Effects of Correlated Interference on the Potential Linear Antenna Gain in CDMA Macrodiversity Systems

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ABSTRACT

It is reported in [1] that in the reverse link of a CDMA macrodiversity system a remarkable L-fold capacity (throughput) increase can be attained by using L antenna elements (AEs) provided that the spread spectrum bandwidth approaches to infinity. In a finite-bandwidth system, however, the increase in capacity as a result of the utilization of multiple AEs will be less than linear due to the presence of the correlated interference effects. In this paper, the spatial correlated interference analysis is presented in order to investigate the effects of the system parameters on the severity of the correlated interference.

The results presented in this paper indicate that the parameter which we defined as the chiplength ([speed of light]/[chip rate]) plays a role in macrodiversity systems (in regards to the correlation effects) similar to the role of carrier wavelength in microdiversity systems. It is observed that in systems where the size of the service region is large enough to enable inter-AE distances many times larger than the chiplength, significant (close to linear) capacity gains can be achieved by employing macrodiversity with many AEs. If, on the other hand, the service region size is not large enough with respect to the chiplength, the returns due to macrodiversity will not be as high.

I. INTRODUCTION

The CDMA reverse-link capacity of a network of antenna elements (AEs) is investigated in [1]. It is reported there that, in the presence of an optimal power control scheme, the capacity (throughput) increases linearly with the number of AEs, and it is further stated that this linear gain is valid irrespective of the user[1] and AE positions (as long as neither AEs nor users are located at the same point).[2] This result is obviously very important due to its fundamental nature.

The key condition in attaining a linear capacity gain in such a macrodiversity system is that the (averaged out) interference components at the branches of the maximal ratio combiner (MRC) have to be uncorrelated. It is demonstrated in [1] that this condition is guaranteed, even if the AEs or the users (interferers) are located close to each other (as long as they are not at the same point), when the spread spectrum bandwidth approaches infinity. In this case, the signal-to-interference ratio (SIR) at the output of the MRC becomes equal to the sum of the branch SIRs [3].

It will be shown in this paper that the consequence of a finite bandwidth, on the other hand, is the possibility of correlated interference, the severity of which would depend on the size of the service region and on the relative positions of the AEs and users. This would degrade the performance by yielding an output SIR which is less than the sum of the branch SIRs, and therefore, would impose a dampening effect on the capacity increase with respect to the number of AEs used. In the limit, if the interference components at the branches become identical, the combiner would reduce to that which has only one effective branch; thus, no gain would be attained since amplification does not increase SIR.

II. CDMA MACRODIVERSITY SYSTEM

The topic of this paper is not the diversity gain achieved by multiple antennas, but rather the gain achieved after the fading is averaged out. It is important to note that when there is no fading, there is also no diversity gain achieved against fading in either micro- or macrodiversity schemes regardless of the number of AEs employed. However, in a CDMA multi-antenna system where the AEs are widely separated (i.e., distributed), there still exists a potential for a further gain which is due to more efficient management of the cochannel interference, depending on the spread spectrum bandwidth, service region size, and AE & user locations. The goal of this paper is to investigate the extent of this gain against interference.

Towards this end, we consider the reverse-link of a network of L omnidirectional AEs which are physically distributed in a service region where fading is averaged out. This can be attained through the employment of other forms of diversity techniques (even microdiversity) in conjunction with the macrodiversity type. The outputs of the AEs are conveyed to a central station (CS) with separate feeders for decoding and combining, through the use of a MRC. We remark that the reverse-link of a CDMA multi-antenna system is inherently different from

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[1] Throughout this paper, user refers to a wireless user.

[2] Less general results are given in [2].

its forward-link due to the fact that in the reverse-link performance gains can be achieved without injecting extra energy into the system.

