On the Performance of Selection Relaying

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***Abstract*—Interest in selection relaying is growing. The recent developments in this area have largely focused on information theoretic analyses such as outage performance. Some of these analyses are accurate only at high SNR regimes. In this paper error rate analyses that are sufficiently accurate over a wide range of SNR regimes are provided. The motivations for this work are that practical systems operate at far lower SNR values than those supported by the high SNR analysis. To enable designers to make informed decisions regarding network design and deployment, it is imperative that system performance is evaluated with a reasonable degree of accuracy over the practical SNR regimes. Simulations have been used to corroborate the analytical results, as close agreement between the two is observed.**

***Index Terms*—Selection relaying, two-hop, diversity gains.**

I. INTRODUCTION AND MOTIVATION

Selection diversity is a fundamental technique that can be transferred over from traditional multiple antenna systems to cooperative relaying systems. As add-on features to network relays, cooperative techniques should not impose strict limitations or require sophisticated hardware. This view of cooperation in relay networks is informed by the fact that future wireless communication standards will be relay-enhanced. The activities in IEEE 802.16 j/m attest to the vital role relays would be playing in future broadband communication networks. Basically, the aim of deploying these relays is to break the link between two communicating nodes into smaller ones (links) with the objective of enabling high data rates and coverage extension. Our view on relay cooperation is that cooperation comes as an add-on feature, which attempts to extract additional benefits on top of this primary objective.

Interest in various forms of cooperative schemes has been growing steadily since the seminal work in cooperation diversity in [1] and [2]. In these works, it is shown that by re-examining the manner in which network nodes relate to each other can provide a cost-effective way of harnessing the advantages (such as diversity gains) of multi-antenna systems without necessarily putting these antennas in one location. This new paradigm has been known in the literature as cooperative relaying or user cooperation diversity schemes, and they are particularly attractive for small-size, antenna-limited wireless devices.

However, in parallel cooperative relaying where each relay is equipped with a single antenna, the potential diversity gains in the network will be destroyed if any of the relays attempts to fully decode its received signal; the performance will be bounded by the single antenna system [3]. Selective relaying techniques are often employed to overcome this problem. In particular the work in [4] discusses selective relaying in the context of single relay that has knowledge of the channel states, helping a source. The present paper considers selection relaying that involves upper layers of the communication protocol. In other words, it is an overlay technique on the routing mechanisms.

The selection relaying schemes analyzed in this work are closely related to those in [5], [6] with the following differences: Here, we provide analysis for the error rate while these earlier works focused on information theoretic analyses. The analysis performed in [5] is for large SNR regime. The implications of always combining the relay-destination path with the source-destination path are not apparent given the conclusions on parallel relays found in [3]. Should the selected relay always cooperate with the source? Our analyzes show that combining the relay path with the source-destination path also provides full diversity. In addition, we show that selecting one path considering the direct path as a virtual relay path also provide full diversity.

Finally, complementary to the outage analyses in [5] and [6], we provide expressions for evaluating the outage and capacity of the selective relaying schemes. Our expressions are accurate in both the low and high SNR regimes. The motivation for the low or medium SNR analysis is the following. It is noted that large SNR analysis has theoretical merits, however, practical systems often operate at lower SNR values. Thus, it is desirable to be able to evaluate system performance to a reasonable degree of accuracy at low SNRs region for the purpose of network design and deployments.

II. SYSTEM MODEL

The system investigated in this paper is shown in Fig. 1. The source, destination, and relays are denoted as S, D, and $\mathrm{R}_r \in \{1, \cdots, N_R\}$, respectively. Each node is equipped with a single antenna. The best relay (produced by some routing schemes) assists S-D communication. A block fading channel model is assumed, where the channel does not change in the block. However, transmitted blocks experience different channel samples that are independently and identically distributed

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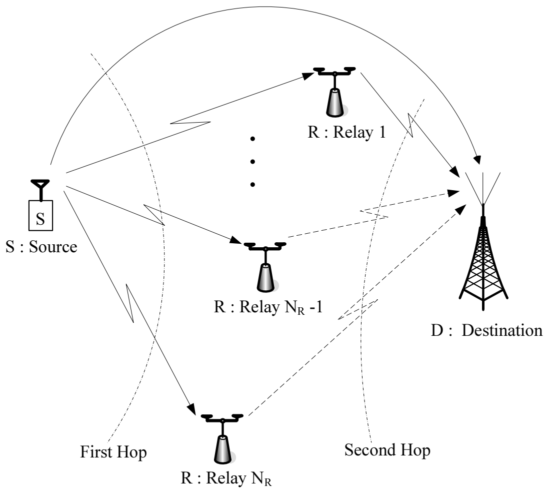

Fig. 1. The multi-relay networks with selective relaying techniques.

