Max-Min Fair Resource Allocation for Multiuser Amplify-and-Forward Relay Networks

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***Abstract*—We investigate the problem of multi-user radio resource allocation for orthogonal frequency division multiple access (OFDMA) amplify-and-forward (AF) relays. In the single-user case, the problem reduces to the well know assignment problem, which maximizes the user rate. For the multi-user case we devise a resource allocation algorithm to achieve max-min fairness. We find max-min fairness since it can provide almost flat ubiquitous coverage. We start by formulating a convex optimization, which takes a parameter that asymptotically makes the optimization produce max-min fair rates. Since the optimization is a convex problem, we are able to devise a sub-optimal gradient-based algorithm to solve it quickly. Simulations show that the algorithm achieves results very close to the optimum solutions due to its gradient origins.**

***Index Terms*—Max-Min Fairness, Amplify-and-Forward Relays, Resource Allocation, Scheduling, Sub-Channel Pairing.**

I. Introduction and Motivation

Next generation wireless networks aim at providing ubiquitous very high data rate coverage. Since wireless channel impairments and transmit power limitations prevent high spectral efficiency even for moderately long links, it is necessary to consider advanced cost-effective radio access networks (RANs), such as relay networks, empowered with fair efficient radio resource management (RRM) techniques, which effectively collect and distribute wireless signals. We investigate multi-user amplify-and-forward (AF) relays, which forward and multiplex data in orthogonal frequency division multiple access (OFDMA). OFDMA-based AF relays store quantized samples of the symbols until they are amplified and transmitted at a later time. These relays are cost-effective, simpler to implement, and introduce less delay in comparison to other decode-and-forward (DF) relay based routers.

We devise an algorithm that provides max-min fairness among the user rates for OFDMA-based AF relay networks. We start by formulating an optimization, which finds rates with a fairness depending on parameter (γ) input to the optimization. We have previously used this optimization to devise an algorithm to achieve generalized proportional fairness (GPF) [1], also known as γ-fairness [2]. In GPF scheduling, γ parameter can be used to gradually change schedules from throughput optimal to proportionally fair. As $\gamma \rightarrow \infty$, it asymptotically changes the optimization to achieve max-min fairness among the rates [2].

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Since the relaxed version of the optimization is a convex problem, we devise a gradient-based algorithm to work in the asymptotic range of the γ parameter. The algorithm allocates radio resources to users in iterations. In each iteration, rates are allocated in accordance with the gradient of the objective function. Since the algorithm is a sub-optimal version of a full convex optimizer, it is expected that its performance would be close to optimum. Simulations show that the algorithm indeed achieves results very close to the optimum solutions.

Previous works show that scheduling for AF relay networks holds great promise [3]–[6]. However, these works mainly consider the single-user setup. In the single-user setup, the AF scheduling problem becomes an assignment problem [4] which can be solved optimally with pairing the sub-channels based on the ordering of the SNRs [5]. This technique also minimizes BER in a high SNR regime [7]. Extensions taking interference information into account are also possible [5], as well as extensions that include power allocation [8].

AF scheduling in the multi-user setup is not as simple as scheduling in the single-user setup. Complications arise from the need to provide end-to-end fairness among user rates. Distributed scheduling and power allocation for uplink OFDMA relaying is examined in [9] with game theoretic approaches, extending [3] to the case of multiple source nodes that compete for sub-channels. Optimization frameworks can be used to achieve multi-user fairness. Proportional fair resource allocation for OFDMA-based DF relay networks through the objective function is considered in [10]. Another approach is to enforce fairness in the constraints of the problem [11]–[13], however in this case the minimum rates must be known in advance and the their feasibility must be checked by another mechanism. In addition, [11]–[13] do not consider the switching possibility of the first-hop sub-carrier to different sub-carrier in the second-hop which limits the capacity.

II. System and Network Model

We consider OFDMA where orthogonal sub-carriers are grouped in time and frequency as resource blocks (RBs), with duration of T_b seconds and a frequency span of W_b Hertz. There are T RBs in the frame and N available sub-channels to be assigned to M users. In the sequel, we assume that the users are connected to the base-station (BS) through a predetermined relay-station (RS) at any given time. A higher layer process determines which M users are connected to the BS through this RS. Results can be easily extended to the multiple RS scenarios.

