Max-Min Fair Resource Allocation for Multiuser Amplify-and-Forward Relay Networks

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VTC2010-Fall
OTTAWA
Connecting the Mobile World

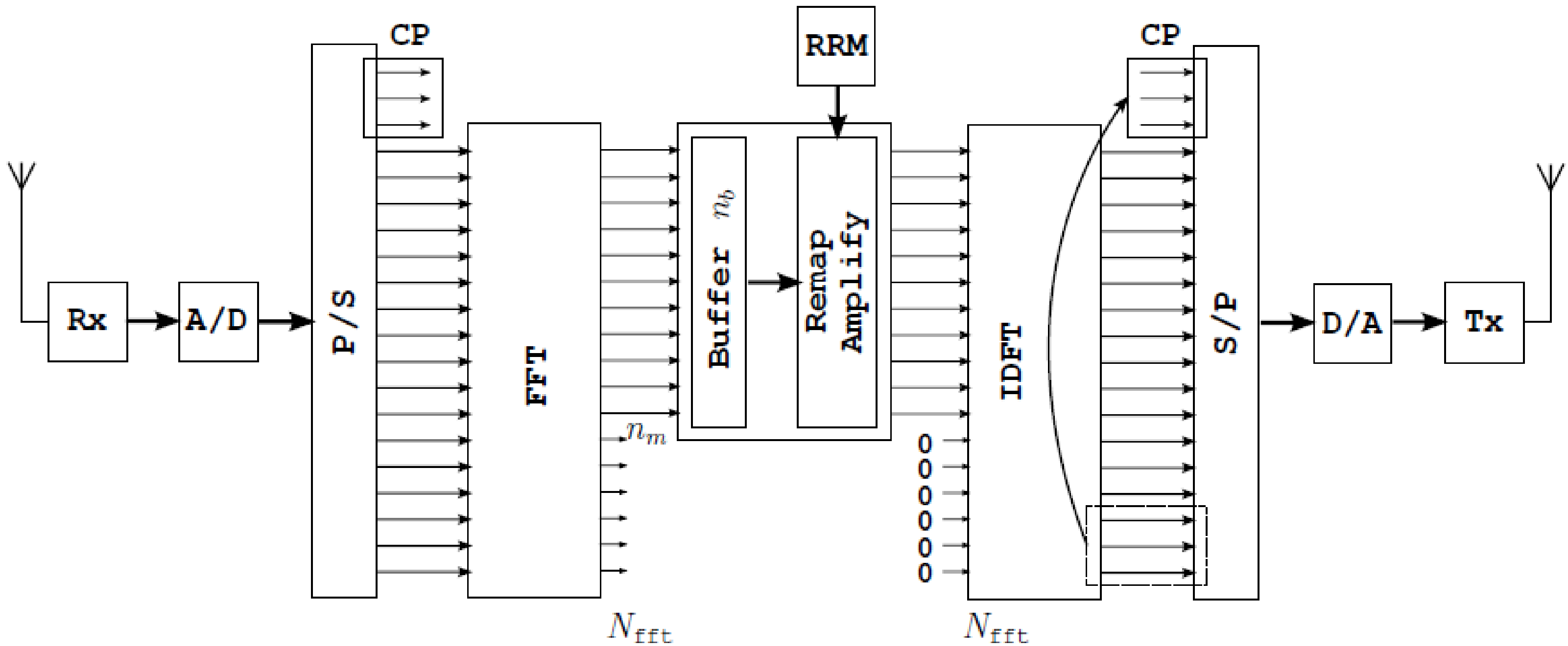
Introduction

- Physical layer rates sound great, but
  - Only work at short distances
  - Necessary to consider advanced RANs (relays)

- Multi-user OFDMA-based AF relays:
  - Layer 1 switching based on OFDMA RBs
  - Faster switching (no header inspection)
  - Simpler implementation (tens of Gbps)

