

A Distributed Framework with a Novel Pricing Model for Enabling Dynamic Spectrum Access for Secondary Users

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Abstract—Wireless Service Providers (WSPs) aim to maximize the revenue from their investment in spectrum lease and infrastructure. In this paper, we present a simple model for enabling temporary wireless access for Secondary (unsubscribed) Users (SUs) along with Primary (subscribed) Users (PUs) to the same Base Station (BS), allowing SU access provided there is unutilized spectrum available at the BS after all the PUs have been served. We develop a distributed framework focusing on the efficient utilization of the spectrum leased by a WSP, in contrast to the centralized approach based on spectrum/spectrum information pooling prominent in literature. This paper includes a detailed signaling framework along with a novel Differentiated Service Code Point (DSCP) based mechanism for distinguishing PUs from SUs at the BS. A novel incentive based pricing model for SUs with an inherent property of resource management at the BS is proposed along with a criterion for autonomous network selection at the SU terminal and a SU terminal initiated price based handoff scheme. The distributed approach proposed in this paper, provides a framework for a single WSP to maximize its profits by allowing SU access, without the need for coordination with other WSPs through a centralized mediating entity.

I. INTRODUCTION

Demand for mobile wireless internet connectivity has been on the rise in recent years. Wireless Service Providers (WSPs) have been investing heavily in the licensed spectrum to satisfy this ever increasing demand, thus making the scarce radio spectrum a very expensive resource. Underutilization of the radio spectrum in both spatial and temporal domains, indicating the inefficient usage of this precious resource, was confirmed through field measurements [1]. Studies such as [1] and [2] revealed the need to facilitate efficient, intelligent and fair use of the radio spectrum, by replacing the current rigid ‘command and control’ spectrum policies with new flexible spectrum policies under the Dynamic Spectrum Access (DSA) paradigm. Secondary Users (SUs), i.e., unsubscribed users equipped with Cognitive Radios (CRs) or Software Defined Radios (SDRs) [3] could gain opportunistic wireless access using DSA techniques with protocols for avoiding interference to the regular functioning of the Primary Users (PUs), i.e., subscribed users in the licensed cellular networks [2], [3].

The centralized approach with spectrum/spectrum information pooling and a regional mediating entity for co-ordination has been prominent in previous work on DSA for infrastructure based networks. Inter-WSP spectrum sharing called ‘DIM-

SUMnet’ was proposed in [4] with protocols and infrastructure based on a centralized spectrum broker as the mediating entity. A dynamic model where the SUs were charged based on the resource and power allocated was described in [5]. WSPs could then obtain the spectrum required for the SUs through a centralized spectrum clearinghouse [5]. Implementation models for transferring spectrum related information from all the WSPs in the area to CR equipped SUs over the Cognitive Pilot Channel (CPC) were described in [6]. A centralized CPC manager [6] was used for the co-ordination of spectrum information from all the WSPs prior to the transmission of this information over the universally harmonized CPC.

In spite of the effectiveness of the centralized approach in [4]–[6] from the spectrum management perspective, due to the regional dynamics of spectrum usage, a mesh based architecture [6] with regional spectrum brokers [4] or centralized spectrum clearinghouses [5] would need to be implemented to facilitate the centralized approach, thus incurring a severe cost burden on WSPs. From the implementation perspective, charging SUs based on power or resource allocated as in [5] could present concerns on tractability due to the variable usage costs and signaling involved for co-ordination with the centralized spectrum clearinghouse. The infrastructure and protocols for allowing SU access as highlighted in [4]–[6] present huge extra costs for the WSPs, apart from the large investments in existing infrastructure and licensed spectrum lease for serving the PUs. Although the centralized approach proposed in [6] is beneficial to the SUs for obtaining spectrum information from all the WSPs on one channel, thus avoiding the scanning of multiple bands at the SU terminal, it is inconceivable to assume that the WSPs would collude and share their licensed spectrum information with competing WSPs.

