On the generalization of decode-and-forward and compress-and-forward for Gaussian relay channels

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Abstract—**In this paper, the generalization of the decode-and-forward (DF) and compress-and-forward (CF) relaying schemes is studied for the case in which Gaussian codebooks are used for signalling over scalar Gaussian memoryless channels. Three SNR regions are identified wherein the generalized DF-CF scheme reduces to either the DF or the CF scheme. In addition, it is shown that there is an SNR region in which the generalized DF-CF scheme can be more advantageous than both schemes.**

I. Introduction

A relay channel refers to a cooperative communication system in which one or more intermediate nodes, known as relays, assist the communication between the source and the destination [1], [2]. Data transmitted by the source is received by both the relay and the destination. The relay does not have its own message and its output depends either deterministically or stochastically on its observed signal. The destination receives a noisy combination of the source and relay signals and processes this combination to decode the intended message.

Despite the envisioned advantages of cooperative communications, the capacity of relay channels, including scalar ones with one relay, remains an open problem, and only partial results are available. Among the various cooperation techniques are the decode-and-forward (DF) and compress-and-forward (CF) techniques [3]. In DF, the relay decodes its observed signal and generates auxiliary information that assists the decoding of the transmitted codeword at the destination. In contrast, in the CF scheme the relay does not decode its observed signal, but compresses it to generate the auxiliary information that facilitates decoding at the destination [3].

In [3] the DF scheme was shown to achieve the capacity of the so-called degraded relay channel in which the signal observed by the destination is a physically degraded [2] version of that observed by the relay. However, in the case of general relay channels that are not degraded, the DF scheme is not necessarily capacity achieving and in many scenarios higher rates can be attained using the CF technique [4]. To explore relaying methodologies that have the potential of achieving higher rates, a generalization that subsumes both DF and CF as special cases was proposed in [3]. In this generalization, the relay combines partial decoding and compression of its received signal to generate the auxiliary information.

An upper bound on the capacity of relay channels is given by the cut-set bound [2]. In [5] this bound and lower bounds derived from CF and the generalization of the DF and CF schemes [3, Theorem 7] have been considered for static channel gains. The application of these schemes have also been extended to Rayleigh fading scalar channels in [4] and to multiple antenna channels in [6]. The DF and CF schemes have also been used in more general multi-terminal networks with multiple sources, relays and destinations [7], [8]. In particular, in [7] the network performance has been investigated for Gaussian and Rayleigh fading channels when some source-relay pairs use DF relaying and the remaining pairs use CF relaying. In [8] the DF scheme was shown to achieve the capacity, not only of degraded single relay networks, but also that of more general degraded networks with multiple relays.

The generalization of the DF and CF schemes encompasses both strategies, and thus, in principle, offers the potential of achieving higher rates than either scheme. Despite being available for more than thirty years, to the best of the authors' knowledge, particular scenarios in which the potential advantages of this generalization are realized have not been studied. Hence, a fundamental question that this paper attempts to answer is whether such scenarios exist. To address this question, we particularize this generalization to Gaussian channels with one relay and analyze the maximum rate it can achieve with Gaussian codebooks in three distinct SNR regions.

Prior to submitting this paper, we became aware of an independent investigation of the generalized DF-CF scheme for Gaussian channels [9]. In that investigation, it was concluded that the generalization does not provide rates higher than DF and CF. However, it appears that certain steps in the analysis in [9] are based on the implicit restriction of the distribution of the codebooks used in the generalized DF-CF scheme to be the same as that of the corresponding codebooks used in the CF scheme.

In contrast with [9], we consider the case in which the generalization of the DF and CF schemes uses independently-parametrized Gaussian codebooks. We identify signal-to-noise (SNR) regions of the source-destination, source-relay and relay-destination links in which the generalization reduces to either DF or CF. In addition, we identify SNR regions in which the generalization can achieve higher rates. In Section IV, an instance of the SNRs and the parameters at which the generalization is more beneficial than both DF and CF is provided. Although the gain offered by the generalization appears to be small, this does not preclude the possibility that higher gains can be achieved in other scenarios; e.g., when the codebooks and/or the channel models are different.

