A Theoretical Characterization of the Multihop Wireless Communications Channel without Diversity

John Boyer, David Falconer, Halim Yanikomeroglu
Broadband Communications and Wireless Systems Centre
Department of Systems and Computer Engineering
Carleton University, 1125 Colonel By Drive
Ottawa, Ontario, Canada K1S 5B6

Abstract – **This paper provides a theoretical characterization of the multihop wireless communications channel without diversity, wherein intermediate terminals relay information without employing spatial diversity techniques. Two channel models are proposed and developed: one where each intermediate terminal digitally decodes and re-encodes the received signal from the immediately preceding terminal and the other where each intermediate terminal simply amplifies the received signal from the immediately preceding terminal. These models are compared, through analysis and simulations, with the singlehop reference channel on the basis of probability of outage and probability of error. Both models achieve significant gains over the singlehop reference channel, with the amplified relaying model matching the performance of the decoded relaying model despite noise propagation.**

I. Introduction

This paper is concerned with a proposed wireless system wherein traditional transmission constraints are removed in order to allow direct communication between mobile terminals. This system gives mobile terminals the ability to relay information when they are neither the initial transmitter nor the final receiver. Relaying systems realize a number of benefits over traditional systems in the areas of deployment, connectivity, adaptability and capacity [5,9].

In order to quantify these benefits and gain a better understanding of the associated issues it is first necessary to provide a mathematical foundation for this system through a derived characterization of the multihop wireless communications channel. This paper proposes two channel models for the case where mobile terminals act as intermediate nodes in wireless communications systems. These are referred to as the *decoded relaying multihop channel* and the *amplified relaying multihop channel*. The decoded relaying multihop channel corresponds to the case where each intermediate terminal digitally decodes and re-encodes the received signal from the immediately preceding terminal before retransmission. The amplified relaying multihop channel corresponds to the case where each intermediate terminal simply amplifies the received signal from the immediately preceding terminal before retransmission. This paper focuses on multihop channels without diversity. Multihop channels with diversity are characterized in [3].

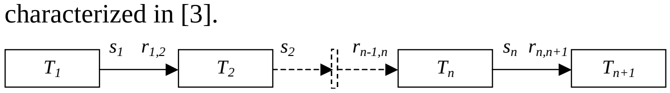

Fig. 1. Generic Multihop Wireless Communications Channel

II. System Model

The system model for multihop channels without diversity is composed of a source terminal, a receiving terminal, and an indeterminate number of potential intermediate relaying terminals. In Fig. 1 the source terminal is identified as T_1, the destination terminal is identified as T_{n+1} and the intermediate terminals are identified as T_2 through T_n, where n is the number of hops along the transmission path.

Let T_S represent the set of source terminals, T_I represent the set of intermediate terminals, and T_D represent the set of destination terminals. Therefore $T_T = T_S \bigcup T_I$ represents the set of all transmitting terminals and $T_R = T_I \bigcup T_D$ represents the set of all receiving terminals. Let $T_{P(i)}$ represent the set of terminals that transmit a signal received by T_i. The notation used in this paper assumes that $T_{P(i)}$ has cardinality equal to 1 in order to support the characterization of scenarios without diversity. For the communications channel illustrated in Fig. 1, $T_S = \{T_1\}$, $T_I = \{T_2, \ldots T_n\}$, $T_D = \{T_{n+1}\}$, $T_T = \{T_1, \ldots T_n\}$, $T_R = \{T_2, \ldots T_{n+1}\}$ and $T_{P(i)} = \{T_{i-1}\}$.

