

# Relay ARQ Strategies for Single Carrier MIMO Broadband Amplify-and-Forward Cooperative Transmission

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# Outline

- 1 Relay ARQ System
- 2 Information-Theoretic Analysis
- 3 Simulation Results
- 4 Conclusion and Perspectives
- 5 Related Works

## Brief Description of the Concept

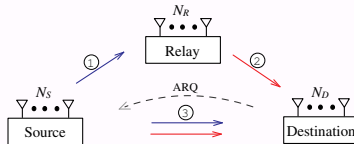


Fig. 1: Relay ARQ System Model

- Channel 1, channel 2, and channel 3 are regarded at  $k$ th transmission as a frequency selective fading MIMO channels having  $L_{SR}$ ,  $L_{RD}$ , and  $L_{SD}$  independent paths, respectively.
- Each path is characterized by its quasi-static flat fading MIMO channel matrix  $\mathbf{H}_l^{AB(k)} \in \mathbb{C}^{N_A \times N_B}$  for  $l \in \{0, \dots, L_{AB} - 1\}$  where  $A \in \{S, R\}$  and  $B \in \{R, D\}$ .
- Relaying works under the framework of half-duplex amplify-and-forward protocol.
- Packet re-transmissions follows the Chase-type ARQ mechanism.
- Each Packet transmission  $k$  within a maximum of  $K$  ARQ rounds spans two consecutive time slots (TS)s.

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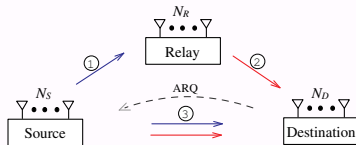


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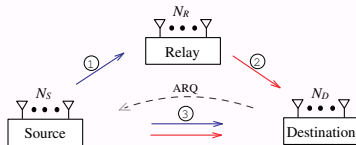


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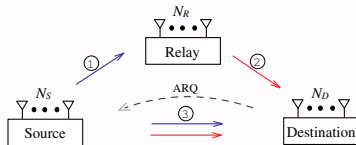


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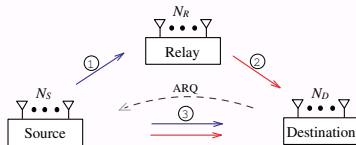


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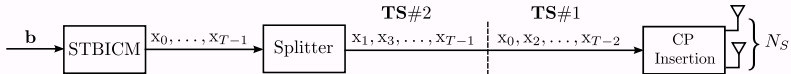


Fig. 2: Source node transmitter scheme.

## Splitting Rule

- Upon the  $1^{st}$  transmission, node  $S$  generates according to an STBICM encoder the symbol packet

$$\mathbf{x} \triangleq [x_0, \dots, x_{T-1}] \in \mathbb{C}^{N_S \times T}. \quad (1)$$

- The symbol vectors  $x_{t'} \in \mathcal{X}^{N_S \times 1}$  for  $t' = 0, \dots, T-1$  are chosen to have equally powered entries, hence satisfying  $\mathbb{E}[x_{t'} x_{t''}^H] = \delta_{t', t''} \mathbf{I}_{N_S}$ .
- It is then splitted into two equally sized  $N_S \times \frac{T}{2}$  sub-packets  $\mathbf{z}_1$  and  $\mathbf{z}_2$  constructed as

$$\begin{cases} z_{1,t} = x_{2t} & , \quad 0 \leq t \leq \frac{T}{2} - 1 \\ z_{2,t} = x_{2t+1} & , \quad 0 \leq t \leq \frac{T}{2} - 1 \end{cases}. \quad (2)$$



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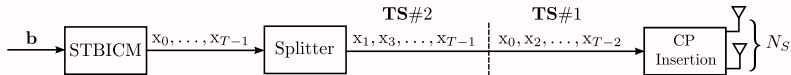


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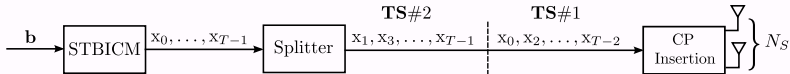


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# Relay ARQ Protocol

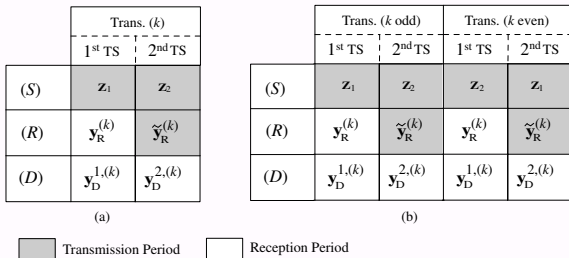


Fig. 3: Relay ARQ Protocol (a), Relay ARQ with Slot-Mapping Reversal (b) for  $k = 1, \dots, K$ .

