Identifying Boundaries of Dominant Regions Dictating Spectrum Sharing Opportunities for Large Secondary Networks

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***Abstract*— An important parameter in determining a spectrum sharing opportunity is the level of interference power that secondary users may generate towards primary users. It is indicated in literature that the aggregate interference power of an infinite network (such as a very large secondary network) is bounded under certain conditions. However, to the best of our knowledge, no work has been devoted to determining the boundary of the dominantly interfering region. In this paper, we identify the smallest portion (dominant region) of the secondary network that would impact spectrum sharing opportunities. Our results show that the dominant region is not necessarily a small region encompassing a few interferers within the proximity of the primary user. Far interferers may tangibly contribute to spectrum sharing decisions when a higher approximation accuracy is required or when a wide exclusion region (within which no secondary users are allowed to transmit) is considered. On the other hand, the dominant region shrinks with the increase in the path-loss exponent or in the level of the interference threshold specified by the primary user or a regulator. Some implications of these results are highlighted. Moreover, the results are anticipated to inspire new ideas for designing MAC protocols for secondary networks.**

***Index Terms*—Spectrum Sharing, Secondary Networks, Cognitive Networks, Aggregate Interference, Interference Probability, Cumulants, Poisson Point Process.**

I. Introduction

The management of the radio spectrum is going through a paradigm shift, allowing secondary users (SUs) to share spectrum bands with primary users (PUs) who hold the licenses of these bands. This spectrum sharing would be permissible under the constraint that activities of SUs in these bands do not harmfully disturb the operation of the PUs [1], [2].

In the context of spectrum sharing, it is important to have a metric that identifies when the interference introduced by SUs is considered to be harmful to the PU. There are some proposed metrics in literature [3]–[7]. In this paper, we use a metric called the interference probability [6]. This metric is based on the probability that the interference caused by SUs' transmissions exceeds a certain threshold. If this probability is less than a prescribed level, then the interference is considered to be non-harmful, and hence the SUs can share the spectrum of the PU. Otherwise, SUs should not share the spectrum unless they modify their transmission parameters.

There could be a single SU trying to access the spectrum, or a group of SUs composing a secondary network. In this paper, we want to address the following question: having an infinite secondary network surrounding a PU, what is the dominant portion of this secondary network that will dictate the spectrum sharing opportunities for the whole secondary network?

Works such as [4], [6] study the spectrum sharing of large secondary networks. Other works such as [8]–[11][1] investigate the aggregate interference of large wireless networks for different applications. It is indicated in [8], [12] and some of the references therein that the interference of an infinite wireless network is bounded when the path-loss exponent is strictly greater than the dimension of the space of the network. Authors in [13] show that the cumulants of the aggregate interference asymptotically approach constant values as the spatial size of the network increases provided that the density of interfering nodes remains constant. While there are some comments in literature (e.g., in [14] and [15]) indicating that the aggregate interference is dominated by the nearby interferers to the victim receiver, there is to the best of our knowledge no work devoted to identifying the boundary of the dominant region. Our contribution comes to fill this gap, especially in the context of spectrum sharing. Results reported in this paper are anticipated to be useful for designing MAC protocols for large wireless secondary networks to create and maintain spectrum sharing opportunities.

The next section describes the system model used in this paper. Section III addresses the characterization of the aggregate interference power in a large secondary network. Then, Section IV provides analysis and discussions on the dominant region. Finally, the paper is concluded with some remarks in Section V.

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[1]These papers are just a few examples; interested readers may refer to the reference lists in [10] and [11].

II. System Model

In this paper, we model a large secondary network as an infinite two-dimensional network. A PU receiver (PU-RX) is located within this network, and its location is assumed to be at the center without loss of generality[2]. The PU-RX could be surrounded by an exclusion region of $r_o \geq 0$ in which there are no active SU transmitters (SU-TXs). The SU-TXs are spatially modeled using a Poisson point process (PPP) with a constant intensity (density) of λ.

