in the average interference power. For most practical values of path loss exponent, the average interference power asymptotically approaches constant levels with the increase in the field size provided that the minimum distance from the field to the primary user is fixed. The paper provides expressions for these constants.
- The ratio of the radial depth of the sensor field to the minimum distance between the field and the primary user is a very important parameter in determining the total average interference power. If this ratio is much smaller than 1, then the average interference power changes linearly with the changes in the radial depth provided that the minimum distance is fixed. On the other hand, if the ratio is much larger than 1, then the average interference power is upper bounded by a constant regardless of the value of the radial depth. An exception to this is when the path loss exponent is 2. In this case, i.e., when the path loss exponent is equal to 2, the average interface power changes logarithmically with the changes in the radial depth.
- The annular sector shape presented in this paper can be used to provide a conservative approximation (an upper bound) for the average interference power of any sensor field with an arbitrary shape.
- This paper also shows how the sensor field can be equivalently represented by a single virtual node producing the same level of the total average interference power.

ACKNOWLEDGEMENT

The authors would like to thank Sebastian Szyszkowicz (Carleton University, Canada) for his valuable comments on the work in this paper.

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