Performance Analysis of SNR-based Selection Combining and BER-based Selection Combining of Signals with Different Modulation Levels in Cooperative Communications

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***Abstract*—Cooperative relaying introduces spatial diversity through the creation of a virtual antenna array. The vast majority of the research in digital cooperative relaying assumes the modulation level used by both the source and relay to be the same. This assumption does not necessarily hold when adaptive modulation is implemented. In conventional selection combining, the branch with the highest SNR is chosen; we refer to this scheme as SNR-based selection combining (SNR-SC). In this paper, we introduce BER-based selection combining (BER-SC), as an alternative to SNR-SC, to be used in cooperative communications when a relay may use a modulation level different than that of the source. We provide BER performance analysis for the SNR-SC and BER-SC schemes and show that BER-SC significantly outperforms SNR-SC, without any increase in complexity. Moreover, we analytically quantify the gain achieved by using BER-SC over SNR-SC through asymptotic approximation. We note that BER-SC and SNR-SC schemes are identical when the received signals belong to the same modulation level.**

Keywords: Cooperative Diversity, Selection Combining, BER Selection Combining, Relay Networks, Diversity Analysis.

I. INTRODUCTION

Cooperative relaying has received tremendous interest in both industry and academia in the recent years. In cooperative relaying, the signals from the source-relay and relay-destination links are properly combined to achieve spatial diversity [1], [2]. Relays can be classified into digital and analog relays. Analog relays amplify and forward the received signal while digital relays decode and forward a regenerated version of the received signal; in this work, digital relaying is considered.

The vast majority of the research in digital cooperative relaying assumes the modulation level used by both the source and relay to be the same. This assumption does not necessarily hold in wireless networks where adaptive modulation is implemented.

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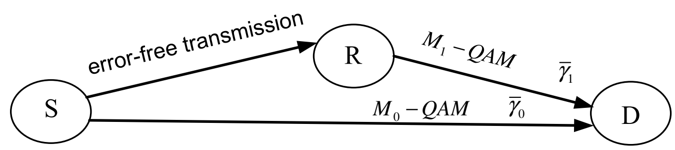

Fig. 1. System model.

In [3] and [4], it is shown that the average throughput of the wireless network can be significantly increased if adaptive modulation and coding is implemented in cooperative relaying. In order to achieve spatial diversity for signals with different modulation levels, selection combining (SC) is used, since this is the least complex diversity combining scheme [3], [4]. Although the literature is rich in the BER performance analysis of conventional SC [5], as far as we know, it is limited to the case of combining signals with the same modulation level. To this end, this paper addresses performance analysis of SC schemes when they are used for combining signals with different modulation levels.

Notation: For a random variable X, $\bar{X} = E\{X\}$ denotes its mean; $Q(x)$ is the right tail of normalized Gaussian probability density function (PDF) given by $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$.

II. SYSTEM MODEL

We consider a three node network consisting of a source (S), a relay (R), and a destination (D), all having one antenna, as shown in Fig. 1.

The transmitting nodes, S and R, transmit on two orthogonal channels, i.e., they do not interfere with each other. For simplicity, we consider time-division multiple access (TDMA) to ensure orthogonal transmission from S and R. In the first time slot, S transmits a packet of C bits to R using M_0-QAM modulation with Gray coding. This packet is overheard by D, because of the broadcast nature of the wireless channel. The

R fully decodes the packet and forwards it to D in the second time slot using M_1-QAM modulation with Gray coding. We focus on these modulation schemes as they are among the most popular schemes in wireless networks [6]. Deciding on which modulation to be used is beyond the scope of this paper and can be found in [3], [4], [7, pp. 54-60]. The D chooses either to decode the signal from S or R. Due to multipath fading, the channel variations in the links S-D and R-D are modeled as independent Rayleigh random variables. In the case of fixed relays deployed by the operator, R can be installed at strategic locations, and as a result, the S-R link can be made very reliable. Therefore, the S-R link is assumed to cause negligible errors, for all practical purposes [3].

The instantaneous SNRs in the links S-D and R-D, which are denoted by γ_0 and γ_1, respectively, are independent exponential random variables. The average SNRs in the links S-D and R-D are denoted by $\bar{\gamma}_0$ and $\bar{\gamma}_1$, respectively.