The SU-TXs can share the spectrum of the PU if their transmissions do not cause harmful interference towards the PU. This harmful interference is quantified using certain metrics [3]–[7]. In this paper, we use the interference probability as the metric to check if the interference experienced by the PU is likely to be harmful or not. According to this metric, a spectrum sharing opportunity exists if

$$P(I_A \geq I_{th}) \leq \beta, \tag{1}$$

where I_A is the aggregate interference power seen by the PU-RX, I_{th} is a threshold value specified by the PU or a regulator, and $\beta \ll 1$ is the maximum allowed interference probability which might be also specified by the PU or the regulator. Equation (1) means that if the probability of the aggregate interference power received by the PU-RX and exceeding I_{th} is less than β, then the interference is considered non-harmful, and hence the SUs can share the spectrum. Otherwise, the SUs should not share the spectrum unless they adapt their transmission parameters. We choose the interference probability metric due to its versatile and fundamental form which relies on the complimentary cumulative distribution function (CCDF) of the interference power. Therefore, our results should be useful even if other metrics are considered provided that those other metrics depend on the distribution of the interference power.

To be able to use (1), the characterization of the distribution of I_A is required. Here, we model I_A as

$$I_A = \sum_{i \in \Lambda} I_i, \tag{2}$$

where I_i is the individual interference power received by the PU-RX due to the transmission of SU-TX i, and Λ is the set of active SU-TXs. The expression in (2) is developed based on the assumption of incoherent addition of interference signals. The individual interference power I_i can be modeled as the multiplication of various deterministic and random variables incorporating different system and channel parameters [10], [13], [16]. In this paper, we model I_i as

$$I_i = X_i g(r_i), \tag{3}$$

where X_i is a positive random variable that could result from the multiplication of various deterministic and random variables, reflecting the effect of transmit power, antenna gains, multipath fading, and shadow fading. We assume that X_i's are independent and identically distributed (i.i.d.), which is a commonly used assumption in many works such as [4], [6], [10], [17]. The function $g(r_i)$ models the distance-dependent attenuation of the received interference power due to the transmission of SU-TX i, which is at a distance r_i from the PU-RX. The distance-dependent attenuation model that we use in this paper is a non-singular model (not approaching infinity when $r_i \to 0$) in the form of [18]:

$$g(r_i) = \begin{cases} k r_i^{-n}, & r_i \geq r_c \\ k r_c^{-n}, & r_i < r_c \end{cases}, \tag{4}$$

where $r_c > 0$ is a critical radius at which the slope of the model changes from a fixed value of attenuation to a distance-dependent attenuation. The parameter n is the path-loss exponent; it is assumed to be strictly greater than 2. The constant parameter k can be assumed to be unity without loss of generality since its effect can be absorbed by X_i. We choose the non-singular model because it is expected to provide more realistic performance figures than the singular models [19].

III. Cumulants of I_A and the Interference Probability

The aggregate interference power from a Poisson field of transmitters is characterized in many works, such as [4], [6], [8]–[10]. These works managed to develop the characteristic function of I_A. However, inverting this characteristic function to a cumulative distribution function (CDF) or a probability density function (PDF) is not feasible, except numerically or for a few special cases. Alternatively, [4], [6], [9], [13], [20] use the characteristic function to calculate cumulants of I_A. These cumulants are then used to approximate the distribution of I_A. Here, we focus more on the results reported in [13], since it provides more comprehensive expressions for cumulants. The cumulants expressions provided in [13] are for a finite circular region of interferers with an inner radius r_o and an outer radius $r_o + L$, similar to region $\mathcal{R}_t$ in Fig. 1. According to [13], the mth cumulants of the I_A can be expressed as

$$\kappa_m(I_A) = \lambda \pi \tilde{\mu}_m(X)[g(r_o)]^m \times \left[\hat{r}^2 \left(1 + \frac{2}{mn-2}\left(1 - \left[\frac{\hat{r}}{r_o + L}\right]^{mn-2}\right)\right) - r_o^2\right], \tag{5}$$

where $\hat{r} = \max(\min(r_c, r_o + L), r_o)$, and $\tilde{\mu}_m(X)$ denotes the mth raw moment of X_i (the subscript i is omitted for convenience and since X_is are i.i.d.). Equation (5) can be used for infinite fields by letting L goes to infinity.

