Interconnection Strategies for Wireless Access Networks*

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Abstract — The Steiner minimal tree (SMT) architecture is proposed for the wired-network infrastructure of wireless access networks. It is demonstrated that the wireless access networks having star or bus logical topologies can be realized within the proposed optimal SMT conduit structure. The SMT architecture results in a significant reduction in conduit length compared to the conventional star type, besides it provides more flexibility and robustness. For the systems which have logical bus topologies with centralized complexity, such as the distributed antenna (DA) systems, the SMT architecture is optimal in both cable and conduit lengths.

1 Introduction

Building an affordable infrastructure for the future wireless access networks, with thousands of cells and numerous processors/switches, is a major challenge. In a wireless access network, there are two main cost factors: the cost of the base stations, and that of the interconnecting infrastructure that links these base stations, which is proportional to the length of the wired-network. The aim of this paper is to investigate interconnection strategies for wireless access networks, in order to have cost-efficient, as well as robust and flexible, architectures.

In microcellular systems, the different ways of distributing the complexity in the system yield various alternatives in which the above mentioned two cost factors conflict. The systems which have low levels of complexity at the microcell base stations (Micro-BS's) have naturally logical star topologies, resulting in greater network lengths. On the other hand, the systems with logical bus topologies, have shorter lengths; however, they suffer from a higher level of complexity at the Micro-BS's, since the signals from different Micro-BS's should be distinguishable at the Central Station (CS) [1].

The length and logical topology of a network are closely related. However, simply changing the logical topology of a network, in order to decrease its length, is not necessarily a solution, since doing so may increase the cost of the base stations. However, the logical topology and the actual conduit layout do not need to have the same architecture. The results presented in this paper apply mainly to the conduit infrastructure which is far more expensive than the material cost of cable, and different logical topologies can be mapped into the optimal SMT conduit architecture presented in this paper, as will be discussed in the last section.

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2 The Steiner Minimal Tree Architecture

Let $P = \{P_1, P_2, ..., P_N\}$ be a set of N points in the plane, denoting N Micro-BS's to be interconnected. If the vertices of the required tree are exactly the points in P, then the shortest network connecting these points is called a minimal spanning tree (MST).

However, it is possible to construct still shorter trees connecting $P_1, ..., P_N$ by adding extra vertices beside the P_i. A tree interconnecting a set of N points in the plane, by adding extra vertices (Steiner points) if necessary, is called a Steiner tree (ST) if it satisfies the following conditions [2]:

1. No two edges meet at less than 120°.
2. There are at most $N-2$ Steiner points.
3. Each Steiner point has exactly three incident edges meeting at 120°.

An SMT is simply the shortest ST (thus, not every ST is an SMT); and indeed, an SMT is the shortest possible interconnecting network [3]. Finally, an ST is called full (FST) if it has exactly $N-2$ Steiner points.

The SMT constructions for $N \geq 5$ is very difficult. The existence of a finite algorithm for SMT construction was proven by Melzak in 1961 [4] which makes use of the following facts [5]: $SMT(P)$ (which denotes the SMT corresponding to set P) may always be decomposed into sets $SMT(P_{n_1})$, $SMT(P_{n_2})$, ..., $SMT(P_{n_l})$, where P_{n_1}, P_{n_2}, ..., P_{n_l} are subsets of P with $| P_{n_i} \cap P_{n_j} | \leq 1 \quad \forall\, n_i \neq n_j$, where $SMT(P_{n_k})$ is an FST for P_{n_k}, and the edges of the $SMT(P_{n_k})$ form a partition of the edges of $SMT(P)$.

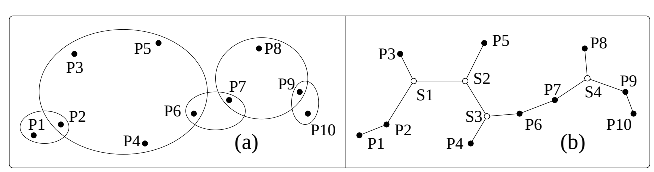


Figure 1: SMT construction for a case of $N = 10$: (a) decomposition, (b) yielding SMT.