Rayleigh random variables. The parameters γ_0 denotes the instantaneous SNRs of S-D, while γ_1 represents the $\min$ (S-R and R-D) for the best relay. The Rayleigh fading model implies that γ_0 is exponential random variable. We let the expected value equal to $\bar{\gamma}_0$. The statistics of γ_1 is found below.

In the half-duplex two-hop protocol, S may or may not be able to communicate directly with the destination. The inability of the source to reach the destination may be due to heavy shadowing. In the first hop, the source transmits while all the N_R listen. In the second hop, the best relay based on the metric that depends on S-R and R-D channels is selected to forward to the destination. The protocol assumes that the network has the mechanisms to select this best relay. To that effect we state that a number of algorithms have been proposed in the literature for performing this task [6]. Thus, the objective of this paper is not to revisit path selection algorithms, but to focus on the analysis of selective relaying schemes. In the analyses, the destination utilizes the signal received through the relay according to selective relaying, SR (the S-D path is not usable at the destination), selective cooperative relaying, SCR (the S-D is utilized at the destination) and all-path selection relaying, ASR (S-D is among the path selection mechanism).

As mentioned above the implementation issues relating to the joint selection relaying schemes have been previously discussed, it is observed that selecting the best relay can be performed at radio network controller, which comes at the expense of increased system overheads. To reduce the overhead and system complexity, [5] considered a simplified relay selection based only on the R-D channel.

III. PERFORMANCE ANALYSIS

A. Probability of Bit Error Calculations: Selective Cooperative Relaying (SCR)

The relay selection, more accurately, the path selection is based on backward and forward channels which is performed jointly. This means we select the relay r^* with $\gamma = \max_r \min(\gamma_{s,r}, \gamma_{r,d})$, where $\gamma_{s,r}$ and $\gamma_{r,d}$ are the S-R and R-D instantaneous SNR. The signal transmitted by this selected relay is combined with the direct path signal using maximal ratio combining. Therefore, the combined SNR at the destination is the sum of the two SNRs. Note that the destination uses the weaker of the first and second hop of the selected relay. In terms capacity, this weaker link constitutes the bottleneck as far as the end-to-end performance is concerned.

Let the instantaneous SNR of the direct path be represented as γ_0 with an average $\bar{\gamma}_0$ and the instantaneous SNR of the relay path be denoted as γ_1 with an average $\bar{\gamma}_1$. Using maximal ratio combining, the combined SNR is then given as $\beta = \gamma_0 + \gamma_1$. Informed by the independence of these random variables, the PDF of β can be obtained through the convolution of the PDFs of γ_1 and γ_0, which is expressed as

$$p(\beta) = p_{\gamma_0}(\gamma_0) \otimes p_{\gamma_1}(\gamma_1) \quad (1)$$

$$p(\beta) = \int_0^{\beta} p_{\gamma_0}(\tau)\, p_{\gamma_1}(\beta - \tau)\, d\tau. \quad (2)$$

The selection of the best relay requires ordered statistics. The first step is to obtain the weaker link between the first hop and second hop of each relay node. These weak links are ordered and the one with the largest SNR is selected as the candidate relay to perform detection and forward to the destination. Given the PDF $f(\gamma)$ and CDF $F(\gamma)$ of the underlying Rayleigh distributed random variable, the PDF of such ordered random variables can be obtained [7] [8] as $p(\gamma) = 2N_R\, f(\gamma)\, [1 - F(\gamma))]\, \left[2F(\gamma) - (F(\gamma))^2)\right]^{N_R - 1}$. Therefore, for the Rayleigh faded links, $f(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(\frac{-\gamma}{\bar{\gamma}}\right)$ and $F(\gamma) = 1 - \exp\left(\frac{-\gamma}{\bar{\gamma}}\right)$. Hence, the PDF of γ_1 can be obtained as,