III. SNR-based Selection Combining and BER-based Selection Combining

In [4], diversity combining of signals with different modulation levels has been dealt with as follow. First, signals with the same modulations are combined using maximal ratio combining (MRC). Then, the signals from the MRC combiners are decoded one-by-one until a packet is decoded correctly, with the help of a cyclic redundancy check (CRC) scheme. In [3], S has full channel state information (CSI) of the links S-D and R-D, and selection diversity is achieved by transmitting only through the link that achieves the highest throughput.

In this work, we will focus on the conventional SNR-based selection combining (SNR-SC) and we will propose the BER-based selection combining (BER-SC) as a better alternative with no additional complexity. In SNR-SC, the receiver decodes the signal only from the branch that has the maximum SNR. When different modulations are employed, the branch that has the maximum SNR may not necessarily be the most reliable branch due to different error-resistance capabilities of the different modulations. Consequently, we introduce BER-SC as a better selection combining scheme in which the receiver decodes the signal from the branch that has the minimum BER.

Using the approximate BER expression for square M-QAM given in [8], the selection criterion can be written as

$$\text{Select branch } i, \text{ where } i = \arg\min_i BER_{M_i}, \tag{1}$$

where

$$BER_{M_i} = c_{M_i} Q\left(\sqrt{2d_{M_i}^2 \gamma_i}\right), \tag{2}$$

and

$$d_{M_i} = \sqrt{\frac{3}{2\left(M_i - 1\right)}}, \text{ and } c_{M_i} = \frac{2\left(1 - 1/M_i\right)}{\log_2 M_i}. \tag{3}$$

Note that BER-SC reduces to SNR-SC in the special case where all the signals belong to the same modulation level. For mathematical tractability, the analysis is limited to the case of single relay. Nevertheless, the analysis can by extended to multiple relays in parallel using similar procedure. We remark that even though our focus in this paper is on square M-QAM modulations, the derived equations are applicable to any modulation scheme that has instantaneous BER in the form $c_{M_i} Q\left(\sqrt{2d_{M_i}^2 \gamma}\right)$.

IV. Performance Analysis of SNR-SC

The instantaneous BER at the output of SNR-SC, given γ_0 and γ_1 can be written as

$$BER_{inst} = \begin{cases} c_{M_0} Q\left(\sqrt{2d_{M_0}^2 \gamma_0}\right), & \gamma_0 \geq \gamma_1 \\ c_{M_1} Q\left(\sqrt{2d_{M_1}^2 \gamma_1}\right), & \gamma_0 < \gamma_1 \end{cases}. \tag{4}$$

The common approach in deriving the average BER is to average the instantaneous BER over the PDF of the output SNR [5]. This approach works when the signals belong to the same modulation level, since in this case the instantaneous BER is a function only of the output SNR. However, this approach does not work in our problem since the instantaneous BER is a piecewise function with intervals that are dependant on the instantaneous SNRs and it can't be expressed as a function of the output SNR only. Consequently, to get the average BER, we average (4) over the joint PDF of γ_0 and γ_1. The joint PDF of γ_0 and γ_1 is the multiplication of the individual PDFs and can be expressed as

$$f\left(\gamma_0, \gamma_1\right) = \begin{cases} \frac{1}{\bar{\gamma}_0} \frac{1}{\bar{\gamma}_1} e^{-\frac{\gamma_0}{\bar{\gamma}_0}} e^{-\frac{\gamma_1}{\bar{\gamma}_1}}, & \gamma_0 \geq 0 \text{ and } \gamma_1 \geq 0 \\ 0, & \text{otherwise} \end{cases}. \tag{5}$$

Using (4) and (5), the average BER can be written as

$$\begin{aligned} BER = & \int_0^\infty \int_0^{\gamma_1} c_{M_1} Q\left(\sqrt{2d_{M_1}^2 \gamma_1}\right) \frac{1}{\bar{\gamma}_0} \frac{1}{\bar{\gamma}_1} e^{-\frac{\gamma_0}{\bar{\gamma}_0}} e^{-\frac{\gamma_1}{\bar{\gamma}_1}} d\gamma_0 d\gamma_1 \\ & + \int_0^\infty \int_{\gamma_1}^\infty c_{M_0} Q\left(\sqrt{2d_{M_0}^2 \gamma_0}\right) \frac{1}{\bar{\gamma}_0} \frac{1}{\bar{\gamma}_1} e^{-\frac{\gamma_0}{\bar{\gamma}_0}} e^{-\frac{\gamma_1}{\bar{\gamma}_1}} d\gamma_0 d\gamma_1. \end{aligned} \tag{6}$$

