Correlated Interference Analysis in CDMA Multi-Antenna Systems

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ABSTRACT

CDMA sectorized distributed antenna system constitutes the logical extent of the soft handoff scheme where all the users communicate with all the antennas in the service region. It has been reported that, in the reverse link, such systems, in comparison to conventional single-antenna systems, have the potential to achieve a capacity increase which is equal to the number of antenna elements used. In order to achieve this gain, the interference picked up by different antenna elements must be uncorrelated. In this paper, we present a spatial correlated interference analysis for the simplest non-trivial system which has 2 antenna elements and 2 users. The effects of system parameters and user & antenna element locations on the correlated interference are investigated. This work lays down a foundational framework that may yield a comprehensive understanding of the performance of CDMA multi-antenna systems.

I. INTRODUCTION AND DEFINITIONS

The CDMA sectorized distributed antenna (SDA) system has recently been proposed as a promising alternative to the conventional wireless access system that employs a single central antenna (base station) [1-2]. An SDA is a multi-antenna system in which many simple antenna elements (AE’s) are connected to a central station (CS) with separate feeders. All of the intelligence is centralized at the CS; thus, there is no signal-specific processing at the AE’s. In the reverse link of an SDA system, a user’s signal is received by all of these AE’s and decoded jointly at the CS. It is worth noting that in an SDA system the AE’s are distributed throughout the coverage area, unlike an antenna array where the AE’s are located only a few wavelengths apart. The SDA scheme is the logical extent of the soft handoff scheme where all the users communicate with all the antennas in the service region, and where the decision making process is centralized.

It has been shown in [1-2] that the total interference experienced in the reverse link of an SDA system with \( L \) AE’s can be as small as \((1/L)\)th of that experienced in a conventional single antenna system. This yields an approximately \( L \)-fold increase in the reverse link SIR (signal-to-interference ratio), which can be transformed into an equivalent increase in the capacity and/or information rate. The reverse link capacity of a network of antennas is also investigated in [3]. Although the approach in that study is different than that used in [1-2], the results are in agreement.

In [1-3], a maximal ratio combiner (MRC) is used at the receiver. In order for an MRC to maximize the output SIR, the interference components at the branches of the combiner must be uncorrelated. If this is the case, then the output SIR turns out to be the algebraic sum of the branch SIR’s. If, however, the interference components are correlated, then the output SIR will be less than the sum of the branch SIR’s. In the limiting case of identical signal and interference components at the branches, there is no gain at all from multi-antenna reception (combining), since amplification does not increase the SIR. Therefore, in multi-antenna systems, it is critical to investigate the presence of correlated interference in the branches of the combiner.

In this paper, we will present a spatial correlated interference analysis for the simplest non-trivial system which has 2 AE’s and 2 users. The effects of system parameters (such as the processing gain and size of the coverage region) and the user & AE locations on the correlated interference will be demonstrated. To the best of the authors’ knowledge, there exists almost no literature in this topic. Therefore, we believe that our work will lay down a foundational framework that may yield a comprehensive understanding of the relationship between the number of AE’s and the yielding system capacity in CDMA multi-antenna systems.

We consider an SDA system with 2 AE’s, AE \( I \) and AE \( II \), and 2 users. We assume that wireless user 1 (\( w_1 \)) is the user of interest. Fig. 1 shows the propagation delays in the system. The total propagation time from \( w_1 \) to the CS through AE \( I \), \( t_{1I,I} \), is equal to the sum of the propagation times in the air, \( t_{1I} \), and in the cable, \( t_{1I,c} \):

\[
t_{1I,I} = t_{1I} + t_{1I,c}.
\]  

We note that \( t_{1I,c} \) does not depend on the location of \( w_1 \). Similarly, \( t_{1II,I} = t_{1II} + t_{1II,c} \). Propagation times

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regarding $w_2$ are defined in a similar way.