$$p(\gamma_1) = N_R\, \frac{\exp(-\frac{\gamma_1}{\bar{\gamma}_1/2})}{\bar{\gamma}_1/2} \left(1 - \exp\left(-\frac{\gamma_1}{\bar{\gamma}_1/2}\right)\right)^{N_R - 1}, \quad (3)$$

and through binomial expansion, the following PDF can be obtained

$$p(\gamma_1) = \sum_{i=1}^{N_R} (-1)^{i-1} \begin{pmatrix} N_R \\ i \end{pmatrix} \frac{2\,i}{\bar{\gamma}_1} \exp\left(-i\, \frac{2\gamma_1}{\bar{\gamma}_1}\right). \quad (4)$$

Using (4) and the PDF of γ_0 (i.e., $\frac{1}{\bar{\gamma}_0} \exp(-\frac{\gamma_0}{\bar{\gamma}_0})$), we have

$$\begin{aligned} p(\beta) &= \int_0^{\beta} \sum_{i=1}^{N_R} (-1)^{i-1} \begin{pmatrix} N_R \\ i \end{pmatrix} \frac{2\,i}{\bar{\gamma}_1 \bar{\gamma}_0} \\ &\times \exp\left(-\frac{i\,\tau}{\bar{\gamma}_1/2}\right) \exp\left(-\frac{\beta - \tau}{\bar{\gamma}_0}\right) d\tau. \end{aligned} \quad (5)$$

By interchanging the integral and summation, (5) can be expressed as

$$\begin{aligned} p(\beta) &= \sum_{i=1}^{N_R} (-1)^{i-1} \begin{pmatrix} N_R \\ i \end{pmatrix} \frac{2\,i}{\bar{\gamma}_1 \bar{\gamma}_0} \\ &\times \exp\left(-\frac{\beta}{\bar{\gamma}_0}\right) \int_0^{\beta} \exp\left(-\left[\frac{i\,\tau}{\bar{\gamma}_1/2} - \frac{\tau}{\bar{\gamma}_0}\right]\right) d\tau. \end{aligned} \quad (6)$$

Finally,

$$\begin{aligned} p(\beta) &= \sum_{i=1}^{N_R} (-1)^{i-1} \begin{pmatrix} N_R \\ i \end{pmatrix} \frac{2\,i}{2i\bar{\gamma}_0 - \bar{\gamma}_1} \\ &\times \left(\exp\left[-\frac{\beta}{\bar{\gamma}_0} \right] - \exp\left[-\frac{2\,i\,\beta}{\bar{\gamma}_1} \right] \right). \qquad (7) \end{aligned}$$

The PDF obtained in (7) can be employed for evaluating the error performance of this relaying scheme with any modulation technique. However, we will demonstrate the evaluation with BPSK as follows:

$$\begin{aligned} BER_{scr} &= \frac{1}{2} \int_0^{\infty} \operatorname{erfc}\left(\sqrt{\beta}\right) p(\beta) d\beta, \\ &= \frac{1}{2} \int_0^{\infty} \operatorname{erfc}\left(\sqrt{\beta}\right) \sum_{i=1}^{N_R} (-1)^{i-1} \begin{pmatrix} N_R \\ i \end{pmatrix} \\ &\times \frac{2\,i}{2i\bar{\gamma}_0 - \bar{\gamma}_1} \left(\exp\left[-\frac{\beta}{\bar{\gamma}_0} \right] - \exp\left[-\frac{2\,i\,\beta}{\bar{\gamma}_1} \right] \right) d\beta \\ &= \sum_{i=1}^{N_R} (-1)^{i-1} \begin{pmatrix} N_R \\ i \end{pmatrix} \\ &\times \frac{i}{2\,(2i\bar{\gamma}_0 - \bar{\gamma}_1)} \left[\bar{\gamma}_0 B_{z_0}[1, 0.5] - \frac{\bar{\gamma}_1}{2\,i} B_{z_1}[1, 0.5] \right], \end{aligned} \qquad (8)$$

where $z_0 = \frac{1}{\bar{\gamma}_0 + 1}$ and $z_1 = \frac{2\,i}{\bar{\gamma}_1 + 2\,i}$ and $B_x[a, b]$ is the incomplete beta function [9].