In this paper, we propose a direct WSP-SU interaction based distributed approach for spectrum trading without a centralized entity as opposed to [4]–[6]. Our proposal is based on the fact that spectrum load changes spatially and temporally with changing PU demand. Thus there would always be an opportunity for WSPs to maximize their profits by selling the current unutilized spectrum, if it exists, directly to SUs temporarily for a fixed price per fixed time window set by the WSPs. The fixed price within this time window could be the price per minute (price/min) or the price per

Megabyte (price/MB) depending on the application class in consideration. Hence the same BS can serve both PUs & SUs on separate channels/slots; with SU access enabled if there remains spectrum available at the BS after all the PUs have been served. In this paper, we develop a detailed signaling framework along with a Differentiated Service Code Point (DSCP) based method for separating PUs from SUs at the BS for ease of Radio Resource Management (RRM) and pricing. The novel pricing model presented in this paper captures the PU demand and spectrum availability at the BS and provides monetary incentives for SUs when utilization at the BS is low. A criterion for SU network selection and a SU terminal initiated price based handoff scheme based on the pricing model are also proposed. The proposed distributive framework is thus based on the efficient utilization of the spectrum band leased by a single WSP, thus allowing it to implement this model without the co-ordination and co-operation of the other WSPs in the area.

The proposed distributed approach can potentially be cost effective for WSPs from the implementation feasibility perspective, in contrast to centralized approach based on spectrum/spectrum information pooling [4]–[6]. However, the SUs need to scan multiple frequency bands for obtaining updated spectrum information from every WSP, prior to network selection or handoff.

This paper is organized as follows: Section II describes the proposed system concept along with the signaling framework and the DSCP based PU-SU differentiation. The pricing model for SU spectrum access is presented in Section III. Section IV describes the SU network selection criterion based on the pricing model and a SU terminal initiated price based handoff scheme. The conclusions and future work are highlighted in Section V.

II. SYSTEM CONCEPT

The wireless telecommunications industry is evolving towards 4G, namely Long Term Evolution (LTE, LTE-A) [7] and WiMAX [8] networks, with Orthogonal Frequency Division Multiplexing (OFDM), packet switching and an all-IP framework. The removal of Radio Network Controllers (RNCs) as in [7], [8] has enhanced the capabilities of the BS for autonomous decision making and resource allocation. In this paper, we assume a single frequency network with either Frequency Division Duplexing (FDD) or Time Division Duplexing (TDD) modes of operation; hence the entire spectrum allocated to a WSP may be used by every BS as shown in Fig. 1. In providing temporary wireless access for SUs to the unutilized spectrum at the BS in a distributed manner, every BS of every WSP would need to broadcast its spectrum information to the SUs in the area.

Figure 1 shows a generic arrangement depicting a SU in the coverage area of the BSs of three different WSPs. The BSs are separated out in Fig. 1 for ease of representation, but they could even be collocated. Frequency band utilization at the BS of every WSP is shown in Fig. 1 with the unutilized part of the spectrum depicted by the dotted orange region

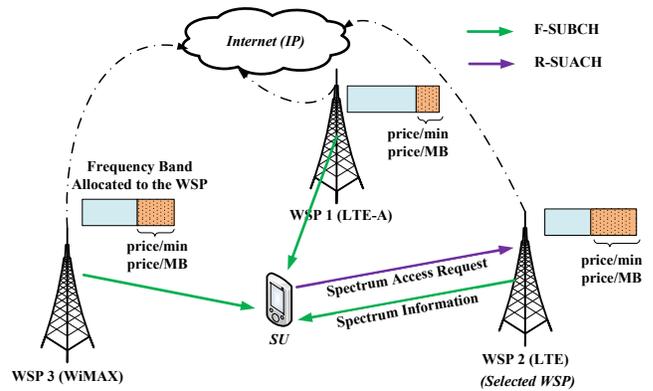


Fig. 1. Network scenario with SU requiring wireless access

and the utilized part of the spectrum by plain blue region. The BSs of every WSP broadcast spectrum information to the SUs in their coverage area over the Forward link Secondary User Broadcast Channel (F-SUBCH), while the Reverse link Secondary User Access Channel (R-SUACH) is used by the SU to send a spectrum access request to the selected WSP.

We assume the SUs to be equipped with a basic multimode SDR terminal with the capability of multiple frequency band operation and reconfigurability. Scanning and detecting the F-SUBCH from every WSP is essential for SU terminal to gather spectrum information for network selection. The SU terminal can then reconfigure to support the Radio Access Technologies (RATs) provided by the WSP selected, thus providing flexibility of operation to the SUs.

SUs requiring temporary wireless access can be classified into two types:

- 1) SU Type A: These SUs are not subscribed to any WSP.
- 2) SU Type B: These are PUs who convert to SUs to gain wireless access to an application class that they are not subscribed to as PUs. For example, a PU subscribed to voice service with a WSP, could gain temporary access as a SU to data/internet service from any WSP.