The baseband transmitters for the two users and baseband receiver for $w_1$ at the CS are illustrated in Fig. 2. Also, the spreading code and the related assumptions are depicted in Fig. 3. At $w_1$’s receiver, the despreading at the $I$th branch is performed by multiplying the received signal by $c_1(t - t_{1I}; \Delta_1)$, where $c_1(t)$ is the spreading code for $w_1$ and $\Delta_1$ is the corresponding (normalized) code phase. $\Delta_1$ is represented in terms of $\tau$ (chip duration); i.e., $c_1(t - t_{1I}; \Delta_1) = c_1(t - t_{1I} - \Delta_1 \tau)$. The situation at the $II$th branch of $w_1$’s receiver is similar; obviously, in this case the despreading is performed by $c_1(t - t_{1II}, t; \Delta_1)$. The background noise is omitted in our analysis.

II. CORRELATION COEFFICIENT ANALYSIS

Let $n_{1I}$ and $n_{1II}$ be the interference components (after despreading) at the branches of the MRC of $w_1$, corresponding to AE’s $I$ and $II$, respectively:

$$n_{1I} = \langle c_1(t - t_{1I}, t; \Delta_1) c_2(t - t_{2I}, t; \Delta_2) \rangle,$$  \hspace{1cm} (3)

where $\langle \rangle$ denotes the inner product (refer to Fig. 3).

We are interested in evaluating the correlation coefficient of the random variables $n_{1I}$ and $n_{1II}$, $\rho$, which is defined as

$$\rho = \frac{\mathbf{E}(n_{1I} n_{1II}) - \mathbf{E}(n_{1I}) \mathbf{E}(n_{1II})}{\sqrt{\mathbf{E}(n_{1I}^2) - \mathbf{E}^2(n_{1I})} \sqrt{\mathbf{E}(n_{1II}^2) - \mathbf{E}^2(n_{1II})}},$$  \hspace{1cm} (4)

where $\mathbf{E}(\cdot)$ denotes the expectation operator.

We also define (normalized) differential code phase, $\Delta_{12}$, as the difference between the spreading code phases of $w_1$ and $w_2$; i.e., $\Delta_{12} = \Delta_1 - \Delta_2$. Without loss of generality, $\Delta_{12}$ is modeled as a uniform random variable in the interval $[0, 1]$.

We note that although $c_1(t) c_2(t) \not= c_1(t - t_0) c_2(t - t_0)$, where $t_0$ denotes some delay, the following equation holds: $\langle c_1(t) c_2(t) \rangle = \langle c_1(t - t_0) c_2(t - t_0) \rangle$. Therefore,

$$\langle c_1(t - t_{1I}) c_2(t - t_{2I}; \Delta_2) \rangle = \langle c_1(t - [t_{1I} - 2] - t_{2I}; \Delta_2) c_2(t) \rangle = \langle c_1(t) c_2(t - [t_{1I} - 2]; \Delta_2) \rangle. \hspace{1cm} (5)$$

As a starting point, we make the assumption that users are synchronized, in other words, $\Delta_1 = \Delta_2 = 0$. The general case, where $\Delta_{12}$ is a uniform random variable in the region $[0, 1]$, will be addressed at the end of this section.

A. Dependence of Correlation Coefficient on Propagation Delays

Equation (5) can be used to simplify $n_{1I}$ and $n_{1II}$, defined by (2) and (3), as

$$n_{1I} = \langle c_1(t - t_{1I}) c_2(t) \rangle, \quad n_{1II} = \langle c_1(t - t_{1II}) c_2(t) \rangle, \hspace{1cm} (6)$$

where,

$$t_{1II} = t_{1I} + t_{t_{1I,c}} - t_{2II}, \quad t_{2II} = t_{1II} + t_{t_{2II,c}} = t_{1I} - t_{2II}. \hspace{1cm} (7)$$

It is observed from (7) and (8) that $t_{1II}$ and $t_{2II}$ depend only on the propagation delays in the air.