Note that γ_1 ($\min(\gamma_{s,r^*}, \gamma_{r^*,d})$) sets the upper-bound on the E2E BER of this selective relaying schemes. However, the numerical examples show that the performance evaluated using these derived expressions are quite tight. The simulation results closely match those obtained using the expressions.

B. Selective Relaying

In this form relaying it is assumed that the direct path is unusable due to deep fade instances or heavy shadowing. Hence, the BER performance can be derived from expression given in (8) by setting $\gamma_0 \to -\infty\, dB$. By doing some manipulations the following error rate expression can be obtained

$$BER_{sr} = \frac{1}{4} \sum_{i=1}^{N_R} (-1)^{i-1} \begin{pmatrix} N_R \\ i \end{pmatrix} B_{z_1}[1, 0.5], \qquad (9)$$

which happens to be a strikingly simple and compact expression.

C. All-Path Selection Cooperating Relaying (ASR)

In this relaying scheme the destination selects one path from the $N_R + 1$ (N_R relay paths and S-D path) for the signal detection. The scheme views the S-D link as a virtual relay path (i.e., the S-R and R-D channels are the same). It then selects a path according to the previous selection scheme. The important distinction from the SCR is that the destination does not need to perform maximal ratio combining. Therefore, the system utilizing the ASR scheme is less complex than that using SCR. Although, full diversity order is obtained the scheme is however, inferior to SCR in terms of coding (or power) gain. The antenna gain advantage of SCR over the ASR is evident by comparing Fig.s 2 and 4. In these two figures, the corresponding curves have the same slope but the curves for SCR have shifts downward (i.e., in the direction of power gain or coding gain).

The ASR combiner follows almost the same principle as the traditional selection combining scheme (of collocated antennas) with the distinction that the broadcast nature of wireless channel is exploited and that the antennas (i.e., the relays) are distributed entities. The combiner in this case can be expressed as $\max\{\min\{\gamma_{S-r^*}, \gamma_{r^*-D}\}, \gamma_{S-D}\}$, where r^* is the best relay. The $\max\min$ formulation is essential to incorporate the fact that the weak link constitutes the bottleneck.

IV. CAPACITY AND OUTAGE PROBABILITY

System capacity and outage probability are information theoretic performance measures. Here, we demonstrate that the analyses in this paper can be extended to calculating these performance measures. The notion of capacity is valid where the channel is ergodic and there are no constraints on the decoding delay on the receiver. These are hardly the case in practical communication systems. The channels do behave in a manner that there are no significant channel variability. In such a slow fading channel condition as is called, there is a non-zero error probability that the channel will be in deep fade. Therefore, it is not possible to send a positive rate through the channel and yet maintain a vanishingly small error probability which explains why in strict sense, the capacity of slow fading channel is zero. It is appropriate in this situation to consider outage probability. We note that outage and capacity are important communication modeling parameters, we therefore provide expressions for evaluating them in the following discussion.

1) Outage probability SCR: An outage is defined as the event where the communication channel does not support a target data rate (see the Appendix for the derivation). In the derivation, we denote $a = 2^{2R} - 1$.

$$\begin{aligned} p_{out,scr} &= \sum_{i=1}^{N_R} (-1)^{i-1} \begin{pmatrix} N_R \\ i \end{pmatrix} \Bigg(1 + \frac{1}{2\,i\bar{\gamma}_0 - \bar{\gamma}_1} \\ &\times \left[\bar{\gamma}_1 \exp\left(-\frac{2\,i\,a}{\bar{\gamma}_1} \right) - 2i\,\bar{\gamma}_0 \exp\left(-\frac{a}{\bar{\gamma}_0} \right) \right] \Bigg), \end{aligned} \qquad (10)$$

where R is the target rate.