The SUs of both types A & B must be preregistered to an online SU database, before they can be allowed temporary wireless access by the WSPs. This database should thus be secure and accessible only to the Authentication Server (AS) of the WSP. The SU database needs to include user information similar to any online user database coupled with critical authentication information including device identity.

A. Signaling Framework for SU - BS Interaction

The signaling framework for SU-BS interaction based on the distributed approach with FDD is illustrated in Fig. 2. The physical channels between the BS and the SU are shown by solid lines, whereas the logical channels between the BS and the AS, and the AS and the Internet (IP), are shown using dashed lines. As shown in Fig. 2, the signaling framework for SU-BS interaction can be classified into three phases based on operational aspects and is described as follows:

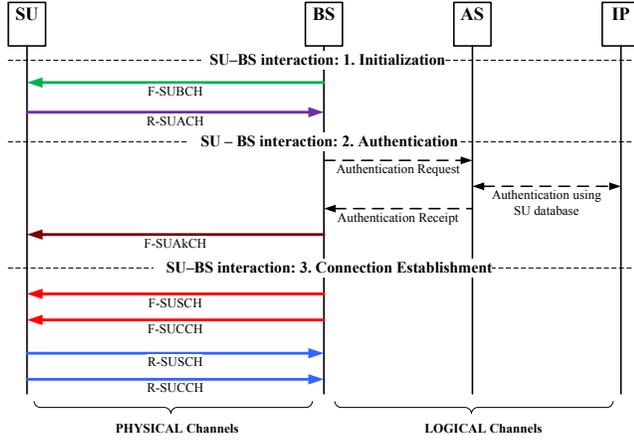


Fig. 2. Signaling framework for SU-BS interaction

1) *Initialization*: The initialization process contains two essential physical channels for BS-SU interaction.

F – SUBCH: The BSs of all WSPs advertise their unutilized spectrum to the SUs in their area over the F-SUBCH as shown in Fig. 1 and Fig. 2. Due to the distributed nature of our scheme, every WSP transmits its F-SUBCH on separate frequencies for SUs in the area. All BSs of a particular WSP could transmit on the same frequency, considering a single frequency network. Spectrum information passed over the F-SUBCH should include the following to assist WSP selection at SU terminal:

- 128 bit IPv6 or 32 bit IPv4 address and 48 bit MAC (Medium Access Control) address of the transmitting BS with open destination address
- BS & WSP identifier and the associated RAT
- Uplink frequency of R-SUACH set by the broadcasting BS for SU response, if chosen by the SU
- Application classes offered by the BS (A_o)
- Fixed price per fixed time window for all the A_o
- Signal to Noise Ratio (SNR) at the SU terminal

R – SUACH: The SUs then select the most appropriate network using the network selection criterion in (5) and send a ‘Spectrum Access Request (SAReq)’ to the selected WSP over the R-SUACH as shown in Fig. 1 and Fig. 2. Information sent by the SU to the selected BS should include the following:

- 128 bit IPv6 or 32 bit IPv4 address and 48 bit MAC address of the SU terminal with packet destination address of the selected BS
- SAReq along with SU authentication information for pricing and billing
- Application class requested by the SU (a_r)

For SUs of type B, a mechanism analogous to twitter [9] for sending real time spectrum updates over the internet or Short Message Service (SMS) could be used as an alternative to the above discussed SU-BS initialization, thus avoiding multi-band scanning at the SU terminal.

2) *Authentication*: Upon receiving the SAReq from the SU, the BS sends a request for authentication to the AS, which

may be collocated at the BS, over the logical channel. On verification of the authentication information from the SU database, a positive or negative acknowledgement, based on authentication, is then sent to the SUs over the Forward link Secondary User Acknowledgement Channel (F-SUAkCH). A temporary service contract is also sent to the successfully authenticated SUs over this channel.

3) *Connection Establishment*: After authentication by the WSP, the SUs will receive temporary wireless access from the WSP over the physical channels similar to the framework in LTE [7] and WiMAX [8]. In the downlink, the BS transmits user data to the SUs over the Forward link Secondary User Shared Channel (F-SUSCH) and control information over the Forward link Secondary User Control Channel (F-SUCCH), while in the uplink, the SUs use Reverse link Secondary User Shared Channel (R-SUSCH) for data and the Reverse link Secondary User Control Channel (R-SUCCH) for control information as shown in Fig. 2.

B. Differentiation between Primary and Secondary Users

Co-existence of PUs and SUs on the same spectrum band at the BS would need differentiation between PUs and SUs for efficient RRM, SU authentication, and implementation of separate pricing mechanisms, described in Section III. In the IP framework considered, the PUs and SUs can be differentiated based on the IP or MAC address; however this implementation would hinder the fast and efficient operation at the BS. We thus propose the use of one unused bit from the DSCP byte of the IP header for quick PU-SU differentiation.