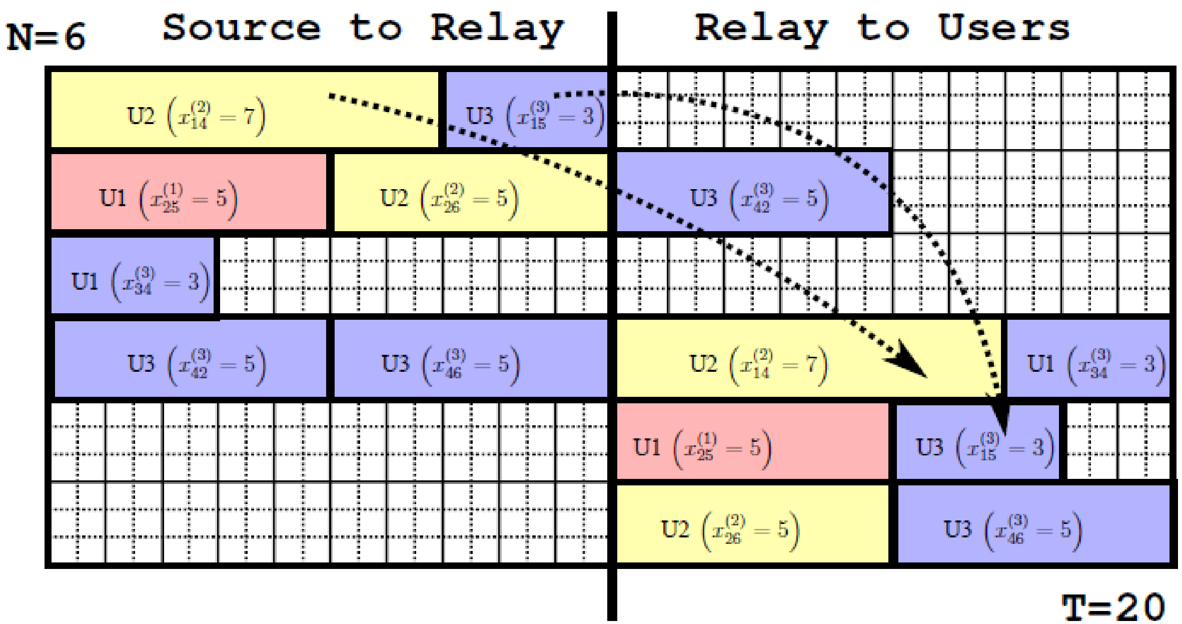
- Our contributions:
  - Extend AF relaying to multiple-user
  - Max-Min resource allocation framework for multi-user OFDMA-based AF relays
  - Fast, near-optimal resource management algorithm

OFDMA-based AF Relay



- Make the “amplify” part more intelligent
- Switch in frequency and time:
  - Buffer stores digitized samples of RBs, before amplification and retransmission
- Needs RRM to find best mappings

Example OFDMA-based AF Relay Schedule



- End-to-end routing done beforehand
- Incoming time-frequency identifies user-destination pair
  - E.g., map incoming to outgoing RBs (e.g., map 7 slots in CH.1 to CH. 7 for U2).
- Our problem: finding “best” $x_{ij}^{(m)}$ - number of RBs on each coupling (i,j)
- Assigning combined RBs on the BS-RS and the RS-UT link, so that end-to-end user fairness is met

Asymptotic Max-Min Fair Scheduling Problem

$$\max_{x_{ij}^{(m)} \in \left\{0,\ldots,\frac{T}{2}\right\}} U_N\left(\ldots, x_{ij}^{(m)}, \ldots\right) = \sum_{m=1}^{M} \frac{1}{1-\gamma} \left(\frac{1}{T_c} \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}^{(m)} x_{ij}^{(m)} \right)^{1-\gamma}$$

$$\sum_{m=1}^{M} \sum_{j=1}^{N} x_{ij}^{(m)} \leq \frac{T}{2} \qquad 1 \leq i \leq N$$

$$\sum_{m=1}^{M} \sum_{i=1}^{N} x_{ij}^{(m)} \leq \frac{T}{2} \qquad 1 \leq j \leq N$$

- When the parameter γ tends to infinity, the allocation becomes max-min.
- Non-linear integer program (hard to solve)
- Real-number relaxation ($0 \leq x_{ij}^{(m)} \leq T/2$) not possible
  - Large problem size (75 000 variables for N=50, M=30)
  - Unusable anyway
- However, real number relaxation gives hints on how to solve the problem and also gives us an upper bound.