2) Capacity of SCR: The ergodic channel capacity is considered. Therefore, averaging the instantaneous channel capacity over the fading distribution has operational meaning. The capacity in bits/s/Hz is given as

$$\begin{aligned} \tilde{C} &= \int_0^{\infty} \frac{1}{2} \log_2(1 + \beta) p(\beta) d\beta \\ &= \sum_{i=1}^{N_R} (-1)^{i-1} \begin{pmatrix} N_R \\ i \end{pmatrix} \\ &\times \left(\frac{2}{(2\,\bar{\gamma}_1 - 4\,i\bar{\gamma}_0) \ln 2} \right) \Bigg[2\,i\bar{\gamma}_0 \exp\left(\frac{1}{\bar{\gamma}_0} \right) \left(E_1\left[\frac{-1}{\bar{\gamma}_0} \right] \right) \end{aligned}$$

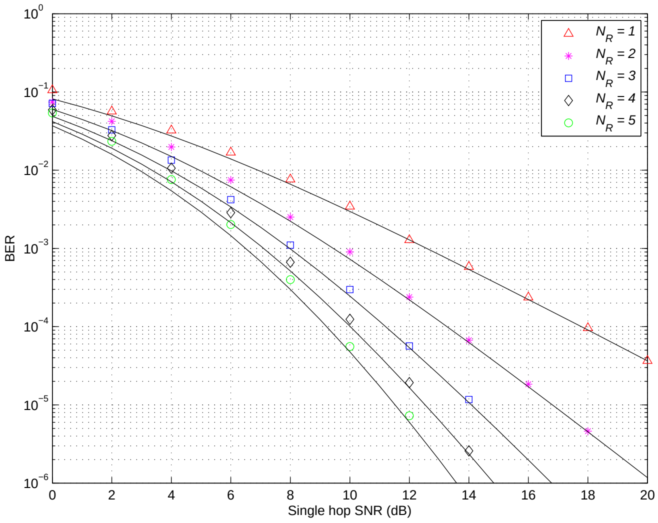

Fig. 2. BER performance of two-hop selective cooperative relaying scheme in Rayleigh fading. The S-D, R-D and S-R have the same average SNR.

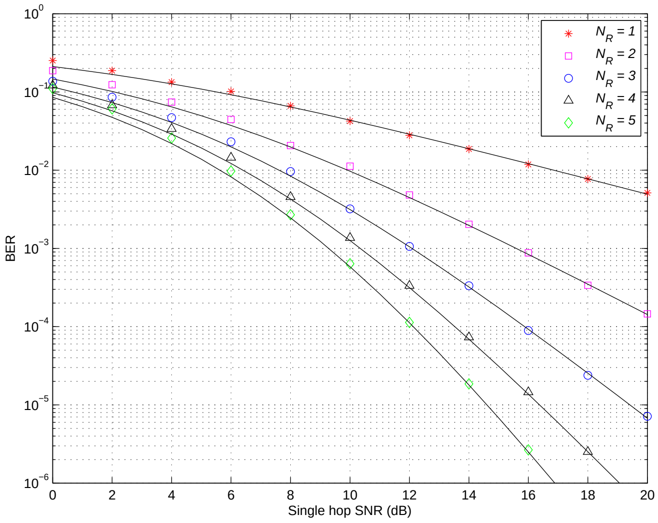

Fig. 3. BER performance of the two-hop selective relaying scheme. The S-D link is not utilized.

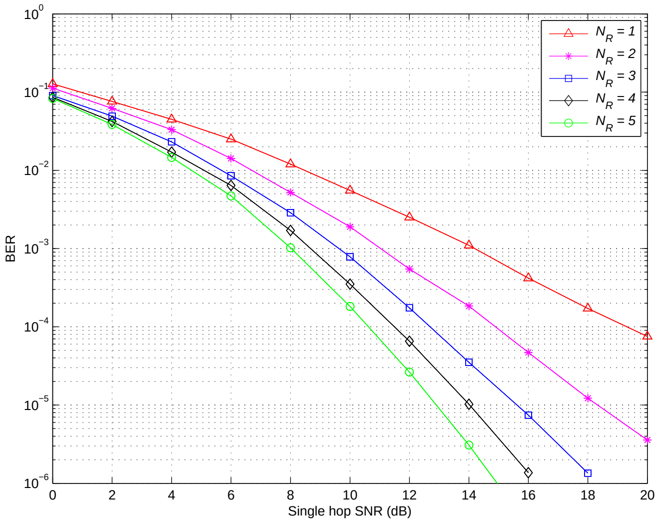

Fig. 4. Simulated BER performance of all-path selection relaying scheme in Rayleigh fading.

