On the Maximum Diversity Order of Wireless Relay Networks: Common Codebook Generation

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Abstract: This paper derives bounds on the maximum achievable diversity order of wireless relay networks that employ common codebook generation. When all terminals must correctly decode it is shown that the maximum achievable diversity order is constrained by the terminal with the minimum number of immediately preceding terminals. When only the destination terminal must correctly decode it is shown that the maximum achievable diversity order is constrained by the cut set with the minimum number of inter-terminal links.

1 Introduction

Full link connectivity between all cooperating [7] terminals in wireless relay networks with multiple relays will generally not be implemented in practice due to excessive channel allocation requirements [4]. Therefore, an important area of analysis is the maximum achievable diversity order of wireless relay networks with arbitrary, but generally less than full, cooperative connectivity. This paper derives bounds on the maximum achievable diversity order of arbitrarily connected wireless relay networks (with any number of relay terminals and any possible combination of links between cooperating terminals) that employ common codebook generation. Common codebook generation refers to cooperative encoding schemes such as repetition coding and space-time coding [5] where the source and all relays share a common codebook. This is in comparison to independent codebook generation, which refers to cooperative encoding schemes where the source and all relays employ independent randomly generated codebooks.

Two classes of relaying method are considered, those requiring all cooperating terminals to correctly decode the transmitted information signal (*comprehensive decoding*) and those requiring only the destination terminal to correctly decode the transmitted information signal (*destination decoding*). The first class includes decoded relaying with error propagation, also known as fixed decode-and-forward relaying and the second class includes decoded relaying without error propagation, also known as adaptive or selective decode-and-forward relaying, and amplified relaying, also known as amplify-and-forward relaying. In general, this paper focuses on physical level cooperative diversity techniques, and does not explicitly consider recently proposed network level cooperative diversity techniques, for example the opportunistic relaying protocol proposed in [2]. However, it is expected that the analysis of this protocol would be similar to the analysis of networks with destination decoding, as at a high level the behavior is the same as the selective decode-and-forward protocol when only a single relay correctly decodes the source transmission.

Previous work has derived the diversity order for specific combinations of cooperative connectivity and relaying method. It is shown in [1] and [5] that the diversity order of a relay network with *N-1* relays connected to the source and destination in parallel with the source and destination also connected is *N* for both decoded relaying without error propagation and amplified relaying. It is shown in [6] that the diversity order of a fully connected relay network with one relay is one for decoded relaying with error propagation, and two for decoded relaying without error propagation and amplified relaying. It is shown in [8] that the diversity order of a fully connected relay network with two relays is three for amplified relaying. It is shown in [9] that the maximum diversity order of a fully connected relay network with *N-1* relays is *N*, and that higher diversity order can only be achieved when clustering results in a change in the underlying channel model assumptions. The current paper presents a more general formulation applicable to cooperative networks with any number of relay terminals and any possible combination of links between cooperating terminals.

Although the results for networks with comprehensive decoding may appear quite intuitive, it is believed that there is value in a more formal derivation that provides definitive results for any number of relay terminals and any possible combination of links between the pairs of terminals. For example, the intermediate high SNR outage probability results are less intuitive, and can be used to help derive the diversity-multiplexing tradeoff [10] of the corresponding networks. Additionally, the results are applicable not only to the decode-and-forward protocol that is generally understood to offer inferior diversity performance to other cooperative diversity protocols [6], but also to cooperative broadcasting protocols that by definition require all cooperating terminals to correctly decode. Finally, it is quite interesting to note the parallels of the formulations based on maximum mutual information across all terminals and maximum mutual information across all cut sets for comprehensive decoding and destination decoding respectively.

2 System Model

The system model used in this paper is similar to the one developed in [3]. Let T_S, T_I, and T_D respectively denote the sets of source, intermediate (relay), and destination terminals, let $T_T = T_S \bigcup T_I$ denote the set of all transmitting terminals, and let $T_R = T_I \bigcup T_D$ denote the set of all receiving terminals.

Let $T_{P(i)}$ denote the set of terminals that transmit a signal received by terminal T_i, which defines the direction and presence of links between pair of terminals. Also, let S_R denote the complete set of distinct cut sets associated with the directed network graph, let L_i denote the set of inter-terminal links associated with a particular cut set S_i, and let each $L_{k,l} \in L_i$ denote the inter-terminal link that joins terminals T_k and T_l across cut set S_i. Finally, let $T_{O(m)}$ denote the set of receiving terminals that have m immediate preceding terminals (i.e. $|T_{P(i)}| = m, \forall T_i \in T_{O(m)}$ where $|T_{P(i)}|$ is the cardinality of $T_{P(i)}$), and let $S_{O(m)}$ denote the set of cut sets that are associated with m inter-terminal links (i.e. $|L_i| = m, \forall S_i \in S_{O(m)}$ where $|L_i|$ is the cardinality of L_i). Notation of the form x_{T_i} is abbreviated to x_i for simplicity of exposition.

