Relay-Assisted Spatial Multiplexing in Wireless Fixed Relay Networks

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***Abstract*— Fixed relays are expected to be a part of future infrastructure-based wireless networks. Besides coverage extension, such relays can form advanced architectures due to the flexibility in their power expenditure and physical size. This paper explores the potential benefits of multi-antenna relays for spatial multiplexing of independent data sources sending data to a common multi-antenna destination such as a base station or an access point. Overall, the system resembles a horizontally coded layered space-time architecture.**

In particular, we consider zero forcing decision feedback (ZF-DF) type MIMO receivers and study their outage performance under various (non-selective and selective) digital relaying protocols. For diversity relaying protocol, we propose two schemes, *Joint ZF-DF* and *Parallel ZF-DF*, for joint processing (combining and decoding) of the direct user signals and the signal from the relay. We show that with the proposed selective diversity relaying protocols and joint ZF-DF processing, the outage probability of the system can be decreased significantly.

I. Introduction

A significant increase in the capacity of wireless channels is possible by deploying multiple antennas at both transmitter and receiver sides if the channel has rich scattering [1], [2]. By spatial multiplexing on the transmitter side, even practical architectures with certain constraints, such as V-Blast [3], can bring much higher spectral efficiencies than the conventional systems. Multiple antennas, either on the transmitter or the receiver side, can also improve link reliability through space-time coding [4] or receive diversity.

Mounting multiple antennas at mobile user terminals might be impractical due to space and cost constraints. However, if the receiver has multiple antennas, it is still possible to obtain multiplexing gains by allowing multiple users to transmit simultaneously. In such a distributed multiplexing scheme, the diversity benefits of space-time coding are not available due to the lack of coordination among user antennas. A straightforward way of increasing diversity is to reduce the number of simultaneously transmitting users in order to have extra degrees of freedom at the receiver. For instance, in a V-BLAST system, each extra receive antenna will increase the diversity order of all users by one. This will, of course, require more bandwidth, since reducing the number of simultaneously transmitting users decreases the effective rate.

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User cooperation was also shown to increase diversity in wireless networks. Several protocols in which users act as relays for each other are proposed in [5], [6]. Diversity obtained this way is usually referred to as cooperative diversity. In practice, user cooperation puts a burden on cooperating users in terms of power. It can also create security vulnerabilities in wireless networks, whose security mechanisms are not designed with peer cooperation in mind. A practical alternative to user cooperation is deploying *fixed relays* as a part of the infrastructure. Originally proposed for coverage extension by decreasing effective path-loss [7], [8], fixed relays can can take the burden of cooperation from mobile users. They are provisioned to have direct access to the power line. Hence their operation is not limited by battery lifetime [9]. Moreover, they will have less severe constraints for cost and size. Hence, they can easily accommodate multiple antennas.

According to the signal processing performed by the relay, relaying schemes can be classified as analog and digital. In analog relaying the relay terminal amplifies the received signal and then retransmits. One disadvantage of analog relaying is the noise and interference enhancement. In digital relaying, the relay detects and possibly decodes the source signal and then regenerates and retransmits it, requiring more processing compared to analog relaying. On the other hand, digital relaying is more suitable for block based processing, which is common in many communication systems, since it is much easier to store digital data compared to analog data.

Diversity relaying capitalizes on independent fading at source-destination and relay-destination channels. The destination is required to combine signals from different channels. In digital diversity relaying, if the relay transmits a data block that is incorrectly detected/decoded, it is likely that there will be an error at the destination. This problem, usually called *error propagation*, limits the diversity order of digital relaying [5], [10], [11]. However, selective protocols that allow the relay to transmit only when it can detect/decode the source signal reliably can be designed with the help of error detection mechanisms at the relay [5], [12], [13]. In this way, error propagation can be significantly reduced and digital relaying can provide full diversity order.

Deploying multi-antenna relays has many advantages. Relay antennas can be used to increase the reliability source-relay channel, which mitigates error propagation in digital relaying [12]. It is also possible to increase relay-destination channel reliability through space-time coding. However, in this paper we are more interested in "multi-stream" relaying, where more than one user streams are relayed simultaneously by forming MIMO channels between the source(s), the relay and the destination.

Most work on relaying is confined to single-antenna terminals. Some multi-stream relaying protocols were recently proposed in [14] and [15]. In [14], authors studied the asymptotic behaviour of the capacity of a network with a multi-antenna source and destination, and a large number of relays. Outage capacities of multi-stream relaying protocols with a multi-antenna source, a multi-antenna relay and a multi-antenna destination were considered in [15]. However, neither of these papers included the direct link from source to destination in their analysis. Combining source and relay signal, where both signals are spatially multiplexed but have different average SNRs, seems to be a new problem that appears only in multi-stream diversity relaying. We reported some simulation results in a limited scope in [16].