## Sub-Packets Slot Mapping is Fixed Fig. 3(a)

- $\mathbf{z}_1$  followed by  $\mathbf{z}_2$  during the first and the second TS, respectively, for all the ARQ rounds.

# Relay ARQ with Slot Mapping Reversal

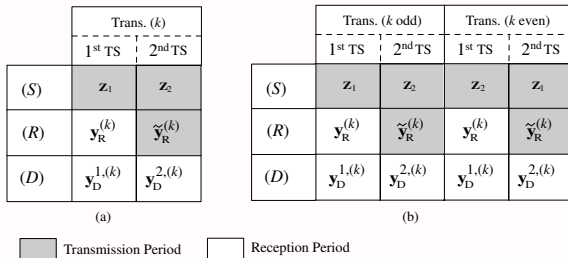


Fig. 3: Relay ARQ Protocol (a), Relay ARQ with Slot-Mapping Reversal (b) for  $k = 1, \dots, K$ .

## Sub-Packets Slot Mapping is Reversed Fig. 3(b)

- Depending on the **transmission index parity**, sub-packets  $\mathbf{z}_1$  and  $\mathbf{z}_2$  are mapped onto either the first or the second time slot.

## Sub-Packets ARQ Transmission Model (I)

During the 1<sup>st</sup> TS of ARQ round  $k$ :

$$\mathbf{y}_{R,t}^{(k)} = \sqrt{E_{SR}} \sum_{l=0}^{L_{SR}-1} \mathbf{H}_l^{SR(k)} \mathbf{z}_{1,(t-l) \bmod \frac{T}{2}} + \mathbf{n}_{R,t}^{(k)} \quad (3)$$

$$\mathbf{y}_{D,t}^{1,(k)} = \sqrt{E_{SD}} \sum_{l=0}^{L_{SD}-1} \mathbf{H}_{1,l}^{SD(k)} \mathbf{z}_{1,(t-l) \bmod \frac{T}{2}} + \mathbf{n}_{D,t}^{1,(k)} \quad (4)$$

- $E_{SR}$  and  $E_{SD}$  are the energies capturing the effects of path loss and shadowing in channel 1 and 3, respectively.
- $\mathbf{n}_{B,t}^{(k)} \sim \mathcal{N}(\mathbf{0}_{N_B \times 1}, N_0 \mathbf{I}_{N_B})$  for  $B \in \{R, D\}$ .
- A cyclic prefix (CP) portion of length  $L_{cp} = \max\{L_{SD}, L_{SR}, L_{RD}\}$  is appended to  $\mathbf{z}_1$  and  $\mathbf{z}_2$  upon their transmission.

AF function at the Relay node:

$$\begin{cases} \tilde{\mathbf{y}}_{R,t}^{(k)} = \gamma \mathbf{y}_{R,t}^{(k)}, & t = 0, \dots, \frac{T}{2} - 1 \\ \gamma = 1/\sqrt{N_S E_{SR} + N_0} \end{cases} \quad (5)$$

## Sub-Packets ARQ Transmission Model (II)

During the  $2^{nd}$  TS of ARQ round  $k$ :

$$\mathbf{y}_{D,t}^{2,(k)} = \sum_{l=0}^{L_{max}-1} \tilde{\mathbf{H}}_l^{(k)} \mathbf{z}_{(t-l) \bmod \frac{T}{2}} + \tilde{\mathbf{n}}_{D,t}^{2,(k)} \quad (6)$$

where

$$\begin{cases} \mathbf{z}_t & \triangleq \begin{bmatrix} \mathbf{z}_{1,t} \\ \mathbf{z}_{2,t} \end{bmatrix} \in \mathcal{X}^{2N_S}, \\ L_{max} & \triangleq \max(L_{SD}, L_{SRD}), \text{ and } L_{SRD} = L_{SR} + L_{RD} - 1, \end{cases} \quad (7)$$

$$\tilde{\mathbf{H}}_l^{(k)} = \left[ \gamma \sqrt{E_{SR} E_{RD}} \underline{\mathbf{H}}_l^{SRD(k)} \quad \sqrt{E_{SD}} \underline{\mathbf{H}}_{2,l}^{SD(k)} \right],$$

$$\tilde{\mathbf{n}}_{D,t}^{2,(k)} = \gamma \sqrt{E_{RD}} \sum_{l=0}^{L_{RD}-1} \mathbf{H}_l^{RD(k)} \mathbf{n}_{R,(t-l) \bmod \frac{T}{2}}^{(k)} + \mathbf{n}_{D,t}^{2,(k)}. \quad (8)$$