These cumulants could be utilized to approximate the distribution of the aggregate interference power. A good approximating distribution that could be used for this purpose is the shifted lognormal distribution. This distribution is initially proposed in [21] and then used in [6] and [13] to approximate the distribution of I_A. The probability density function of the shifted lognormal random variable can be expressed as [13]

[2]While we consider a single PU-RX in our model, our results are also applicable for cases when there are many PU-RXs provided that the spectrum sharing decision is based on the PU-RXs experiencing the worst interference.

$$f_Z(z) = \frac{1}{s(z-b)\sqrt{2\pi}} e^{-(\ln(z-b)-u)^2/2s^2}, z > b, \quad (6)$$

where s, b, and u can be calculated using the cumulants of I_A according to the following equations:

$$\begin{aligned} s^2 =& \ln \tau, \\ u =& \tfrac{1}{2} \ln \frac{\kappa_2(I_A)}{\tau(\tau-1)}, \\ b =& \kappa_1(I_A) - \sqrt{\frac{\kappa_2(I_A)}{\tau-1}}, \\ \tau =& \left[\upsilon + \sqrt{\upsilon^2-1}\right]^{1/3} + \left[\upsilon - \sqrt{\upsilon^2-1}\right]^{1/3} - 1, \\ \upsilon =& 1 + \tfrac{1}{2}\rho^2, \\ \rho =& \frac{\kappa_3(I_A)}{[\kappa_2(I_A)]^{3/2}}. \end{aligned} \quad (7)$$

Based on this approximation, the interference probability of I_A can be expressed as

$$P(I_A \geq I_{th}) \simeq Q\left(\frac{\ln(I_{th}-b)-u}{s}\right). \quad (8)$$

IV. The Dominant Region

As mentioned before, the objective of this paper is to determine the boundary of the dominant region. Therefore, we present two different approaches for identifying the dominant interfering region. There might be other approaches, but these two are deemed to be sufficient to explain the concept. The first approach is based on the cumulants while the other one is based on the interference probability. In this paper, we focus on the case where there is an exclusion region around the PU with $r_o \geq r_c$. Other cases can be investigated in a similar way starting from substituting $\hat{r}$ and $g(r_o)$ in (5) with their corresponding expressions.

A. A Cumulant-Based Approach

Let us define the dominant region for the mth cumulant to be the minimum region around the PU-RX that approximately produces the same mth cumulant as the one obtained for the whole network. To quantify this approximation, we assume that the PU-RX is insensitive to, or can tolerate, a maximum relative approximation error ϵ_κ in the calculation of cumulants. Denoting the mth cumulant of the interference from the whole network as $\kappa_m(I_A, \mathcal{R}_t)$, and the mth cumulant of the interference from the dominant region only as $\kappa_m(I_A, \mathcal{R}_d)$, the approximation error can be expressed as

$$\frac{\kappa_m(I_A, \mathcal{R}_t) - \kappa_m(I_A, \mathcal{R}_d)}{\kappa_m(I_A, \mathcal{R}_t)} \leq \epsilon_\kappa. \quad (9)$$

where ϵ_κ denotes the maximum acceptable error in approximating $\kappa_m(I_A, \mathcal{R}_t)$ by $\kappa_m(I_A, \mathcal{R}_d)$. The error should be very small, i.e., $\epsilon_\kappa \ll 1$. The expressions for $\kappa_m(I_A, \mathcal{R}_t)$ and $\kappa_m(I_A, \mathcal{R}_d)$ are obtained from (5) by letting $L \to \infty$ and $L = L_d$, respectively. By inserting the expressions of $\kappa_m(I_A, \mathcal{R}_t)$ and $\kappa_m(I_A, \mathcal{R}_d)$ into (9) and performing some mathematical manipulations, we get