Recently, it has been shown in [10]–[12] that the generalized DF-CF scheme might yield higher rates when decoding tech-

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niques other than that proposed in [3, Theorem 7] are utilized. When those techniques are employed and the codebooks are restricted to be Gaussian, the maximum achievable rate of the corresponding mixed DF-CF sceheme in various SNR regions can be analyzed using the approach provided herein.

II. Generalization of DF and CF for General Gaussian Relay Channels

We consider the relay channel in [3]. The Gaussian version of this channel is depicted in Figure 1, wherein X_1 and X_2 denote the transmitted signal of the source and the relay, respectively, and Y_1 and Y denote the observed signals at the relay and the destination, respectively. For this channel, we will study the maximum rate provided by the generalized DF-CF relaying scheme developed in [3, Theorem 7] when the codebooks are restricted to be Gaussian.

To begin with, in this section the rate expressions obtained in [3] for general discrete memoryless channels will be particularized to general scalar power-constrained memoryless Gaussian channels when Gaussian codebooks are used for signalling. We begin by providing a descriptive overview of this generalization for general discrete memoryless channels. We will use $\mathcal{X}_i$ to denote the codebook containing the length-n codewords, $\{x_{i,k}\}$, $i = 1, 2$. The auxiliary codebooks that will be used for generating $\mathcal{X}_1$ and $\mathcal{X}_2$ are defined similarly.

A. The general discrete memoryless channel

Codebook generation: Two auxiliary codebooks $\mathcal{U}$ and $\mathcal{V}$ are available at both the source and the relay. Each $v \in \mathcal{V}$ corresponds to a partition of $\mathcal{U}$ and a partition of $\mathcal{X}_2$. Each $u \in \mathcal{U}$ corresponds to a partition of the codebook $\mathcal{X}_1$. At the relay, an estimation codebook $\hat{\mathcal{Y}}_1$ is generated with partitions corresponding to each (u, x_2) pair, where $x_2 \in \mathcal{X}_2$, and $\mathcal{U}$, $\mathcal{X}_1$ and $\mathcal{X}_2$ are the unions of their respective partitions.

Encoding: We will use superscripts to denote the block index. At the i-th block, the source and the relay determine $v^{(i)}$ to be the auxiliary codeword that corresponds to the index of the partition of $u^{(i-1)}$ used in the $(i-1)$-th block. The source wishes to send the message $w^{(i)}$, which can be expressed as a pair $(w^{(i)\prime}, w^{(i)\prime\prime})$. Knowing $v^{(i)}$, the source chooses the particular $u^{(i)}(v^{(i)})$ to encode $w^{(i)\prime}$ and chooses $x_1^{(i)}(u^{(i)}|v^{(i)})$ to encode $w^{(i)\prime\prime}$. At each block, the relay assists the destination by sending information about the previous block. At the i-th block, the relay knows $u^{(i-2)}$ and $x_2^{(i-1)}$. The relay uses $u^{(i-2)}$ to determine $v^{(i-1)}$, and upon receiving $y_1^{(i-1)}$, it uses $v^{(i-1)}$ to determine $u^{(i-1)}$. Knowing the pair $(u^{(i-1)}, x_2^{(i-1)})$, the relay determines the corresponding partition of $\hat{\mathcal{Y}}_1$, which is given by the index of $(u^{(i-1)}, x_2^{(i-1)})$. The relay then chooses $\hat{y}_1^{(i-1)}$ in this partition that is jointly typical with its received signal $y_1^{(i-1)}$. The index of $\hat{y}_1^{(i-1)}$ in the partition defined by $(u^{(i-1)}, x_2^{(i-1)})$ yields the index of $x_2^{(i)}$.

Decoding: The destination uses joint typicality to successively decode the components of the transmitted codewords and subsequently the message $w^{(i)} = (w^{(i)\prime}, w^{(i)\prime\prime})$. At the i-th block, the destination observes $y^{(i)}$ and uses it to decode $v^{(i)}$ and subsequently $x_2^{(i)}$. Using $(v^{(i)}, v^{(i-1)}, y^{(i-1)})$, the destination decodes $u^{(i-1)}$ and thus obtains $w^{(i-1)\prime}$. The destination uses the quintuple $(v^{(i-1)}, u^{(i-1)}, x_2^{(i-1)}, x_2^{(i)}, y^{(i-1)})$ to determine $\hat{y}_1^{(i-1)}$, from which $w^{(i-1)\prime\prime}$ is recovered.