Each terminal T_i transmits a discrete-time signal with complex baseband amplitude given by

$$s_i = \sqrt{\varepsilon_i}(\alpha_i + \beta_i), \quad (1)$$

where ε_i is the transmitted power, α_i is the complex amplitude of the information symbol over a given signaling interval, and β_i is propagated noise. This model normalizes the transmitted signal such that $|\alpha_i|^2 + E[|\beta_i|^2] = 1$. For terminals which do not propagate noise, $\beta_i = 0$ and $|\alpha_i|^2 = 1$.

Each terminal T_i then receives from each immediately preceding terminal $T_k \in T_{P(i)}$ over the link $L_{k,i}$ a discrete-time signal with complex baseband amplitude given by

$$r_{k,i} = a_{k,i}\sqrt{\varepsilon_k}(\alpha_k + \beta_k) + z_{k,i}, \quad (2)$$

where $a_{k,i}$ captures the effects of distance-dependant attenuation, shadowing, and fading between T_k and T_i, and $z_{k,i}$ is a zero-mean Gaussian random variable with variance $N_{k,i}$ that captures the combined effects of local thermal noise and other interference. For the case of mutually independent flat slow Raleigh fading each $a_{k,i}$ is modeled as an iid complex Gaussian random variable with variance $\sigma^2_{k,i}$.

The link SNR at terminal T_i from each immediately preceding terminal $T_k \in T_{P(i)}$ is given by

$$\psi_{k,i} = SNR\mu_{k,i}\left|a_{k,i}\right|^2, \tag{3}$$

where $SNR = \varepsilon_0 / N_0$ is a reference SNR and $\mu_{k,i} = SNR^{-1}\varepsilon_k / N_{k,i}$ is a scaling factor for each link SNR with respect to the reference SNR. The propagated noise β_k is not present in result (3) because the link SNR by definition does not include propagated noise (i.e. $\beta_k = 0$ and $\left|\alpha_k\right|^2 = 1$) and is an upper bound on the SNR for relaying methods that involve propagated noise. Although the results (2) and (3) do not imply any particular channel allocation, when the received signals from all of the terminals belonging to the set $T_{P(i)}$ are combined it requires either orthogonal channel allocation or techniques that compensate for the superposition of signals from multiple previous terminals on a single channel. It is assumed that all relays operate in half-duplex mode and that a network with N transmitters in general requires $2 \le K \le N$ orthogonal channels, resulting in a rate factor of $1/K$. The system model does not imply any particular method by which the set of relay terminals, or set of active links between the pairs of terminals, are chosen. It is equally applicable to possible methods where the links are set up in advance via some static allocation scheme, where the links are dynamically chosen for each signal transmission based on current channel conditions, and where all available links (those not experiencing significant shadowing or obstruction) are always used, among other possible methods.

The method used in the remainder to calculate the diversity order based on the $SNR \to \infty$ behavior of the probability of the maximum average mutual information falling below a target rate R is similar to that used in [5]. Different from [1], for practicality the model is constrained to non-overlapping symbol periods. The codebook generation scheme considered in this paper is one where the source and all relays share a common randomly generated iid circularly symmetric, complex Gaussian codebook. This common codebook generation encompasses many practical encoding schemes, including both repetition coding and space-time coding. For common codebook generation the maximum average mutual information when multiple inter-terminal links are combined takes the form of a log-sum, $I = \log\left(1 + \sum \psi_i\right)$. This is in comparison to a codebook generation scheme where the source and all relays employ independent randomly generated iid circularly symmetric, complex Gaussian codebooks. This independent codebook generation corresponds to utilizing parallel channels, and provides an information theoretic lower bound on the probability of outage of all achievable codebook generation schemes. For independent codebook generation the maximum average mutual information when multiple inter-terminal links are combined takes the form of a sum-of-logs, $I = \sum \log(1+\psi_i)$.