To simplify the above expression, we define the following function:

$$\begin{aligned} H\left(x; a, b, c\right) &= \int a Q\left(\sqrt{2bx}\right) \tfrac{1}{c} e^{-\frac{x}{c}} dx \\ &= -0.5a\sqrt{\tfrac{bc}{1+bc}} \left(1 - 2Q\left(\sqrt{\tfrac{(1+bc)}{c} 2x}\right)\right) \\ &\quad - 0.5aQ\left(\sqrt{2bx}\right) e^{-\frac{x}{c}}, \end{aligned} \tag{7}$$

where the previous integration is evaluated in [9, Appendix A]. Moreover, we define the following function:

$$\begin{aligned} J\left(x; a, b, c, d\right) &= \int H\left(x; a, b, c\right) \tfrac{1}{d} e^{-\frac{x}{d}} dx \\ &= -\int 0.5a\sqrt{\tfrac{bc}{1+bc}} \tfrac{1}{d} e^{-\frac{x}{d}} dx \\ &\quad + 2a \int \sqrt{\tfrac{bc}{1+bc}} Q\left(\sqrt{\tfrac{(1+bc)}{c} 2x}\right) \tfrac{1}{d} e^{-\frac{x}{d}} dx \\ &\quad - \int aQ\left(\sqrt{2bx}\right) e^{-\frac{x}{c}} \tfrac{1}{d} e^{-\frac{x}{d}} dx \\ &= 0.5a\sqrt{\tfrac{bc}{1+bc}} e^{-\frac{x}{d}} + H\left(x; a\sqrt{\tfrac{bc}{1+bc}}, \tfrac{(1+bc)}{c}, d\right) \\ &\quad - H\left(x; a\tfrac{c}{c+d}, b, \tfrac{cd}{c+d}\right). \end{aligned} \tag{8}$$

The average BER can be expressed in terms of the function $H\left(x;a,b,c,d\right)$ as

$$\begin{aligned}BER=&\int_0^\infty c_{M_1}Q\left(\sqrt{2d_{M_1}^2\gamma_1}\right)\left(1-e^{-\frac{\gamma_1}{\bar\gamma_0}}\right)\tfrac{1}{\bar\gamma_1}e^{-\frac{\gamma_1}{\bar\gamma_1}}d\gamma_1\\&+\int_0^\infty\left(H\left(\infty;c_{M_0},d_{M_0}^2,\bar\gamma_0\right)-H\left(\gamma_1;c_{M_0},d_{M_0}^2,\bar\gamma_0\right)\right)\\&\times\tfrac{1}{\bar\gamma_1}e^{-\frac{\gamma_1}{\bar\gamma_1}}d\gamma_1.\end{aligned}\tag{9}$$

By evaluating the previous definite integral, the average BER can be expressed in terms of the functions $H\left(x;a,b,c,d\right)$ and $J\left(x;a,b,c,d\right)$ as