Using the above codebook construction and the encoding-decoding procedure, it was shown in Theorem 7 in [3] that the following rate is achievable:

$$R = \sup\Big\{\min\{I(X_1; Y, \hat{Y}_1|X_2, U) + I(U; Y_1|X_2, V), \quad I(X_1, X_2; Y) - I(\hat{Y}_1; Y_1|X_2, X_1, U, Y)\}\Big\}, \tag{1a}$$

subject to the constraint

$$I(\hat{Y}_1; Y_1|Y, X_2, U) \leq I(X_2; Y|V), \tag{1b}$$

where the supremum is taken over the probability mass functions of the form $p(u, v, x_1, x_2, y, y_1, \hat{y}_1) = p(v)p(u|v)p(x_1|u)p(x_2|v)p(y, y_1|x_1, x_2)p(\hat{y}_1|x_2, y_1, u)$.

B. The scalar Gaussian memoryless channel

In this section we will particularize the construction of the previous section to the case depicted in Figure 1 wherein the codebooks and the channels are Gaussian. The gain of the source-destination link is normalized, and the gains of the source-relay and the relay-destination links are denoted by a and b, respectively. The received signal at the relay, Y_1, and

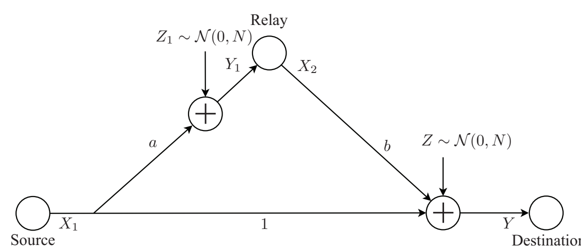

Fig. 1. Relay Channel.

the destination, Y, can be expressed as

$$Y_1 = aX_1 + Z_1, \tag{2}$$

$$Y = X_1 + bX_2 + Z. \tag{3}$$

The additive Gaussian noise at the relay is denoted by $Z_1 \sim \mathcal{N}(0, N)$ and that at the destination is denoted by $Z \sim \mathcal{N}(0, N)$. The transmit power at the source and relay are denoted by P_1 and P_2, respectively. We also define the following SNRs: $\gamma_0 \triangleq \frac{P_1}{N}$, $\gamma_1 \triangleq \frac{a^2 P_1}{N}$, and $\gamma_2 \triangleq \frac{b^2 P_2}{N}$.

To construct the codebooks described in the previous section, we define the following power partitions $\{\alpha_i\}_{i=0}^2$ to be used at the source, and $\{\beta_j\}_{j=0}^1$ to be used at the relay, where $\alpha_i, \beta_j \geq 0$, $i = 0, 1, 2$, $j = 0, 1$, $\sum_{i=0}^2 \alpha_i = 1$, and $\sum_{j=0}^1 \beta_j = 1$. For Gaussian codebooks, we have $V \sim \mathcal{N}(0, 1)$, and we define $V_1 = \sqrt{\alpha_0 P_1} V$, and $V_2 = \sqrt{\beta_0 P_2} V$. Consider the construction of the auxiliary codebook $\mathcal{U}$ described in Section II-A. From this construction, a codeword $u \in \mathcal{U}$ can be expressed as the superposition of a codeword $v_1 \in \mathcal{V}_1$, containing the partition information and another codeword that contains incremental information, which we denote by $x_{11} \in \mathcal{X}_{11}$. The codebooks $\mathcal{X}_1$, $\mathcal{X}_2$ can be constructed similarly using incremental codebooks $\mathcal{X}_{12}$ and $\mathcal{X}_{22}$, respectively. For the estimation codebook, $\hat{\mathcal{Y}}_1$, the partition information is contained in the superposition of the $\mathcal{X}_2$ and $\mathcal{U}$, and the incremental information can be expressed in terms of another Gaussian codebook. However, since $\hat{Y}_1$ represents an estimation of Y_1, it will be convenient to use the approach in [5] and [7] to express the incremental information as the superposition of Y_1 and a statistically independent estimation noise Z'. In particular,