3 Networks with Comprehensive Decoding

The probability of outage of relay networks employing relaying methods with comprehensive decoding is at least the probability of outage at any cooperating terminal in the network, since all cooperating terminals are required to correctly decode. The maximum average mutual information at terminal T_i (between terminal T_i and all immediately preceding terminals $T_{P(i)}$) is given by

$$I_i = \frac{1}{K}\log\left(1 + SNR \sum_{T_k \in T_{P(i)}} \mu_{k,i}\left|a_{k,i}\right|^2\right), \tag{4}$$

and the probability of outage at T_i is the probability of the maximum average mutual information falling below a target rate R, and is given by

$$p_i^{out}(SNR, R) = \Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR}-1}{SNR}\right]. \tag{5}$$

Since outage events at different cooperating terminals are independent, the total probability of outage at any cooperating terminal in the network is given by

$$p^{out}(SNR, R) = 1 - \prod_{T_i \in T_R}\left(1 - \Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR}-1}{SNR}\right]\right), \tag{6}$$

which when expanded to show all possible terminal outage event combinations is expressed as

$$\begin{aligned} p^{out}(SNR, R) = & \sum_{T_i \in T_R}\left(\Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR}-1}{SNR}\right]\right) \\ & - \sum_{\substack{T_i, T_j \in T_R \\ T_i \neq T_j}}\left(\Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}\left|a_{k,i}\right|^2 < \frac{2^{KR}-1}{SNR}\right]\Pr\left[\sum_{T_k \in T_{P(j)}} \mu_{k,j}\left|a_{k,j}\right|^2 < \frac{2^{KR}-1}{SNR}\right]\right) \\ & + \sum \textit{other terms with an odd \# of outage events} - \sum \textit{other terms with an even \# of outage events} \end{aligned} \tag{7}$$

This can be further expanded to separate out terms involving terminals with the minimum number of immediately preceding terminals M_T, and taken to the limit as $SNR \to \infty$ to result in

$$\frac{p^{out}(SNR,R)}{\left(\frac{2^{KR}-1}{SNR}\right)^{M_T}} = \begin{pmatrix} \sum\limits_{\substack{T_i \in T_R \\ T_i \in T_{O(M_T)}}} \left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_T} \Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}|a_{k,i}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to (M_T!)^{-1} \prod_{T_k \in T_{P(i)}} \left(\mu_{k,i}\sigma_{k,i}^2\right)^{-1}} \right) \\ + \sum\limits_{\substack{T_i \in T_R \\ T_i \notin T_{O(M_T)}}} \left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_T} \Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}|a_{k,i}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0} \right) \end{pmatrix}$$
$$- \begin{pmatrix} \sum\limits_{\substack{T_i, T_j \in T_R \\ T_i \neq T_j \\ T_i \in T_{O(M_T)}}} \left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_T} \Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}|a_{k,i}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to (M_T!)^{-1} \prod_{T_k \in T_{P(i)}} \left(\mu_{k,i}\sigma_{k,i}^2\right)^{-1}} \underbrace{\Pr\left[\sum_{T_k \in T_{P(j)}} \mu_{k,j}|a_{k,j}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0} \right) \\ + \sum\limits_{\substack{T_i, T_j \in T_R \\ T_i \neq T_j \\ T_i \notin T_{O(M_T)}}} \left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_T} \Pr\left[\sum_{T_k \in T_{P(i)}} \mu_{k,i}|a_{k,i}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0} \underbrace{\Pr\left[\sum_{T_k \in T_{P(j)}} \mu_{k,j}|a_{k,j}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0} \right) \end{pmatrix} . \quad (8)$$
$$+ \sum \underbrace{\text{other terms with an odd \# of outage events}}_{\to 0} - \sum \underbrace{\text{other terms with an even \# of outage events}}_{\to 0}$$
$$\to \sum_{\substack{T_i \in T_R \\ T_i \in T_{O(M_T)}}} \left(\frac{1}{M_T!} \prod_{T_k \in T_{P(i)}} \frac{1}{\mu_{k,i}\sigma_{k,i}^2} \right)$$

where the asymptotic approximation uses CDF results for the sum of independent exponential random variables [5]. The probability of outage at high SNR can therefore be approximated by

$$p^{out}(SNR,R) \approx \sum_{\substack{T_i \in T_R \\ T_i \in T_{O(M_T)}}} \left(\frac{1}{M_T!} \prod_{T_k \in T_{P(i)}} \frac{1}{\mu_{k,i}\sigma_{k,i}^2} \right) \left(\frac{2^{KR}-1}{SNR} \right)^{M_T}, \quad (9)$$

and the maximum diversity order [10] of the network is given by

$$d = -\lim_{SNR \to \infty} \frac{\log p^{out}(SNR,R)}{\log SNR} = M_T = \min_{T_i \in T_R} \left\{ \left| T_{P(i)} \right| \right\}, \quad (10)$$

the minimum number of immediately preceding terminals across all terminals in the network. Since at least one relay receives only from the source, the maximum diversity order of the network is $M_T = 1$. Fig. 1 shows an example network, annotated with the achievable diversity order of each terminal.