In this paper, we analyze three multi-stream digital relaying protocols for the spatial multiplexing of M mobile users. Conventional Digital Relaying (CDR) is digital relaying without diversity combining at the destination. Non-selective Digital Relaying (NDR) and Selective Digital Relaying (SDR) are extensions of well-known decode-and-forward and selective decode-and-forward [5] to spatially multiplexed signals. In NDR, the relay retransmits, regardless of the quality of user-relay channels. In SDR, the relay is allowed to transmit only when it can decode all the user streams without error. We analyze the impact of these strategies on the outage performance of the system.

We first investigate ways of combining such signals using a V-Blast receiver. We define two methods based on Zero Forcing with Decision Feedback (ZF-DF). In *Joint ZF-DF*, two output signals are stacked and ZF-DF is applied to this equivalent system. In the second method, which we call *Parallel ZF-DF*, the data of a user is estimated independently from the direct and relay output signals. These two estimates are combined to detect and decode the stream.

We show that, in accordance with the results for single-stream relaying [5], selective diversity relaying combined with joint ZF-DF detection can improve the outage performance significantly, even if the direct signals have lower average SNRs than the relayed signal.

A. Notation

Superscripts T and H are used for transpose and Hermitian conjugate of matrices, respectively. $diag\{x_1, x_2, \ldots, x_n\}$ stands for an $n \times n$ diagonal matrix with given elements on its diagonal. $\mathbf{I}_n$ and $\mathbf{0}_{m,n}$ denote the $n \times n$ identity matrix and $m \times n$ zero matrix, respectively. $\mathbf{A}(i_1 : i_2, j_1 : j_2)$, with $i_1 \leq i_2$ and $j_1 \leq j_2$, represents the submatrix of $\mathbf{A}$ composed of rows $i = i_1, i_1+1, \ldots, i_2$ and columns $j = j_1, j_1+1, \ldots, j_2$. $\mathbf{A}(i,j)$ denotes the element at the i-th row and the j-th column of $\mathbf{A}$. A real Gaussian vector with zero mean and identity covariance matrix is called a standard real Gaussian random vector. A circularly symmetric complex Gaussian vector with zero mean and identity covariance matrix is called standard Gaussian random vector. A central chi-square random variable with n degrees of freedom is denoted by $\chi^2(n)$.

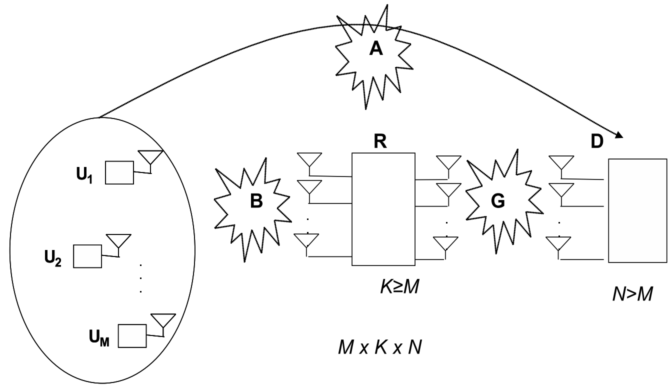

Fig. 1. $M \times K \times N$ system: M single antenna users, a relay with K antennas and a destination with N antennas.

II. System model

We consider a system with M mobile users, each with a single antenna and a destination with N antennas ($N \geq M$). A fixed relay with K antennas ($K \geq M$) assists the communication between the users and the destination. We call such a system as an $M \times K \times N$ system. All channels, User-Relay channel ($\mathbf{B}$), User-Destination channel ($\mathbf{A}$), and Relay-Destination channel ($\mathbf{G}$), are assumed to experience independent Rayleigh fading. We also assume slow block fading, which implies that all channels stay unchanged for two block durations, where each block duration is equal to L symbol periods. The channel state information is available only at the receiver side for all three links.

We assume that user i has a fixed target rate R_i and encodes its data independently using a single input single output (SISO) encoder whose rate depends on R_i but its codeword length is fixed and equal to L. The block length L is assumed to be large enough so that a decoding error occurs if and only if $log_2(1 + SNR) < R_i$. Users transmit synchronously but without any cooperation. Both the relay and the destination use ZF-DF receivers. The system is similar to a horizontally coded point-to-point MIMO system, which is sometimes referred to as H-blast [17]. To simplify the analysis, we assume that the order of decoding is the same at the relay and the destination and it is independent of the channel realizations.