The DSCP byte in the IP header is responsible for indicating the Quality of Service (QoS) requested by a user. Figure 3 shows the DSCP byte defined by the Internet Engineering Task Force (IETF) for the Diffserv architecture. The first 6 bits of this byte are used to define the QoS for a particular user, while the last two bits are reserved for future use. These last two bits are currently unused and are default set to 0 [10].

We propose the use of the rightmost bit in the DSCP byte for fast differentiation between PU and SU at the BS. This rightmost bit can be set to ‘1’ in the IP packets for the SUs to help differentiate them from the PUs as shown in Fig. 3. The decimal value of the DSCP byte thus is odd in case of SU and even for the PU. Security mechanisms will be needed to be developed for the above proposal to be implemented.

III. PRICING MODEL FOR SECONDARY USERS

In this section, a novel incentive based pricing model is presented that relates the spectrum utilization at the BS, PU demand and the fixed price charged to the PUs by the

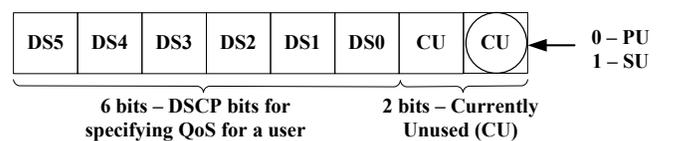


Fig. 3. DSCP byte [10] and the proposed use of the rightmost bit for SUs

WSPs. Spectrum utilization characteristics at the BS have been observed to change with time and location [1]. We define the Spectrum Utilization Factor (SUF) at the BS for WSP i denoted by $\alpha_{i,t}$ to capture the dynamics of this variable utilization, where $\alpha_{i,t} \in [0, 1]$. The SUF can be defined as the total spectrum utilization at the BS, which depends on the number of users (PUs & SUs) currently being served at the BS. Thus assuming a single frequency network, the SUF value $\alpha_{i,t} = 0$ indicates no utilization, while $\alpha_{i,t} = 1$ indicates complete utilization of the spectrum leased by the WSP i at that BS.

In this paper, a single application class is considered for the pricing model, but this model can also be extended to multiple application classes. The PUs pay a fixed price to the WSPs based on their application class requirements for guaranteed wireless access. We define p_i as the fixed price paid by PUs to the WSP i . According to our proposal, the SUs are potentially granted temporary wireless access, if and only if there is unutilized spectrum ($\alpha_{i,t} < 1$) at the BS, and thus WSPs require a separate pricing model for the SUs considering the SUF at the BS. The incentive based pricing model for SUs in this section, presents a monetary incentive with respect to p_i to attract SUs when SUF is low, while a monetary penalty is applied when SUF is high, consequently providing inherent resource management of PUs & SUs at the BS. The terminology relevant to the pricing model with respect to spectrum utilization at the BS of WSP i is as follows:

- $\alpha_{i,pu}$: PU utilization,
 - $\alpha_{i,h}$: Spectrum reserved for handoff,
 - $\alpha_{i,th}$: Threshold for spectrum utilization; $\alpha_{i,th} = 1 - \alpha_{i,h}$,
 - $\alpha_{i,su}$: SU utilization; $\alpha_{i,su}$ exists iff $\alpha_{i,pu} < \alpha_{i,th}$,
 - $\alpha_{i,t}$: Total utilization, i.e., SUF; $\alpha_{i,t} = \alpha_{i,pu} + \alpha_{i,su}$,
 - $\alpha_{i,ic}$: Incentive cutoff limit beyond which $s_i > p_i$,
- where $\alpha_{i,pu}, \alpha_{i,h}, \alpha_{i,th}, \alpha_{i,su}, \alpha_{i,t}, \alpha_{i,ic} \in [0, 1]$.

The SU price s_i can be related to fixed PU price p_i and the variable nature of spectrum utilization at the BS as follows:

$$s_i = f_i(\alpha_{i,t}) \times p_i, \quad (1)$$

where $s_i, f_i(\alpha_{i,t})$ and p_i are non-negative real numbers.