The above described macrodiversity system is different from the conventional macrodiversity types (such as soft handoff) in two major ways: first, the outputs of all the AEs in the service region are involved in the combining process rather than a subset; and secondly, combining technique is the maximal ratio type rather than selection combining. As a consequence of these two factors, more energy is collected and utilized from each user. Such a system constitutes the logical limit of all practical macrodiversity implementations.

Here are some further system-level assumptions:

- The service region is isolated in the sense that there is no interference coming from outside.
- The system is interference limited (thermal-type background noise is omitted).
- Only a single-class service is considered for the sake of simplicity, but the results and discussions can readily be extended to systems with multi-class services.
- An optimal power control scheme which yields SIR-balancing is employed [4],[1],[5]. Towards this end, it is assumed that (i) all the link gains are known, (ii) power levels are adjusted precisely and instantaneously, and (iii) there are no constraints on the transmit power levels.

III. SPATIALLY CORRELATED INTERFERENCE ANALYSIS

A. 2 AEs with 2 Users

The correlated interference analysis for the simplest non-trivial macrodiversity system which has 2 AEs ($L=2$) and 2 users ($K=2$) is presented in [6]. In this section we summarize the relevant results given in [6].

For this special case, the correlation coefficient (ρ) for the mean interference components (after despreading) at the two branches of the MRC of the user of interest were obtained, first as a function of the propagation delays in the air and then as a function of the distances involved in the service region. Hence, for a given user location, the portions of the service region in which other user(s) would cause correlated interference to the given user were determined; this region is called the caution zone for the given user. Then the boundaries of the caution zone is approximated by a pair of hyperbolas.

Fig. 1 shows the actual caution zone (shaded region) and its approximation (the area between the hyperbolas labelled as "-1" and "1"), for a system where the service region is a square with side length $s = 400$ meters and chip rate $R_c = 10$ Mcps. ρ is less than 0.5 in the shaded regions outside the approximated caution zones, therefore, the performance degradation introduced by such an approximation is expected to be insignificant. In the rest of this paper the term caution zone refers to the approximate caution zone.

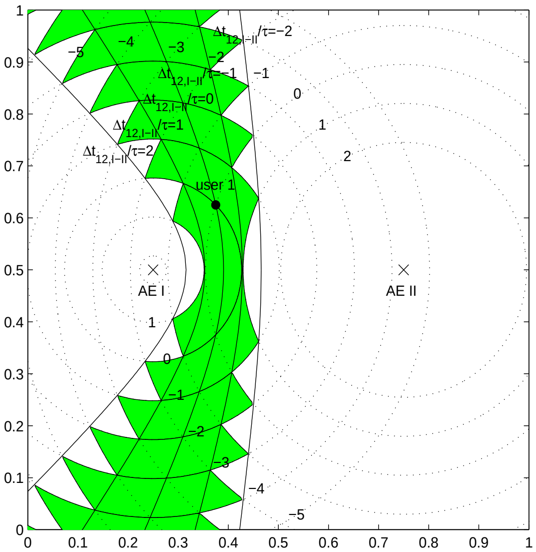


Figure 1: The actual and approximate caution zones for user 1 with $s=400$ and $R_c=10$ Mcps.

It is observed that, for a given user, the caution zone depends on the location of that particular user, as well as the AE locations, service region size, and the chip rate. Increasing the chip rate, and placing the AE's as far as possible from each other, results in a reduction in the correlation between the interference components at the combiner branches.

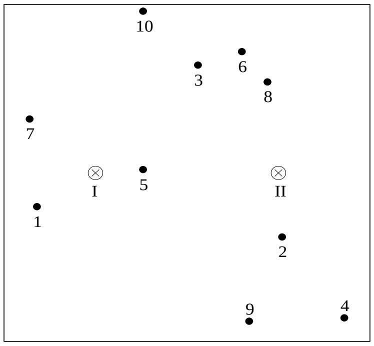
B. 2 AEs with Many Users

In this section and the next one, we further the investigation of the spatial correlation analysis by considering first 2 AEs with many users, and then, the most general case of many AEs with many users. Due to the computational complexity, however, we work with an intermediate performance metric, which we call percent correlation, instead of the correlation coefficient itself.