The RS is an OFDMA-based AF relay, which multiplexes user data after receiving them from the BS. The received signal is sampled and processed through fast-Fourier transform (FFT) to obtain the received modulation symbols, which are then stored in the RS's buffer. The RS may re-map the RBs from sub-channel to a different sub-channel, before performing the inverse FFT to obtain the output signal, similar to single-user "chunk-based" sub-carrier coupling [4]. Note that if we overlook this switching capability, a good sub-channel may be bottle-necked by a deep faded sub-channel which limits the capacity. RS has at most $T/2$ RBs on each sub-channel, before re-transmitting them to users, since RBs must be assigned in pairs. Multiple users may have RBs on the same sub-channel in the same frame.

The number of bits carried in an RB depends on the adaptive modulation and coding used in the combined transmission over the two hops. We denote the number of bits transmitted in an RB, allocated for user m, on sub-channel i by the BS and retransmitted on channel j by the RS with $b_{ij}^{(m)}$. In the sequel, we call $b_{ij}^{(m)}$ the rate of sub-channel coupling (i, j) for user m, where (i, j) refers to "coupled" transmission from BS to the RS, on sub-channel i, with the transmission from the RS to the user m on sub-channel j.

The end-to-end user rate depends on the RB allocation in the frame

$$r_m = \frac{1}{T_b} \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}^{(m)} x_{ij}^{(m)}, \tag{1}$$

where r_m is the rate of user m in bits-per-second, $x_{ij}^{(m)}$ is the number of RBs assigned to user m on sub-channel coupling (i, j). The RB allocations $x_{ij}^{(m)}$ are the unknowns that the RRM algorithm is searching for. These allocations also implicitly define a schedule, since the order in which users transmit is unnecessary for defining a schedule.

III. Max-min Scheduling for AF Relay Networks

We now formulate the max-min rate sub-channel allocation for OFDMA-based AF relay networks and solve it with a gradient-based algorithm. We start by formulating an optimization, which asymptotically produces max-min fair rates and also has convex relaxed counterpart

$$\max_{x_{ij}^{(m)}} \quad \sum_{m=1}^{M} \frac{1}{1-\gamma} \left(\frac{1}{T_b} \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}^{(m)} x_{ij}^{(m)} \right)^{1-\gamma} \tag{2a}$$

$$\text{s.t.} \quad \sum_{m=1}^{M} \sum_{j=1}^{N} x_{ij}^{(m)} \leq \frac{T}{2}, \quad 1 \leq i \leq N, \tag{2b}$$

$$\sum_{m=1}^{M} \sum_{i=1}^{N} x_{ij}^{(m)} \leq \frac{T}{2}, \quad 1 \leq j \leq N, \tag{2c}$$

$$x_{ij}^{(m)} \in \left\{ 0, \dots, \frac{T}{2} \right\}, \quad 1 \leq i, j \leq N, 1 \leq m \leq M, \tag{2d}$$

where M is the number of users, N is the number of sub-channels, T_b is the time duration of the resource block, $b_{ij}^{(m)}$ is number of bits that can be carried in an RB for the sub-channel coupling (i, j) of user m, $x_{ij}^{(m)}$ is the number of RBs assigned to user m on sub-channel coupling (i, j), and γ is the parameter which asymptotically produces max-min fair rates.

The objective function

$$U_N(\cdots, x_{ij}^{(m)}, \cdots) = \sum_{m=1}^{M} \frac{1}{1-\gamma} \left(\frac{1}{T_b} \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}^{(m)} x_{ij}^{(m)} \right)^{1-\gamma}$$

is the γ-fair utility function, which we have used previously to obtain generalized proportionally fair rates for $0 \leq \gamma \leq 1$ [1]. Here we use the well known fact that as $\gamma \to \infty$ the optimum rates produced by the objective function are max-min fair [2]. The constraints (2b) and (2c) ensure that the total number of allocated blocks does not exceed what is available in the frame. The constraint (2d) ensures that the allocation is integral.

The integrality of time allocations, which is exhibited by the discrete nature of constraint (2d) makes the problem computationally hard. However, if the integrality of time allocations is relaxed, by replacing the constraint (2d) with

$$0 \leq x_{ij}^{(m)} \leq \frac{T}{2}, \quad 1 \leq i, j \leq N, 1 \leq m \leq M, \tag{3}$$

the optimization becomes a convex problem. However, since we are dealing with asymptotic values of $\gamma \to \infty$ it is not possible to use an off-the-shelf convex solver. Nevertheless, the optimization can solve by replacing the objective function with the regular max-min objective

$$\max_{x_{ij}^{(m)}} \quad \min_{m} \quad \frac{1}{T_b} \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}^{(m)} x_{ij}^{(m)}, \tag{4}$$

and then using an off-the-shelf linear program solver.