$$\begin{aligned} & - \bar{\gamma}_1 \exp\left(\frac{2\,i}{\bar{\gamma}_1}\right)\left(E_1\left[\frac{-2\,i}{\bar{\gamma}_1}\right]\right)\Bigg] \\ = & \sum_{i=1}^{N_R} (-1)^{i-1}\left(\frac{1}{(\bar{\gamma}_1 - 2\,i\bar{\gamma}_0)\ln 2}\right)\binom{N_R}{i} \\ \times & \left[-2\,i\bar{\gamma}_0 \exp\left(\frac{1}{\bar{\gamma}_0}\right) E_1\left[\frac{1}{\bar{\gamma}_0}\right] + \bar{\gamma}_1 \exp\left(\frac{2\,i}{\bar{\gamma}_1}\right) E_1\left[\frac{2\,i}{\bar{\gamma}_1}\right]\right]. \end{aligned} \tag*{(11}$$

where $E_1[\cdot]$ is the exponential integral [9]. This capacity analysis also generalizes the single relay treatment in [10] to arbitrary number of relays.

3) Outage Probability for SR: The outage probability for SR can be expressed as,

$$\begin{aligned} p_{out,sr} = & \sum_{i=1}^{N_R} (-1)^{i-1}\binom{N_R}{i} \\ \times & \left(1 - \exp\left[-\frac{2i(2^{2\,R}-1)}{\bar{\gamma}}\right]\right). \end{aligned} \tag{12}$$

4) Capacity for the SR: The capacity of this scheme can be derived in a similar way as derived for SCR above. The expression is given as

$$\begin{aligned} \tilde{C} = & \sum_{i=1}^{N_R} (-1)^{i-1}\binom{N_R}{i}\frac{1}{\ln 2} \\ \times & \exp\left(\frac{2i}{\bar{\gamma}}\right) E_1\left(\frac{2\,i}{\bar{\gamma}}\right) \quad \text{bits/s/Hz}. \end{aligned} \tag{13}$$

V. NUMERICAL EXAMPLES

Fig.s 2, 3 and 4 show the bit error rate of selective cooperative relaying, selective relaying and all path selection relaying schemes, respectively. The BPSK modulation is used in all the links. The Rayleigh slow fading channel is assumed. In the SCR, the S-D and S-R, and R-D have the same average channel gains. All receiving nodes have the same noise statistics. From Fig.s 2 and 3, simulation results shown in symbols, match closely with the analytical ones in solid curves. From Fig. 2, it can be seen that diversity order equal to $N_R + 1$ is obtained for N_R relay network. This order of diversity can be calculated from the slope of the curves. The same diversity order can be calculated from Fig. 4. However, Fig. 2 presents a superior power gain advantage over Fig. 4.

The derived formulas for capacity are plotted in Fig. 5. The figure compares the capacity of SCR and SR schemes. The capacity for SCR is shown in solid curves and SR in dotted. The advantage of utilizing the direct path is also obvious from this figure, where over 11% increase in capacity is obtained over the capacity of SR scheme. A general observation is that the capacity saturates quickly with the number of relays.

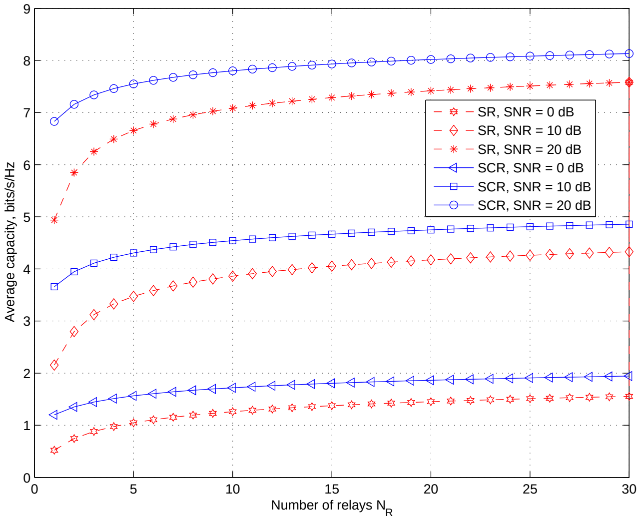

Fig. 5. Average capacity of selective relaying and selective cooperative relaying in Rayleigh fading for different average SNR.