In a system with K users, for each user i (w_i), we determine whether the remaining $K-1$ users are in the caution zone for that particular user. By this way, we construct a $K \times K$ correlation matrix, $\mathbf{U} = \{u_{ij}\}$ such that

$$u_{ij} = \begin{cases} 1, & \text{if } w_j \text{ is in the caution zone for } w_i, \\ 0, & \text{otherwise.} \end{cases} \qquad (1)$$

We note that $u_{ii} = 0$, $\forall i$, since a user does not create interference to itself. Also, it can be shown that if w_j is



U: Correlation Matrix

	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	0	0	1	0	0	0
2	0	0	0	1	0	0	0	1	0	0
3	0	0	0	0	0	0	0	0	0	0
4	0	1	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	1	0
7	1	0	0	0	0	0	0	0	0	0
8	0	1	0	1	0	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0

Figure 2: A system with $L=2$ and $K=10$, and the corresponding correlation matrix, $\mathbf{U}$, for $s=400$ meters ($D=200$ meters) and $R_c=10$ Mcps.

in the caution zone for w_i, then w_i must be in the caution zone for w_j.

In Fig. 2, a system with $L=2$ and $K=10$ is illustrated and the corresponding $\mathbf{U}$ matrix is given. In Fig. 2, the caution zones are not drawn. But, if we were to draw the caution zone for w_8, for instance, then w_2 and w_4 would be in that caution zone. Consequently, $u_{82}=1$, $u_{84}=1$, and all the other entries in the 8th row of the $\mathbf{U}$ matrix are 0's.

In the worst case, all of the $K-1$ entries in a row of $\mathbf{U}$ will be 1's, and in the best case, all of those entries will be 0's. Based on this observation, we define the percent correlation for w_i, ϕ_i, as follows

$$\phi_i = \frac{1}{K-1}\sum_{j=1}^{K} u_{ij} \times 100, \quad \forall i. \tag{2}$$

It is worth noting that although ϕ is not equal to the correlation coefficient, there is a direct relation between them; $\phi=0\%$ corresponds to the $\rho=0$ case, and $\phi=100\%$ to the $\rho=1$ case.

C. Many AEs with Many Users

Obviously, the most interesting case is the most general type of L AEs with K users. We assume that the AEs are evenly distributed on the service region, so that the coordinates of the jth AE on the unit square region can be represented by the pair

$$\left(\frac{2[(j-1) \bmod \sqrt{L}]+1}{2\sqrt{L}}, 1-\frac{2\lceil j/\sqrt{L}\rceil - 1}{2\sqrt{L}}\right). \tag{3}$$

In the above, $\lceil . \rceil$ denotes the ceiling function.

Now, for each user, a total of $L(L-1)/2$ caution zones exist, each of which corresponds to a particular AE pair. Therefore, the $\mathbf{U}$ matrix is composed of $L(L-1)/2$ submatrices (one for each antenna pair), with sizes $K\times K$. onsequently, the correlation matrix, $\mathbf{U}$, is 3-dimensional with size $K \times K \times [L(L-1)/2]$. Such a system, for the case of $L=4$ and $K=10$, is illustrated in Fig. 3.

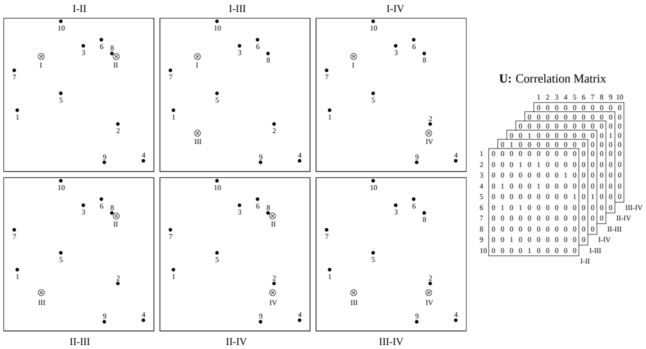


Figure 3: A system with $L=4$ and $K=10$, and the corresponding correlation matrix, $\mathbf{U}$, for $s=400$ meters ($D=200$ meters) and $R_c=10$ Mcps.

