The Ergodic and Outage Capacities of Distributed Antenna Systems in Generalized-K Fading Channels

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Motivation

The introduction of distributed antenna systems (DASs) in cellular systems (in general, distributed wireless systems) has raised the following research problems:

- In a DAS, each distributed antenna port (DAP) has different channel gain since uplink/downlink signals between the user terminals and the different antenna ports experience different multipath fading, shadowing and path loss characteristics.
- In a DAS system, each DAP has its own power constraint whereas in collocated MIMO systems, a common power constraint is imposed.
- The dedicated links used to connect the DAPs to the base station (BS) or the central processing unit might be subject to delay or other constraints.

Distributed Antenna Systems

- DASs were first proposed for indoor communications to improve coverage.
- Later, proposals on the use of DASs in code division multiple access (CDMA) systems appeared, where the distributed antennas are integrated in a way that creates deliberate multipath for the temporal processing at the receiver.
- Recent proposals for integrating DASs in current cellular systems are attractive for high data rate requirements.

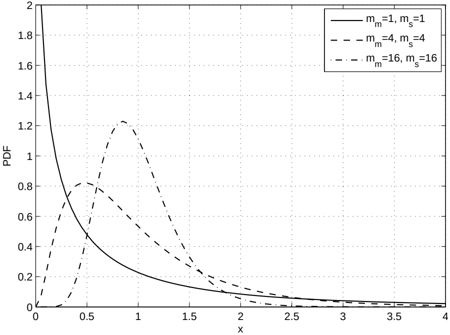
Wireless Channel Modeling

- In wireless channels, the three phenomena of multipath fading, shadowing, and path loss take place concurrently.
- Multipath fading is widely modeled by the Nakagami distribution.
- Shadowing is usually modeled by the lognormal distribution. However, other models such as Gamma distribution and Weibull distribution are used as more tractable models.
- Composite fading models are needed for collocated antenna systems with low mobility and for distributed wireless systems.

The Generalized-K Model

The use of the Nakagami model for multipath fading and the Gamma model for shadowing leads to the generalized-K (GK) model.

$$p_\gamma(x) = \frac{2b^{m_m+m_s}}{\Gamma(m_m)\Gamma(m_s)} x^{(m_m+m_s)/2-1} K_{m_s-m_m}(2b\sqrt{x})$$



DAS Model

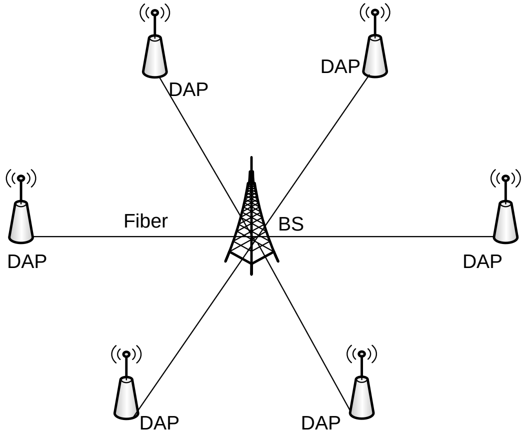


Figure: A typical DAS with six DAPs per cell.

DAS Model (Cont.)

The received signal at the BS, through the set of the cooperating (participating) DAPs, can be expressed as

$$\mathbf{y} = \begin{pmatrix} \sqrt{\frac{w_1}{r_1}} h_1 \\ \vdots \\ \sqrt{\frac{w_{N_{cp}}}{r_{N_{cp}}}} h_{N_{cp}} \end{pmatrix} x + \mathbf{n},$$

where N_{cp} is the number of cooperating DAPs and w_i is the shadowing component. The normalized distance r can be expressed as

$$r_i = \left(\frac{d_{BS}}{d_1}\right)^{\beta} \left(\frac{d_1}{d_i}\right)^{\beta}, \ i = 1, \ldots, N_{cp}.$$

The ratios $\left(\frac{d_{BS}}{d_1}\right)^{\beta}$ and $\left(\frac{d_1}{d_i}\right)^{\beta}$ are used in the selection criterion of the cooperating DAPs.

The Performance of DASs in Cellular Networks

- The ergodic capacity and the information outage probability (for delay-sensitive applications) are the common measures for both single-cell and multi-cell scenarios.
- In past literature, the use of lognormal-based composite fading models has led to numerical or simulation-based approaches.