Gradient approach

- We devise an algorithm based on the gradient of the objective function.
- Consider Taylor's expansion of the network utility:

$$U_N\left(\dots, x_{ij}^{(m)}+1, \dots\right) \approx U_N\left(\dots, x_{ij}^{(m)}, \dots\right) + \frac{\partial}{\partial x_{ij}^{(m)}} U_N\left(\dots, x_{ij}^{(m)}, \dots\right)$$

- Where the derivative is:

$$\frac{\partial}{\partial x_{ij}^{(m)}} U_N\left(\dots, x_{ij}^{(m)}, \dots\right) = \frac{1}{T_c} \frac{b_{ij}^{(m)}}{\left(\frac{1}{T_c}\sum_{i=1}^{N}\sum_{j=1}^{N} b_{ij}^{(m)} x_{ij}^{(m)}\right)^{\gamma}}$$

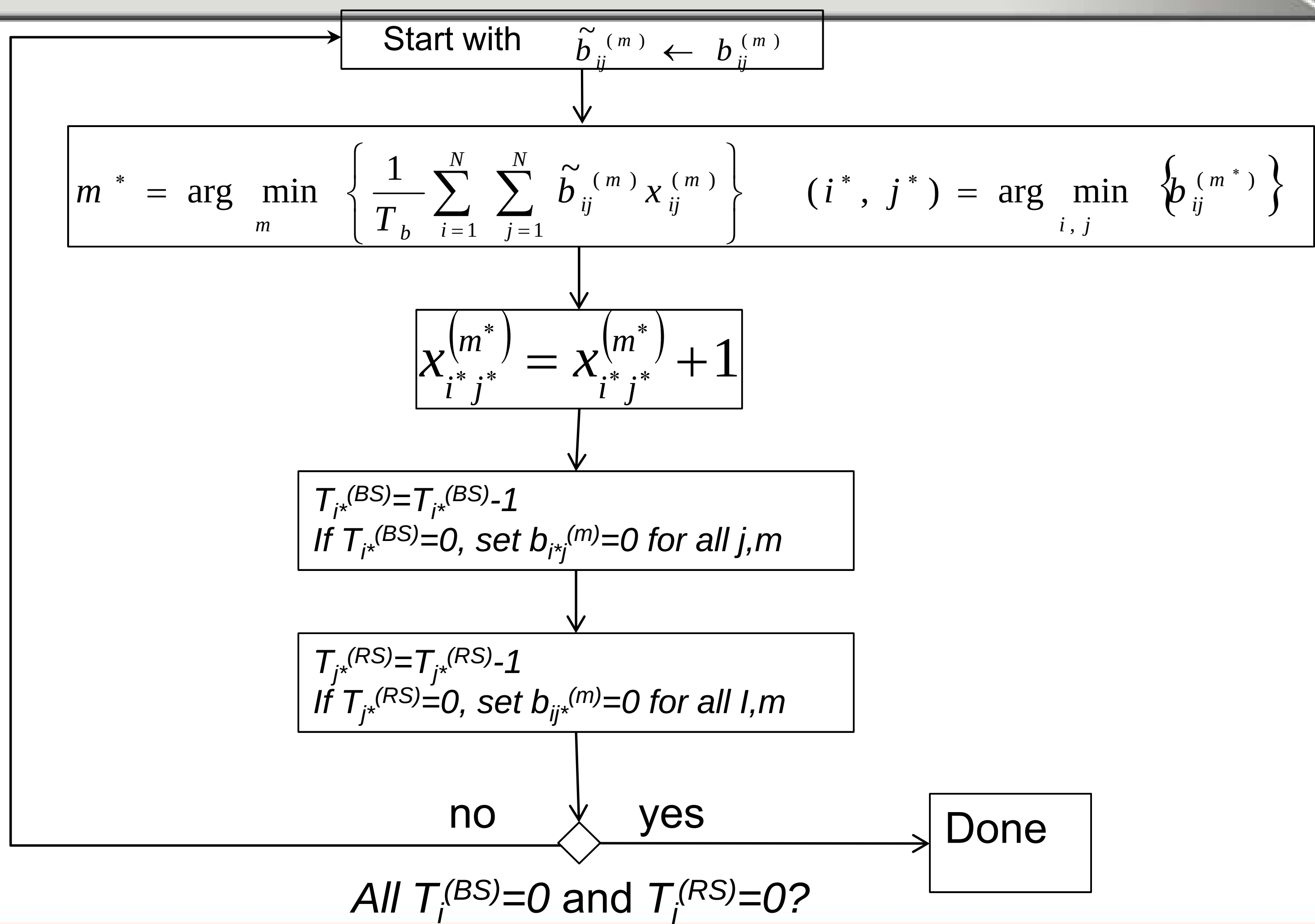
- The maximum change in the objective function, that can be obtained from increasing one time-allocation by one, is obtained by adding time allocation in the direction of the steepest gradient