Fig.s 1(a) and (b) show such a decomposition and the yielding SMT, respectively, for a case of $N = 10$. Though the total number of possible decompositions is finite, their number will grow exponentially. Indeed, the SMT con-

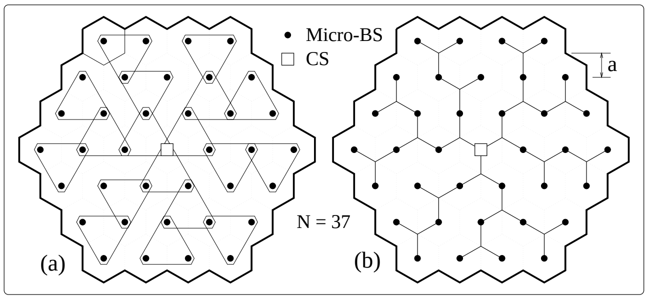


Figure 2: SMT construction in hexagonal layout: (a) decomposition, (b) yielding SMT.

struction for a general set P is NP-complete [6].

3 SMT Construction for Special Layouts

SMT construction in hexagonal and square layouts is worth investigating due to the practical importance of these layouts. Let us first introduce the concept of the Steiner ratio. For a given arbitrary set of points P, if L_{MST} and L_{SMT} denote the lengths of the MST and SMT, respectively, then, the following theorem holds [3]:

> **Steiner Ratio Theorem:** *ρ (Steiner ratio) $= \inf_P(L_{\mathrm{SMT}}(P)/L_{\mathrm{MST}}(P)) = \sqrt{3}/2$, where "inf" is taken over all possible sets of P.*

Using the above theorem the maximum increase in cable length for constructing an MST instead of the SMT can be calculated as

$$\frac{1-\sqrt{3}/2}{\sqrt{3}/2} \times 100\% \simeq 15\%. \quad (1)$$

3.1 Hexagonal Layout

We start by examining the special case of a hexagonal service region in the hexagonal layout.

> **Theorem:** *For a hexagonal service region in the hexagonal layout, the SMT construction is achieved by decomposing the vertices into subsets of 3 points which form equilateral triangles.*

There are many possible ways of forming these 3-point subsets, one of them is illustrated Fig. 2 for the case of $N = 37$.

> *Proof:* In a hexagonal service region, for a given n, there are N=3n(n-1)+1 points; for instance, in Fig. 2, $n = 4$ and $N = 37$. Let L_0 be the length of the tree structure illustrated in Fig. 2(b). Since there are a total of $(N-1)/2$ 3-point subsets with length $3a$,
>
> $$L_0 = \frac{N-1}{2}3a. \quad (2)$$
>
> Since a tree with N vertices has precisely $N-1$ edges, L_{MST} can be found as
>
> $$L_{\mathrm{MST}} = (N-1)\sqrt{3}a. \quad (3)$$
>
> Then,
>
> $$L_0/L_{\mathrm{MST}} = \frac{\sqrt{3}}{2}. \quad (4)$$
>
> But this is equal to ρ, the Steiner Ratio; therefore, $L_{\mathrm{SMT}} = L_0$. This result shows that the decomposition strategy in the form of 3-point subsets depicted in Fig. 2 yields SMT's. □

Based on the above proof, many other interconnecting structures for arbitrary service region shapes can also be shown to be SMT's as long as N is an odd number and the exact decomposition of set P into 3-point subsets is possible as shown in Fig.s 3(a) and (b). If N is an even

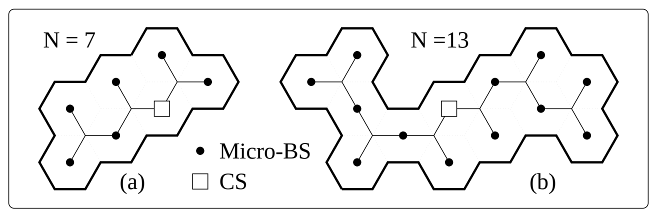


Figure 3: SMT's for general service region shapes with (a) $N = 7$, (b) $N = 13$.

number, however, this exact decomposition into subsets of 3-points cannot be realized.