$$\begin{aligned}BER=&\{H\left(\gamma_1;c_{M_1},d_{M_1}^2,\bar\gamma_1\right)\\&-H\left(\gamma_1;c_{M_1}\tfrac{\bar\gamma_0}{\bar\gamma_0+\bar\gamma_1},d_{M_1}^2,\tfrac{\bar\gamma_0\bar\gamma_1}{\bar\gamma_0+\bar\gamma_1}\right)\\&-H\left(\infty;c_{M_0},d_{M_0}^2,\bar\gamma_0\right)e^{-\frac{\gamma_1}{\bar\gamma_1}}\\&-J\left(\gamma_1;c_{M_0},d_{M_0}^2,\bar\gamma_0,\bar\gamma_1\right)\}\big]_{\gamma_1=0}^{\gamma_1=\infty}\\=&H\left(\infty;c_{M_1},d_{M_1}^2,\bar\gamma_1\right)\\&-H\left(\infty;c_{M_1}\tfrac{\bar\gamma_0}{\bar\gamma_0+\bar\gamma_1},d_{M_1}^2,\tfrac{\bar\gamma_0\bar\gamma_1}{\bar\gamma_0+\bar\gamma_1}\right)\\&+H\left(\infty;c_{M_0},d_{M_0}^2,\bar\gamma_0\right)\\&-J\left(\infty;c_{M_0},d_{M_0}^2,\bar\gamma_0,\bar\gamma_1\right)\\&-H\left(0;c_{M_1},d_{M_1}^2,\bar\gamma_1\right)\\&+H\left(0;c_{M_1}\tfrac{\bar\gamma_0}{\bar\gamma_0+\bar\gamma_1},d_{M_1}^2,\tfrac{\bar\gamma_0\bar\gamma_1}{\bar\gamma_0+\bar\gamma_1}\right)\\&+J\left(0;c_{M_0},d_{M_0}^2,\bar\gamma_0,\bar\gamma_1\right).\end{aligned}\tag{10}$$

Finally, we evaluate (10) using (7) and (8). After considerable simplifications, the average BER can be explicitly written as

$$\begin{aligned}BER=&\tfrac{1}{2}c_{M_0}\left(1-\sqrt{\tfrac{d_{M_0}^2\bar\gamma_0}{1+d_{M_0}^2\bar\gamma_0}}\right)+\tfrac{1}{2}c_{M_1}\left(1-\sqrt{\tfrac{d_{M_1}^2\bar\gamma_1}{1+d_{M_1}^2\bar\gamma_1}}\right)\\&-\tfrac{1}{2}c_{M_0}\tfrac{\bar\gamma_1}{\bar\gamma_0+\bar\gamma_1}\left(1-\sqrt{\tfrac{d_{M_0}^2\bar\gamma_2}{1+d_{M_0}^2\bar\gamma_2}}\right)\\&-\tfrac{1}{2}c_{M_1}\tfrac{\bar\gamma_0}{\bar\gamma_0+\bar\gamma_1}\left(1-\sqrt{\tfrac{d_{M_1}^2\bar\gamma_2}{1+d_{M_1}^2\bar\gamma_2}}\right),\end{aligned}\tag{11}$$

where $\bar\gamma_2\triangleq\frac{\bar\gamma_0\bar\gamma_1}{\bar\gamma_0+\bar\gamma_1}$.

As a sanity check, we evaluate the previous expression for the special case when the signals belong to the same modulation level M as

$$BER=\tfrac{1}{2}c_M\left(1-\sqrt{\tfrac{d_M^2\bar\gamma_0}{1+d_M^2\bar\gamma_0}}-\sqrt{\tfrac{d_M^2\bar\gamma_1}{1+d_M^2\bar\gamma_1}}+\tfrac{\sqrt{d_M^2\bar\gamma_2}}{\sqrt{1+d_M^2\bar\gamma_2}}\right)\tag{12}$$

where $\bar\gamma_2\triangleq\frac{\bar\gamma_0\bar\gamma_1}{\bar\gamma_0+\bar\gamma_1}$. Note that (12) is identical to [5, Eq. 9.210], even though they were derived in very different ways. This suggests that [5, Eq. 9.210] can be viewed as a special case of our derived BER expression for SNR-SC.

V. Performance Analysis of BER-SC

The instantaneous BER at the output of BER-SC, given γ_0 and γ_1, can be written as

$$BER_{inst}=\begin{cases}c_{M_0}Q(\sqrt{2d_{M_0}^2\gamma_0}), & BER_{M_0}\le BER_{M_1}\\ c_{M_1}Q(\sqrt{2d_{M_1}^2\gamma_1}), & BER_{M_0}>BER_{M_1}\end{cases},\tag{13}$$

where BER_{M_0} and BER_{M_1} are given by (2). Similar to the procedure in the previous section, we average (13) over the joint PDF of γ_0 and γ_1. However, intervals of the piecewise function in (13) contains non-linear functions, which makes it difficult to perform the integration. Consequently, we resort to approximating the intervals by linear functions through the use of the following approximation

$$\begin{cases}c_{M_0}Q(\sqrt{2d_{M_0}^2\gamma_0})\approx Q(\sqrt{2d_{M_0}^2\gamma_0}), & \gamma_0>>1\\ c_{M_1}Q(\sqrt{2d_{M_1}^2\gamma_1})\approx Q(\sqrt{2d_{M_1}^2\gamma_1}), & \gamma_1>>1\end{cases}.\tag{14}$$

The high accuracy of the previous approximation is due to the exponential nature of the Q function which results in having its argument as the dominant factor. As it will be shown later, such an approximation significantly simplifies the analysis, while sustaining reasonable accuracy.