## Sub-Packets ARQ Transmission Model (III)

At the end of the second slot node  $D$  builds up (jointly) the augmented size signal vector

$$\mathbf{y}_{D,t}^{equ(k)} \left\{ \begin{bmatrix} \mathbf{y}_{D,t}^{1,(k)} \\ \tilde{\mathbf{y}}_{D,t}^{2,(k)} \end{bmatrix} \right. = \sum_{l=0}^{L_{max}-1} \mathbf{H}_l^{equ(k)} \mathbf{z}_{(t-l) \bmod \frac{T}{2}} + \mathbf{n}_{D,t}^{equ(k)}, \quad (9)$$

in which the  $k$ -parity  $2N_D \times 2N_S$  equivalent MIMO channel matrix  $\mathbf{H}_l^{equ(k)}$  has been carefully introduced with the following form

$$\begin{cases} \mathbf{H}_l^{equ(k)} = \begin{bmatrix} \mathbf{A} & \mathbf{0}_{N_D \times N_S} \\ \mathbf{B} & \mathbf{C} \end{bmatrix}, & k \text{ odd} \\ \mathbf{H}_l^{equ(k)} = \begin{bmatrix} \mathbf{0}_{N_D \times N_S} & \mathbf{A} \\ \mathbf{C} & \mathbf{B} \end{bmatrix}, & k \text{ even} \end{cases} \quad (10)$$

where,

$$\mathbf{A} = \sqrt{E_{SD}} \underline{\mathbf{H}}_{1,l}^{SD(k)}, \quad (11)$$

$$\mathbf{B} = \gamma \sqrt{E_{SR} E_{RD}} L^{-1} \underline{\mathbf{H}}_l^{SRD(k)}, \quad (12)$$

$$\mathbf{C} = \sqrt{E_{SD}} L^{-1} \underline{\mathbf{H}}_{2,l}^{SD(k)}. \quad (13)$$

## Sub-Packets ARQ Transmission Model (III)

In a joint manner signal vector  $\mathbf{y}_{D,t}^{equ(k)}$  is grouped with all the previously received signals  $\mathbf{y}_{D,t}^{equ(k-1)}, \dots, \mathbf{y}_{D,t}^{equ(1)}$  to decode the data packet.

### K ARQ rounds Transmission Model

This leads to the  $2N_{Dk} \times 2N_s$  block transmission model given by

$$\underbrace{\begin{bmatrix} \mathbf{y}_{D,t}^{equ(1)} \\ \vdots \\ \mathbf{y}_{D,t}^{equ(k)} \end{bmatrix}}_{\mathbf{y}_{D,t}^{equ,k}} = \sum_{l=0}^{L_{max}-1} \underbrace{\begin{bmatrix} \mathbf{H}_l^{equ(1)} \\ \vdots \\ \mathbf{H}_l^{equ(k)} \end{bmatrix}}_{\mathbf{H}_l^{equ,k}} \mathbf{z}_{(t-l) \bmod \frac{T}{2}} + \underbrace{\begin{bmatrix} \mathbf{n}_{D,t}^{equ(1)} \\ \vdots \\ \mathbf{n}_{D,t}^{equ(k)} \end{bmatrix}}_{\mathbf{n}_{D,t}^{equ,k}}. \quad (14)$$



# Outage Probability

## Definition (Pertaining to $K=1$ )

The outage probability at a given signal-to-noise ratio (SNR)  $\rho$ , denoted by  $P_{out}$ , refers to the probability half of the information rate  $\mathcal{I}$  (the factor  $\frac{1}{2}$  comes from the fact that one channel use of the equivalent received signal model (9) corresponds to two temporal channel uses), between transmitted block  $\underline{\mathbf{z}}$  and received block  $\underline{\mathbf{y}}_D^{equ,1}$ , is below a target rate  $\mathcal{R}$ ,

$$P_{out}(\rho, \mathcal{R}) = \Pr \left\{ \frac{1}{2} \mathcal{I} \left( \underline{\mathbf{z}}; \underline{\mathbf{y}}_D^{equ,1} \mid \left\{ \mathbf{H}_l^{equ,1} \right\}, \rho \right) < \mathcal{R} \right\} \quad (15)$$

where

$$\underline{\mathbf{z}} = \begin{bmatrix} \mathbf{z}_1 \\ \vdots \\ \mathbf{z}_{\frac{T}{2}} \end{bmatrix}, \text{ and } \underline{\mathbf{y}}_D^{equ,1} = \begin{bmatrix} \mathbf{y}_{D,1}^{equ,1} \\ \vdots \\ \mathbf{y}_{D, \frac{T}{2}-1}^{equ,1} \end{bmatrix}.$$