The term $f_i(\alpha_{i,t})$ captures the variability of spectrum utilization at the BS, and can be considered as the normalized SU price with respect to the fixed p_i . The normalized SU price $f_i(\alpha_{i,t})$ is based on the log barrier penalty function [11] with an inherent property that $f_i(\alpha_{i,t}) \rightarrow \infty$ as $\alpha_{i,t} \rightarrow \alpha_{i,th}$. This property helps restricts SU access at the BS as $\alpha_{i,t}$ at the BS increases and can be observed in Fig. 4. The normalized SU price $f_i(\alpha_{i,t})$ is defined as follows:

$$f_i(\alpha_{i,t}) = \begin{cases} -\ln\left(1 - \left(\frac{\alpha_{i,t}}{\alpha_{i,th}}\right)^{n_i}\right) & \text{if } \alpha_{i,t} < \alpha_{i,th} \\ \infty & \text{if } \alpha_{i,t} \geq \alpha_{i,th} \end{cases}, \quad (2)$$

where n_i is a positive real number representing the Incentive Cutoff Factor (ICF). The ICF defines the part of spectrum with

a monetary incentive for the SUs based on the $\alpha_{i,ic}$ & $\alpha_{i,th}$ set at the BS by the WSP i such that when $\alpha_{i,t} < \alpha_{i,ic}$ we have $s_i < p_i$. The value of ICF can be obtained by solving (2) for $\alpha_{i,t} = \alpha_{i,ic}$ and $f_i(\alpha_{i,t}) = 1$ as follows:

$$n_i = \frac{\ln(1 - e^{-f_i(\alpha_{i,t})})}{\ln(\alpha_{i,t}) - \ln(\alpha_{i,th})} = \frac{\ln(1 - e^{-1})}{\ln(\alpha_{i,ic}) - \ln(\alpha_{i,th})}. \quad (3)$$

Figure 4 illustrates the variable nature of $f_i(\alpha_{i,t})$ and thus of s_i in relation to PU demand and fixed reference $p_i = 1$. The part of the spectrum available for SU access ($\alpha_{i,pu}, \alpha_{i,th}$) is also shown in Fig. 4 with a monetary incentive for the SUs when $\alpha_{i,t} \in (\alpha_{i,pu}, \alpha_{i,ic})$, while a monetary penalty when $\alpha_{i,t} \in [\alpha_{i,ic}, \alpha_{i,th}]$.

In addition to the variable nature of s_i , the WSPs need to have a mechanism for further price adjustment, i.e., for profit maximization and for competing with the prices set by the other WSPs in the region. To aid this purpose, we introduce a Price Leveling Factor (PLF) denoted by m_i , for giving the WSPs the flexibility to adjust the SU prices according to their requirements, keeping the configuration parameters $\alpha_{i,ic}$ & $\alpha_{i,th}$ set by the WSPs unchanged. The adjusted SU price \bar{s}_i can be given as follows:

$$\bar{s}_i = (f_i(\alpha_{i,t}))^{m_i} \times p_i, \quad (4)$$

where m_i is a non negative real number.

At $m_i = 0$, we get $\bar{s}_i = p_i$ indicating that as m_i increases \bar{s}_i reduces in the interval of incentive ($\alpha_{i,pu}, \alpha_{i,ic}$), while \bar{s}_i shoots up in the interval of penalty $[\alpha_{i,ic}, \alpha_{i,th}]$. Figure 5 illustrates the price adjustment feature introduced by the PLF and the effect of different values of the PLF on the normalized SU price $f_i(\alpha_{i,t})$ in the intervals of incentive and penalty. The configuration parameters used for Fig. 5 are the same as in Fig. 4, i.e., $\alpha_{i,th} = 0.9$ & $\alpha_{i,ic} = 0.7$ and the normalized SU price $f_i(\alpha_{i,t})$ is plotted with respect to a fixed reference PU price $p_i = 1$. As observed from (4) and Fig. 5, the PLF controls the slope and hence variance of $f_i(\alpha_{i,t})$ and in turn of \bar{s}_i in the incentive or penalty intervals.