The analyses in this work can be applicable to the systems that employ coding and systems that do not, such as in sensor networks where the node may have only detect and forward capability. In the latter scenario, the error rate is a valid performance criterion while in the former, where coding is used (applicable to decode-and-forward relaying) the outage or capacity is the reasonable performance measure.

VI. CONCLUSION

Most recent studies of selective relaying have focused on information theoretic analyses. Bounds or high SNR regime outage analyses have been presented in most of these studies. In contrast to earlier works, our contributions provide error rate analyses that are reasonably accurate over a large range of SNR regimes, most importantly low/medium SNR region. It is worth noting that practical systems operate at considerably lower SNR values than the SNR range where large SNR analysis is accurate. It is important that network designers are able to evaluate system performance with a reasonable degree of accuracy to help them make accurate decisions on network design and system deployment.

APPENDIX

This section presents the derivation of the outage of the cooperative selective relaying scheme.

$$\begin{aligned} p_{out,scr} &= \Pr\left(I < R\right) \\ &= \Pr\left(\log\left(1+\beta\right) < 2\,R\right) \\ p_{out,scr} &= \int_{-\infty}^{a} \sum_{i=1}^{N_R} (-1)^{i-1} \binom{N_R}{i} \\ &\times \frac{2\,i}{2i\bar{\gamma}_0 - \bar{\gamma}_1} \left(\exp\left[-\frac{\beta}{\bar{\gamma}_0}\right] - \exp\left[-\frac{2\,i\,\beta}{\bar{\gamma}_1}\right] \right) d\,\beta, \end{aligned} \tag{14}$$

where $a = 2^{2\,R} - 1$. Now by interchanging the integral and summation operations, the integration can be performed giving the following:

$$\begin{aligned} &= \sum_{i=1}^{N_R} (-1)^{i-1} \binom{N_R}{i} \frac{2\,i}{2i\bar{\gamma}_0 - \bar{\gamma}_1} \\ &\times \int_{0}^{a} \left(\exp\left[-\frac{\beta}{\bar{\gamma}_0}\right] - \exp\left[-\frac{2\,i\,\beta}{\bar{\gamma}_1}\right] \right) d\,\beta, \\ &= \sum_{i=1}^{N_R} (-1)^{i-1} \binom{N_R}{i} \frac{2\,i}{2i\bar{\gamma}_0 - \bar{\gamma}_1} \left(\bar{\gamma}_0 - \frac{\bar{\gamma}_1}{2\,i} \right. \\ &+ \left. \frac{1}{2\,i} \left[\bar{\gamma}_1 \exp\left[-\frac{2\,ia}{\bar{\gamma}_1}\right] - 2i\bar{\gamma}_0 \exp\left[\frac{-a}{\bar{\gamma}_0}\right] \right] \right). \end{aligned} \tag{15}$$

Finally,

$$\begin{aligned} p_{out,SCR} &= \sum_{i=1}^{N_R} (-1)^{i-1} \binom{N_R}{i} \left(1 \right. \\ &+ \left. \frac{1}{2\,i\bar{\gamma}_0 - \bar{\gamma}_1} \left[\bar{\gamma}_1 \exp\left(-\frac{2\,ia}{\bar{\gamma}_1}\right) - 2i\bar{\gamma}_0 \exp\left(\frac{-a}{\bar{\gamma}_0}\right) \right] \right). \end{aligned} \tag{16}$$

This completes the derivation of the outage for the selective cooperative relaying scheme. The outage for the selective relaying scheme (without the direct path) can be obtained in a similar way through the PDF in (4). It can also be derived from (16). This outage is given as

$$\begin{aligned} p_{out,sr} &= \int_{-\infty}^{2^{2R}-1} p(\gamma)\, d\gamma \\ &= \sum_{i=1}^{N_R} (-1)^{i-1} \binom{N_R}{i} \left(1 - \exp\left[-\frac{2\,ia}{\bar{\gamma}}\right] \right). \end{aligned} \tag{17}$$

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