4 Networks with Destination Decoding

The probability of outage of relay networks employing relaying methods with destination decoding is at least the probability of outage at any cut set in the network, since an outage event at any cut set will result in an outage event at the destination. The maximum average mutual information at cut set S_i (across all the inter-terminal links associated with cut set S_i) is given by

$$I_i = \frac{1}{K}\log\left(1 + SNR\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2\right), \tag{11}$$

and the probability of outage at S_i is the probability of the maximum average mutual information falling below a target rate R, and is given by

$$p_i^{out}(SNR,R) = \Pr\left[\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right]. \tag{12}$$

Since outage events at different cut sets are not necessarily independent due to the possibility of shared inter-terminal links, the total probability of outage at any cut set in the network is given by

$$p^{out}(SNR,R) = \Pr\left[\bigcup_{S_i\in S_R}\left(\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right)\right], \tag{13}$$

which when expanded to show all possible terminal outage event combinations is expressed as

$$\begin{aligned}p^{out}(SNR,R) = &\sum_{S_i\in S_R}\left(\Pr\left[\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right]\right)\\ &-\sum_{\substack{S_i,S_j\in S_R\\ S_i\neq S_j}}\left(\begin{aligned}&\Pr\left[\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right]\\ &\times\Pr\left[\left(\sum_{L_{k,l}\in L_j}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right)\Big|\left(\sum_{L_{k,l}\in L_i}\mu_{k,l}\left|a_{k,l}\right|^2 < \frac{2^{KR}-1}{SNR}\right)\right]\end{aligned}\right)\\ &+\sum \textit{other terms with an odd \# of outage events} - \sum \textit{other terms with an even \# of outage events}\end{aligned} \tag{14}$$

This can be further expanded to separate out terms involving cut sets with the minimum number of inter-terminal links M_S, and taken to the limit as $SNR \to \infty$ to result in

$$\frac{p^{out}(SNR,R)}{\left(\frac{2^{KR}-1}{SNR}\right)^{M_S}} = \left(\begin{array}{l} \sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_S} \Pr\left[\sum_{L_{k,l}\in L_i} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to (M_S!)^{-1} \prod_{L_{k,l}\in L_i} (\mu_{k,l}\sigma_{k,l}^2)^{-1}} \right) \\ + \sum_{\substack{S_i \in S_R \\ S_i \notin S_{O(M_S)}}} \left(\underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_S} \Pr\left[\sum_{L_{k,l}\in L_i} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0} \right) \end{array} \right)$$

$$- \left(\begin{array}{l} \sum_{\substack{S_i, S_j \in S_R \\ S_i \neq S_j \\ S_i \in S_{O(M_S)}}} \left(\begin{array}{l} \underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_S} \Pr\left[\sum_{L_{k,l}\in L_i} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to (M_S!)^{-1} \prod_{L_{k,l}\in L_i} (\mu_{k,l}\sigma_{k,l}^2)^{-1}} \\ \times \underbrace{\Pr\left[\left(\sum_{L_{k,l}\in L_j} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right) \mid \left(\sum_{L_{k,l}\in L_i} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right)\right]}_{\to 0} \end{array} \right) \\ + \sum_{\substack{S_i, S_j \in S_R \\ S_i \neq S_j \\ S_i \notin S_{O(M_S)}}} \left(\begin{array}{l} \underbrace{\left(\frac{2^{KR}-1}{SNR}\right)^{-M_S} \Pr\left[\sum_{L_{k,l}\in L_i} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right]}_{\to 0} \\ \times \underbrace{\Pr\left[\left(\sum_{L_{k,l}\in L_j} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right) \mid \left(\sum_{L_{k,l}\in L_i} \mu_{k,l}|a_{k,l}|^2 < \frac{2^{KR}-1}{SNR}\right)\right]}_{\to 0} \end{array} \right) \end{array} \right). \quad (15)$$

$$+ \sum \underbrace{other\ terms\ with\ an\ odd\ \#\ of\ outage\ events}_{\to 0} - \sum \underbrace{other\ terms\ with\ an\ even\ \#\ of\ outage\ events}_{\to 0}$$