3.2 Square Layout

To construct SMT's in a layout other than the hexagonal one is quite challenging. In this section we discuss a square layout with square service regions of $N = n^2$ Micro-BS's, with inter-BS distance a. For the special case of $n = 2^k$ (or $N = 2^{2k}$), $k = 1, 2, \ldots$, the SMT is constructed by decomposing the set P into 4-point subsets which form squares [7]. The case for $n = 8$ is shown in Fig. 4(a) and the resulting tree structure is illustrated in Fig. 4(b). There are a total of $(N-1)/3$ such subsets, with each FST for each subset having a length $(\sqrt{3}+1)a$. Therefore,

$$L_{\mathrm{SMT}} = \frac{N-1}{3}(\sqrt{3}+1)a. \quad (5)$$

Since, $L_{\mathrm{MST}} = (N-1)a$,

$$\frac{L_{\mathrm{SMT}}}{L_{\mathrm{MST}}} = \frac{\sqrt{3}+1}{3} = 0.9107 > \rho = \frac{\sqrt{3}}{2} = 0.8660. \quad (6)$$

Hence, the Steiner ratio theorem cannot be used to prove that this structure is an SMT.

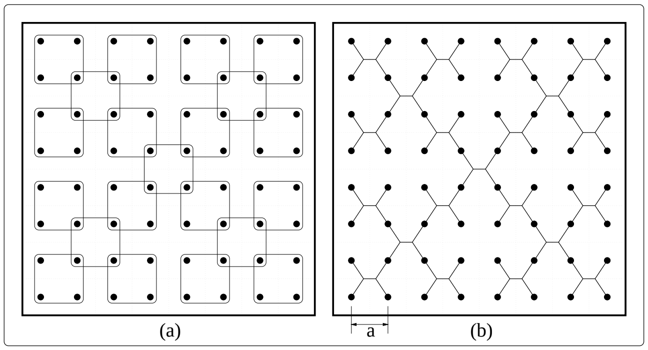
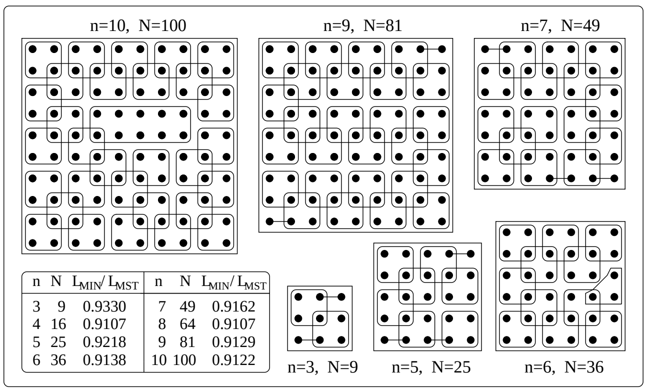


Figure 4: SMT construction in a square layout for the special case of a square service region with $n = 8$: (a) Decomposition, (b) yielding SMT.



n	N	L_{MIN}/L_{MST}	n	N	L_{MIN}/L_{MST}
3	9	0.9330	7	49	0.9162
4	16	0.9107	8	64	0.9107
5	25	0.9218	9	81	0.9129
6	36	0.9138	10	100	0.9122

Figure 5: Decomposition strategies for square service region in square layout.

Fig. 5 shows the appropriate decompositions that yield the minimum[1] lengths for square service regions.[2] The shortest ST's currently known, for an extended set of points in a square layout, can be found in [7].

4 Conduit Length Comparisons between Star and SMT Architectures

The discussions presented in this section and in the following one are based on the hexagonal layout, however, the conclusions can also be extended to the square layout.

Fig. 6(a) shows $L_{\mathrm{STAR}}/L_{\mathrm{SMT}}$, for the case of hexagonal service regions with the CS in the center of the service region (see Appendix for the calculation of L_{STAR}). The remarkable gain in using SMT architecture versus star type is obvious.

In addition, the conduit length in a star architecture depends on the location of the CS in the service region; L_{STAR} yields a minimum value if the CS is in the center of the service region, but, it may increase considerably if the CS is located in some other part of the service region. However, this is not the case for SMT and also for MST architectures. Fig.s 7(a) and (b) illustrate the star and SMT architectures, respectively, for the extreme case where the CS is in the corner; the corresponding MST architecture is also illustrated in Fig. 7(c). As observed from Fig. 6(b), $L_{\mathrm{STAR}}/L_{\mathrm{SMT}}$ further increases for such a case.

