SIR-Balanced Macro Power Control for the Reverse Link of CDMA Sectorized Distributed Antenna System*

Halim Yanikomeroglu  Elvino S. Sousa  
Department of Electrical and Computer Engineering — University of Toronto  
E-mails: \{halim.sousa\}@comm.toronto.edu

**ABSTRACT**

The CDMA sectorized distributed antenna (SDA) is a novel antenna architecture which yields an increase in the reverse link capacity, in the order of the number of antenna elements used [1]. In an SDA system, a power control algorithm that balances the SIR should be considered, since the conventional power-balanced power control algorithm results in considerable disparities among the SIR levels of different users. However, SIR-balancing for the SDA system is more complicated than that for the conventional central antenna systems due to macrodiversity. We use a power control algorithm which we refer to as SIR-balanced macro power control (SBMPC). SBMPC can be viewed as a special case of the power control algorithm introduced in [2] in the context of CDMA macrodiversity radio networks. In SBMPC, the set of equations to be solved are nonlinear (due to diversity) which makes the solutions for SIR-balancing algorithms, given in the literature, inapplicable. Therefore, we propose an iterative solution to the SBMPC algorithm which always converges. Because of the non-smooth convergence characteristics of the iterations, finding a suitable termination criterion for the iterations is a nontrivial problem. We suggest a multi-stage criterion which yields very low disparities among the SIR levels of different users for reasonably low number of iterations. Although the SBMPC algorithm and its iterative solution addressed in this paper are presented in the context of SDA systems, they may have wide applications. One such application is the power control problem in cellular systems employing macro diversity.

I. INTRODUCTION AND SYSTEM MODEL

The CDMA sectorized distributed antenna (SDA) system has recently been proposed as a promising alternative to the conventional wireless access systems that employ central antennas (CA’s) [1]. We will summarize the main features of the SDA system here.

In an SDA system, many simple antenna elements (AE’s) are connected to a central station (CS) with separate feeders, as illustrated in Fig. 1. All of the intelligence is centralized at the CS; thus, there is no signal-specific processing at the AE’s. In the reverse link of an SDA system, a user’s signal is received by all of these AE’s. As a result of the separate connection to the CS, the multiple access interference, can be reduced by approximately a factor of the number of AE’s, $L$, and therefore, the reverse link SIR (signal-to-interference ratio) can be increased approximately $L$ times compared to a DA system; this increase can be transformed into an equivalent increase in the capacity and/or information rate.

The SDA system is, in essence, still of a DA type, since all the salient characteristics of the DA are preserved (for CDMA DA systems, see [3]–[5]). The AE’s are omnidirectional — the term *sectorized* is used for this system because of the conceptual parallelism with sectorization in the conventional cellular systems.

We consider a flat fading channel model. In such a case, a total of $L$ distinguishable signals (one from each AE) would be received at the CS from each user. Rake receiver and maximal ratio combining are employed at the CS, as shown in Fig. 1. Then, the output SIR at the CS for any user $i$, $\Gamma_i$, would be $\Gamma_i = \sum_{j=1}^{L} \Gamma_{ij}$, $i \in \{1, \ldots, K\}$, where $K$ is the number of users, and $\Gamma_{ij}$ is the SIR at the $j$th finger of the receiver corresponding to user $i$.

We use a matrix notation similar to the ones used in [5] and [1]. The powers of the received signals at the CS are represented by a $K \times L$ matrix $P = \{P_{ij}\}$ such that $P_{ij} = G_{ij} \bar{P}_i$, where $\bar{P}_i$ is the $i$th user’s transmit power, and $G_{ij}$ is the link gain between user $i$ and AE $j$, as shown in Fig. 2.
In an SDA system, the intercell interference is expected to be much less significant compared to a CA system [1]. Therefore, along with the background noise, the intercell interference is also omitted. Then, a user's SIR at a finger of the corresponding Rake receiver would be the ratio of the signal power at that finger to that of the intracell interference.