$$\left(\frac{r_o}{r_o + L_d}\right)^{mn-2} \leq \epsilon_\kappa, \quad (10)$$

which leads to

$$L_d \geq \left(\epsilon_\kappa^{\frac{1}{2-mn}} - 1\right) r_o. \quad (11)$$

The dominant region is defined in this paper as the minimum L_d that satisfies the inequality in (11). From (11), it is observed that the dominant region is a function of the cumulant order, i.e., m. As m increases, the dominant region shrinks. The dominant region for the mean (i.e., κ_1) is bigger than the one for the variance, i.e., κ_2. Therefore, if a region provides a satisfactory approximation for the mean of I_A, then it also provides satisfactory approximations for higher cumulants. It is also clear from (11) that a wider exclusion region (r_o) leads to a larger value of L_d. Moreover, (11) indicates that the higher the path-loss exponent is, the smaller the dominant region becomes.

As an example, if $r_o = 1$ m and $n = 3$, approximating the whole network by a region of $L_d = 10$ m generates a maximum approximation error of 0.091 for the mean. The same region, i.e., $L_d = 10$ m, provides a smaller approximation error of 10^{-4} for the variance. Based on (11), Table I shows the values of L_d calculated for different values of maximum approximation errors in the first three cumulants. The table reflects the effect of the exclusion region and the path-loss exponent on L_d.

It is worth noting that the dominant region from the cumulants' perspective does not depend on transmit power, antenna gains, or fading distributions. The results on the dominant region from the perspective of cumulants give us some insight on the dynamics of the aggregate interference. The results also give some hints on what portion of the secondary network would dictate the spectrum sharing opportunities. However, a more proper investigation of the dominant region of the secondary network from the perspective of spectrum sharing should be based on the interference probability or on some other appropriate harmful interference metrics.

B. An Interference Probability-Based Approach

From the perspective of spectrum sharing, the dominant region could be defined as the minimum portion of a secondary network that would provide approximately the same spectrum sharing conclusions as those drawn by considering the whole secondary network. In this paper, we use the interference probability as the metric for the spectrum sharing opportunity. This interference probability is mainly based on the CCDF of I_A. However, Fig. 2 and Fig. 3 demonstrate that the CCDF of I_A converges to a limiting distribution as L increases[3]. The convergence occurs faster for smaller r_o. Inspired by this

[3]Fig. 2 and Fig. 3 show the upper tail of the CCDF, which is more relevant to the interference probability.

convergence, there is a dominant region that would satisfy the following condition:

$$\frac{P_{int}(I_{th}, \mathcal{R}_t) - P_{int}(I_{th}, R_d)}{P_{int}(I_{th}, \mathcal{R}_t)} \leq \epsilon, \tag{12}$$

where $P_{int}(I_{th}, \mathcal{R}_t)$ is the interference probability considering the aggregate interference from the whole field and a threshold level of I_{th}. $P_{int}(I_{th}, \mathcal{R}_d)$ is the interference probability considering the interference coming from $\mathcal{R}_d$ only with the same I_{th}.

Assuming that the aggregate interference power of $\mathcal{R}_t$, and the one of $\mathcal{R}_d$ are approximated by shifted lognormal random variables, (12) can be expressed as

$$1 - \frac{Q\left(\frac{\ln(I_{th} - b(L_d)) - u(L_d)}{s(L_d)}\right)}{Q\left(\frac{\ln(I_{th} - b) - u}{s}\right)} \leq \epsilon. \tag{13}$$