Single-cell Scenario

The ergodic capacity of DASs in a single-cell can be expressed as

$$C_{erg} = E\left[\log_2\left(1 + \mathrm{SNR}\left(\frac{d_{BS}}{d_1}\right)^{\beta}\sum_{i=1}^{N_{cp}}\left(\frac{d_1}{d_i}\right)^{\beta} z_i w_i\right)\right],$$

where $z_i = |h_i|^2 \ \forall\ i = 1, \dots, N_{cp}$, and SNR is the input signal-to-noise ratio P/σ_n^2. Similarly, the information outage probability can be expressed as

$$P_{out}(R) = Pr\left[\log_2\left(1 + \mathrm{SNR}\left(\frac{d_{BS}}{d_1}\right)^{\beta}\sum_{i=1}^{N_{cp}}\left(\frac{d_1}{d_i}\right)^{\beta} z_i w_i\right) \leq R\right].$$

On the Statistics of the Sum of GK RVs

The sum of GK RVs is essential for the performance analysis of DASs.

- Using the H-function distribution umbrella, the distribution of the product, powers, and quotient of independent GK RVs can be obtained.
- The distribution of the sum of independent H-function RVs is not in general another H-function distribution (handling the products of special functions is possible only for limited cases). Existing results are involved, even for the i.i.d. case.
- Developing approximate distributions is of interest.

On the Distribution for the Sum of Independent GK RVs

The approximation of the PDF of the sum of independent GK RVs by another GK PDF is motivated by the fact that the GK PDF for large values of m_m or m_s can be well-approximated by a Gamma RV and it is well-known that the sum of such independent Gamma RVs is another Gamma RV.

On the Distribution for the Sum of Independent GK RVs

The parameters of the approximating single GK PDF can be obtained using the moment matching method as

$$m_{m,\zeta} = \frac{(1+a) + \sqrt{(1+a)^2 + \frac{4}{\tilde{c} m_m m_s} k_1 a}}{2(1 + a + \frac{1}{m_s})} \tilde{c} m_m,$$

$$m_{s,\zeta} = \frac{m_{m,\zeta}}{a},$$

$$\Omega_{0,\zeta} = \sum_{j=1}^{N} c_j \Omega_{0,j},$$

where $k_1 = (m_m + m_s + 1)$, $a = \frac{m_m}{m_s}$ and $\tilde{c} = \left(\sum_{j=1}^{N} c_j\right)^2 / \sum_{j=1}^{N} c_j^2$.

Extension to the Weighted Sum of Independent GK RVs

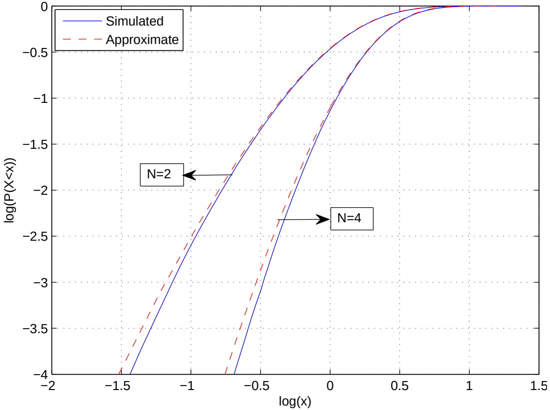


Figure: The log-log (to base 10) CDF plots for the weighted sum of generalized-K RVs and the approximating generalized-K RV for different values of N with $m_m = m_s = 2$ and $c_1 = 1, c_2 = 0.75, c_3 = 0.5,$ and $c_4 = 0.5$.

Extension to the Weighted Sum of Independent GK RVs

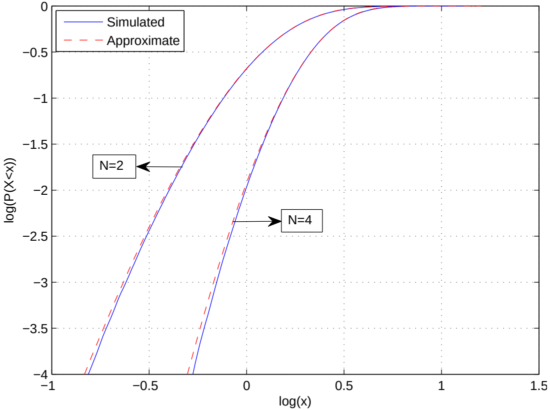


Figure: The log-log (to base 10) CDF plots for the weighted sum of generalized-K RVs and the approximating generalized-K RV for different values of N with $m_m = m_s = 4$ and $c_1 = 1, c_2 = 0.75, c_3 = 0.5,$ and $c_4 = 0.5$.