II. POWER BALANCED POWER CONTROL (PBPC) IN CDMA SYSTEMS

We consider only the reverse link power control (PC) problem in this paper. Many different PC algorithms have been suggested and examined in the literature. A commonly used type in practice (such as in IS-95), due to its computational simplicity and satisfactory performance, is the power-balanced power control (PBPC) algorithm. In PBPC, each row of the received power vector, \( \mathbf{P} \), sums to a constant which we will denote by \( c: \hat{P}_i \sum_{j=1}^{L} G_{ij} = c, \forall i \). That is, for each user, the total received signal power from all the AE’s is the same.

Although PBPC algorithm balances the signal power, it does not guarantee the same interference for all users. Therefore, when PBPC is employed, in general, \( \Gamma_k \neq \Gamma_l \), for \( k,l \in \{1,2,\ldots,K\} \) and \( k \neq l \).

CA Systems: The CA system can be considered as a special case of the DA type with \( L = 1 \). In this case, the required transmitted power for a user \( i \) would be \( P_i = c/G_{i1} \). Then,

\[
\Gamma_{i,CA,PBPC} = N \left( \sum_{k=1}^{K} G_{i1} \hat{P}_k \right) - G_{i1} \hat{P}_i = \frac{N}{K-1},
\]

where \( N \) denotes the CDMA processing gain. The above expression is independent of the user index \( i \). So, in a CA system, PBPC yields the same SIR value for all the users.\(^2\)

DA Systems: In DA systems the PC dynamic range is much smaller compared to CA types. However, PC is still essential in a DA system, even in a system with a very large number of AE’s [5].

In a DA system with PBPC, SIR for a user \( i \) can be calculated as

\[
\Gamma_{i,DA,PBPC} = \sum_{j=1}^{L} \Gamma_{ij} = N \sum_{j=1}^{L} \left( \frac{G_{ij} \hat{P}_i}{\sum_{k=1}^{K} G_{kj} \hat{P}_k} \right) - G_{ij} \hat{P}_i.
\]

Since, the following approximation holds

\[
\left( \sum_{k=1}^{K} G_{kj} \hat{P}_k \right) - G_{ij} \hat{P}_i \approx \sum_{k=1}^{K} G_{kj} \hat{P}_k = \sum_{k=1}^{K} c = Kc,
\]

especially for large \( K \) values, Eqn. 2 can be approximated as

\[
\Gamma_{i,DA,PBPC} \approx N \sum_{j=1}^{L} \frac{G_{ij} \hat{P}_i}{Kc} = \frac{N}{K}.
\]

Hence, every user has approximately the same SIR value. This is shown to be true by extensive simulations in [5].

![Figure 2: The link gain model in an SDA cell.](image)

![Figure 3: CDF of \( \Gamma_{SDA,PBPC} \) for \( L=4, K=5 \), and \( L=4, K=100 \).](image)

SDA Systems: Since PBPC works well in DA systems, it would be logical to investigate the performance of this PC scheme in SDA systems.

In an SDA system, since a substantial portion of the multiple access interference is rejected, \( \Gamma_i \) takes the following form:

\[
\Gamma_{i,SDA,PBPC} = \sum_{j=1}^{L} \Gamma_{ij} = N \sum_{j=1}^{L} \left( \frac{G_{ij} \hat{P}_i}{\sum_{k=1}^{K} G_{kj} \hat{P}_k} \right) - G_{ij} \hat{P}_i.
\]

When PBPC is employed, it is, in general, not accurate to approximate the denominator of the above expression (inside the summation) by the first term which is in parentheses. Moreover, even if this were possible, it would not

\(^1\)If, however, the intercell interference is required to be included in the analysis, then the effective number of users in an SDA cell can be considered to be \( K_e = K/(1+\beta) \), where \( \beta \) is a constant that denotes the effect of intercell interference as a fraction of the intracell type, with \( 0 < \beta \leq 1 \). We anticipate that as \( L \) increases, \( \beta \) would become closer to 0.