Each terminal T_i transmits a signal given by

$$s_i = \sqrt{\varepsilon_i}\, a_t + \beta_i, \tag{1}$$

where ε_i is the transmitted power, a_t is the binary information symbol at time interval t, and β_i is propagated noise. The propagated noise term in (1) is zero for source terminals as well as for intermediate terminals that employ decoded relaying. Each terminal T_i then receives a signal given by

$$r_{P(i),i} = \alpha \sqrt{(L_{P(i),i} / d_{P(i),i}^{\,p})}\, R_{P(i),i} (\sqrt{\varepsilon_{P(i)}}\, a_t + \beta_{P(i)}) + z_{P(i),i}, \tag{2}$$

where α^2 is the free space signal power attenuation factor between the transmitting terminal and an arbitrary reference distance, $d_{P(i),i}$ is the inter-terminal distance relative to the reference distance, p is the propagation exponent, $L_{P(i),i}$ is a zero-mean lognormal random variable with variance $\sigma^2_{L_{P(i),i}}$, $R_{P(i),i}$ is a complex gaussian (Rayleigh) random variable with mean power $E[R^2_{P(i),i}] = 1$, and $z_{P(i),i}$ is a zero-mean additive white gaussian noise random variable with variance N_0. Using this model, the received signal to noise ratio at T_i is given by

$$\gamma_{P(i),i} = \frac{\alpha^2 \varepsilon_{P(i)}}{(d_{P(i),i}^{p} / L_{P(i),i} |R_{P(i),i}|^2) N_0 + \alpha^2 |\beta_{P(i)}|^2}, \qquad (3)$$

where $|R_{P(i),i}|^2$ is an exponential random variable with mean $2\sigma_{R_{P(i),i}}^2 = 1$. The probability of outage due to lognormal shadowing when $\beta_{P(i)} = 0$ is given in [7] by

$$\Pr[\gamma_{P(i),i} < \gamma] = Q\left(\frac{10 \log(\overline{\gamma_{P(i),i}} / \gamma)}{\sigma_{L_{P(i),i}}}\right), \qquad (4)$$

where γ is an arbitrary signal to noise ratio that must be met in order to maintain communication. The calculation of probability of error is dependent on the modulation scheme employed. For the special case of BPSK, the probability of error under fading conditions when $\beta_{P(i)} = 0$ is given in [6] by

$$P_e(\gamma_{P(i),i}) \approx \frac{1}{2\overline{\gamma_{P(i),i}}}, \ \overline{\gamma_{P(i),i}} >> 1. \qquad (5)$$

III. Decoded Relaying

The decoded relaying multihop channel corresponds to the case where each intermediate terminal digitally decodes and re-encodes the received signal from the immediately preceding terminal before retransmission. This digital relaying channel does not propagate noise along the multihop channel, introduces the possibility of decoding error at each intermediate terminal, and experiences delay due to intermediate terminal decoding as well as signal propagation.

The channel model is given by (1) through (3) with $\beta_{P(i)} = 0$. The received signal to noise ratio at T_i as a result of the signal from $T_{P(i)}$ is given by

$$\gamma_{P(i),i} = \frac{\alpha^2 \varepsilon_{P(i)}}{(d_{P(i),i}^{p} / L_{P(i),i} |R_{P(i),i}|^2) N_0}. \qquad (6)$$

The total probability of outage for the decoded relaying multihop channel is given by

$$P_o = 1 - \prod_{T_i \in T_R} (1 - \Pr[\gamma_{P(i),i} < \gamma]), \qquad (7)$$

where $\Pr[\gamma_{P(i),i} < \gamma]$ is the probability of outage at terminal T_i given a received signal to noise ratio of $\gamma_{P(i),i}$. This value can be upper-bounded by

$$P_o \le \sum_{T_i \in T_R} \Pr[\gamma_{P(i),i} < \gamma]. \qquad (8)$$

The total probability of decoding error for the decoded relaying multihop channel is given by

$$P_e = 1 - \prod_{T_i \in T_R} (1 - P_e(\gamma_{P(i),i})), \qquad (9)$$

where $P_e(\gamma_{P(i),i})$ is the probability of decoding error at terminal T_i given a received signal to noise ratio of $\gamma_{P(i),i}$. This value can be upper-bounded by

$$P_e \le \sum_{T_i \in T_R} P_e(\gamma_{P(i),i}). \qquad (10)$$

IV. Amplified Relaying

The amplified relaying multihop channel corresponds to the case where each intermediate terminal simply amplifies the received signal from the immediately preceding terminal before retransmission. This analog relaying channel propagates noise along the multihop channel, introduces the possibility of decoding error only at the destination terminal, and experiences delay due solely to signal propagation.