Without loss of generality we will first assume that $0 \leq t_{1II} \leq \tau$; we will consider the most general case later in this section. Now, $n_{1I}$ can be computed as

$$n_{1I} = \sum_{k=0}^{N-1} \left[ (\tau - t_{1II}) c_{1,k} c_{2,k} + t_{1II} c_{1,k} c_{2,k+1} \right]. \hspace{1cm} (9)$$

We assume that $\{c_{ik}\}$ are independent identically distributed equi-probable Bernoulli random variables with values equal to $\pm 1$. With this assumption, it is easy to show that

$$\mathbf{E}(n_{1I}) = 0, \quad \mathbf{E}(n_{1I}^2) = N \left( (\tau - t_{1II})^2 + t_{1II}^2 \right). \hspace{1cm} (10)$$
It is observed from (9) that in order for $n_{1I}$ and $n_{1II}$ to be correlated, $n_{1II}$ should have components in the form of $c_{1,k} c_{2,k}$ and/or $c_{1,k} c_{2,k+1}$. It is not difficult to show that for $t_{12II} < -\tau$ or $t_{12II} > 2\tau$, $n_{1II}$ does not have such components. We will investigate the remaining range for $t_{12II}$ in three intervals, namely, $-\tau \leq t_{12II} \leq 0$, $0 \leq t_{12II} \leq \tau$, and $\tau \leq t_{12II} \leq 2\tau$.

If $-\tau \leq t_{12II} \leq 0$, then it can be shown that

$$n_{1II} = \sum_{k=0}^{N-1} \left[ -t_{12II} c_{1,k} c_{2,k-1} + (t_{12II} + \tau) c_{1,k} c_{2,k} \right]. \quad (11)$$

The mean and variance of $n_{1II}$ are found as

$$E(n_{1II}) = 0, \quad E(n_{1II}^2) = N \left( (t_{12II} + \tau)^2 + t_{12II}^2 \right). \quad (12)$$

Now, using (9) and (11), the correlation, $E(n_{1I} n_{1II})$, can be computed as

$$E(n_{1I} n_{1II}) = N (\tau - t_{12II}) (t_{12II} + \tau). \quad (13)$$

Substituting (10), (12), and (13), in (4), $\rho$ is obtained as

$$\rho = \frac{(\tau - t_{12II}) (t_{12II} + \tau)}{\sqrt{(\tau - t_{12II})^2 + t_{12II}^2} \sqrt{(t_{12II} + \tau)^2 + t_{12II}^2}}. \quad (14)$$

The calculations are similar for the other intervals and

$$\rho$$ is plotted for various values of $t_{12II}$, in the range of $[0, \tau]$, in Fig. 4. As it is already discussed, if $0 < t_{12II} < \tau$ then the range of $t_{12II}$ for nonzero $\rho$ is $(-\tau, 2\tau)$. Also, the 3-dimensional plot of $\rho$ in the analyzed intervals of $0 \leq t_{12II} \leq \tau$, and $-\tau \leq t_{12II} \leq 2\tau$ is shown in Fig. 5. We note that in Fig. 5, the intersection of the vertical $t_{12II}$ plane with the 3-dimensional plot at certain values of $t_{12II}$ yield the curves shown in Fig. 4.

The results obtained so far for the case of $0 \leq t_{12II} \leq \tau$, can be generalized for $t_{12II}$ values in any interval. In Fig. 6, the general expression for $\rho$ is plotted with respect to $t_{12II}$ and $t_{12II}$ [4].

It is observed from Fig. 6 that

$$t_{12II} = n\tau \quad \rightarrow \quad \left\{ \begin{array}{ll}
\rho = 0, & \text{for } t_{12II} \leq (n-1)\tau \text{ and } t_{12II} \geq (n+1)\tau \\
0 < \rho < 1, & \text{for } (n-1)\tau < t_{12II} < (n+1)\tau,
\end{array} \right. \quad (15)$$

where $n$ is an integer.