$$U = V_1 + X_{11}, \qquad X_1 = U + X_{12}, \tag{4a}$$
$$X_2 = V_2 + X_{22}, \qquad \hat{Y}_1 = Y_1 + X_2 + U + Z', \tag{4b}$$

where $Z' \sim \mathcal{N}(0, N')$. With the construction in Section II-A, V, X_{11} and X_{22} are mutually statistically independent, and U and X_{12} are statistically independent; X_1 and X_2 are correlated through V. The source uses the power fraction $\alpha_0 P_1$ to transmit V_1, the power fraction $\alpha_1 P_1$ to transmit X_{11} and the power fraction $\alpha_2 P_1$ to transmit X_{12}. The relay uses the power fraction $\beta_0 P_2$ to transmit V_2 and the power fraction $\beta_1 P_2$ to transmit X_{22}.

III. Application of the Generalized DF-CF scheme to Gaussian channels: Achievable Rate expressions and analysis

Using the construction of the Gaussian codebooks with the SNRs and power partitions in the previous section, we have:

Proposition 1: Restricting the codebooks of the generalized DF-CF scheme to be Gaussian, the following rate is achievable for scalar Gaussian memoryless channels.

$$R_G^* = \max_{\{\alpha_i\}_{i=0}^2, \{\beta_j\}_{j=0}^1, \gamma'} \min\{R_1, R_2\}, \tag{5a}$$

subject to

$$\gamma' \geq \frac{(1 + \alpha_2\gamma_0 + \alpha_2\gamma_1)(1 + \alpha_1\gamma_0 + \alpha_2\gamma_0)}{(1 + \alpha_2\gamma_0)\beta_1\gamma_2}, \tag{5b}$$

$$\sum_{i=0}^{2} \alpha_i = 1, \ \sum_{j=0}^{1} \beta_j = 1, \tag{5c}$$

$$\alpha_i \geq 0, \ \beta_j \geq 0, \ \forall \ i, j, \tag{5d}$$

where

$$R_1 = \mathcal{C}\Big(\frac{\alpha_2\gamma_1}{1+\gamma'} + \alpha_2\gamma_0\Big) + \mathcal{C}\Big(\frac{\alpha_1\gamma_1}{1+\alpha_2\gamma_1}\Big), \tag{6}$$

$$R_2 = \mathcal{C}\Big(\gamma_0 + \gamma_2 + 2\sqrt{\alpha_0\beta_0\gamma_0\gamma_2}\Big) - \mathcal{C}\Big(\frac{1}{\gamma'}\Big), \tag{7}$$

where $\mathcal{C}(x) \triangleq \frac{1}{2}\log_2(1+x)$, $x \geq 0$, and $\gamma' \triangleq N'/N$.

Proof: See Appendix A in [13]. ■

Note that when $\alpha_2 = 0$ and $\beta_1 = 0$, the rate in (5a) reduces to the maximum achievable rate of the DF scheme; i.e.,

$$\begin{aligned} R_G^*\Big|_{\substack{\alpha_2=0\\ \beta_1=0}} &= \max_{\rho_0} \min\Big\{\mathcal{C}\big(\rho_1\gamma_1\big), \mathcal{C}\big(\gamma_0 + \gamma_2 + 2\sqrt{\rho_0\gamma_0\gamma_2}\big)\Big\} \\ &= R_{DF}^*, \end{aligned} \tag{8}$$

where $\sqrt{\rho_0}$ is the correlation coefficient of X_1 and X_2 in the DF scheme, and ρ_0 and ρ_1 satisfy $\sum_{i=0}^{1}\rho_i = 1$; cf. [3, Theorem 1]. For Gaussian channels the maximum rate of the DF scheme is achieved with Gaussian codebooks [7, Proposition 2]. Furthermore, when $\alpha_2 = 1$ and $\beta_1 = 1$, the rate in (5a) reduces to the maximum achievable rate of the CF scheme with Gaussian codebook construction, R_{CF}^*; that is,

$$R_G^*\Big|_{\substack{\alpha_2=1\\ \beta_1=1}} = \mathcal{C}\Big(\gamma_0 + \frac{\gamma_1\gamma_2}{1+\gamma_0+\gamma_1+\gamma_2}\Big) = R_{CF}^*. \tag{9}$$

However, for CF, it is generally not known if Gaussian codebooks achieve the maximum rate.