[1] Among all the constructions in Fig. 5, only the pattern for the case $n = 3$ has been proven to be a SMT [7]; therefore, the expression L_{MIN} is used to denote the lengths rather than L_{SMT}.

[2] In Fig. 5, there is a 3- and a 10-point subset, for the cases $n = 6$ and $n = 10$, respectively. The construction of FST's for these particular subsets are not demonstrated here.

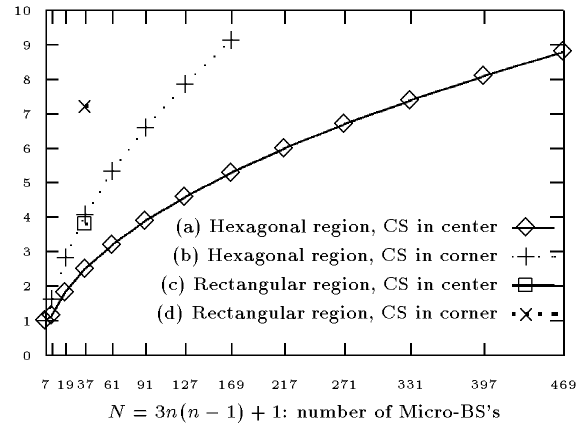


Figure 6: L_{STAR}/L_{SMT} vs. number of Micro-BS's.

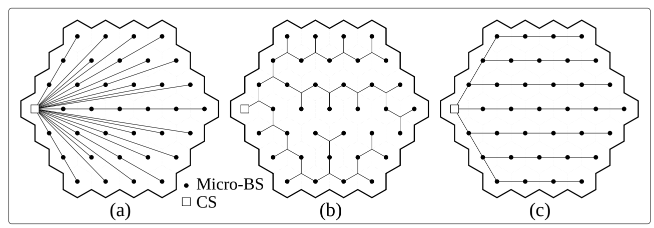


Figure 7: (a) Star, (b) SMT and (c) MST architectures for the case where the CS is in the corner of the service region.

Furthermore, L_{STAR} also depends on the shape of the service region; for a fixed area, if the service region has the shape of a hexagon (which approximates a circle), then the required conduit length is less than that of any other service region shape. The effect of service region shape on L_{SMT}, however, is marginal. In Fig. 8, rectangular service regions in the hexagonal layout with $N = 37$ are illustrated. If the CS is in the center of the service region, then

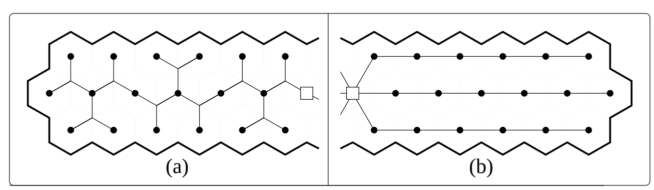


Figure 8: Rectangular cells in hexagonal layout with $N = 37$ Micro-BS's: (a) SMT, and (b) MST architectures with the CS in the center.

$L_{\mathrm{STAR}}/L_{\mathrm{SMT}}$ (see Fig. 8(a)) would be 3.80 (Fig. 6(c)); this is considerably higher than 2.51 (Fig. 6(a)), which is the corresponding ratio for a hexagonal service region. In Fig. 8(b), an MST architecture, for which the conduit length is entirely independent of the service region shape, is also shown.

Obviously, for star architecture the combined effect of the service region shape and the CS location has an even greater impact on the conduit length. For instance, for the case where the CS is located in the corner of a rectangular service region with $N = 37$, $L_{\mathrm{STAR}}/L_{\mathrm{SMT}}$ increases to 7.21 (Fig. 6(d)); this value is significantly higher than 2.51, which is the corresponding ratio for the case where the CS is in the center of a hexagonal service region.

5 Interconnection of Central Stations

One way to establish the interconnection of the CS's is to link a CS and the corresponding Micro-BS's in the form of an SMT, and then connect the CS's and the main switch, again in an SMT form, as shown in Fig. 9(a). We call this structure a *double SMT* architecture. Such a structure can be thought of as a tree with two levels (Fig. 9(b)). It is worth noting that although both levels of this interconnecting tree are constructed as SMT's, the overall tree is not an SMT, nor even an ST. Therefore, the *double SMT* architecture does not yield the minimal length. Nevertheless, it is still shorter than a double star type, simply because the SMT in each level of the tree is shorter than the corresponding star type.