\(^2\)In the derivation of Eqn. 1, an ideal case with unlimited PC dynamic range is assumed. In a practical system where there are severe propagation anomalies, to transmit at the required power level may, at least time to time, not be possible. In those cases, SIR for different users would be different and the performance would deteriorate [5].
yield an SIR expression independent of user index, because the
summation \( \sum_{k=1}^{K} G_{kj} \hat{P}_k \) depends on the AE index \( j \),
and thus, does not yield a constant. Therefore, in an SDA
system employing PBPC, SIR for different users may vary
significantly. The cumulative distribution functions (CDF’s)
(corresponding to \( \Gamma_{\text{SDA,PBPC}} \)’s) are shown in Fig. 5 for the
cases of \( L = 4, K = 5, \) and \( L = 4, K = 100. \) As expected, the
disparity among the \( \Gamma_\text{i} \)’s is more significant in unloaded (low
\( K/L \) ratio) systems compared to that in loaded (high \( K/L \)
ratio) ones.

III. SIR-BALANCED PC IN CA SYSTEMS

Since the system performance depends on the SIR, not on
the received power level, a PC scheme which balances SIR,
rather than the received power, would expected to perform
better. Moreover, this is a fair PC scheme since it yields the
same SIR level for all users. In fact, PC algorithms which balance
the SIR value have been studied in the literature, for conversational cellular systems with CA’s, under the name SIR-balancing.

The concept of SIR-balancing is first introduced in [7] in the
context of interference management in satellite communications. These results are broadened in [8]-[10] and applied to non-fading cellular DS-CDMA systems. Subsequently, an optimal global PC algorithm that minimizes outage probability using SIR-balancing with cell removal is demonstrated in [11]; this PC algorithm is applicable mainly for TDMA
and FDMA systems. The main drawback of this algorithm is that it requires continuous access to all radio paths in the system; therefore, its use in practical cellular systems is very
difficult. Nevertheless, the results produce estimates for the
optimal performance. An almost-distributed algorithm that
converges to the optimal one is discussed in [12]. In that
study, each base station (and user) would control its own
transmit power based on only limited knowledge about the
link gain matrix. It is shown in [13] that maximizing the
minimum SIR level, or equivalently minimizing the maximum
one, is equivalent to SIR-balancing.

In [7]-[12], the problem of SIR-balanced PC, which we will
refer to as SBPC, is first presented in matrix notation, then
transformed into an eigenvalue problem, and finally, solved
analytically using matrix algebra. However, there is a major
difference between those cases and the SDA system that we
are analyzing: in the SDA system there is diversity reception
which complicates the PC problem significantly.

In the conventional cellular systems, in the absence of macro
diversity, each user communicates with only one base station.
Obviously, this is significantly different than the SDA system
where each user communicates with all the AE’s. Consequently, modelling of such a conventional cellular system is
quite different than that of an SDA type. If we use the same
\( L \) and \( K \) values for both systems, then for the conventional
cellular case, there would be \( L \) different cells each with one

CA, compared to one cell with \( L \) AE’s in the case of an SDA
system. Also, the number of users in each of the conventional
cells would be \( K/L \), assuming uniform user distribution.

IV. SIR-BALANCED MACRO PC (SBMPc)

In this section, a PC algorithm that balances SIR in SDA
systems is given; we refer to this algorithm as SIR-balanced
macro PC (SBMPc). \(^4\)

SBMPc Algorithm: find \( \{ \hat{P}_i \}^K_{i=1} \), subject to

\[
\Gamma_i, \text{SDA,SBMPc} = N \sum_{j=1}^{L} \frac{G_{ij} \hat{P}_i}{\left( \sum_{k=1}^{K} G_{kj} \hat{P}_k \right)} - G_{ij} \hat{P}_1 = \gamma, \quad \forall i. \tag{6}
\]

Because of the outmost summation (\( \sum_j \)) in Eqn. 6, which
appears as a result of diversity combining, the set of equations
given in Eqn. 6 become nonlinear. Therefore, the eigenvalue
method described in the previous section does not apply here.