By utilizing (14), the intervals of the piecewise function given in (13) can be simplified and the instantaneous BER at the output of BER-SC can be well-approximated as

$$BER_{inst}\approx\begin{cases}c_{M_0}Q(\sqrt{2d_{M_0}^2\gamma_0}), & d_{M_0}^2\gamma_0\le d_{M_1}^2\gamma_1\\ c_{M_1}Q(\sqrt{2d_{M_1}^2\gamma_1}), & d_{M_0}^2\gamma_0>d_{M_1}^2\gamma_1\end{cases}.\tag{15}$$

Using (15) and (5), the average BER can be well-approximated as

$$\begin{aligned}BER\approx&\int_0^\infty\int_0^{\frac{d_{M_1}^2}{d_{M_0}^2}\gamma_1}c_{M_1}Q(\sqrt{2d_{M_1}^2\gamma_1})\tfrac{1}{\bar\gamma_0}\tfrac{1}{\bar\gamma_1}e^{-\frac{\gamma_0}{\bar\gamma_0}}e^{-\frac{\gamma_1}{\bar\gamma_1}}d\gamma_0d\gamma_1\\&+\int_0^\infty\int_{\frac{d_{M_1}^2}{d_{M_0}^2}\gamma_1}^\infty c_{M_0}Q(\sqrt{2d_{M_0}^2\gamma_0})\tfrac{1}{\bar\gamma_0}\tfrac{1}{\bar\gamma_1}e^{-\frac{\gamma_0}{\bar\gamma_0}}e^{-\frac{\gamma_1}{\bar\gamma_1}}d\gamma_0d\gamma_1.\end{aligned}\tag{16}$$

The integrations in (16) can be evaluated using the procedure explained in the previous section; this results in the average BER which can be expressed in terms of the functions $H(x;a,b,c,d)$ and $J(x;a,b,c,d)$ as

$$\begin{aligned}BER\approx&H(\infty;c_{M_1},d_{M_1}^2,\bar\gamma_1)\\&-H(\infty;c_{M_1}\tfrac{d_{M_0}^2\bar\gamma_0}{d_{M_0}^2\bar\gamma_0+d_{M_1}^2\bar\gamma_1},d_{M_1}^2,\tfrac{d_{M_0}^2\bar\gamma_0\bar\gamma_1}{d_{M_0}^2\bar\gamma_0+d_{M_1}^2\bar\gamma_1})\\&+H(\infty;c_{M_0},d_{M_0}^2,\bar\gamma_0)-J(\infty;c_{M_0},d_{M_1}^2,\tfrac{d_{M_0}^2}{d_{M_1}^2}\bar\gamma_0,\bar\gamma_1)\\&-H(0;c_{M_1},d_{M_1}^2,\bar\gamma_1)+J(0;c_{M_0},d_{M_1}^2,\tfrac{d_{M_0}^2}{d_{M_1}^2}\bar\gamma_0,\bar\gamma_1)\\&+H(0;c_{M_1}\tfrac{d_{M_0}^2\bar\gamma_0}{d_{M_0}^2\bar\gamma_0+d_{M_1}^2\bar\gamma_1},d_{M_1}^2,\tfrac{d_{M_0}^2\bar\gamma_0\bar\gamma_1}{d_{M_0}^2\bar\gamma_0+d_{M_1}^2\bar\gamma_1}).\end{aligned}\tag{17}$$