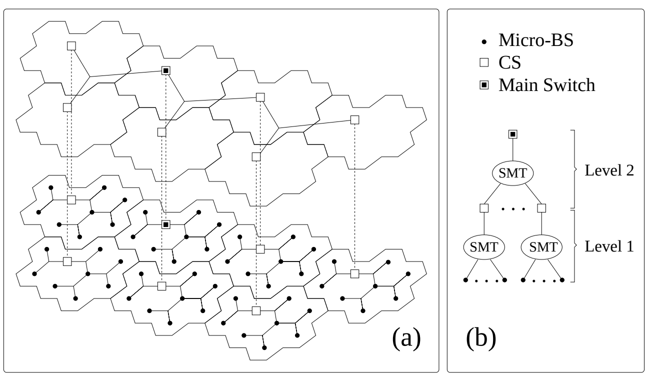


Figure 9: *Double SMT* architecture: (a) details of the architecture, (b) two-level tree representation.

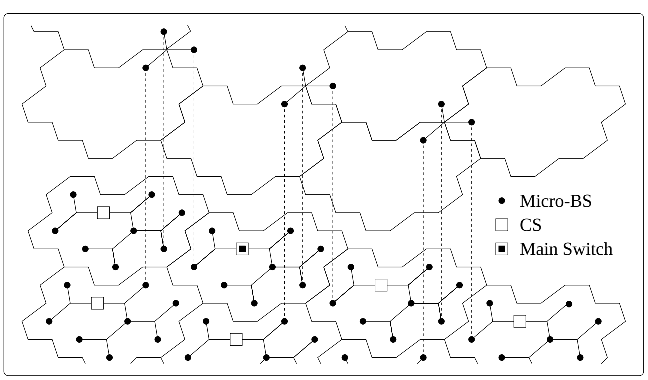


Figure 10: *Extended SMT* architecture.

An improved structure is shown in Fig. 10 where the overall network is a single SMT, hence it yields the minimum length. We call this structure an *extended SMT* architecture.

6 Concluding Remarks

Both star and bus logical topologies can be realized within the optimal SMT conduit structure, as shown in Fig 11.

It is observed from Fig 11(a) that mapping a network with logical star topology into the SMT conduit structure results in some increase in the actual cable length. The total cable length for a network with logical star topology mapped into the SMT conduit structure can be calculated as

$$L = 2n(n-1)(2n-1) = \frac{2}{3}(N-1)\sqrt{1+\frac{4}{3}(N-1)}, \quad (7)$$

for a hexagonal service area in the hexagonal layout, with the CS in the center of the region. If the ratio of Eqn. 7 to Eqn. 12, or to Eqn. 14, is taken, the increase in the cable length, for using SMT conduit architecture, instead of the star type, can be obtained to be less than 27%, $\forall n$. Taking the results shown in Fig. 6 into account, we conclude that the savings from the more expensive conduit layout justify this increase in the cable length.

It is worth noting that both the conduit and cable lengths for the network shown in Fig. 11(b) are minimal, since in this case the actual cable architecture is realized in SMT form as well. This structure may correspond to a distributed antenna (DA) system, which may be a promising alternative to a microcellular system in certain environments [8]. In a DA system, since the same signal is transmitted from and received by all the antenna elements, the system has a logical bus topology with a very low level of complexity at the antenna elements. Therefore, SMT architecture for DA systems yield savings in both cable and conduit lengths.

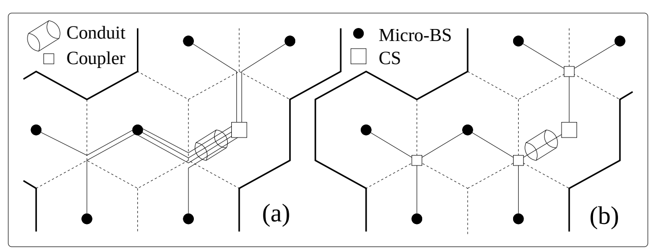


Figure 11: Mapping of (a) logical star, and (b) logical bus topologies into the SMT conduit architecture.