Max-Min Algorithm

Proposition 1. *For γ sufficiently large, assigning a time slot to the user with the minimum current rate on its best sub-channel coupling is equivalent to assigning resources to the user with largest gradient (5).*

- The algorithm works in iterations, until all RBs are assigned. In each iteration RBs are assigned to the user with the lowest rate and then to the sub-channel coupling where the RBs have the highest AMC.
- When all the RBs on an RS or a BS sub-carrier are allocated, the bit allocations for those channels are set to 0.

Max-Min Algorithm



Max-Min Algorithm

Algorithm 1 ALGORITHM-MM($b_{ij}^{(m)}, M, N, T$)

$\forall i, j, m : \quad \tilde{b}_{ij}^{m} \leftarrow b_{ij}^{m}$

Initialize: $1 \leq i, j \leq N : T_i^{(\mathrm{BS})} = T/2,\ T_j^{(\mathrm{RS})} = T/2$

while $\exists\, T_i^{(\mathrm{BS})} > 0$ **and** $\exists\, T_j^{(\mathrm{RS})} > 0$ **do**

  $m^* \leftarrow \arg \min_{1 \leq m \leq M} \left\{ \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}^{(m)} x_{ij}^{(m)} \right\}$

  $(i^*, j^*) \leftarrow \arg \max_{1 \leq i,j \leq N} \tilde{b}_{i,j}^{(m^*)}$

  $x_{i^* j^*}^{(m^*)} \leftarrow x_{i^* j^*}^{(m^*)} + 1$

  $T_{i^*}^{(\mathrm{BS})} \leftarrow T_{i^*}^{(\mathrm{BS})} - 1$

  $T_{j^*}^{(\mathrm{RS})} \leftarrow T_{j^*}^{(\mathrm{RS})} - 1$

  if $T_{i^*}^{(\mathrm{BS})} = 0$ **then**

    $\tilde{b}_{i^* j}^{m} \leftarrow 0, \quad 1 \leq m \leq M, \quad 1 \leq j \leq N$

  end if

  if $T_{j^*}^{(\mathrm{RS})} = 0$ **then**

    $\tilde{b}_{i j^*}^{m} \leftarrow 0, \quad 1 \leq m \leq M, \quad 1 \leq i \leq N$

  end if

end while

Simulation Parameters

BS-RS channel	Rician, K=10 dB
BS-RS shadowing	Log-normal, variance 3 dB
BS-RS Doppler shift	4 Hz
RS-users channel	Rayleigh
RS-users shadowing	Log-normal, variance 5 dB
RS-users Doppler shift	37 Hz
Path loss	38.4 +2.35 log 10(d) dB
Sub-carrier bandwidth	10.9375 kHz

Sub-carriers per RB	18
Number of users	M = 30
Number of sub-channels	N = 50
Slots per frame	T = 20
Cell radius	1000 m
BS-RS distance	500 m
Transmit power	40 dBm BS, 30 dBm RS
Antenna gain	10 dB BS, 5 dB RS, 0 dB Users
Noise figure	2 dB RS, 2 dB Users
Monte-Carlo Scenarios	80000