# Outage Probability

## Generalization

To extend the previous formula on our ARQ relay system, we use the renewal theory as well as the observation that allows us to view the presented Chase-type ARQ mechanism, with a maximum number of rounds  $K$ , as a repetition coding scheme over  $K$  parallel sub-virtual channels. Accordingly, given the equivalent MIMO-ARQ channel model (14), (15) can be re-written as

$$P_{out}(\rho, \mathcal{R}) = \Pr \left\{ \frac{1}{2K} \mathcal{I} \left( \underline{z}; \underline{\mathbf{y}}_D^{equ,K} \mid \left\{ \mathbf{H}_l^{equ,K} \right\}, \rho \right) < \mathcal{R}, \overline{\mathcal{A}}_1, \dots, \overline{\mathcal{A}}_{K-1} \right\},$$

where  $\overline{\mathcal{A}}_k$  represents the event that a NACK feedback is sent back to the source node  $S$  at round  $k = 1, \dots, K - 1$ .

# Average Throughput

The average throughput formula corresponding to the transmission over the equivalent Relay ARQ MIMO channel is given by

$$\eta = \frac{\mathbb{E}[R]}{\mathbb{E}[\nu]}. \quad (16)$$

- $R$  is a discrete random variable equals either to  $\mathcal{R}$  when successful packet decoding is detected within the  $K$  rounds or 0 otherwise.
- In an outage sense, these two values are taken with probabilities  $1 - P_{out}(\rho, \mathcal{R})$  and  $P_{out}(\rho, \mathcal{R})$ , respectively.
- $\nu$  is a RV counting the number of rounds consumed to transmit one packet.

Thus, the **average throughput** (16) can be re-expressed as

$$\eta = \mathcal{R}_\nu (1 - P_{out}(\rho, \mathcal{R})) \quad (17)$$

where  $\mathcal{R}_\nu = \mathcal{R}/\mathbb{E}[\nu]$ .

# Scenario 1

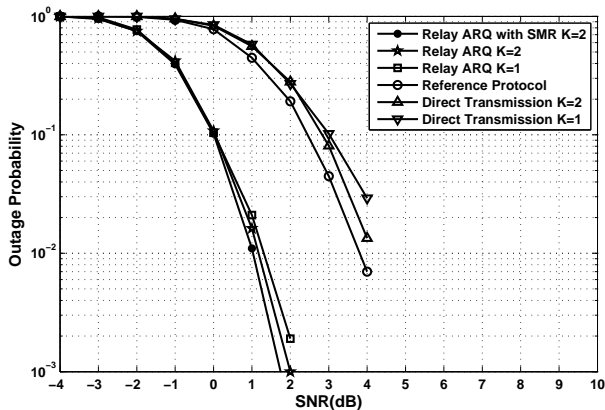


Fig. 4: Outage probability versus SNR for  $l_{SR} = 0.3$ ,  $N_S = N_R = N_D = 2$ ,  $L_{SR} = L_{RD} = L_{SD} = 3$ , and  $\kappa = 3$ .

## Scenario 2

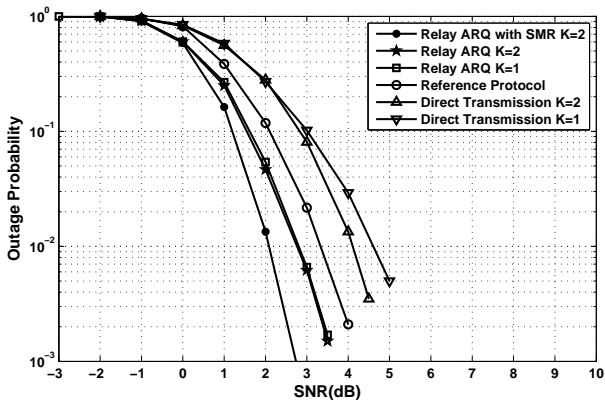


Fig. 5: Outage probability versus SNR for  $l_{SR} = 0.7$ ,  $N_S = N_R = N_D = 2$ ,  $L_{SR} = L_{RD} = L_{SD} = 3$ , and  $\kappa = 3$ .