Due to the real number relaxation, solutions of the linear max-min optimization are an upper bound on the performance of any integer solution. However, the solution of the relaxed problem can only be used to measure the optimality since it consists of real numbers, which violates the integrality required by the actual problem. Therefore, we devise a sub-optimal algorithm with relatively simple complexity for the maximization.

We note that a part of the novelty in our formulation is that the problem becomes a convex optimization when the constraints (2d) are relaxed to include real-number solutions. The relaxed optimization can then be solved with a gradient-based sub-optimal algorithm. On the other hand, approaching the problem with the linear objective (4) does not produce an obviously good heuristic.

The gradient-based algorithm is obtained by observing that the maximum change in the objective function, that can be obtained from increasing any one of available $x_{ij}^{(m)}$s by one, is obtained by increasing the time allocation of sub-channel coupling (i, j) and user m with the steepest ascent direction (largest derivative) [1]. This fact can be observed from the

Taylor's expansion of the objective function

$$U_N(\cdots, x_{ij}^{(m)}+1, \cdots) \approx U_N(\cdots, x_{ij}^{(m)}, \cdots) + \frac{\partial}{\partial x_{ij}^{(m)}} U_N(\cdots, x_{ij}^{(m)}, \cdots),$$

where

$$\frac{\partial}{\partial x_{ij}^{(m)}} U_N(\ldots, x_{ij}^{(m)}, \ldots) = \frac{b_{ij}^{(m)}}{\left(\sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}^{(m)} x_{ij}^{(m)} \right)^{\gamma}}$$

is strictly positive.

Using the previous observation we devised an iterative greedy algorithm to solve the GPF optimization [1], where in each iteration, the user with the highest partial derivative is allocated an RB on its highest available sub-channel coupling:

$$(i^*, j^*, m^*) \leftarrow \arg \max_{\substack{1\le m\le M, \\ 1\le i,j\le N}} \frac{b_{ij}^{(m)}}{\left(\sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}^{(m)} x_{ij}^{(m)} \right)^{\gamma}}. \quad (5)$$

We now devise a gradient-based algorithm to solve the problem for $\gamma \to \infty$, which corresponds to the max-min rate allocation. Our algorithm is based on the results of the following proposition:

Proposition 1. *For γ sufficiently large, assigning a time slot to the user with the minimum current rate on its best sub-channel coupling is equivalent to assigning resources to the user with largest gradient (5).*

Proof: Define the best sub-channel coupling for user m with

$$\hat{b}_{ij}^{(m)} \triangleq \max_{1\le i,j\le N} \left\{ b_{ij}^{(m)} \right\},$$

the rate of user $\underline{m}$ with the lowest rate among all users with

$$r_{\underline{m}} \triangleq \min_{1\le m\le M} \left\{ \frac{1}{T_b} \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}^{(m)} x_{ij}^{(m)} \right\},$$

and a threshold on γ

$$\gamma_0 \triangleq \max_{1\le m\le M} \left\{ \log\left(\frac{\hat{b}_{ij}^{(m)}}{\hat{b}_{ij}^{(\underline{m})}} \right) \Big/ \log\left(\frac{r_m}{r_{\underline{m}}} \right) \right\}.$$

Since $\forall m \neq \underline{m}$, we have $0 < \log\left(\frac{r_m}{r_{\underline{m}}}\right)$, the following is true, subject to $\forall \gamma \geq \gamma_0, \forall m \neq \underline{m}$.

$$\gamma \log\left(\frac{r_m}{r_{\underline{m}}} \right) \geq \log\left(\frac{\hat{b}_{ij}^{(m)}}{\hat{b}_{ij}^{(\underline{m})}} \right).$$

Taking the exponent of both sides of the inequality,

$$\frac{(r_m)^{\gamma}}{(r_{\underline{m}})^{\gamma}} \geq \frac{\hat{b}_{ij}^{(m)}}{\hat{b}_{ij}^{(\underline{m})}} \quad \leftrightarrow \quad \frac{\hat{b}_{ij}^{(m)}}{(r_m)^{\gamma}} \leq \frac{\hat{b}_{ij}^{(\underline{m})}}{(r_{\underline{m}})^{\gamma}}.$$