The Performance of DASs in Cellular Networks

Using the approximation for the distribution of the weighted sum of GK RVs, the approximate expressions for both the ergodic capacity and the outage probability can be computed.

We may express the ergodic capacity as

$$C_{erg} = \frac{1}{\Gamma\left(m_{m,\zeta}\right)\Gamma\left(m_{s,\zeta}\right)} G_{2,4}^{4,1}\left[\frac{m_{m,\zeta}m_{s,\zeta}}{\Omega_{0,\zeta}\mathrm{SNR}_e}\Big|_{0,0,m_{m,\zeta},m_{m,\zeta}}^{0,1}\right],$$

where $G_{m,n}^{p,q}$ denotes the Meijer's function and $\mathrm{SNR}_e = \frac{P}{\sigma_n^2}\left(\frac{d_{BS}}{d_1}\right)^{\beta}$.

The information outage probability can be expressed as

$$P_{out} = \frac{G_{1,3}^{2,1}\left[\frac{m_{m,\zeta}m_{s,\zeta}}{\Omega_{0,\zeta}}\frac{2^R-1}{\mathrm{SNR}_e}\Big|_{m_{m,\zeta},m_{s,\zeta},0}^{1}\right]}{\Gamma\left(m_{m,\zeta}\right)\Gamma\left(m_{m,\zeta}\right)}.$$

Single-cell Scenario

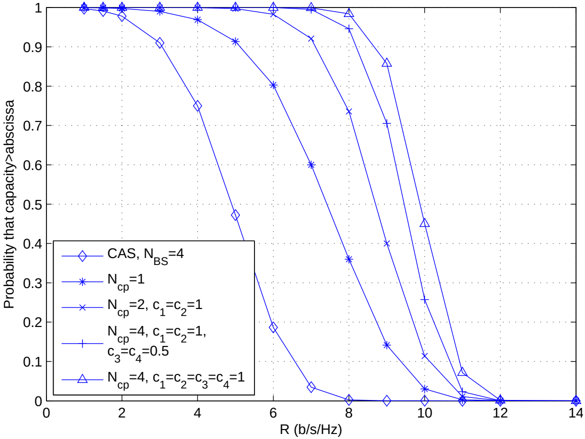


Figure: The plot of the outage probability versus the target rate for different numbers of cooperating DAPs at $P/\sigma^2 = 10 \text{ dB}$.

Single-cell Scenario (Cont.)

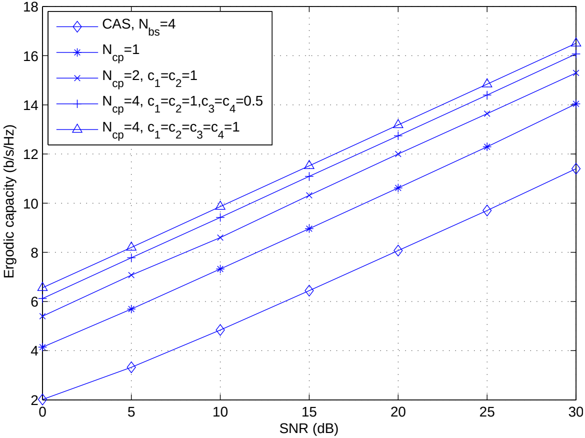


Figure: The plot of the ergodic capacity for different number of cooperating DAPs.

Conclusions

- The introduction of distributed wireless systems has motivated new research problems and/or revisiting other ones.
- The *H*-function umbrella is used to derive the PDFs for the product and the quotient of independent GK RVs.
- The PDF of the sum and the weighted sum of independent GK RVs is well-approximated by another GK PDF. This significantly simplifies further performance analysis.
- Approximate closed-form expressions for the ergodic capacity and information outage probability have been derived for both single-cell and multi-cell scenarios.
- The capacity gain in DASs is mainly due to the reduced access distance and the macrodiversity gain beyond $N_{cp} = 3$ is marginal.

Thank You!

Question?