Finally, we evaluate the previous expression using (7) and (8). After considerable simplifications, the average BER can be explicitly approximated as

$$\begin{aligned}BER\approx&\tfrac{1}{2}c_{M_0}\left(1-\sqrt{\tfrac{d_{M_0}^2\bar\gamma_0}{1+d_{M_0}^2\bar\gamma_0}}\right)+\tfrac{1}{2}c_{M_1}\left(1-\sqrt{\tfrac{d_{M_1}^2\bar\gamma_1}{1+d_{M_1}^2\bar\gamma_1}}\right)\\&-\tfrac{1}{2}\tfrac{c_{M_0}d_{M_1}^2\bar\gamma_1+c_{M_1}d_{M_0}^2\bar\gamma_0}{d_{M_0}^2\bar\gamma_0+d_{M_1}^2\bar\gamma_1}\left(1-\sqrt{\tfrac{\bar\gamma_2}{1+\bar\gamma_2}}\right),\end{aligned}\tag{18}$$

where $\bar\gamma_2\triangleq\frac{d_{M_0}^2\bar\gamma_0d_{M_1}^2\bar\gamma_1}{d_{M_0}^2\bar\gamma_0+d_{M_1}^2\bar\gamma_1}$.

Once again, as a sanity check, we evaluate (18) for the special case when the signals to be combined belong to the same modulation level M as

$$BER = \frac{1}{2}c_M\left(1 - \sqrt{\frac{d_M^2\bar{\gamma}_0}{1+d_M^2\bar{\gamma}_0}} - \sqrt{\frac{d_M^2\bar{\gamma}_1}{1+d_M^2\bar{\gamma}_1}} + \sqrt{\frac{\bar{\gamma}_2}{1+\bar{\gamma}_2}}\right) \tag{19}$$

where $\bar{\gamma}_2 \triangleq d_M^2\frac{\bar{\gamma}_0\bar{\gamma}_1}{\bar{\gamma}_0+\bar{\gamma}_1}$. Note that (19) is identical to both (12) and [5, Eq. 9.210]; this is expected since BER-SC reduces to SNR-SC for this special case.

VI. Comparison between SNR-SC and BER-SC

Although the derived BER for SNR-SC and BER-SC given by (11) and (18), respectively, are very useful in estimating the BER performance, it is not straightforward to use them to quantify the gain achieved by using BER-SC over SNR-SC. Consequently, we derive simple asymptotic BER expressions for both schemes and we quantify the asymptotic gain of BER-SC.

We start by writing the average SNRs in the BER expressions given by (11) and (18) as $\bar{\gamma}_0 = \sigma_0^2\mathbf{SNR}$ and $\bar{\gamma}_1 = \sigma_1^2\mathbf{SNR}$. The goal is to get simple expressions for the BER as **SNR** goes to infinity. By using Taylor series expansion and truncating the higher order terms, the following asymptotic approximation can be made [10]

$$1 - \sqrt{\frac{x}{x+1}} \approx \frac{1}{2x} - \frac{3}{8x^2}, \text{ as } x \to \infty. \tag{20}$$

Applying the previous approximation in (11) and going through considerable manipulations and simplifications, we get the following asymptotic BER expression for SNR-SC

$$BER \approx \left(G^{SNR-SC}\mathbf{SNR}\right)^{-2}, \text{ as } \mathbf{SNR} \to \infty, \tag{21}$$

where $G^{SNR-SC} = \frac{4}{\sqrt{3}}\left(\frac{c_{M_0}d_{M_1}^4+c_{M_1}d_{M_0}^4}{d_{M_0}^4d_{M_1}^4}\right)^{-\frac{1}{2}}\sigma_0\sigma_1$. The constant G^{SNR-SC} represents the SNR gain achieved by SNR-SC. Similarly, the asymptotic BER expression for BER-SC can be written as

$$BER \approx \left(G^{BER-SC}\mathbf{SNR}\right)^{-2}, \text{ as } \mathbf{SNR} \to \infty, \tag{22}$$

where $G^{BER-SC} = \frac{4}{\sqrt{3}}\left(c_{M_0}+c_{M_1}\right)^{-\frac{1}{2}}d_{M_0}d_{M_1}\sigma_0\sigma_1$.