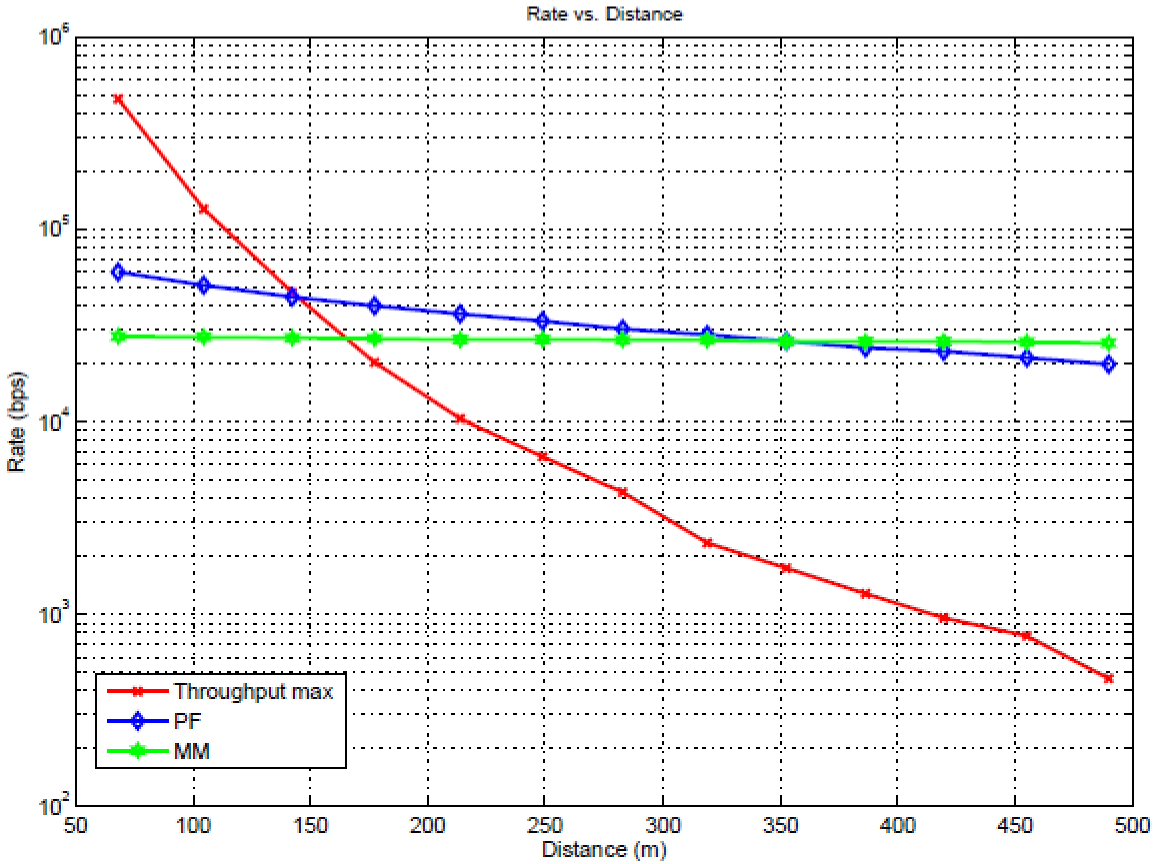
Sub-optimality gap

Distance from optimum:

$$\Delta_H \triangleq \frac{\left|U_N(\cdots, \tilde{x}_{ij}^{(m)}, \cdots) - U_N(\cdots, \hat{x}_{ij}^{(m)}, \cdots)\right|}{\left|U_N(\cdots, \hat{x}_{ij}^{(m)}, \cdots)\right|},$$

Using the measured sub-optimality gap from the simulation, we find that the output of the algorithm is on average within 8% of the upper bound found (with standard deviation of 1.6%)