**B. Dependence of Correlation Coefficient on User and AE Locations**

Our goal in this section is to find the region for $w_2$ which will result in a non-zero $\rho$, for a given set of AE and $w_1$ locations. The time domain requirements for this region is already known from (15). Therefore, the task is to find the equivalent of (15) in terms of distance.
Figure 6: Correlation coefficient as a function of $t_{12I}$ and $t_{12II}$, for the case of synchronous users ($\Delta_{12}=0$).

As it is well known, the propagation time, $t$, depends on the distance, $d$, as $t = d/c$, where $c$ is the speed of light which will be taken as $3 \times 10^8$ m/sec. Therefore, from (7) and (8), $t_{12I}$ and $t_{12II}$ can be stated in terms of distances as

$$t_{12I} = (d_{1I} - d_{2I})/c, \quad t_{12II} = (d_{1II} - d_{2II})/c, \quad (16)$$

where $d_{ij}$ is the distance between $w_i$ and AE $j$, $i \in \{1, 2\}$, $j \in \{I, II\}$.

We will work on a unit square area with side 1 meter. Distances in this unit service area will be denoted by $h$. In other words, $h$ is the normalized distance which is related to the actual distance, $d$, as

$$d = hs, \quad (17)$$

where $s$ is the scaling factor. For instance, if $s=500$, this means that the service area has the shape of a square with side length 500 meters. Also, we define the chip rate, $R_c$, as $R_c = 1/\tau$.

We will first find the region corresponding to $n\tau < t_{12I} < (n+1)\tau$. This region is a function of the locations of $w_1$ and $w_2$, and AE $I$ (but, not AE $II$). Then, we will find the region corresponding to $(n-1)\tau < t_{12II} < (n+2)\tau$, which is a function of the locations of $w_1$ and $w_2$, and AE $II$ (but, not AE $I$).

We start by analyzing the $n=0$ case. Using (16) and (17), we can write

$$0 < t_{12I} = t_{1I} - t_{2I} < \tau \quad \rightarrow \quad 0 < (h_{1I} - h_{2I}) < c\tau/s,$$

$$-\tau < t_{12II} = t_{1II} - t_{2II} < 2\tau \quad \rightarrow \quad$$

Figure 7: For the case of synchronous users ($\Delta_{12}=0$), the shaded area shows the following region: $0 < (h_{1I} - h_{2I}) < 0.075$ and $-0.075 < (h_{1II} - h_{2II}) < 0.15$ ($s = 400$ and $R_c = 10$ Mcps).

$$-c\tau/s < (h_{1II} - h_{2II}) < 2c\tau/s. \quad (18)$$

It is observed from (15) that the intersection of these two regions yields the non-zero $\rho$ region for $w_2$, for $n = 0$. In a similar way, the corresponding regions for other $n$ values can also be found. We call the union of all such regions the caution zone for $w_1$; because, the interference resulting from a user in this zone, at the 1st branch of $w_1$’s combiner at the CS, will be correlated with the corresponding interference at the 2nd branch.

As an example, we consider a system with $s=400$ and $R_c = 10$ Mcps (Mega chips/sec). Then, $c\tau/s = 0.075$. We assume that AE’s $I$, $II$, and $w_1$ are placed at the coordinates $(.25,.5)$, $(.75,.5)$, and $(.375,.625)$, respectively, on the unit service area. The intersection of the regions given in (18) is depicted in Fig. 7. The integers on the circles in Fig. 7 indicate the $(t_{1I}-t_{2I})/\tau$ and $(t_{1II}-t_{2II})/\tau$ values for any $w_2$ location on those circles.

Similarly, using (15), (16) and (17), the corresponding regions for other $n$ values can also be found. We realize that for $n \geq 1$, the conditions given in (18) do not yield an overlapping region. In Fig. 8 the caution zone for $w_1$ is shown, which is the union of the regions corresponding to $n=-6,-5,\ldots,0$.

It can be shown that, for the case of asynchronous users, (15) can be modified accordingly [4]. In Fig. 9, the caution zone for $w_1$ is shown for the case of $\Delta_{12} = 0.5$.