In Appendix C-C in [13], we show that the normalized variance of the Gaussian estimation noise, γ', that yields the maximum achievable rate of the CF scheme in (9) is

$$\gamma_{CF}'^* = \frac{1+\gamma_0+\gamma_1}{\gamma_2}. \tag{10}$$

We will analyze the achievable rates yielded by the optimization problem in (5) for three distinct cases; viz., $\gamma_1 > \gamma_0(1+\gamma_2)$, $\gamma_1 \leq \gamma_0$, and $\gamma_2 \to \infty$.

A. The case of $\gamma_1 > \gamma_0(1+\gamma_2)$

In this region the gain of the source-relay link is greater than the gain of the source-destination link; i.e., $a > 1$ in Figure 1. In this case, we have the following theorem.

Theorem 1: When the channel gains in Figure 1 satisfy $\gamma_1 > \gamma_0(1+\gamma_2)$, the generalized DF-CF scheme with Gaussian codebooks reduces to the DF scheme. In particular,

$$R_G^* = R_{DF}^*. \tag{11}$$

Proof: To prove this theorem, we consider the Karush-Kuhn-Tucker (KKT) conditions corresponding to an alternate formulation of (5). Since neither (5) nor the reformulated problem is convex, the KKT conditions are only necessary for optimality [14]. To analyze these conditions, we prove in Appendix D in [13] that, when $\gamma_1 > \gamma_0(1+\gamma_2)$, the optimal γ' must satisfy

$$\gamma' > \frac{\gamma_0}{\gamma_1 - \gamma_0}. \tag{12}$$

Using (12), the KKT system yields two solutions. For the first solution $R_G^* = R_2 \leq R_1$, and the corresponding $\gamma' = \infty$. By analyzing the power partitions, $\{\alpha_i\}_{i=0}^2$ and $\{\beta_j\}_{j=0}^1$, we show that $R_1 = 0$, which implies that $R_G^* = 0$. Hence, this solution does not yield the maximum rate that can be obtained by the generalized DF-CF scheme, and R_G^* must be given by the second solution. For this solution

$$R_G^* = R_1 \leq R_2, \tag{13}$$

and $\alpha_2 = \beta_1 = 0$. However, from (8), this setting for the power partitions corresponds to the DF relaying scheme, which implies the statement of the theorem.

The details of the proof are given in Appendix E in [13]. ■

Theorem 1 indicates that if the channel gain of the source-relay link is sufficiently large, compared to the other two links, the generalized DF-CF scheme with Gaussian codebooks does not yield rates higher than those achieved by the DF scheme.

B. The case of $\gamma_1 \leq \gamma_0$

When $\gamma_1 \leq \gamma_0$, the channel gain of the source-relay channel is less than or equal to that of the source-destination channel, which corresponds to the case of $a \leq 1$ in Figure 1. The main result of this section is provided in the following theorem.

Theorem 2: When the channel gains in Figure 1 satisfy $\gamma_1 \leq \gamma_0$, the generalized DF-CF scheme with Gaussian codebooks reduces to the corresponding CF scheme. In particular,

$$R_G^* = R_{CF}^*. \tag{14}$$

Proof: To prove this theorem, we use the construction of the Gaussian codebooks in Section II to derive an upper bound on R_1 in (6) for any scalar Gaussian memoryless relay channel. In particular, in Appendix F in [13], we show that

$$R_1 \leq R_2 - I(X_2, U; Y) + I(X_2; Y|V) + I(U; Y_1|X_2, V).$$

Equality holds when the condition in (1b) holds with equality; that is, when

$$I(\hat{Y}_1; Y_1|Y, X_2, U) = I(X_2; Y|V). \tag{15}$$

In fact, it was shown in [12] that restricting (1b) to hold with equality does not incur loss in the achievable rate. Using this observation it can be seen that constructing the codebooks in such a way that (15) is satisfied ensures that both R_1 in (6) and R_2 in (7) are maximized; i.e., enforcing the constraint in (1b) to hold with equality is without loss of optimality.