The channel model is composed of a set of individual transmission channels given by (1) through (3). Assuming that each intermediate terminal can track both lognormal shadowing and Rayleigh fading, the amplification factor at each intermediate terminal T_i is given by

$$A_i = \frac{\varepsilon_i}{\alpha^2 (\varepsilon_{P(i)} + |\beta_{P(i)}|^2) L_{P(i),i} |R_{P(i),i}|^2 / d_{P(i),i}^{p} + N_0}. \qquad (11)$$

Although this amplification factor is exact, the resulting mathematical characterization of the channel model is extremely complex. In order to simplify the amplification factor, an approximation can be made where the noise terms are removed from the denominator of (11). This approximation yields an upper bound on the amplification factor, which is tight provided that the signal to noise ratio at the terminal under consideration is significantly greater than 1. The amplification factor at each terminal is then given by

$$A_i \le \frac{\varepsilon_i d_{P(i),i}^{p}}{\alpha^2 \varepsilon_{P(i)} L_{P(i),i} |R_{P(i),i}|^2}, \qquad (12)$$

and the received signal to noise ratio at the destination terminal T_D is given by

$$\gamma_{P(D),D} \approx \frac{\alpha^2}{\sum_{T_i \in T_R} (d_{P(i),i}^{p} / \varepsilon_{P(i)} L_{P(i),i} |R_{P(i),i}|^2) N_0}. \qquad (13)$$

The additional terms in the denominator of (13) where $i \in T_I$ are the result of the propagated noise term in (3), given by

$$|\beta_{P(D)}|^2 = \sum_{T_i \in T_I} (\varepsilon_{P(D)} d_{P(i),i}^{p} / \alpha^2 \varepsilon_{P(i)} L_{P(i),i} |R_{P(i),i}|^2) N_0. \qquad (14)$$

Alternatively, the received signal to noise ratio at the destination terminal can be expressed as

$$\gamma_{P(D),D}^{-1} \approx \sum_{T_i \in T_R} \psi_{P(i),i}^{-1}, \qquad (15)$$

where $\psi_{P(i),i}$ is the received signal to noise ratio $\gamma_{P(i),i}$ at terminal T_i with $\beta_{P(i)} = 0$.

A. Sum of Inverse Exponential Components

The channel model for the amplified relaying multihop channel can be simplified by approximating the sum of inverse exponential components contained in (15) [5]. This approximation does not take into account the lognormal shadowing characteristics of the channel and therefore provides a characterization that is constrained to small-scale channel effects.

The total received signal to noise ratio at the destination terminal T_n can be upper-bounded by

$$\gamma_{P(D),D} \le \min_{T_i \in T_R}\{\psi_{P(i),i}\}. \tag{16}$$

Since $\psi_{P(i),i}$ are independent exponential random variables, the minimum is also an exponential random variable with mean $(\sum_{T_i \in T_R} \overline{\psi_{P(i),i}}^{-1})^{-1}$, where $\overline{\psi_{P(i),i}}$ is the expected value of $\psi_{P(i),i}$. The total received signal to noise ratio at the destination terminal can now be approximated by

$$\gamma_{P(D),D} \approx \frac{\alpha^2 \left|R_{Z(D)}\right|^2}{\sum_{T_i \in T_R} (d_{P(i),i}^{p} / \varepsilon_{P(i)} L_{P(i),i}) N_0}, \tag{17}$$

where $\left|R_{Z(D)}\right|^2$ is an exponential random variable with mean $2\sigma_{R_{Z(D)}}^2 = 1$.