Spatial Distribution of Rates



CDF of Rates

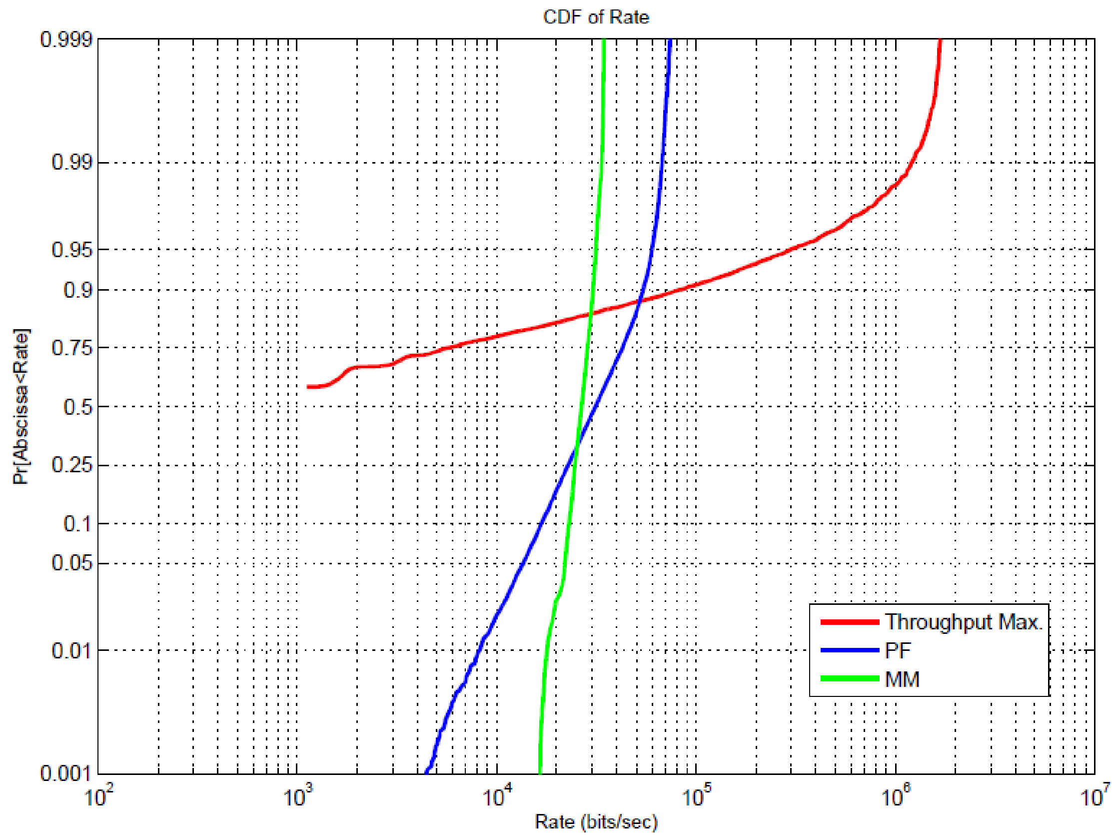
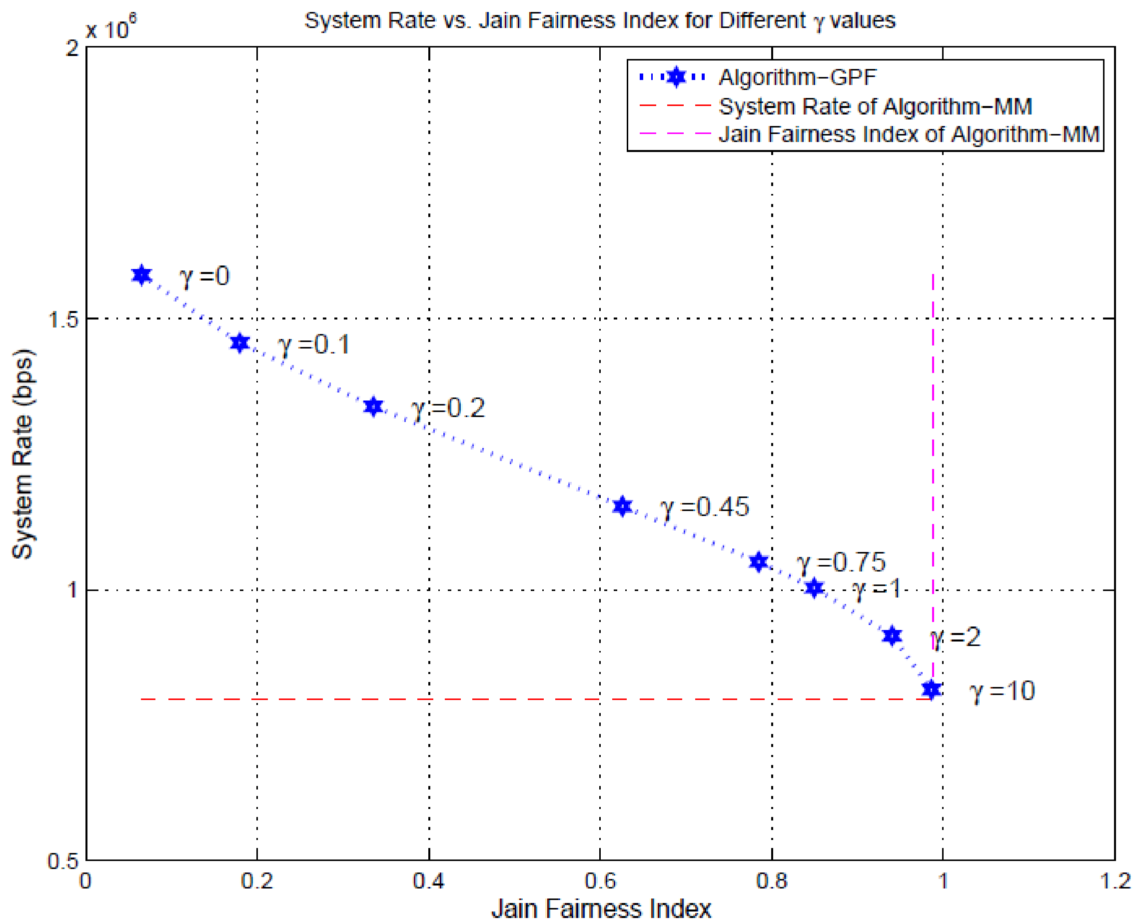