We use the notations $b(L_d)$, $u(L_d)$, and $s(L_d)$ to indicate that these parameters are obtained from the cumulants of I_A coming from $\mathcal{R}_d$. To identify the boundary of the dominant region, (13) should be solved for L_d with the equality. This dominant region should be sufficient to investigate the spectrum sharing opportunities for the whole secondary networks. To demonstrate this, let us assume that the PU or the regulator specify that the spectrum of the PU can be shared by the secondary network if $P(I_A \geq I_{th}) \leq 0.1$, where $I_{th} = 0.0233$ (a normalized value with respect to the transmit power, antenna gains and other constant parameters). If the PU is insensitive to, or can tolerate an error in the interference probability of 1% or less (i.e., $\epsilon \leq 0.01$), then according to Fig. 4 the minimum L_d satisfying $\epsilon \leq 0.01$ is 1700 m which defines the boundary of the corresponding dominant region. Fig. 4 is plotted based on (13) for system and channel parameters indicated in the caption of the figure including an exclusion region of 10 m. If the PU can tolerate a larger error, e.g., $\epsilon \leq 0.1$, then $L_d = 150$ m for the same I_{th} and the same system and channel parameters. If I_{th} is specified at a higher value, e.g., 0.0604, then L_d decreases to 650 m and 50 m for $\epsilon = 0.01$ and 0.1 respectively.

The effect of r_o on the dominant region can be demonstrated by comparing Fig. 4 and Fig. 5.[4] While Fig. 4 shows CCDF curves for $r_o = 10$ m, Fig. 5 shows CCDF curves for $r_o = 1$ m. From Fig. 5 and for $I_{th} = 0.374$ (corresponding to $P_{int}(I_{th}, \mathcal{R}_t) = 0.1$), $L_d = 34.6$ m for $\epsilon = 0.01$ and 2.44 m for $\epsilon = 0.1$. These values of L_d are smaller than those obtained from Fig. 4 when r_o=10 m and $I_{th} = 0.0233$, which corresponds to $P_{int}(I_{th}, \mathcal{R}_t) = 0.1$.

In summary, the dominant region decreases as ϵ increases. On the other hand, it increases as r_o increases. These observations are similar to the findings on the dominant region from the perspective of cumulants in the previous subsection. However, the dominant region from the perspective of the interference probability also depends on the value of I_{th}, which does not appear in the formulations of the cumulant-based dominant region.

[4]The values of I_{th} in these figures are chosen to correspond to identical interference probabilities, i.e., the upper curves in these figures correspond to 0.1 and the lower curves correspond to 0.01.

C. Implications of the Dominant Region

Following are some implications of the results obtained for the dominant region.

- In interference limited systems, there might be a tendency to assume that performance measures of the system are affected by a few interferers around a victim receiver. While this assumption could be valid for many scenarios, it might be invalid for other scenarios. Therefore, it is advisable to verify this assumption by identifying the dominant region around the victim receiver.
- Simulations of the interference and spectrum sharing opportunities in large networks can be significantly simplified, without degrading the accuracy, by simulating the dominant region only not the whole network.
- A PU-RX who is within a finite secondary network but away from the edge of the network by a distance of L_d or more is practically receiving the same level of interference as if it is located at the center of the secondary network. Hence, a PU-RX has almost identical influence on spectrum sharing decisions regardless of its location within the secondary network as long as it is away from the edge by a minimum distance of L_d.
- Any deployments of SU-TXs outside the dominant region has no effect on the spectrum sharing decisions provided that the density of SU-TXs outside the dominant region does not exceed the density of the SU-TXs within the dominant region.
- We anticipate that our results in this paper can inspire new ideas for designing MAC protocols for secondary networks. Due to space limitations, detailed discussions about these ideas and related examples are devoted for future submissions.

V. Conclusions

In this paper, we investigated the dominant region that dictates the spectrum sharing opportunities for large secondary networks. We identified the boundary of this dominant region. We showed that as the exclusion region increases the dominant region increases as well. However, the dominant region shrinks with the increase in the path-loss exponent. The higher the required accuracy is, the wider the dominant region becomes. In addition, we demonstrated that the boundary of the dominant region shrinks with the increase of the interference threshold level. Results reported in this paper are anticipated to be useful for designing MAC protocols and algorithms for large wireless secondary networks to create and maintain spectrum sharing opportunities.