B. Sum of Inverse Lognormal Components

The channel model for the amplified relaying multihop channel can be further simplified by approximating the power sum of inverse lognormal components contained in (15). This approximation does not take into account the Rayleigh fading characteristics of the channel and therefore provides a characterization that is constrained to large-scale channel effects. The following derivation uses Wilkinson's method [1,8]. Let the approximation of the power sum of log normal components be given by

$$\begin{aligned} L_{Z(D)} &= \sum_{T_i \in T_R} L_{P(i),i} \\ &= \sum_{T_i \in T_R} e^{Y_{P(i),i}} \end{aligned}, \tag{18}$$

where $L_{Z(D)}$ is a log normal random variable with mean $\mu_{L_{Z(D)}}$ and standard deviation $\sigma_{L_{Z(D)}}$. Each lognormal component $L_{P(i),i}$ is independent with mean $\mu_{L_{P(i),i}} = \ln(d_{P(i),i}^{p} / \varepsilon_{P(i)})$ and standard deviation $\sigma_{L_{P(i),i}} = \Omega$, where Ω is typically between 6 and 12 dB or between 1.4 and 2.8 in the natural logarithmic scale [1]. Matching the first moment of $L_{Z(D)}$ gives

$$\begin{aligned} u_1 &= \sum_{T_i \in T_R} e^{\ln(d_{P(i),i}^{p} / \varepsilon_{P(i)}) + \Omega^2/2} \\ &= e^{\Omega^2/2} \sum_{T_i \in T_R} (d_{P(i),i}^{p} / \varepsilon_{P(i)}) \end{aligned}, \tag{19}$$

and matching the second moment of $L_{Z(D)}$ gives

$$\begin{aligned} u_2 &= \sum_{T_i \in T_R} e^{2\ln(d_{P(i),i}^{p} / \varepsilon_{P(i),i}) + 2\Omega^2} + 2 \sum_{T_i \in T_I} \sum_{\substack{T_j \in T_R, \\ j>i}} e^{\ln(d_{P(i),i}^{p} / \varepsilon_{P(i)}) + \ln(d_{P(j),j}^{p} / \varepsilon_{P(j)}) + \Omega^2} \\ &= e^{2\Omega^2} \sum_{T_i \in T_R} (d_{P(i),i}^{p} / \varepsilon_{P(i)})^2 + 2e^{\Omega^2} \sum_{T_i \in T_I} \sum_{\substack{T_j \in T_R, \\ j>i}} (d_{P(i),i}^{p} d_{P(j),j}^{p} / \varepsilon_{P(i)} \varepsilon_{P(j)}) \end{aligned}. \tag{20}$$

The total received signal to noise ratio at the destination terminal can now be approximated by

$$\gamma_{P(D),D} \approx \frac{\alpha^2 \left|R_{Z(D)}\right|^2}{L_{Z(D)} N_0}, \tag{21}$$

where $L_{Z(D)}$ is a log normal random variable and the mean and standard deviation of the corresponding gaussian random variable are given by

$$\mu_{L_{Z(D)}} = \frac{(20\ln(u_1) - 5\ln(u_2))}{\ln(10)}, \text{ and} \tag{22}$$

$$\sigma_{L_{Z(D)}} = \frac{\sqrt{100\ln(u_2) - 200\ln(u_1)}}{\ln(10)}. \tag{23}$$

The probability of outage for the amplified relaying multihop channel at the destination terminal is given by

$$P_o = \Pr[\gamma_{P(D),D} < \gamma] = Q\left(\frac{10\log(\overline{\gamma_{P(D),D}} / \gamma)}{\sigma_{L_{Z(D)}}}\right), \tag{24}$$

where $\Pr[\gamma_{P(D),D} < \gamma]$ is the probability of outage at the destination terminal given a received signal to noise ratio of $\gamma_{P(D),D}$.