$$\to \sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\frac{1}{M_S!} \prod_{L_{k,l}\in L_i} \frac{1}{\mu_{k,l}\sigma_{k,l}^2} \right)$$

where the asymptotic approximation again uses CDF results for the sum of independent exponential random variables [5]. The probability of outage at high SNR can therefore be approximated by

$$p^{out}(SNR,R) \approx \sum_{\substack{S_i \in S_R \\ S_i \in S_{O(M_S)}}} \left(\frac{1}{M_S!} \prod_{L_{k,l}\in L_i} \frac{1}{\mu_{k,l}\sigma_{k,l}^2} \right) \left(\frac{2^{KR}-1}{SNR}\right)^{M_S}, \quad (16)$$

and the maximum diversity order [10] of the network is given by

$$d = -\lim_{SNR\to\infty} \frac{\log p^{out}(SNR,R)}{\log SNR} = M_S = \min_{S_i \in S_R} \{|L_i|\}, \quad (17)$$

the minimum number of inter-terminal links across all cut sets in the network. This is equivalent to the number of disjoint paths through the network joining the source and destination. Fig. 2 shows an example network, annotated with the achievable diversity order of each cut set that is relevant given the directed connectivity. We note that [9] also uses a cut set bound argument, although the approach taken is different and is limited to fully connected relay networks.

5 Conclusion

This paper has derived bounds on the maximum achievable diversity order of arbitrarily connected wireless relay networks that employ common codebook generation. When all cooperating terminals must correctly decode it is shown that the maximum achievable diversity order is constrained by the minimum number of immediately preceding terminals across all receiving terminals in the network, which is one. Furthermore, inter-terminal links that are not associated with terminals with the minimum number of immediately preceding terminals do not asymptotically (at high SNR) affect the probability of outage. When only the destination terminal must correctly decode the maximum achievable diversity order is constrained by the minimum number of inter-terminal links across all cut sets in the network, which is the number of disjoint paths through the network. Furthermore, inter-terminal links that are not associated with cut sets with the minimum number of inter-terminal links do not asymptotically (at high SNR) affect the probability of outage. These results are intuitively satisfying as it is natural to think of the diversity order of a network as being equivalent to the minimum number of inter-terminal links that have to "fail" for the network to "fail". Finally, it is important to note that diversity order is only one aspect of the total probability of outage or error of relay networks. The presented results clearly indicate that although the diversity order of various configurations of cooperative connectivity between terminals in multihop relay networks may be the same, the total probability of outage or error will in general be different. This must be kept in mind when comparing the performance of relay networks with different cooperative connectivity operating at specific target rates and signal to noise ratios.

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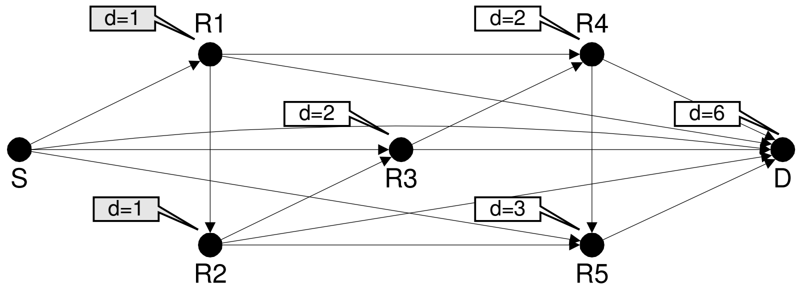

Fig. 1. Diversity Order of Example Network with Comprehensive Decoding ($M_T = 1$)

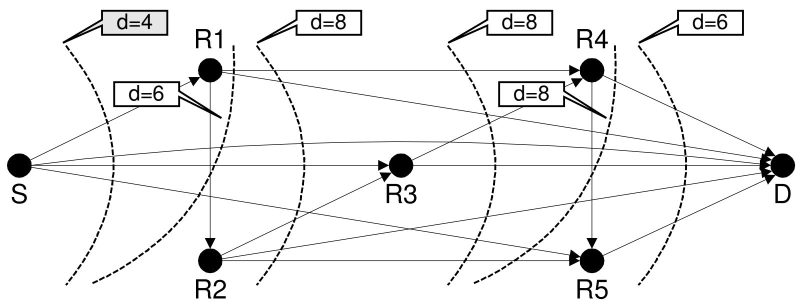

Fig. 2. Diversity Order of Example Network with Destination Decoding ($M_S = 4$)