Algorithm 1 ALGORITHM-MM($b_{ij}^{(m)}$, M, N, T)

1: $\forall i,j,m: \quad \tilde{b}_{ij}^{m} \leftarrow b_{ij}^{m}$
Initialize: $1 \le i,j \le N : T_i^{(\mathrm{BS})} = T/2, T_j^{(\mathrm{RS})} = T/2$
2: **while** $\exists\, T_i^{(\mathrm{BS})} > 0$ **and** $\exists\, T_j^{(\mathrm{RS})} > 0$ **do**
3: $\quad m^* \leftarrow \arg \min_{1\le m\le M} \left\{ \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}^{(m)} x_{ij}^{(m)} \right\}$
4: $\quad (i^*, j^*) \leftarrow \arg \max_{1\le i,j\le N} \tilde{b}_{i,j}^{(m^*)}$
5: $\quad x_{i^*j^*}^{(m^*)} \leftarrow x_{i^*j^*}^{(m^*)} + 1$
6: $\quad T_{i^*}^{(\mathrm{BS})} \leftarrow T_{i^*}^{(\mathrm{BS})} - 1$
7: $\quad T_{j^*}^{(\mathrm{RS})} \leftarrow T_{j^*}^{(\mathrm{RS})} - 1$
8: $\quad$ **if** $T_{i^*}^{(\mathrm{BS})} = 0$ **then**
9: $\quad\quad \tilde{b}_{i^*j}^{m} \leftarrow 0, \quad 1 \le m \le M, \quad 1 \le j \le N$
10: $\quad$ **end if**
11: $\quad$ **if** $T_{j^*}^{(\mathrm{RS})} = 0$ **then**
12: $\quad\quad \tilde{b}_{ij^*}^{m} \leftarrow 0, \quad 1 \le m \le M, \quad 1 \le i \le N$
13: $\quad$ **end if**
14: **end while**

Since by definition of $\hat{b}_{ij}^{(m)}$

$$\frac{b_{ij}^{(m)}}{(r_m)^{\gamma}} \leq \frac{\hat{b}_{ij}^{(m)}}{(r_m)^{\gamma}} \leq \frac{\hat{b}_{ij}^{(\underline{m})}}{(r_{\underline{m}})^{\gamma}},$$

we have

$$\max_{1\le m\le M} \max_{1\le i,j\le N} \frac{b_{ij}^{(m)}}{(r_m)^{\gamma}} \leq \frac{\hat{b}_{ij}^{(\underline{m})}}{(r_{\underline{m}})^{\gamma}}$$

for $\forall \gamma \geq \gamma_0$, proving the proposition. ■

Using the proposition, we see that finding the largest derivative is equivalent to assigning time to the user with the minimum current rate to its best sub-channel coupling. Based on this fact, we devise a gradient-based algorithm to find the max-min fair allocation of user rates (ALGORITHM-MM).

The algorithm works in iterations to allocate time to achieve max-min fairness among user rates. Steps 3-4 perform the search according to the proposition: first the minimum rate user is found (Step 3), then its best sub-channel coupling is found (Step 4). Variables $T_i^{(BS)}$ and $T_j^{(RS)}$ keep track of the available slots on each sub-channel for the BS and the RS transmissions, respectively to keep the allocation feasible according to constraints (2b) and (2c). After each iteration $T_i^{(BS)}$ and $T_j^{(RS)}$ are updated if any slots are allocated on their channels (Steps 6-7). The bits-per-slot values $\tilde{b}_{ij}^{(m)}$ are also updated (set to zero) according to the availability of RBs, to ensure that allocated slots are not considered in the next iteration (Steps 8-12). Note that $b_{ij}^{(m)}$ does not change as the algorithm runs and is used to find the user rates so far, on the other hand $\tilde{b}_{ij}^{(m)}$, changes as the algorithm runs and is used to find the best coupling for a selected user in each iteration. $\tilde{b}_{ij}^{(m)}$ reflect the allocation in previous steps.