The total probability of decoding error for the amplified relaying multihop channel is given by

$$P_e = P_e(\gamma_{P(D),D}), \tag{25}$$

where $P_e(\gamma_{P(D),D})$ is the probability of decoding error at the destination terminal given a received signal to noise ratio of $\gamma_{P(D),D}$.

V. Simulation Results

In order to visualize the discussion, the results presented thus far are applied in two simulations and compared against the reference channel on the basis of probability of outage and probability of error. A BPSK modulation scheme is used for simplicity of exposition. The example multihop channel is composed of $n+1$ terminals: source T_1, intermediate T_2 through T_n and destination T_{n+1}. The coordinates of the channel are normalized with respect to the distance between the source and destination terminals such that $d_{1,n+1} = 1$. The propagation exponent is $p = 4$. The lognormal shadowing components are independent with zero-mean and variance $\sigma^2_{L_{P(i),i}} = 12$ dB. The Rayleigh fading components are independent with mean power $E[R^2_{P(i),i}] = 1$. The threshold signal to noise ratio for outage calculations is $\gamma = 6$ dB. For the purpose of simplifying the comparison, and without loss of generality, the free space signal power attenuation factor is $\alpha^2 = 1$. Optimal power distribution is assumed for each of the channel models with the total power constrained to the reference power ε_0.

For the first simulation the intermediate terminals are fixed so that they divide the direct path between the source and destination terminals into *n* equal length segments. This serves to validate the theory presented thus far as well as illustrate the power gain that can be realized under an optimal placement of the intermediate terminals with respect to the source and destination terminals. For the second simulation the single intermediate terminal is placed at a set of locations uniformly distributed across a unit square. The source and destination terminals are located at (0,0) and (1,0) respectively. The intermediate terminal ranges from 0 to 1 along the x-axis and -½ to ½ along the y-axis. This serves to illustrate the robustness of the channel models with respect to distance from the optimal placement of the intermediate terminal.

Figs. 2-5 show the simulated outage and error performance of the decoded relaying multihop channel and the amplified relaying multihop channel respectively. The theoretical characterizations (8), (10), (24), and (25) are represented by dotted lines and indicate good agreement with the simulated results. Figs. 6-7 show the variation of the error performance of the decoded relaying multihop channel and the amplified relaying multihop channel with respect to the position of the intermediate terminal. A horizontal plane indicates the error performance of the singlehop reference channel. The graphs represent the theoretical characterization presented in (10) and (25) and indicate that the performance gain with respect to the reference channel is fairly sensitive to the relative position of the intermediate terminal.

VI. Conclusion

Not surprisingly, the multihop channels significantly outperform the singlehop reference channel. What is somewhat unexpected, however, is that the amplified relaying multihop channel experiences performance gains that are equal to the decoded relaying multihop channel. On closer examination, it is evident that this result arises from the inverse relationship between received signal to noise ratio and probability of error over Rayleigh fading channels. In non-fading gaussian channels where the relationship between received signal to noise ratio and probability of error is inverse exponential the decoded relaying channel outperforms the amplified relaying channel.

These results attest to the importance of good decisions when selecting intermediate terminals. Although the performance gain is significant when the intermediate terminal is positioned close to the midpoint between the source and destination terminals, the gain becomes negligible and in some cases negative as the distance with respect to that midpoint position increases. Further discussion related to the problem of selecting intermediate terminals is presented in [10].

Although a good comparison is provided in terms of probability of outage and probability of error, there are a number of other factors that are important to consider as well. These factors include the delay characteristics of the channel, transmitting and receiving in the same channel, interference distribution, power control, and node complexity. Discussion of these and other important issues relevant to potential implementations is included in [2] and [4].

The results presented in this paper provide a firm foundation for the characterization of multihop channels without diversity. The mathematical characterizations outlined are very tractable and enable the quick comparison of the proposed channels with the singlehop reference channel. The results suggest that there are significant advantages to be gained from employing multihop channels, and indicate a number of interesting areas for further development. In the future, these results will be applied to generalized architectures for cellular and ad-hoc systems in order to provide a comparison in terms of coverage, relay usage distribution, interference distribution, and system capacity.