However, we can find the transmit power vector, \( \hat{P} \), by solving
Eqn. 6 iteratively. Note that there are \( K \) equations, but
\( K+1 \) unknowns, namely, \( \gamma \) and \( \{ \hat{P}_i \}^K_{i=1} \), Therefore, \( \Gamma_i \)’s will be
obtained within a multiplicative constant. We fix \( \hat{P}_1 \) and
find \( \{ \hat{P}_i \}^K_{i=2} \) as proportional to \( \hat{P}_1 \). Therefore, \( \hat{P}_1 \) at the \( \nu \)th
step of iterations is \( \hat{P}_1^{(\nu)} = \hat{P}_1^{(0)} \).

First, we rearrange Eqn. 6, and denote the \( i \)th equation by
\( \mathcal{E}_i \), as follows:

\[
\mathcal{E}_i: \quad \gamma = N \sum_{j=1}^{L} \frac{G_{ij} \hat{P}_i}{\left( \sum_{k=1}^{K} G_{kj} \hat{P}_k \right)} - G_{ij} \hat{P}_1, \tag{7}
\]

\[
\mathcal{E}_i : \quad \hat{P}_i = \frac{N}{\gamma} \sum_{j=1}^{L} \frac{G_{ij}}{\left( \sum_{k=1}^{K} G_{kj} \hat{P}_k \right)}, \quad i \in [2, \ldots, K]. \tag{8}
\]

Now, we can state the iterative solution.

Iterative Solution for SBMPc Algorithm:

\[
\hat{P}^{(0)} = \left\{ \hat{P}_i^{(0)} \right\} = \left\{ \left( \sum_{j=1}^{K} G_{ij} \right)^{-1} \right\}, \quad \forall i,
\]

\[
\gamma^{(\nu+1)} = \mathcal{E}_1 \left( \hat{P}^{(\nu)} \right),
\]

\[
\hat{P}_1^{(\nu+1)} = \hat{P}_1^{(\nu)},
\]

\[
\hat{P}_i^{(\nu+1)} = \mathcal{E}_i \left( \gamma^{(\nu+1)}, \hat{P}^{(\nu)} - \left( \hat{P}_1^{(\nu)} \right) \right), \quad i \in [2, \ldots, K]. \tag{9}
\]

The algorithm starts with an initial vector which is calculated
according to PBPC. \(^5\) In the above algorithm, \( \mathcal{E}_i(.) \) denotes
inserting the variables in the brackets into \( \mathcal{E}_i \) given in
Eqs 7 and 8.

\(^4\) SBMPc can be viewed as a special case of the PC algorithm introduced in [2] in the context of CDMA macrodiversity radio networks.

\(^5\) Without loss of generality, the constant \( c \) is taken to be 1.
Obviously, one very important concern is the termination criterion for the iterations. This nontrivial issue is discussed in the next section.

V. SIMULATIONS

Simulations are carried out to determine the convergence characteristics of the iterative solution for the SBMPC algorithm introduced in the previous section. The objective is to find a termination criterion that would yield low disparities among the SIR levels of different users for a reasonable number of iterations (NoI’s).

Let us denote the final step of iterations by \( \nu_t \), i.e., NoI = \( \nu_t \). The yielding SIR vector, \( \Gamma^{(\nu_t)} = \{ \Gamma_i^{(\nu_t)} \}_{i=1}^{K} \), can be calculated by using \( \hat{\mathbf{P}}^{(\nu_t)} \) (which is obtained from Eqn. 9) in Eqn. 6. The SIR error for a user \( i \) after \( \nu_t \) steps of iteration, \( \Delta_i^{(\nu_t)} \), is defined as a measure that shows the difference between \( \Gamma_i^{(\nu_t)} \) and the optimal value, \( \gamma^* \): \[
\Delta_i^{(\nu_t)} = 10 \log \left( \frac{\Gamma_i^{(\nu_t)}}{\gamma^*} \right) \text{ dB.} \tag{10}
\]