Fig. 2. Distribution of network resources.

System rate vs. Jain fairness index



Summary

- We observed OFDMA-based AF relay allows for buffering and scheduling of transmissions at different times and sub-channels.
- We devised a framework to allocate resource blocks for fair rates.
- We devised a near-optimal gradient-based algorithm.
- How the allocation and sub-channel switching can be done in a fair optimal manner.
- Simulations show how the cell edge users are traded with best users.
- Max-min provides the most ubiquitous coverage with a Jain Index close to 1.

Thank you!

Proof of the Proposition

Proposition 1. *For γ sufficiently large, assigning a time slot to the user with the minimum current rate on its best sub-channel coupling is equivalent to assigning resources to the user with largest gradient (5).*

Proof: Define the best sub-channel coupling for user m with

$$\hat{b}_{ij}^{(m)} \triangleq \max_{1\leq i,j\leq N} \left\{ b_{ij}^{(m)} \right\},$$

the rate of user $\underline{m}$ with the lowest rate among all users with

$$r_{\underline{m}} \triangleq \min_{1\leq m\leq M} \left\{ \frac{1}{T_b} \sum_{i=1}^{N} \sum_{j=1}^{N} b_{ij}^{(m)} x_{ij}^{(m)} \right\},$$

and a threshold on γ

$$\gamma_0 \triangleq \max_{1\leq m\leq M} \left\{ \log\left(\frac{\hat{b}_{ij}^{(m)}}{\hat{b}_{ij}^{(\underline{m})}} \right) \Big/ \log\left(\frac{r_m}{r_{\underline{m}}} \right) \right\}.$$

Since $\forall m \neq \underline{m}$, we have $0 < \log\left(\frac{r_m}{r_{\underline{m}}}\right)$, the following is true, subject to $\forall \gamma \geq \gamma_0, \forall m \neq \underline{m}$.