When (15) holds, we have

$$R_1 = R_2 - I(X_2, U; Y) + I(X_2; Y|V) + I(U; Y_1|X_2, V). \tag{16}$$

Hence, to proceed with the proof, we invoke the construction of the Gaussian codebooks in Section II-B into (16). We then show that when $\gamma_1 \leq \gamma_0$,

$$I(X_2, U; Y) - I(X_2; Y|V) - I(U; Y_1|X_2, V) < 0,$$

which implies that when $\gamma_1 \leq \gamma_0$, $R_1 \leq R_2$; that is, in this SNR region, R_1 is the constraining rate.

Hence, to maximize the rate that can be achieved by the generalization of the DF and CF schemes, it suffices to maximize the right hand side of (16). Doing so, in Appendix G in [13], we show that the maximum is achieved if and only if $\alpha_2 = 1$ and $\beta_1 = 1$, which, using (9), implies the statement of the theorem. ■

Theorem 2 suggests that when the codebooks are Gaussian and the channel gain of the source-relay link is less than that of the source-destination link, the generalized DF-CF scheme does not provide a higher rate than the CF scheme.

C. The case of $\gamma_2 \to \infty$, $0 < \gamma_0, \gamma_1 < \infty$

In this section we investigate the case in which the relay-destination link is perfect; that is, the case in which the gain of the source-relay channel and the source power are finite, but $b^2 P_2$ is infinite, where b is the gain of the relay-destination link in Figure 1. For this case we have the following result.

Theorem 3: When $\gamma_2 \to \infty$, the generalized DF-CF scheme with Gaussian codebooks reduces to the corresponding CF scheme and achieves the capacity of the relay channel; i.e.,

$$R_G^* = R_{CF}^* = C, \tag{17}$$

where C is the capacity of the relay channel.

Proof: To prove this theorem, we show that, for $\gamma_2 \to \infty$, R_1 in (6) is maximized when $\alpha_2 = \beta_1 = 1$. Using (9), we have that the maximum rate that the generalized scheme can achieve with these power partitions is equal to the maximum rate that the CF scheme when the codebooks are Gaussian. To complete the proof of the first statement of the theorem we show that these settings yield $R_2 \geq R_1$. To show that the CF scheme with Gaussian codebooks achieves the channel capacity in this asymptotic case, we show that R_{CF}^* attains the cut-set bound with Gaussian codebooks. Invoking Proposition 2 in [7] yields the second statement of the theorem.

See Appendix H in [13] for the details of the proof. ■

The optimality of the CF scheme with Gaussian codebooks in the asymtptoic case of $\gamma_2 \to \infty$ has also been observed in [5] using a somewhat different approach.

Theorem 3 implies that for any finite source-relay channel gain, if the relay power is sufficiently higher than the source power, superimposing the DF and the CF schemes with Gaussian codebooks does not yield additional gain.

Hence, in each of the three regions, when the codebooks are restricted to be Gaussian, the maximum rate that the generalized DF-CF scheme can achieve does not exceed that can be achieved by either the DF or the CF scheme.

IV. Numerical Results

In this section, we will provide numerical results that show the performance of the generalized DF-CF scheme with Gaussian codebooks in different SNR regions for the Gaussian channel depicted in Figure 1. For comparison, the cut-set bound in [3] is also plotted.