By comparing (21) and (22), we observe that both schemes achieve diversity order of 2, i.e., the full diversity (as expected). However, the SNR gain achieved by BER-SC is higher than that of SNR-SC. We define the asymptotic gain (AG) in dB, achieved by BER-SC over SNR-SC as

$$\begin{aligned} AG &= 10\log_{10}\left(\frac{G^{BER-SC}}{G^{SNR-SC}}\right) \\ &= 10\log_{10}\left(\frac{\frac{4}{\sqrt{3}}\left(c_{M_0}+c_{M_1}\right)^{-\frac{1}{2}}d_{M_0}d_{M_1}\sigma_0\sigma_1}{\frac{4}{\sqrt{3}}\left(\frac{c_{M_0}d_{M_1}^4+c_{M_1}d_{M_0}^4}{d_{M_0}^4d_{M_1}^4}\right)^{-\frac{1}{2}}\sigma_0\sigma_1}\right) \\ &= 5\log_{10}\left(\frac{c_{M_0}\frac{d_{M_1}^2}{d_{M_0}^2}+c_{M_1}\frac{d_{M_0}^2}{d_{M_1}^2}}{c_{M_0}+c_{M_1}}\right). \end{aligned} \tag{23}$$

It is interesting to note that AG is independent of the average SNRs and it merely depends on the modulation levels of the signals to be combined. By substituting (3) in (23), we evaluate AG for different scenarios as shown in Table I. Note that AG=0 dB when $M_0 = M_1 = M$, since SNR-SC and BER-SC are equivalent in this scenario. The maximum AG is achieved when $M_0 = 2$ and $M_1 = 64$ and it is equal to 5.95 dB and the minimum AG is achieved when $M_0 = 2$ and $M_1 = 4$. In general, the gain increases as the difference increases between the modulation levels of the signals to be combined.

Table I.
Asymptotic gain achieved by BER-SC over SNR-SC for different scenarios.

Scenario	Asymptotic gain (dB)
M_0=2, M_1=4	0.48
M_0=2, M_1=16	3.19
M_0=2, M_1=64	5.95
M_0=4, M_1=16	1.77
M_0=4, M_1=64	4.45
M_0=16, M_1=64	1.47

VII. Analytical and Simulation Results

In Fig.s 2 and 3, we plot the BER performance of a single relay network using numerical simulation as well as the BER expression for SNR-SC and BER-SC, respectively. It is clear from the figures that there is an excellent agreement between the derived BER expressions and the simulation results which validates the mathematical derivations and justifies the approximations made.

To confirm the accuracy of the asymptotic approximation given by (21) and (22), and the AG given by (23), we plot the exact and the asymptotic BER for different scenarios in Fig. 4. It is clear that the asymptotic expression is tight for high SNRs which also confirms the accuracy of the calculated AGs for the different scenarios. It is worth repeating that such an asymptotic approximation is used merely for quantifying the gain of BER-SC over SNR-SC (as shown in Table 1) and it shouldn't be used as an approximate BER, since such an approximation is loose in the low SNR regime, as shown in Fig. 4.

Since the gains calculated in Table I are valid only asymptotically, we plot the gains achieved by BER-SC over SNR-SC for different BER in Fig. 5. The gains are obtained by numerically inverting the BER formulas for BER-SC and SNR-SC given by (11) and (18), respectively. For all scenarios, the gains increase as the BER decreases (i.e., **SNR** increases) and they saturate at the asymptotic gain values given in Table I. This again validates the asymptotic analysis given in Section VIII. Moreover, most of the gains are attained for BER values of 10^{-3} or less, which are reasonable BER values for uncoded schemes.

VIII. Conclusions

In this paper, we have introduced BER-SC that significantly outperforms SNR-SC in BER performance in combining signals with different modulation levels. This performance gain comes at no penalty in complexity. Moreover, we have derived closed-form BER expressions for both SNR-SC and BER-SC. The derived expression for SNR-SC is more general than the existing expression in literature [5, Eq. 9.210] that applies only to combining signals with the same modulation level. In addition, we have analytically quantified the significant asymptotic gain achieved by using BER-SC over SNR-SC.