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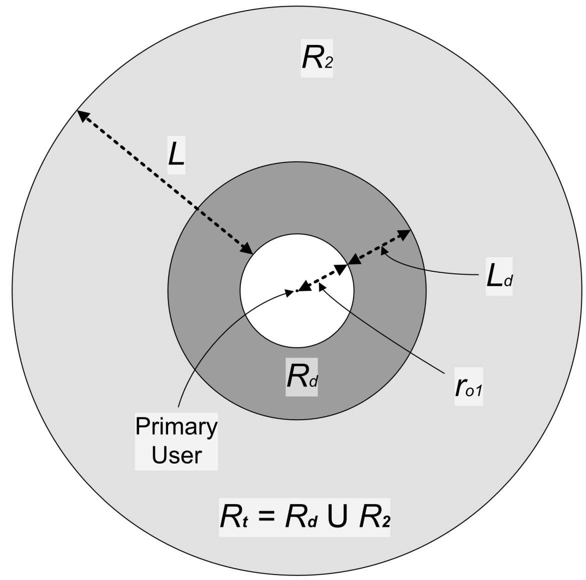

Fig. 1. Layout of a partitioned secondary network. The dominant region $\mathcal{R}_d$ has an inner radius of r_o and an outer radius $r_o + L_d$. The inner white circle is the exclusion region. The region $\mathcal{R}_t = \mathcal{R}_d \cup \mathcal{R}_2$ corresponds to the whole secondary network.

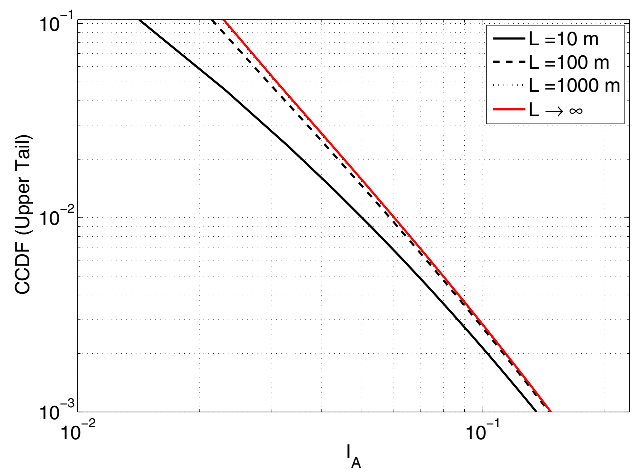

Fig. 2. The upper tail of the CCDF of I_A based on a shifted lognormal approximation for different values of L ($r_o = 10$ m, $r_c = 1$ m, $\lambda = 0.01$ nodes/m^2, $n = 3$, Rayleigh fading, and 6 dB shadowing).

TABLE I

L_d CALCULATED FOR DIFFERENT VALUES OF MAXIMUM APPROXIMATION ERRORS IN THE FIRST THREE CUMULANTS. CALCULATIONS ARE REPEATED FOR DIFFERENT VALUES OF r_o AND n TO REFLECT THEIR EFFECTS.

r_o	n	ϵ_κ	L_d for κ_1	L_d for κ_2	L_d for κ_3
1 m	3	0.1	9 m	0.78 m	0.39 m
		0.01	99 m	2.16 m	0.93 m
		0.001	999 m	4.62 m	1.68 m
10 m	3	0.1	90 m	7.78 m	3.89 m
		0.01	990 m	21.62 m	9.31 m
		0.001	9990 m	46.23 m	16.83 m
10 m	4	0.1	21.62 m	4.68 m	2.59 m
		0.01	90.00 m	11.54 m	5.85 m
		0.001	306.23 m	21.62 m	9.95 m