References

[1] A. Abu-Dayya and N. Beaulieu, "Outage probabilities in the presence of correlated lognormal interferers," *IEEE Trans. on Vehicular Technology*, vol. 43, no. 1, 1994.

[2] J. Boyer, *Multihop Wireless Communications Channels*, Master's Thesis, summer 2001.

[3] J. Boyer, D. Falconer, and H. Yanikomeroglu, "A theoretical characterization of multihop wireless communications channels with diversity," submitted to *IEEE Globecom*, 2001.

[4] J. Boyer, D. Falconer, and H. Yanikomeroglu, "A theoretical characterization of multihop wireless communications channels," to appear in Proc. of *Canadian Workshop on Information Theory*, 2001.

[5] J. Laneman and G. Wornell, "Energy-efficient antenna sharing and relaying for wireless networks," Proc. of *IEEE Wireless Networking and Communications Conference*, 2000.

[6] J. Proakis, *Digital Communications*, McGraw-Hill, Inc., New York, Third edition, 1995.

[7] T. Rappaport, *Wireless Communications: Principles & Practice*, Prentice Hall, Inc., New Jersey, 1996.

[8] A. Safak, "Statistical analysis of the power sum of multiple correlated log-normal components," *IEEE Trans. on Vehicular Technology*, vol 42, no. 1, 1993.

[9] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity – parts I and II," submitted to *IEEE Trans. on Communications*.

[10] V. Sreng, D. Falconer, and H. Yanikomeroglu, "Capacity enhancement through relaying in cellular radio systems," unpublished.