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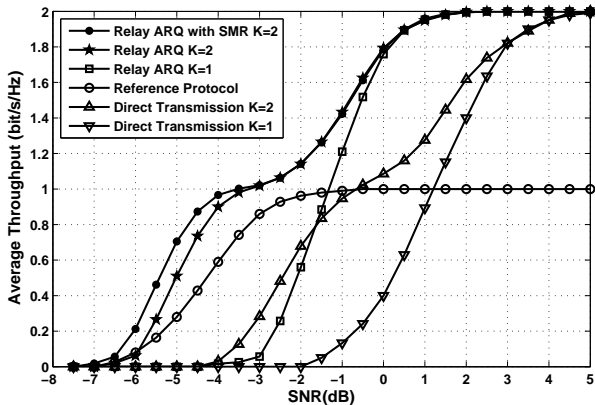


Fig. 6: Average throughput versus SNR for  $l_{SR} = 0.3$ ,  $N_S = N_R = N_D = 2$ ,  $L_{SR} = L_{RD} = L_{SD} = 3$ , and  $\kappa = 3$ .

## Scenario 2

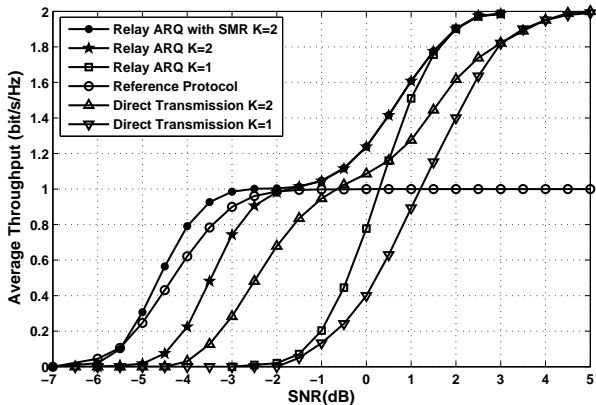


Fig. 7: Average throughput versus SNR for  $l_{SR} = 0.7$ ,  $N_S = N_R = N_D = 2$ ,  $L_{SR} = L_{RD} = L_{SD} = 3$ , and  $\kappa = 3$ .

# Conclusion

- **New throughput-efficient relay ARQ techniques are investigated.**
- The half-duplex constraint has been turned from a disadvantage causing a multiplexing gain loss to an advantage providing significant improvement in average throughput & outage probability performance.
- Relay ARQ with slot mapping reversal provides considerable gain in terms of both outage prob. & average throughput over the entire SNR region.



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# Perspectives

- **It is recommended to design practical turbo receivers that can approach the previous theoretical limits.**
- Analytical results of the outage probability and average throughput instead of Monte-Carlo based simulations should be investigated.
- Extension of the proposed techniques to a multi-user environment where several relays are deployed.

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## Related Works

- Houda Chafnaji, Tarik Ait-Idir, Halim Yanikomeroglu, and Samir Saoudi, "Analysis of Packet Combining for Single Carrier Multi-Relay Broadband Systems," in Proc., IEEE International Workshop on Signal Processing Advances in Wireless Communications, SPAWC 2010, Marrakech, Morocco, Jun. 2010.
- Houda Chafnaji, Halim Yanikomeroglu, Tarik Ait-Idir, and Samir Saoudi, "Turbo Packet Combining Techniques for Multi-Relay-Assisted Systems over Multi-Antenna Broadband Channels," in Proc., ACM International Wireless Communications and Mobile Computing Conference, IWCMC 2010, Caen, France, Jun. 2010.
- Tarik Ait-Idir, and Samir Saoudi, "Turbo Packet Combining Strategies for the MIMO-ISI ARQ Channel," IEEE Transactions on Communications, vol. 57, no. 12, pp. 3782-3793, Dec. 2009.
- Houda Chafnaji, Tarik Ait-Idir, Halim Yanikomeroglu, and Samir Saoudi, "Joint Turbo Equalization for Relaying Schemes over Frequency-Selective Fading Channels," in Proc., ACM International Wireless Communications and Mobile Computing Conference, IWCMC 2009, Leipzig, Germany, Jun. 2009.