III. APPROXIMATION OF THE CAUTION ZONES

We start by defining the differential delay, $\Delta t_{12I-II}$, as

$$\Delta t_{12I-II} = t_{12I} - t_{12II} = (t_{1I} - t_{2I}) - (t_{1II} - t_{2II}), \quad (19)$$
which can be rewritten as $\Delta t_{12,I-II} = (t_{11} - t_{11'}) - (t_{21} - t_{21'})$. We note from (19) that $\Delta t_{12,I-II}$ is independent of the differential code phase, $\Delta t_{12}$, that is, $\Delta t_{12,I-II}$ is the same for both synchronous and asynchronous users.

We notice from (15) that $\rho$ depends on the actual values of $t_{121}$ and $t_{212}$, rather than their difference, $\Delta t_{12,I-II}$. In fact, it is observed from Fig. 6 that $\rho$ exhibits periodicity with respect to $\Delta t_{12,I-II}$ when $-2\tau < \Delta t_{12,I-II} < 2\tau$. The correlation analysis would have been much simpler if $\rho$ were constant (rather than being periodic) with respect to $\Delta t_{12,I-II}$. In that case, a (2-dimensional) plot of $\rho$, as a function of $\Delta t_{12,I-II}$ would have been sufficient.

Let us investigate the dependence of $\rho$ on $\Delta t_{12,I-II}$ more closely. It is clear from Figs. 4-6 that $t_{121} = t_{212}$ corresponds to the worst case and $|t_{121} - t_{212}| > 2\tau$ guarantees the uncorrelatedness. In other words, it can be stated using (19) that

$$|\Delta t_{12,I-II}|/\tau = 0 \longrightarrow \rho = 1, \quad (20)$$

$$|\Delta t_{12,I-II}|/\tau \geq 2 \longrightarrow \rho = 0. \quad (21)$$

One other important observation from Fig. 4 is that

$$1 \leq |\Delta t_{12,I-II}|/\tau < 2 \longrightarrow 0 \leq \rho \leq 0.5. \quad (22)$$

Obviously, it is desirable to have $\rho = 0$. However, it is important to note that even in the case of a $\rho$ very close to unity, analytically there would still be some gain from diversity combining, although this gain would be minuscule. Even in the limiting case of $\rho = 1$, although there would be no gain from diversity combining, there would be no harm either. In a practical system, however, there are other factors to be taken into consideration besides SIR. If $\rho$ is close to unity, the insignificant returns in SIR would not justify the increased processing, complexity, and thus, cost.

It would be efficient if a threshold value for $\rho$ is determined, say $\rho_o$, such that it can be argued that if $0 \leq \rho \leq \rho_o$, then there would be benefit from diversity, however, if $\rho_o < \rho \leq 1$, then practically there would not be much gain from diversity. We note that in order for the output SIR in an MRC to be the sum of the branch SIR's, $\rho$ should be equal to 0. Therefore, for $\rho = \rho_o$, as described above, the output SIR will be less than the sum of the branch SIR's.

We choose a conservative value for $\rho_o$: $\rho_o = 0.5$. Based on (20) and (22), setting $\rho_o$ to 0.5 is equivalent to the following inequality:

$$|\Delta t_{12,I-II}|/\tau > 1. \quad (23)$$

It is obvious from the discussion so far that $|\Delta t_{12,I-II}|/\tau$ is a critical quantity in the correlation analysis.

Fig. 10 shows the $|\Delta t_{12,I-II}|/\tau = -2, -1, 0, 1$, and 2 lines along with the caution zone for $\Delta t_{12} = 0$, respectively, for the example case ($s=400, R_c = 10$ Mcps). For any $\Delta t_{12} \neq 0$ value, these lines will still be the same [4]. Fig. 10 is in conformity with (21), in other words, $\rho = 0$ for $|\Delta t_{12,I-II}|/\tau > 2$. Also from Fig. 10, (19) is confirmed, that is, $|\Delta t_{12,I-II}|/\tau$ does not depend on $\Delta t_{12}$.