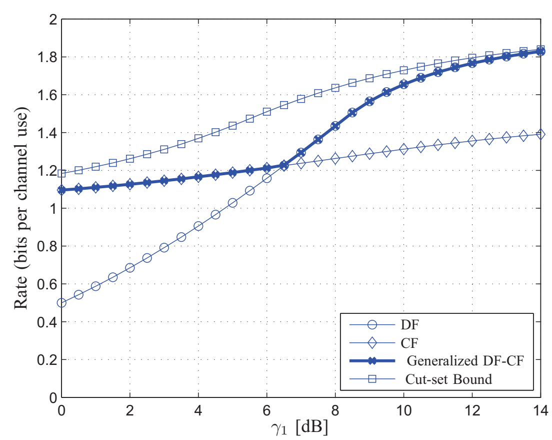

Fig. 2. Maximum achievable rate of the generalized DF-CF scheme.

To generate the numerical results, we calculated the maximum achievable rate of the DF using the expression in (8). From this expression, it can be shown that the square of the optimal correlation coefficient, ρ_0^*, is either 0, or it results in equating the minimization arguments. For the CF scheme, the maximum achievable rate when the codebooks are Gaussian is given in (9); cf. [13, Appendix C]. Finally, for the generalized DF-CF scheme, we use the KKT necessary optimality

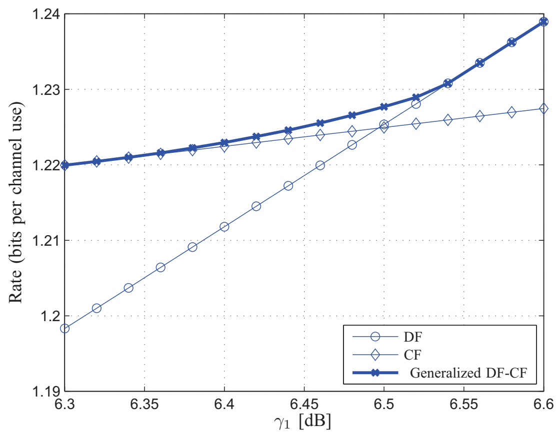

Fig. 3. A magnified version of Figure 2. Generalization outperforms the DF and CF schemes.

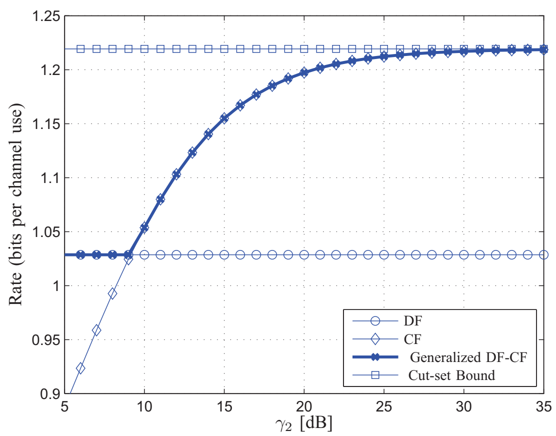

Fig. 4. Generalization reduces to the CF scheme as $\gamma_2 \to \infty$.

conditions corresponding to the formulation in (5) to reduce the search for the optimal power partitions.The cut-set bound is calculated using the expression in [3, Theorem 4].

In Figures 2 and 3, the SNR of the source-destination and the relay-destination links are set to be $\gamma_0 = 5$ dB and $\gamma_2 = 5.5$ dB, respectively. For this setting, $\gamma_0(1+\gamma_2) = 11.5$ dB.

From Figure 2 it can be seen that, in agreement with Theorem 1, for $\gamma_1 > 11.5$ dB, R_G^* coincides with the maximum rate of the DF scheme. Similarly, in agreement with Theorem 2, for $\gamma_1 \leq 5$ dB, R_G^* coincides with the maximum achievable rate of the CF scheme with Gaussian codebooks.

To further investigate the performance of the generalized DF-CF scheme with Gaussian codebooks when $\gamma_0 < \gamma_1 \leq \gamma_0(1+\gamma_2)$, in Figure 3 we plot a magnified version of the region in which $\gamma_1 \in [6.3, 6.6]$ dB. This figure indicates that the generalized DF-CF scheme can yield a rate gain in the region at which the DF and CF schemes yield approximately the same rate. For instance, at $\gamma_1 \approx 6.496377$ dB, $R_{DF}^* = R_{CF}^* \approx 1.22486$ bits per channel use (bpcu), whereas $R_G^* \approx 1.22748$ bpcu, which is obtained by the following power partitions: $\alpha_0 = 0.004$, $\alpha_1 \approx 0.762621$, $\alpha_2 = 1 - \alpha_1 - \alpha_0$, $\beta_0 \approx 0.179$, and $\beta_1 = 1 - \beta_0$.