For a w_i, the most disadvantageous situation will occur if all of the remaining $K-1$ users are in all of the $L(L-1)/2$ caution zones for w_i. Obviously, this is an event with a very low likelihood! Such a situation will yield $(K-1)[L(L-1)/2]$ 1's in the 2-dimensional ith row of the $\mathbf{U}$ matrix. Based on this observation, ϕ_i can be stated as

$$\phi_i = \frac{1}{(K-1)L(L-1)/2}\sum_{k=1}^{L(L-1)/2}\sum_{j=1}^{K} u_{ijk} \times 100, \quad \forall i. \tag{4}$$

We note that (4) reduces to (2) for $L=2$.

IV. SIMULATION RESULTS

Simulations have been run to obtain the ϕ values for various combinations of the system parameters with the assumption of uniform user distribution. For each such combination, a total of approximately 40,000 ϕ values are collected and the corresponding cumulative distribution function is plotted.

As stated throughout this paper, in a finite-bandwidth macrodiversity system, the increase in capacity as a result of the utilization of multiple AEs will be less than linear. In other words, a capacity penalty will be incurred due to the presence of the correlated interference effects. The simulation results given in this section do not quantify this penalty; but, they rather show the degree of interference, in terms of ϕ, with respect to the key system parameters (namely, the spread spectrum bandwidth [chip rate], service region size, and inter-AE distance). Obviously, a high ϕ value implies a greater departure from the linear capacity gain. In the limiting case of $\phi = 100\%$, there will be almost no capacity gain from using multiple AEs instead of a single AE.

It is worth noting that the increase in capacity that we refer to is always with respect to the capacity of a system with a single AE. For instance, if the spread spectrum bandwidth is increased in a single AE system, then the capacity will also increase accordingly. The refered capacity increase is the additional gain obtained by using multiple AEs in such a system instead of only one AE.

It is well known that in microdiversity systems, in order to attain the microdiversity gain, the inter-AE distance should be at least a few times larger than the wavelength of the carrier, $\lambda = c/f$, where f denotes the carrier frequency [7]. In a CDMA macrodiversity system, a similar quantity which we will refer to as "chiplength" can be defined as follows:

$$\Lambda = c/R_c \text{ meters.} \quad (5)$$

It will become apparent in the following simulation results that the role Λ plays in macrodiversity systems (in regards to the correlation effects) is indeed similar to that λ plays in microdiversity types.

Before presenting the simulation results, we would like to make a remark for interpreting the results: if the number of users is identical in two systems which are compared, then the users in the system with higher (lower) ϕ values will experience a lower (higher) balanced SIR value. In order to compensate for (exploit) this effect, the number of users in that system has to be reduced (can be increased).

A. Percent Correlation and (Inter-AE Distance)/(Chiplength)

We define the inter-AE distance, D, as the distance between the two closest AEs. The relation between s and D is obvious: the maximum value for D is determined by the size of the service region. In Fig. 4, the relation between D and Λ is shown by fixing the number of AEs to 4. Each of the three sets of curves in this figure, labeled as I, II, and III, corresponds to a different D/Λ ratio: 16.7, 6.67, and 1.67, respectively. Fig. 4(a) and (b) provide two different interpretations for the above given D/Λ values. In Fig. 4(a), Λ is kept constant at 30 meters (that is, $R_c = 10$ Mcps), and the size of the service region is changed by reducing s from 1000 to 400 to 100. Note that since the AEs are uniformly distributed, decreasing the service region size results in decreasing the inter-AE distance as well. In Fig. 4(b), on the other hand, the size of the service region is kept constant ($s = 400$), and Λ is increased from 12 to 30 to 120 meters (that is, R_c is decreased from 25 to 10 to 2.5 Mcps).