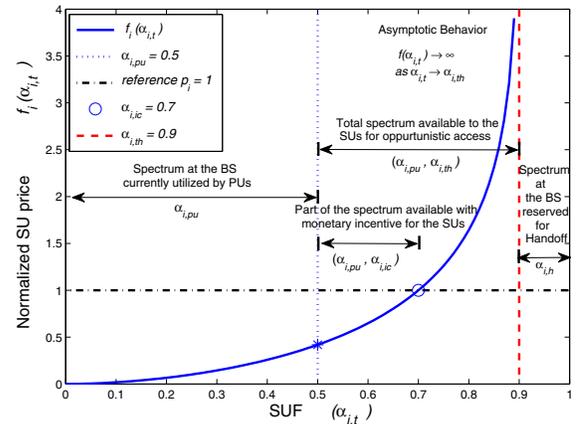


Fig. 4. Variable nature of the incentive based SU pricing model

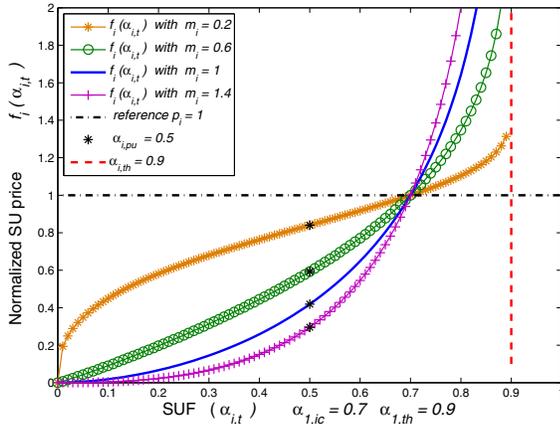


Fig. 5. Effect of different PLFs on the normalized SU price

IV. NETWORK SELECTION AND HANDOFF

In this section, we describe a criterion for network selection at the SU terminal based on the pricing model along with a SU terminal initiated price based handoff scheme.

A. Criterion for Network Selection at the SU Terminal

The network selection criterion at the SU terminal needs to be designed for autonomous WSP selection, without any user intervention. We assume that the application class requested by the SU is offered by the WSP i , i.e., $a_r \in A_o$. The SUs would want their SU terminal to select a network, which provides temporary wireless access with the least SU price s_i but the best QoS. The SU price s_i is the monetary price paid by the SU for temporary wireless access as discussed in the pricing model. The QoS can be judged through the link spectral efficiency η_i which is based on the SNR γ_i observed at the SU terminal for the BS-SU link, and is given as $\eta_i = \log_2(1 + \gamma_i)$, where $\gamma_i > 0$. The utility function defining the SU price to QoS ratio for autonomous network selection at the SU terminal is as follows:

$$\text{Minimize } U_{SU,i} = \left(\frac{s_i}{\eta_i} \right). \quad (5)$$

The above utility function implicitly considers the available bandwidth at the BS for SUs through the SU price s_i , which is low when the spectrum utilization at the BS is low. The SU terminal thus connects to a WSP with the most resources available and the best possible SU-BS link, ensuring the best possible data-rate for the application class in consideration.

B. SU Terminal Initiated Price based Handoff Scheme

On selection of the WSP, based on network selection criterion in (5), the SU at time t_1 is charged a fixed monetary price $s_i(t_1)$ for a fixed time window $(t_2 - t_1)$ set by the WSP i . At time instant t_2 , when the time window expires, the new SU price $s_i(t_2)$ is calculated considering the updated $\alpha_{i,t}$ at the BS. The SU thus will have to pay the new price $s_i(t_2)$ for this next time window. Thus there can be a possibility of

price based handoff, provided the new price $s_i(t_2)$ is higher than previous price $s_i(t_1)$ by a cost threshold C_{th} as follows:

$$s_i(t_2) \geq (1 + C_{th}) \times s_i(t_1), \quad (6)$$

where C_{th} is a positive real number. The value of this cost threshold C_{th} is a SU specific parameter and must be specified by the SU along with the application class required, before network selection. For example, setting $C_{th} = 0.2$, triggers a price based handoff if there is a 20% increase in $s_i(t_1)$. Once the price based handoff is triggered, the SU terminal will proceed to scan all the F-SUBCH in the area, and then using the network selection criterion in (5) select the most suitable WSP.

V. CONCLUSIONS AND FUTURE WORK

A distributed framework for enabling dynamic SU access in heterogeneous wireless networks employing licensed spectrum is described in this paper. The system framework and pricing model demonstrates the revenue potential for WSPs and spectrum access opportunity for SUs by utilization of the spectrum unutilized at the BS of a WSP. This scheme is also amenable for centralized coordinated access but it is rather unlikely for the WSPs to share spectrum related information. The distributed approach proposed could allow a single WSP to provide SU access, without co-operation with other WSPs in the area and hence avoiding centralized coordinating entities. The proposed system can thus be considered as an intermediate step between current infrastructure based networks employing licensed spectrum serving only PUs and the CR based infrastructure free networks of the future.

We are currently evaluating techniques to extend the pricing model and network selection criterion proposed here to a multiple WSP environment.

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