The algorithm's complexity depends on the implementation of the search in Step 4. We do not get into the specifics of

TABLE I
SIMULATION PARAMETERS

Parameter	Value
BS-RS Channel	Rician, K=10 dB [15]
BS-RS Shadowing	Log-normal, variance 3 dB
BS-RS Doppler shift	4 Hz
RS-Users Channel	Rayleigh [15]
RS-Users Shadowing	Log-normal, variance 5 dB
RS-Users Doppler shift	37 Hz
Path loss	$38.4 + 2.35 \log 10(d)$ dB
Sub-carrier bandwidth	10.9375 kHz
Sub-carriers per Sub-channel	18
Number of users	$M = 30$
Number of sub-channels	$N = 50$
Slots per frame	$T = 20$
Cell radius	1000 m
BS-RS Distance	500 m
Transmit Power	40 dBm BS, 30 dBm RS
Antenna Gain	10 dB BS, 5 dB RS, 0 dB Users
Noise Figure	2 dB RS, 2 dB Users

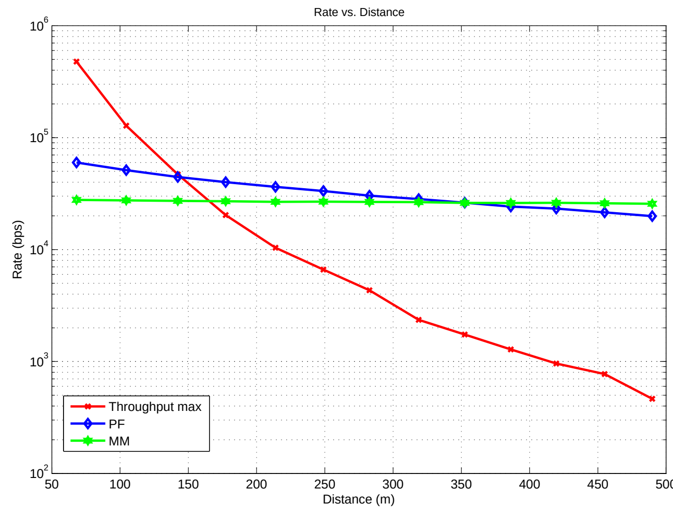

Fig. 1. Cumulative distribution function.

the algorithm's implementation. However, we note that the search step can be implemented with multiple sorted lists holding SNRs; namely M lists for second-hop SNR measured at users, and one list for first-hop SNR measured at RS. It takes $N\log(N)$ steps to sort each list. With the sorted lists, in Step 4, we know the best available coupling for each user without any computation. We find the user with the minimum rate in M steps. Note that linked list can be sorted in one step after Step 10 or Step 13, by putting the modified rate to the end of the list, since it is zero. Taking into account that there are $N\frac{T}{2}$ RBs in each hop, the algorithm goes through $N\frac{T}{2}$ iterations. Finally the complexity of the algorithm is $\mathcal{O}((M+1)N\log(N) + MN\frac{T}{2})$.

A similar algorithm is also used in the context of OFDMA cellular networks without relays [14], where in each iteration the user with the minimum rate is allocated resources on its best sub-channel. Nevertheless, the contribution in this paper is showing that this type of resource allocation is an asymptotic version of a more general GPF allocation. Since our algorithm is a sub-optimal version of the convex utility-based max-min fair resource allocation problem, we have an explanation of why the allocations derived by the algorithm are so close to optimum. Also, our algorithm is in the context of OFDMA-based AF relay networks, unlike [14] where it is used in the context of cellular networks.

IV. SIMULATION RESULTS

We ran two sets of Monte-Carlo simulation for evaluation of the proposed algorithm. In both simulations, we consider a network of 30 users connected to the BS through a predetermined RS. In each iteration of the Monte-Carlo simulation we randomly "drop" the users with a uniform density in the area around the relay. From the users' locations, we calculate each user's path-loss to the relay and use a detailed channel model to find the number of bits carried in an RB on each sub-channel for 20 frames. Details of the simulation parameters are shown in Table I.

In the first set of simulations we measure the sub-optimality of the algorithm. There are a total of 40 drops for a total of 800 distinct inputs to the optimization. Even this modest number of drops took about 20 hours to run due to the time it takes to find the upper bound values.

For each drop, we calculate users' time allocations using the relaxed optimization ($\hat{x}_{ij}$) and time allocations with the proposed algorithm ($\tilde{x}_{ij}$). Since the relaxed optimization is the upper bound on the integer solution of the problem, the sub-optimality gap is bounded by

$$\Delta_H \triangleq \frac{\left|U_N(\cdots,\tilde{x}_{ij}^{(m)},\cdots) - U_N(\cdots,\hat{x}_{ij}^{(m)},\cdots)\right|}{\left|U_N(\cdots,\hat{x}_{ij}^{(m)},\cdots)\right|},$$

where $|\cdot|$ is the absolute value of its operand. This gap shows how much the low complexity allocation is far from the upper bound on the optimum "total satisfaction".