In order to investigate Fig. 4 closely, let us assume that the number of users is fixed ($K = 50$, 100, 200, or 400). The curves in Fig. 4 confirm our expectation that for a given number of AEs, decreasing D/Λ (i.e., decreasing the service region size while keeping the spread spectrum bandwidth fixed, or decreasing the spread spectrum bandwidth while keeping the size of the service region fixed) yields greater correlation between the interference components received at different AEs. Furthermore, the increase in correlation is relatively more significant when D/Λ is reduced from 6.67 to 1.67, in comparison to when it is reduced from 16.7 to 6.67. This suggests a nonlinear relationship between D/Λ and ϕ.

It is also observed from Fig. 4 that for a given D/Λ value, the effect of the number of users on ϕ is marginal in all three cases (I, II, and III).[3] This is due to the fact that the user locations are taken to be 2-dimensional uniform random variables. Statistically, the percentage of users that are in a caution zone are determined by the size of that caution zone (in percentage) in comparison to the total size of the service region. Therefore, increasing the number of users reduces the statistical variations, and as K increases the tails of the distributions become less significant. However, the median values remain almost the same in each set, as expected.

B. Percent Correlation and Inter-AE Distance

The change in ϕ with respect to the number of AEs is investigated in Fig. 5 for a fixed s/Λ value of 13.3. In a fixed service region, increasing L means decreasing the inter-AE distance. In this figure, the ratio of the number of users per AE is also kept constant ($K/L = 25$).[4] It is observed, as expected, that ϕ increases with the increasing L; however this increase in correlation is quite mild. For instance, the increase in the median values of ϕ for the $L=25$ and $K=625$ case, in comparison to the $L=4$ and $K=100$, and to the $L=16$ and $K=400$ cases, are less than 7% and around 1%, respectively. This would mean that the output SIR for a system with $L=25$ and $K=625$ will only be slightly lower than that with $L=4$ and $K=100$.

The conclusion from this figure is that in systems with relatively high s/Λ values, even though the capacity increase with the increasing number of AEs will be less than linear, this increase will not saturate rapidly. Therefore, there is room for significant capacity gains with the use of multiple AEs in such systems.

C. Percent Correlation and Multiplexing

Finally, in Fig. 6, the effects of pooling the resources are shown. In this figure, Λ and K/L are kept fixed at 30 meters and 25 users/AE, respectively, and the size of the coverage region is changed in such a way that the inter-AE distance (or the number of AEs per unit area) is kept the same. It is observed from Fig. 6 that a higher L and K pair yields a lower ϕ value due to statistical multiplexing. Thus, if a system with $L = 4$ can accomodate 100 users, then that with $L = 25$ (arranged in the way shown in Fig. 6) can accomodate more than 625 users.

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[3] Obviously, as K increases (decreases) while the other parameters remain unchanged, the balanced SIR value that the users will experience will decrease (increase) accordingly.

[4] We note based on Fig. 4 that if K is varied for a given L value, the tails of the distribution will be affected, but the median value will not change noticeably.

V. SUMMARY AND CONCLUSIONS

In a CDMA macrodiversity system the reverse link capacity increases linearly with the number of AEs employed, provided that the interference signals picked up by AEs are uncorrelated; otherwise, the capacity returns will be more modest. In this paper, spatially correlated interference analysis is presented, in order to investigate the effects of the system parameters on the severity of the correlated interference. Due to the computational complexity, however, a performance metric called percent correlation is used instead of the correlation coefficient itself. The direct relation between the correlation coefficient and percent correlation is obvious.