The implication of (23) is that the caution zone (the shaded area) in Fig 10 can be approximated by the area between the $|\Delta t_{12,I-II}|/\tau = -1$ and 1 lines. We know from (22) that $\rho$ is less than 0.5 in the shaded regions outside the approximated caution zones, therefore, the performance degradation introduced by such an approximation is expected to be insignificant.
Figure 10: The $\Delta t_{12, I-II}/\tau = -2, -1, 0, 1$, and 2 lines along with the caution zone for $w_1$, for the case of synchronous users ($\Delta t_{12} = 0$), with $s = 400$ and $R_c = 10$ Mcps.

Note that $\Delta t_{12, I-II}/\tau = -1$ and 1 lines are hyperbolas. If we draw all the hyperbolas for which $\Delta t_{12, I-II}/\tau$ is an integer, then we construct an hyperbolic grid. The “origin” of the hyperbolic grid is the hyperbola on which $w_1$ is located.

IV. THE EFFECTS OF SYSTEM PARAMETERS ON THE CAUTION ZONE

From (17) and (19), $|\Delta t_{12, I-II}|/\tau$ can be written as

$$\frac{|\Delta t_{12, I-II}|}{\tau} = \frac{sr_c}{c} \left| (h_{1I} - h_{1II}) - (h_{2I} - h_{2II}) \right|. \quad (24)$$

In the right hand side of (24), $c$ is a constant, and $h_{ij}$, $i \in \{1, 2\}$, $j \in \{I, II\}$, depends on the user and AE locations. Therefore, the product $sr_c$ is the most important system parameter. It is also noticed that for a certain value of the product $sr_c$, the actual values of $s$ and $R_c$ do not matter. Hence, as long as the correlation analysis is concerned, a system with $s = 400$ and $R_c = 10$ Mcps is equivalent to the one with $s = 1000$ and $R_c = 4$ Mcps.

For a large $sr_c$ value, the hyperbolic grid will be denser; in other words, the hyperbolas, for which $|(h_{1I} - h_{1II}) - (h_{2I} - h_{2II})|sr_c / c$ is an integer, will be closer to each other. Since the approximate caution zone is the area between the hyperbolas -1 and 1, this area will be smaller. Therefore, it is desirable to have a large $sr_c$ value [4].

The density and orientation of the hyperbolic grid also depend on the AE locations in the service area. For a given $sr_c$ value, the density of the hyperbolic grid will increase with the increasing distance between the AE’s. Therefore, to minimize the caution zone (and thus, the correlation effect), AE’s must be placed as far apart as possible — an intuitively satisfying result.

V. SUMMARY AND CONCLUDING REMARKS

The objective of this paper was to present a spatial correlated interference analysis in CDMA multi-antenna systems. Towards that end, we analyzed the simplest non-trivial case of $L = 2$ with $K = 2$, in order to develop some insight into the problem of correlated interference. For this special case, we were able to obtain the correlation coefficient as a function of the distances involved in the service region. Hence, for a given user location, we were able to determine the portions of the service region in which other user(s) would cause correlated interference to the given user; we called this region the caution zone for the given user. We then found an approximate expression (and an approximate caution zone) in simpler terms.

We observed that the caution zone, for a given user, depends on the location of that particular user, as well as the AE locations, service region size, and the chip rate. Increasing the chip rate, and placing the AE’s as far as possible from each other, results in a reduction in the correlation between the interference components at the combiner branches. This implies that for reasonably high chip rates (wideband-CDMA) and for service regions that are not very small, the level of correlation should be low in a system with 2 AE’s and many users. Further investigation is required to make similar conclusions for systems with greater numbers of AE’s.

This initial analysis may eventually enable us to develop a comprehensive understanding of the benefits of the CDMA multi-antenna systems. Currently, this work is being extended to analyze the most general case of many AE’s with many users to answer various questions, such as the optimal number and location of AE’s for a given irregular service region.

References