Using the measured integrality gap from our simulations, we find that the optimum value of our algorithm is on average within 8% of the upper bound found with the linear program with the standard deviation of 1.6%.

In the second set of simulations, we measure the performance of the algorithm with the total 1000 distinct inputs. We also simulate the performance of the algorithm, which allocates rates to maximize the system throughput and the algorithm, which allocates the rates to achieve proportional fairness (PF) between the rates. The algorithm that maximizes the system throughput corresponds to the naïve extension of the single-user algorithm [5].

Figure 1 shows the system rate allocated to users as a function of distance from the relay. We observe that as we move from the MM allocation, to the PF allocation, to the maximum throughput allocation, more system resources are assigned to users closer to the RS, increasing the system throughput. The flattest rate coverage is achieved by the max-

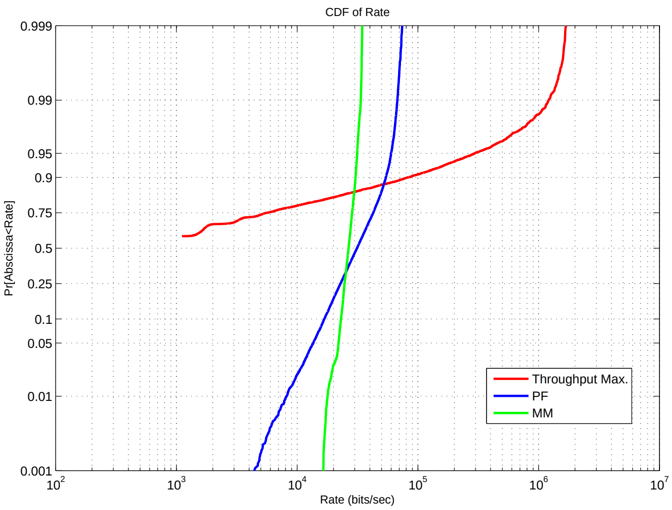

Fig. 2. Distribution of network resources.

min algorithm.

Figure 2 shows the CDF of rate for different algorithms. We see that as we move from throughput maximization to PF to MM, the CDF tends to the step function. The step function CDF corresponds to the almost uniform distribution of resources in the network. We note that 67% of the users get no allocation of resources for maximum throughput allocation.

Table II shows the rate of the lowest 5^{th} percentile of the users (the cell-edge) and the highest 5^{th} percentile of the users. We see that as we move from PF to MM, the cell-edge rate increase, at the expense of the top users. As expected, the rates of the lowest 5^{th} percentile and the highest 5^{th} percentile converge to each other for MM, since max-min fair rates are almost uniformly distributed. To further compare the impact of allocations on the fairness, we use Jain's fairness index

$$\mathcal{J} = \frac{\left(\sum_{i=1}^{M} r_m\right)^2}{\left(M \sum_{i=1}^{M} r_m^2\right)}, \tag{6}$$

which measures how similar rates are. Table II also shows the system rate and Jain's fairness measure. For $\mathcal{J}$ close to one the rates are the most similar, so the system is in extreme fair case, and for $\mathcal{J}$ close to $\frac{1}{M}$, the rates are least similar so the system is in extreme unfair case. As expected, for MM, where the system is fairer and transferring resources to the weaker users, the system throughput decreases.

TABLE II
SYSTEM SATISFACTION AND USER SATISFACTION

	Throughput Max.	PF	MM
05 percentile (Kbps)	0.000	13.600	22.010
95 percentile (Mbps)	0.2945	0.0600	0.0220
Throughput (Mbps)	1.580	1.003	0.800
Jain's index	0.06531	0.84920	0.98770

V. CONCLUSION

We devise a gradient-based max-min fair resource allocation algorithm for multi-user OFDMA-based AF relay networks. We use max-min fairness to provide an almost flat rate to the users. We start with a formulation, which has a convex relaxed version and takes a parameter that asymptotically moves the optimization to find the max-min fair rates. Then we devise a sub-optimal gradient-based algorithm to find the rates close to the optimum solution for the asymptotic values of the parameter. Our simulations show that the proposed algorithm is very close to the optimum solution and that its allocations are fairer than the allocations by proportionally fair or throughput optimal algorithms and translates to the flattest coverage.

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