The simulation results presented in this paper indicate that the parameter which is defined as the chiplength plays a role in macrodiversity systems (in regards to the correlation effects) similar to the role of carrier wavelength plays in microdiversity systems. In this respect, it is observed that the inter-AE distance should be many times larger than the chiplength. It can be concluded that in systems where this condition is satisfied, significant (close to linear) capacity (throughput) gains can be achieved by employing macrodiversity with many AEs. If, on the other hand, the service region size is not large enough to enable such inter-AE distances, then the returns due to macrodiversity will not be as high.

Since chiplength is inversely related to the spread spectrum bandwidth, a wider bandwidth will enable the efficient utilization of multiple AEs in a macrodiversity system, in addition to wideband CDMA's many other benefits reported in the literature (such as, attaining microdiversity through multipath resolution). These results are in agreement with those presented in [1]: in the limiting case of infinite bandwidth ($R_c \to \infty$) the caution zone for a user reduces to a line, and the probability of other users being on this line approaches zero.

Further research is required for translating the correlated interference level into the actual capacity loss incurred.

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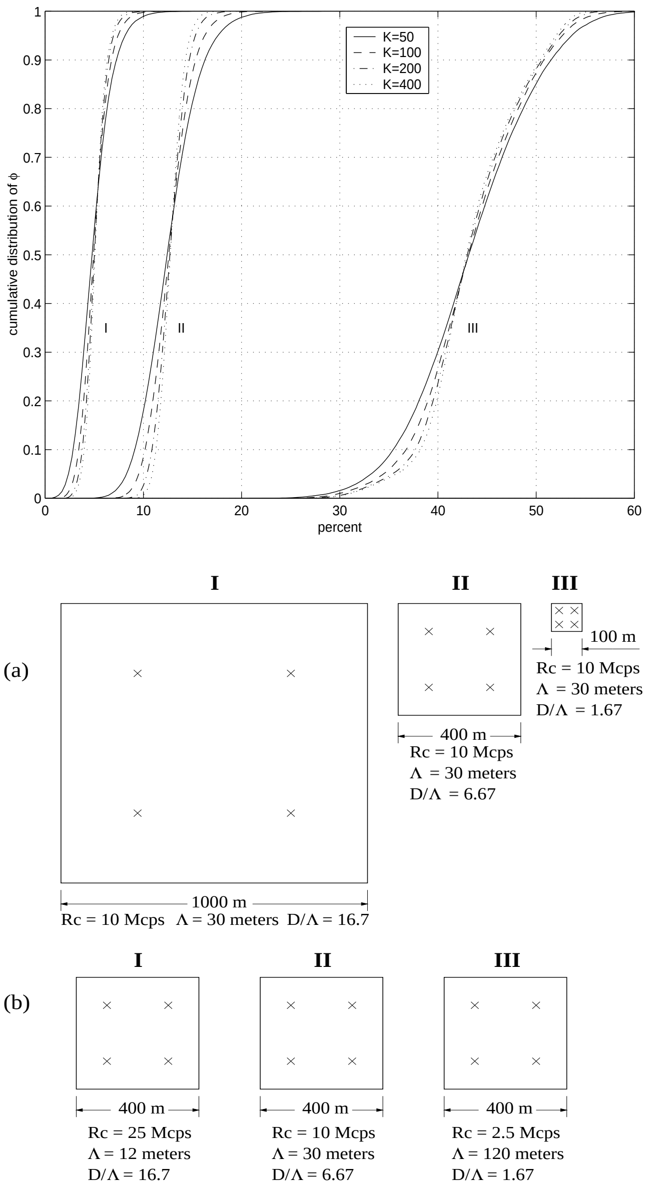

Figure 4: Cumulative distribution function of ϕ for various D/Λ values: interpretation (a) chip rate is kept constant and service region size is changed; interpretation (b) service region size is kept constant and chip rate is changed.

