Dominant Regions Dictating Spectrum Sharing Opportunities

Muhammad Aljuaid\textsuperscript{1} and Halim Yanikomeroglu\textsuperscript{2}

\textsuperscript{1} Saudi Aramco, Dhahran, Saudi Arabia
\textsuperscript{2} Carleton University, Ottawa, Canada

The 21st Annual IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC 2010)
Outline

- Introduction
  1. Cumulant-based Characterization of the Aggregate Interference Power
  2. Impact of the Spatial Size of the Secondary Network on Spectrum Sharing
  3. Dominant Regions Dictating Spectrum Sharing Opportunities
- Conclusions
Introduction

- There is an exponential growth in the number of wireless systems and devices.
- Radio spectrum is a scarce resource; however, it is under-utilized.
- Spectrum management is going through a paradigm shift.
- Secondary users (SUs) could share the spectrum with primary users (PUs) under the following condition:
  - SUs don’t introduce “harmful interference" towards PUs.
- Different metrics are proposed to gauge the harmful interference.
Interference Probability
A harmful interference metric [Ghasemi08] and [Win09]

- Non-harmful interference:
  \[ P(I_A \geq I_{th}) \leq \beta \]
  \( \Rightarrow \) spectrum sharing allowed

- Harmful interference:
  \[ P(I_A \geq I_{th}) > \beta \]
  \( \Rightarrow \) spectrum sharing NOT allowed
System Model

- Aggregate Interference:
  \[ I_A = \sum_{i \in \Lambda} I_i = \sum_{i \in \Lambda} g(r_i)X_i \]

- Distance-Dependant Attenuation
  \[ g(r_i) = \begin{cases} 
  kr_i^{-n}, & r_i \geq r_c \\
  kr_c^{-n}; \text{ constant}, & r_i < r_c 
  \end{cases} \]

- Other system and channel parameters
  \[ X_i = \prod_l X_{i,l} \]

\( \lambda \): Density of active nodes
\( n \): Path loss exponent
\( X_i \)'s are i.i.d.
I. Cumulant-based Characterization of the Aggregate Interference Power

Motivations

- Characteristic function is known.
- No closed-form expressions for PDF/CDF.
- Numerical inversion is possible, however, cumulants approach is more attractive.
I. Cumulant-based Characterization of the Aggregate Interference Power

Lit. Review

- A number of recent papers in literature have dealt with cumulants of the aggregate interference but under specific scenarios.
  
  **Lichte10** considers the first cumulant, i.e., the mean.
  **Chan01** provides an integral form to compute the cumulants for out-of-cell interference in a CDMA networks.
  **Menon05,06** deal with cumulants for non-fading scenarios.
  **Ghasemi08** considers an infinite field with a very small exclusion region.

- Extending these results and generalizing them for a wide range of scenarios are of great importance and advantage to study the spectrum sharing in large secondary networks.
I. Cumulant-based Characterization of the Aggregate Interference Power

Results

\[ I_A = \sum_{i \in N} g(r_i)X_i \]

\[ \kappa_m(I_A) = N_{eff}(m)[g(r_o)]^m \tilde{\mu}_m(X) \]

\[ N_{eff}(m) = \lambda A_{eff}(m) \]

\[ A_{eff}(m) = \frac{1}{2} \theta \left[ r_{eff}^2(m) - r_0^2 \right] \]

\[ r_{eff}(m) = \hat{r} \sqrt{1 + \frac{2}{mn-2} \left( 1 - \left[ \frac{\hat{r}}{r_o + L} \right]^{mn-2} \right)} \]

\[ \hat{r} = \max \left( \min \left( r_c, r_o + L \right), r_o \right) \]

\( \lambda \): Density of active nodes
\( n \): Path loss exponent
\( X_i \)'s are i.i.d.
\( \tilde{\mu}_m(X) = E[X_i^m] \)
I. Cumulant-based Characterization of the Aggregate Interference Power

Cumulant-based Approximation of the Distribution of $I_A$
III. Impact of the Spatial Size of the Secondary Network on Spectrum Sharing

Motivations and Lit. Review

- Previous works such as [Menon05], [Pinto07], [Ghasemi08] and [Ofcom08] studied the effect of different system parameters on spectrum sharing opportunities.

- However, a parameter that has received little attention is the spatial size of the secondary network.

- Usually, the spatial size is assumed to be infinite, e.g., [Menon05], [Menon06], [Ghasemi08] and [Win09].

- Results developed for infinite networks might be too pessimistic leading to missing spectrum sharing opportunities.

- Impact of spatial size of the secondary network on spectrum sharing opportunities?
III. Impact of the Spatial Size of the Secondary Network on Spectrum Sharing

Impact of the Spatial Size on Cumulants of $I_A$
III. Impact of the Spatial Size of the Secondary Network on Spectrum Sharing

Impact of the Spatial Size on the CCDF of $I_A$

Simulation

![Graph showing the CCDF of $I_A$ for different spatial sizes](image)

- $L=10$ meters
- $L=100$ meters
- $L=1000$ meters
- $L=10000$ meters
There are some comments in literature (e.g., [Etkin06] and [Weber07]) indicating that the aggregate interference is dominated by the nearby interferers to the victim receiver.

There is to the best of our knowledge no work devoted to precisely identifying the boundary of the dominant region.

A contribution is required to fill this gap, especially in the context of spectrum sharing.
IV. Dominant Regions Dictating Spectrum Sharing Opportunities

\[ R_t = R_d \cup R_2 \]
Dominant Regions Dictating Spectrum Sharing Opportunities

- **Cumulant-Based Approach:**
  \[
  \frac{\kappa_m(I_A, R_t) - \kappa_m(I_A, R_d)}{\kappa_m(I_A, R_t)} \leq \epsilon_{\kappa}
  \]

- **Interference Probability-Based Approach:**
  \[
  \frac{P_{int}(I_{th}, R_t) - P_{int}(I_{th}, R_d)}{P_{int}(I_{th}, R_t)} \leq \epsilon
  \]
IV. Dominant Regions Dictating Spectrum Sharing Opportunities

Relative Error ($\varepsilon$) in $P_{\text{int}}$

$L_d (m)$

$I_{th} = 0.0233$

$I_{th} = 0.0604$

$R_t = R_d \cup R_z$
Observations

Results reflect the following:

- The dominant region is not necessarily a small region encompassing a few interferers within the proximity of the primary user.
- Far interferers may tangibly contribute to spectrum sharing decisions when a higher approximation accuracy is required or when a wide exclusion region is considered.
- On the other hand, the dominant region shrinks with the increase in the path-loss exponent or in the level of the interference threshold specified by the primary user or a regulator.
Implications

- Simulations of the interference and spectrum sharing opportunities in large networks can be significantly simplified by simulating the dominant region only not the whole network.

- A PU-RX who is within a finite secondary network but away from the edge of the network by a distance of $L_d$ or more is practically receiving the same level of interference as if it is located at the center of the secondary network.

- A PU-RX has almost identical influence on spectrum sharing decisions regardless of its location within the secondary network as long as it is away from the edge by a minimum distance of $L_d$.

- Any deployments of SU-TXs outside the dominant region has no effect on the spectrum sharing decisions provided that the density of SU-TXs outside the dominant region does not exceed the density of the SU-TXs within the dominant region.
Summary

- Introduced cumulant-based characterization of the aggregate interference power
- Discussed the impact of the spatial size of the secondary network on spectrum sharing
- Identified the smallest portion (dominant region) of the secondary network that would impact spectrum sharing opportunities
Thank you