$$\gamma \log\left(\frac{r_m}{r_{\underline{m}}} \right) \geq \log\left(\frac{\hat{b}_{ij}^{(m)}}{\hat{b}_{ij}^{(\underline{m})}} \right).$$

Taking the exponent of both sides of the inequality,

$$\frac{(r_m)^\gamma}{(r_{\underline{m}})^\gamma} \geq \frac{\hat{b}_{ij}^{(m)}}{\hat{b}_{ij}^{(\underline{m})}} \quad \leftrightarrow \quad \frac{\hat{b}_{ij}^{(m)}}{(r_m)^\gamma} \leq \frac{\hat{b}_{ij}^{(\underline{m})}}{(r_{\underline{m}})^\gamma}.$$

Since by definition of $\hat{b}_{ij}^{(m)}$

$$\frac{b_{ij}^{(m)}}{(r_m)^\gamma} \leq \frac{\hat{b}_{ij}^{(m)}}{(r_m)^\gamma} \leq \frac{\hat{b}_{ij}^{(\underline{m})}}{(r_{\underline{m}})^\gamma},$$

we have

$$\max_{1\leq m\leq M} \max_{1\leq i,j\leq N} \frac{b_{ij}^{(m)}}{(r_m)^\gamma} \leq \frac{\hat{b}_{ij}^{(\underline{m})}}{(r_{\underline{m}})^\gamma}$$

for $\forall \gamma \geq \gamma_0$, proving the proposition. ■

Versatility of the γ Parameter

- Maximize throughput ?

$$\sum_{m=1}^{M}\frac{1}{1-\gamma}\left(\frac{1}{T_c}\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}^{(m)}x_{ij}^{(m)}\right)^{1-\gamma} \underset{\gamma=0}{=} \sum_{m=1}^{M}\frac{1}{T_c}\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}^{(m)}x_{ij}^{(m)}$$

- Proportional fairness?

$$\sum_{m=1}^{M}\frac{1}{1-\gamma}\left(\frac{1}{T_c}\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}^{(m)}x_{ij}^{(m)}\right)^{1-\gamma} \underset{\gamma\to 1}{=} \sum_{m=1}^{M}\ln\left(\frac{1}{T_c}\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}^{(m)}x_{ij}^{(m)}\right)$$

- Max-min fairness?

$$\max\sum_{m=1}^{M}\frac{1}{1-\gamma}\left(\frac{1}{T_c}\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}^{(m)}x_{ij}^{(m)}\right)^{1-\gamma} \underset{\gamma\to\infty}{=} \max\quad \min_{m=1..M}\sum_{m=1}^{M}\frac{1}{T_c}\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}^{(m)}x_{ij}^{(m)}$$

TABLE II

SYSTEM SATISFACTION AND USER SATISFACTION

	Throughput Max.	PF	MM
05 percentile (Kbps)	0.000	13.600	22.010
95 percentile (Mbps)	0.2945	0.0600	0.0220
Throughput (Mbps)	1.580	1.003	0.800
Jain's index	0.06531	0.84920	0.98770

Background

- Providing very high data rate coverage, when and where required, is a formidable goal, requiring dense cost-effective radio access network (RAN) architectures. Since path loss, fading, and transmit power limitations prevent high spectral efficiency even for moderately long links, it is necessary to consider advanced RANs, such as relay networks, which effectively collect and distribute wireless signals. However, to achieve the full potential of the advanced RANs, efficient RRM techniques are also necessary to match the demand with limited wireless resources in a fair way.

- The invention considers RANs with multi-user enabled digital amplify-and-forward (AF) relays, which multiplex user data with cut-through switching. Digital AF relays buffer quantized samples of the symbols until they are amplified and transmitted at a later time. Cut-through switching forwards data without examining network layer headers, and is possible due to the synchronicity of Orthogonal Frequency Division Multiple Access (OFDMA) relay networks.

- Current RRM approaches for AF relay networks consider a single user scenario. For OFDMA- based relays, one approach is to match sub-carriers at the input and the output to maximize throughput. In the case of multiple-users, this approach breaks down, since it may starve out some of the users.

95th percentile rate vs. 5th percentile rate