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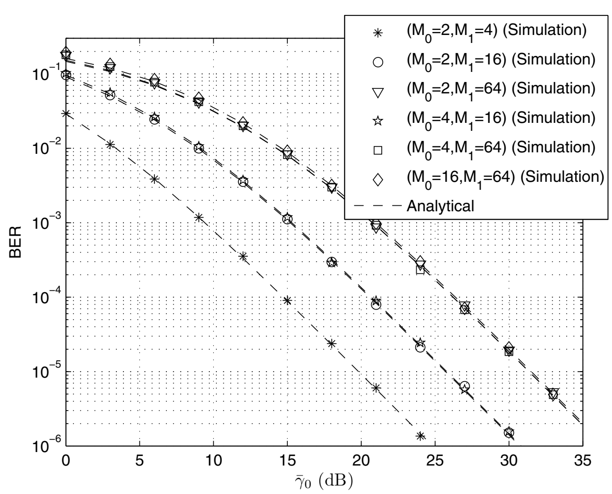

Fig. 2. BER performance of SNR-SC, assuming $\bar{\gamma}_1 = \bar{\gamma}_0 + 10$ dB. It is clear from the figure that there is an excellent agreement between the simulation results and the derived BER expression given by (11).

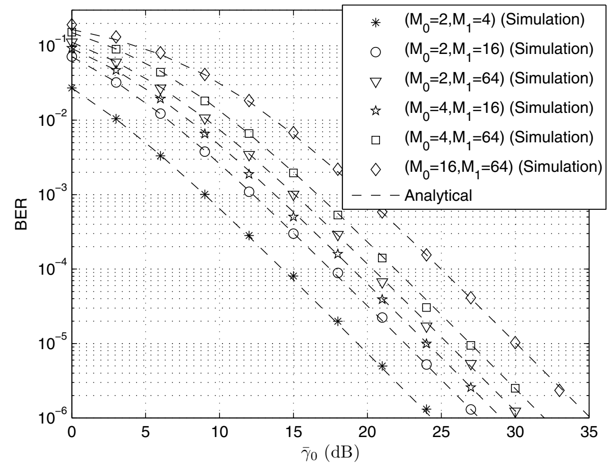

Fig. 3. BER performance of BER-SC, assuming $\bar{\gamma}_1 = \bar{\gamma}_0 + 10$ dB. It is clear from the figure that there is an excellent agreement between the simulation results and the derived BER expression given by (18).

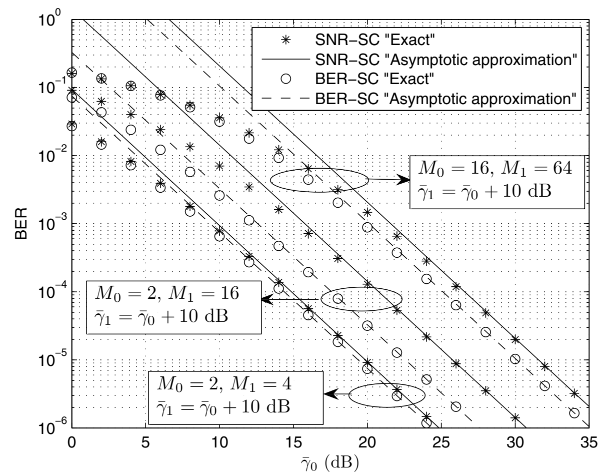

Fig. 4. Asymptotic BER performance of SNR-SC and BER-SC.

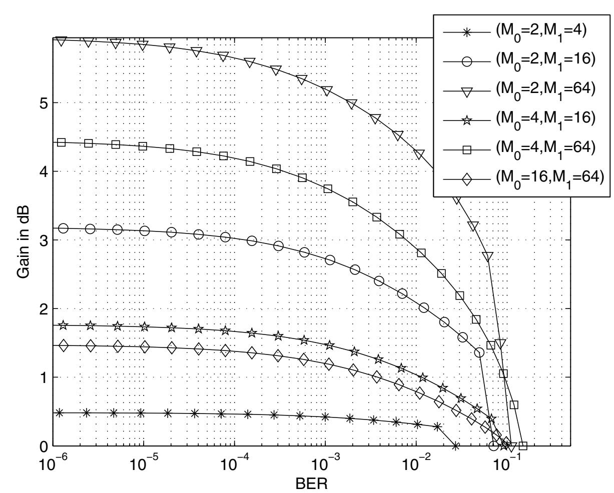

Fig. 5. Gain (in dB) achieved by BER-SC over SNR-SC, assuming $\bar{\gamma}_1 = \bar{\gamma}_0 + 10$ dB.