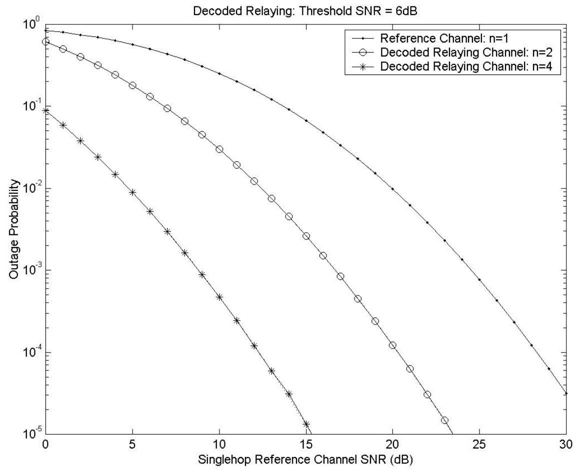

Fig. 2. Probability of Outage for Decoded Relaying Multihop Channel

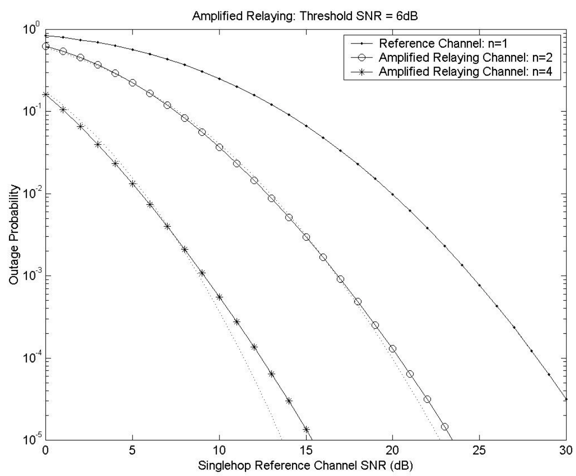

Fig. 3. Probability of Outage for Amplified Relaying Multihop Channel

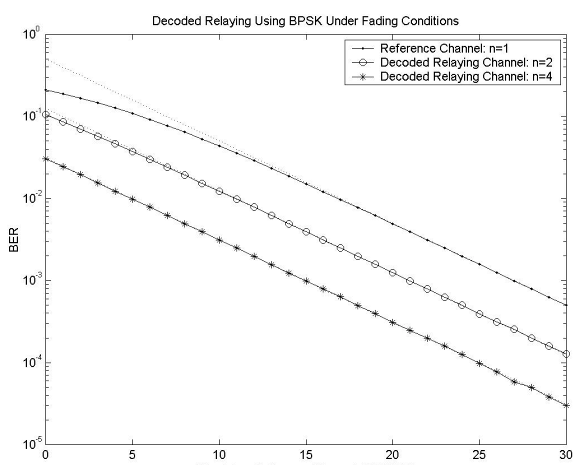

Fig. 4. Probability of Error for Decoded Relaying Multihop Channel

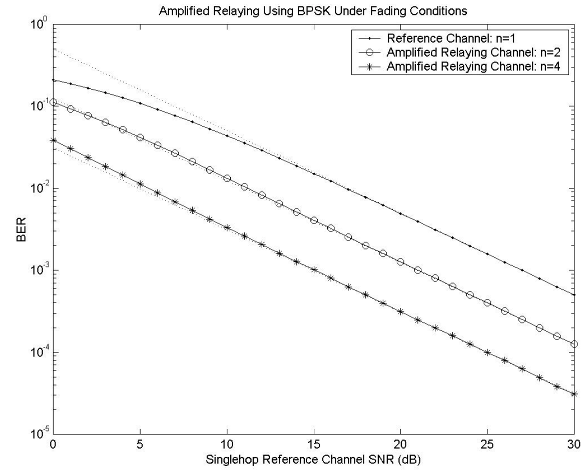

Fig. 5. Probability of Error for Amplified Relaying Multihop Channel

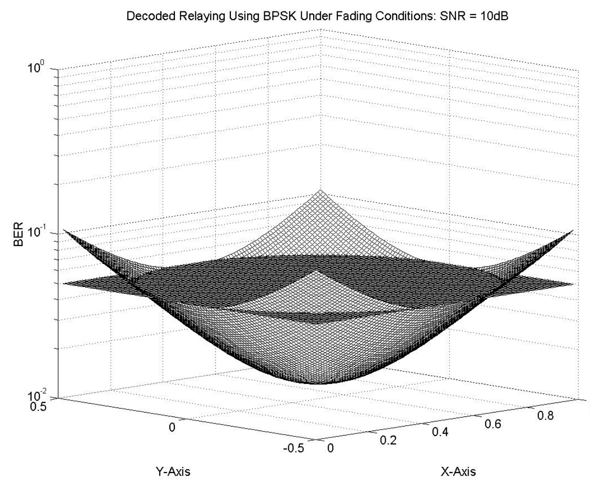

Fig. 6. Probability of Error for Decoded Relaying Multihop Channel

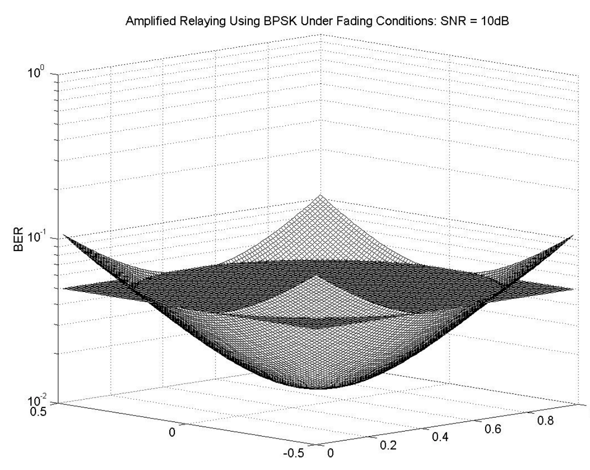

Fig. 7. Probability of Error for Amplified Relaying Multihop Channel