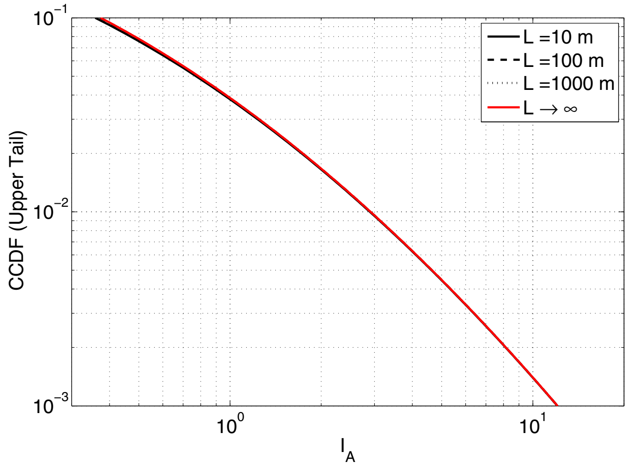

Fig. 3. The upper tail of the CCDF of I_A based on a shifted lognormal approximation for different values of L ($r_o = 1$ m, $r_c = 1$ m, $\lambda = 0.01$ nodes/m^2, $n = 3$, Rayleigh fading, and 6 dB shadowing).

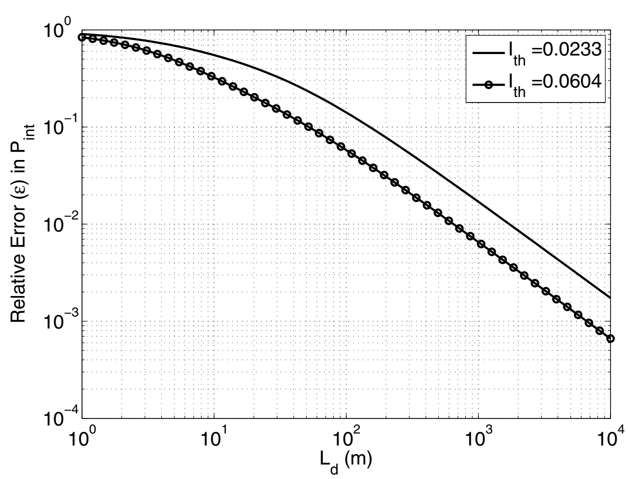

Fig. 4. Relative approximation error in the interference probability for different values of I_{th} ($r_o = 10$ m, $r_c = 1$ m, $\lambda = 0.01$ nodes/m^2, $n = 3$, Rayleigh fading, and 6 dB shadowing). The values of I_{th} are chosen to correspond to the interference probability of 0.1 and 0.01, respectively, that a PU-RX would experience from an infinite secondary network.

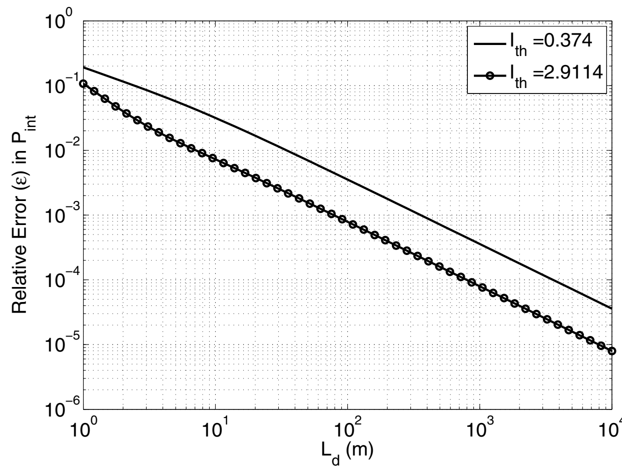

Fig. 5. Relative approximation error in the interference probability for different values of I_{th} ($r_o = 1$ m, $r_c = 1$ m, $\lambda = 0.01$ nodes/m^2, $n = 3$, Rayleigh fading, and 6 dB shadowing). The values of I_{th} are chosen to correspond to the interference probability of 0.1 and 0.01, respectively, that a PU-RX would experience from an infinite secondary network.