A. Simulation Parameters

For simulations, a single SDA cell with \( L \) AE’s and \( K \) users is considered. The AE’s are assumed to be uniformly placed in the cell which has a square shape with side length \( a \) meters. It is assumed that \( \sqrt{L} \) is an integer, and that the AE’s are elevated \( h \) meters above the user level. Based on these assumptions, the AE locations can be represented by the vector \( \mathbf{V} = [V_1, V_2, \ldots, V_i, \ldots, V_L]^T \), where \( V_i \)'s are triplet entries, in meters, denoting the coordinates of AE’s:

\[
V_i = \left( \frac{2[(i-1) \mod \sqrt{L}] + 1}{2 \sqrt{L}} a, (1 - \frac{2i/\sqrt{L} - 1}{2 \sqrt{L}})a, h \right). \tag{11}
\]

In the above, \([,]\) denotes the ceiling function. In the simulations, \( L = 4 \) and \( L = 16 \) cases are considered with \( h = 0.02a \).

The users are placed randomly throughout the SDA cell. The first two coordinates of the user locations are determined by two independent uniform random variables in the range \([0, a]\), and the third coordinate is always kept at zero. The simulations are run for various numbers of users with a processing gain of \( N = 128 \).

As stated earlier, the intercell interference is omitted in our analysis. We further assume that the multipath fading is averaged out; in this case, the performance of the system will only depend on the local average received power. Based on these assumptions, the link gains are modeled as \( G_{ij} = A_{ij}/d_{ij}^\alpha \), where \( d_{ij} \) is the distance between user \( i \) and AE \( j \), \( \alpha \) is the distance power law coefficient, and \( A_{ij} \) is a random quantity that models the power variation due to shadowing.

\( A_{ij} \)'s are assumed to be independent, identically distributed log-normal random variables with 0 dB expectation and 8 dB standard deviation. Also, \( \alpha = 4 \) is used in the simulations.

In the simulations, for a certain set of random user locations, first, the \( \mathbf{G} \) matrix is formed, and then the balanced SIR value is calculated through the iterative solution of the SBMPC algorithm introduced in the previous section. This process is repeated for different user location sets in order to collect enough data to plot accurate CDF’s for NoI’s and \( \Delta \).

B. Termination Criterion for Iterations

One straightforward termination criterion would be

\[
\text{stop if } 10 \log \left( \frac{\gamma^{(\nu+1)}}{\gamma^{(\nu)}} \right) \leq \epsilon \text{ dB,} \tag{12}
\]

where \( \epsilon \) is sufficiently close to 0. However, this criterion does not yield satisfactory results, because of the nature of the convergence characteristics of the iterations, which varies according to \( L \) and \( K \) values, and shows some anomalies.

Two types of problems are encountered; we refer to them as type I and type II. A type I problem occurs when

\[
10 \log \left( \frac{\gamma^{(\nu+1)}}{\gamma^{(\nu)}} \right) \leq \epsilon \text{ dB,} \ldots,
\]

\footnote{The optimal values of \( \hat{\mathbf{P}} \) and \( \gamma \) are denoted by \( \hat{\mathbf{P}}^* \) and \( \gamma^* \), respectively.}

Figure 4: A set of user locations that cause a type I problem, for the case of \( L = 4, K = 5 \), and the resulting irregular oscillations in the SIR (● and X denote the AE and user locations, respectively).
10 \log \left( \frac{\gamma^{(n+1)}}{\gamma^{(n)}} \right) \leq \epsilon \text{ dB}, \\
\text{but, } 10 \log \left( \frac{\gamma^{(n+1)}}{\gamma^{(n-1)}} \right) > \epsilon \text{ dB}, 
\tag{13}

for some \( n \geq 1 \). If the termination criterion given in Eqn. 12 is used, a type I problem will cause a premature stop, and this will yield high \( \Delta \) values. Such a case is shown in Fig. 4 where \( \epsilon \) is taken to be 0.04 dB. It is observed from this figure that although 10 \[ \log \left( \frac{\gamma^{(6)}}{\gamma^{(5)}} \right) \leq \epsilon \text{ dB} \] and 10 \[ \log \left( \frac{\gamma^{(7)}}{\gamma^{(6)}} \right) \leq \epsilon \text{ dB} \], 10 \[ \log \left( \frac{\gamma^{(8)}}{\gamma^{(7)}} \right) > \epsilon \text{ dB} \] (which corresponds to \( n = 2 \)).