Finally, in Figure 4 we verify Theorem 3 when $\gamma_0 = 1$ dB and $\gamma_1 = 5$ dB. For these SNR settings, the rate achieved by the DF scheme corresponds to the first term in the argument of the minimization on the right hand side of (8), and hence, is independent of γ_2. In accordance with Theorem 3, it can be seen from Figure 4 that as γ_2 becomes sufficiently large, the maximum achievable rate of the generalized DF-CF scheme coincides with the maximum achievable rate of the CF scheme, which coincides with the cut-set bound.

V. Conclusion

In this paper, we investigated the generalized DF-CF scheme developed in Theorem 7 in [3] when Gaussian codebooks are used for signalling over scalar power-constrained Gaussian memoryless channels. We have shown that the generalization:

- reduces to the CF scheme if the SNR of the source-relay link is less than that of the source-destination link or if the SNR of the relay-destination link is sufficiently high;
- reduces to the DF scheme when the SNR of the source-relay link is sufficiently higher than the SNR of the source-destination link;
- can be more advantageous than both DF and CF schemes for intermediate source-relay link SNRs at which the DF and CF schemes yield approximately the same rates.

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References

[1] E. C. van der Meulen, “Three-terminal communication channels,” *Adv. Appl. Probab.*, vol. 3, pp. 120–154, 1971.
[2] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.
[3] T. M. Cover and A. A. El Gamal, “Capacity theorems for the relay channel,” *IEEE Trans. Inf. Theory*, vol. IT-25, pp. 572–584, Sept. 1979.
[4] A. Høst-Madsen and J. Zhang, “Capacity bounds and power allocation for wireless relay channels,” *IEEE Trans. Inf. Theory*, vol. 51, pp. 2020–2040, June 2005.
[5] A. A. El Gamal, M. Mohseni, and S. Zahedi, “Bounds on capacity and minimum energy-per-bit for AWGN relay channels,” *IEEE Trans. Inf. Theory*, vol. 52, pp. 1545–1561, Apr. 2006.
[6] B. Wang, J. Zhang, and A. Høst-Madsen, “On the capacity of MIMO relay channels,” *IEEE Trans. Inf. Theory*, vol. 51, pp. 20–43, Jan. 2005.
[7] G. Kramer, M. Gastpar, and P. Gupta, “Cooperative strategies and capacity theorems for relay networks,” *IEEE Trans. Inf. Theory*, vol. 51, pp. 3037–3063, Sept. 2005.
[8] P. Razaghi and W. Yu, “Parity forwarding for multiple-relay networks,” *IEEE Trans. Inf. Theory*, vol. 55, pp. 158–173, Jan. 2009.
[9] N. Ramalingam and Z. Wang, “On the rate achievable for Gaussian relay channels using superposition forwarding,” Oct. 2010. Available at: http://arxiv.org/abs/1010.3071.
[10] S. R. Bhaskaran, “Forward decoding over a relay channel,” *Proc. IEEE Int. Symp. Inf. Theory*, pp. 2673–2677, July 2008.
[11] H.-F. Chong, M. Motani, and H. Krishna, “Generalized backward decoding strategies for the relay channel,” *IEEE Trans. Inf. Theory*, vol. 53, pp. 394–401, Jan. 2007.
[12] H.-F. Chong and M. Motani, “On achievable rates for the general relay channel,” *IEEE Trans. Inf. Theory*, vol. 57, pp. 1249–1266, Mar. 2011.
[13] K. Luo, R. H. Gohary, and H. Yanikomeroglu, “On the generalization of decode-and-forward and compress-and-forward for Gaussian relay channel,” Jan. 2011. Available at: http://www.sce.carleton.ca/~wenluo/publications/ITW_2011/itw_2011_full.pdf.
[14] D. P. Bertsekas, *Nonlinear programming*. Nashua, NH: Athena Scientific, 2nd ed., 1999.