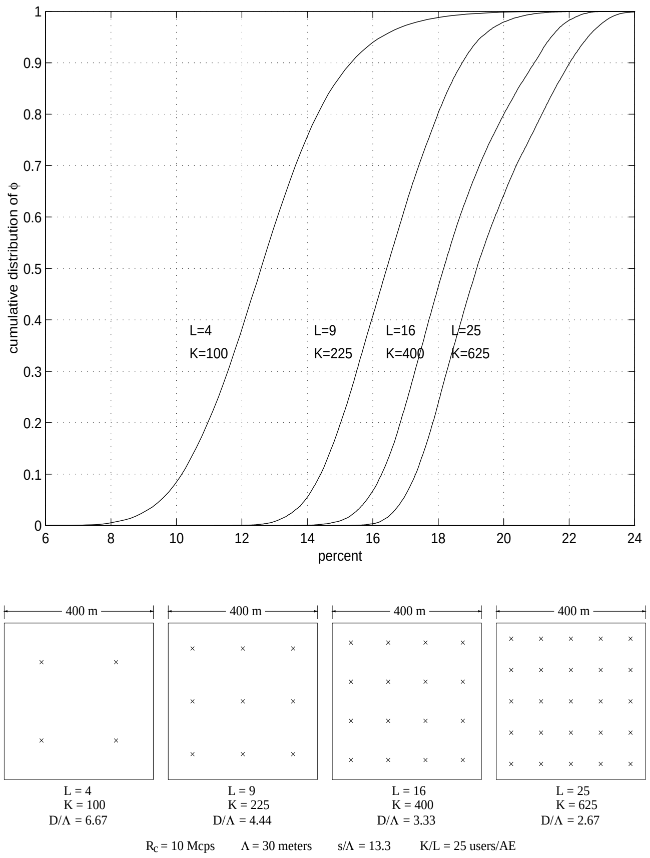

Figure 5: Cumulative distribution function of ϕ for various inter-AE distances.

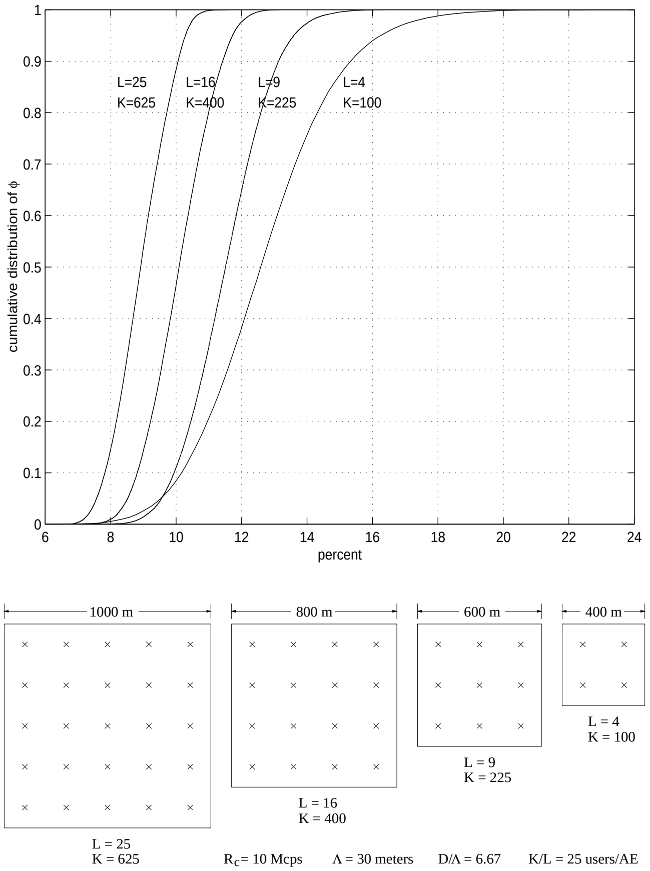

Figure 6: Cumulative distribution function of ϕ that shows the effects of pooling the resources.