For an arbitrary layout, the most reasonable strategy may be to use an MST structure; because, there are straightforward algorithms for constructing an MST, however, SMT construct is an NP-complete problem. Although there is an increase in the conduit length of the MST architecture over that of the SMT type of at most 15%, both architectures still offer significant reduction in conduit length along with flexibility and robustness, compared to the star architecture.

A Appendix

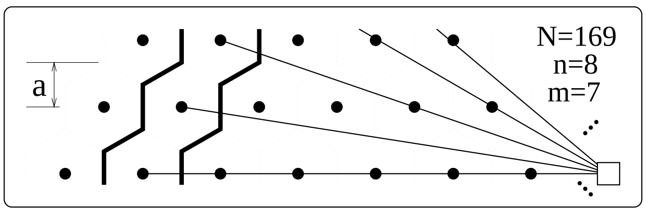


Figure 12: Concentric rings of Micro-BS's around the CS.

In a hexagonal service region with parameter n, there are a total of $n-1$ concentric rings of Micro-BS's around the CS. Let L_m denote the total length of the links between the Micro-BS and the CS for the m-th ring; such a case with $n=8$ and $m=7$ is illustrated in Figure 12. Therefore,

$$L_{\mathrm{STAR}} = \sum_{m=2}^{n} L_m. \quad (8)$$

For the sake of simplicity, let $a=1$. Then, making use of the symmetry of the layout, L_m can be calculated for the odd- and even-numbered layers as

$$\begin{aligned} L_m &= 12 \sum_{i=1}^{\frac{m-3}{2}} \sqrt{\left[\frac{3}{2}(m-1)\right]^2 + \left[(2i)\frac{\sqrt{3}}{2}\right]^2} \\ &+ 6\left[2(m-1)\frac{\sqrt{3}}{2} + \frac{3}{2}(m-1)\right], \quad m: \text{odd}; \quad (9) \end{aligned}$$

$$\begin{aligned} L_m &= 12 \sum_{i=1}^{\frac{m-2}{2}} \sqrt{\left[\frac{3}{2}(m-1)\right]^2 + \left[(2i-1)\frac{\sqrt{3}}{2}\right]^2} \\ &+ 6\left[2(m-1)\frac{\sqrt{3}}{2}\right], \quad m: \text{even}. \quad (10) \end{aligned}$$

We first consider the case where n is an odd number. Now, L_{STAR} can be calculated by making the following substitution

$$m = \left\{ \begin{array}{ll} 2j+1, & m\text{: odd} \\ 2j, & m\text{: even} \end{array} \right\}, \quad j = 1, \ldots, \frac{n-1}{2} \quad (11)$$

in Eqn.s 9 and 10, and then by using Eqn.s 9 and 10 in Eqn 8:

$$\begin{aligned} L_{\mathrm{STAR}} &= 3\sqrt{3}n(n-1) + \frac{9}{4}(n^2-1) \\ &+ 12 \sum_{j=2}^{\frac{n-1}{2}} \sum_{i=1}^{j-1} \left[\sqrt{9j^2+3i^2} + \sqrt{9(j-\frac{1}{2})^2 + 3(i-\frac{1}{2})^2}\right], \\ & n\text{: odd}. \quad (12) \end{aligned}$$

For the case where n is an even number, instead of Eqn. 11, the following substitution should be made in Eqn.s 9 and 10:

$$m = \left\{ \begin{array}{lll} 2j+1, & j = 1, \ldots, (n-1)/2, & m\text{: odd} \\ 2j, & j = 1, \ldots, n/2, & m\text{: even.} \end{array} \right. \quad (13)$$

Following the same procedure described for the n: odd case, L_{STAR} can be calculated as

$$\begin{aligned} L_{\mathrm{STAR}} &= 3\sqrt{3}n(n-1) + \frac{9}{4}n(n-2) \\ &+ 12 \sum_{j=2}^{\frac{n-2}{2}} \sum_{i=1}^{j-1} \sqrt{9j^2+3i^2} \\ &+ 12 \sum_{j=2}^{\frac{n}{2}} \sum_{i=1}^{j-1} \sqrt{9(j-\frac{1}{2})^2 + 3(i-\frac{1}{2})^2}, \quad n\text{: even}. \quad (14) \end{aligned}$$

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