Type I problems are observed when the users are unevenly located in the cell, as shown in Fig. 4.

A type II problem occurs, on the other hand, when the convergence rate of the iterations is very small. This causes the NoI’s to be very high (however, such cases yield small \( \Delta \) values). Type II problems are observed when each user is close to an AE.

For an \( \epsilon \) arbitrarily close to 0, all the anomalies of type I can be handled. But this would yield a very high NoI for both types of problems (and an unnecessarily high accuracy). Therefore, the solution is not making \( \epsilon \) close to 0, but rather considering a multi-stage termination criterion.

The flow chart of such a multi-stage termination criterion is shown in Fig. 5. We note that the “update \( \gamma \)” and “update \( \tilde{P} \)” lines in the flow chart are performed according to Eqns 7 and 8, respectively. After the final stop, \( \Gamma = \{ \Gamma_1 \} \) is calculated using Eqn. 6.

### C. Observations

Simulations are carried out for a wide range of \( L \) and \( K \) values (\( L = 4 \) and \( K = 5 \) to 100, and \( L = 16 \) and \( K = 20 \) to 400). In all the cases, the iterations have always converged;

![Flow chart of the multi-stage termination program](image)

![CDF of NoI’s](image)

that is, \( \lim_{\nu \to \infty} \tilde{P}_\nu = \tilde{P}^* \), and \( \lim_{\nu \to \infty} \gamma_\nu = \gamma^* \).

Initial transmit power vectors, \( \tilde{P}_0 \), other than that given in Eqn. 9 are also considered, and it is observed that the resulting NoI’s are similar (although, \( \tilde{P}_0 \) given in Eqn. 9 yields slightly lower NoI’s). Therefore, the iterative solution algorithm is very robust with respect to the choice of \( \tilde{P}_0 \); in other words, whatever \( \tilde{P}_0 \) is, the iterative algorithm converges to \( \tilde{P}^* \).

CDF’s for NoI’s and \( \Delta \) are given in Figs 6-7 for \( \epsilon = 0.1 \) dB, and \( m = 1, 3, \) and 5, where \( m \) denotes the number of stages in the multi-stage stop criterion (see Fig. 5). It is observed from these figures that NoI’s are lower for the loaded systems (\( L = 4, K = 100 \) and \( L = 16, K = 400 \)) compared to the unloaded ones (\( L = 4, K = 5 \), and \( L = 16, K = 400 \)). This demonstrates efficiency since the required computation at each iteration step for the loaded systems is much more than that for the unloaded ones (due to the size of the link gain.
matrix $G$).

We note from Fig. 6 that there is significant improvement in the statistics of $\Delta$, when a 3-stage stop criterion is used instead of a 1-stage type. This is due to the fact that most of the type I problems are eliminated by using a 3-stage stop criterion. On the other hand, NoI's increase for $m=3$ case compared to the $m=1$ case (see Fig. 7), as expected, however, the increase is not substantial. Therefore, $m=3$ constitutes a good compromise.

VI. CONCLUDING REMARKS

In this paper, a nonlinear SIR-balanced PC algorithm, which is referred to as the SIR-balanced macro PC (SBMPC) due to its handling of macro diversity, is developed for CDMA SDA systems. An efficient iterative solution that always converges with reasonably low numbers of iteration steps is also discussed.

In the iterative solution of the SBMPC algorithm, it is assumed that we have instantaneous access to the entire link gain matrix $G$. Since all the AE's are connected to a CS, this is not an unrealistic assumption.

Although the SBMPC algorithm and its iterative solution addressed in this paper are presented in the context of SDA systems, they may have wide applications. One such application is the power control problem in cellular systems employing macrodiversity.

References