III. Multi-stream Relaying Protocols

We consider three digital relaying protocols: Conventional DR (CDR), Non-selective DR (NDR), and Selective DR (SDR). In all protocols, transmission takes place in two equal time slots, each having L symbol periods. During the first time slot, M active mobile users transmit simultaneously in a synchronous manner. Then, in the second slot, the relay transmits using at most M antennas.

In all protocols, relay decodes the signals from the users, and then reencodes its estimates and retransmits the resulting block $\hat{\mathbf{X}}$ in the second slot. It uses the same SISO encoder as user i for regenerating this user's signal. The streams of different users are spatially multiplexed and transmitted from randomly assigned antennas.

- **Conventional Digital Relaying (CDR):** In CDR, the destination decodes based on only the relay signal.
- **Non-Selective Digital Relaying (NDR):** In this protocol, relay retransmits regardless of the outcome of the transmission in the first time slot. The destination decodes based on the direct and the relayed signals.
- **Selective Digital Relaying (SDR):** The relay transmits only if it can decode all M streams reliably. Otherwise, it remains silent[1].

IV. Outage Analysis

For completeness, let us first review the outage probability of the direct transmission from users to the destination, which uses a Zero Forcing-Decision Feedback (ZF-DF) receiver.

A. Direct Transmission

The system is described by

$$\mathbf{Y_d} = \mathbf{A}\mathbf{W_d}\mathbf{X} + \mathbf{N}, \tag{1}$$

where $\mathbf{X} \in \mathbb{C}^{M\times L}$ is the transmit signal block of M users, $\mathbf{Y_d} \in \mathbb{C}^{N\times L}$ is the received block. $\mathbf{A}$ is the channel matrix with independent, circularly symmetric complex gaussian elements representing i.i.d. Rayleigh fading, $\mathcal{CN}(\mathbf{0},\mathbf{I})$. $\mathbf{N} \in \mathbb{C}^{N\times L}$ is Gaussian noise with temporally and spatially independent elements having distribution $\mathbf{N}(i,j) \sim \mathcal{CN}(0,1)$. $\mathbf{W_d} = diag\{\sqrt{\eta_{1,d}},\ldots,\sqrt{\eta_{M,d}}\}$ and $\eta_{i,d}$ is the average SNR of user i's direct signal at the destination.

We assume that the user streams are decoded according to their indices. Each time, the total received signal is projected onto a subspace orthogonal to the streams that are yet to be detected. From [18], the resulting output SNR for the i-th user detected can be obtained as:

$$\rho_{\mathbf{i}} = \frac{1}{2}\eta_{i,d}\, z(2(N-M+i)), \tag{2}$$

where $\mathbf{z}(m)$ is a chi-square random variable with m degrees of freedom. After all the data block is projected, user i is decoded. Then, its codeword is regenerated and its effect is cancelled from the total signal. It is well known that this procedure is equivalent to Gram-Schmidt orthogonalization of the channel matrix [19]. Assuming that $\mathbf{A}$ has linearly independent columns, which happens with very high probability, it can be uniquely decomposed as [20]:

$$\mathbf{A} = \mathbf{Q_A}\mathbf{R_A}$$

where $\mathbf{Q_A} \in \mathbb{C}^{M\times M}$ is unitary, satisfying $\mathbf{Q_A^H}\mathbf{Q_A} = \mathbf{I}_M$ and $\mathbf{R_A} \in \mathbb{C}^{M\times N}$ is an upper triangular matrix whose diagonal elements are positive. Then, $\mathbf{R_A}(M,M)\mathbf{W}(M,M)$ corresponds to the output SNR of the first stream decoded.[2]. Moreover, the output SNRs obtained by this procedure are independent, given that the decoding order is independent of matrix $\mathbf{A}$ [21].

We define the system outage event as the union of individual user outages. From the independence of output SNRs, we can write:

$$\begin{aligned} P_o^{u\to d}(M,N,\mathbf{W_d}) &= 1-\prod_{i=1}^{M} Pr\{\log(1+\rho_i) > R_i\} \\ &= 1-\prod_{i=1}^{M}\left[1-F_{2(N-M+i)}(\gamma_i)\right], \end{aligned} \tag{3}$$

where R_i is the target rate of user i. $F_k(.)$ denotes the cdf of the chi-square distribution with k degrees of freedom. Using (2), γ_i is obtained as $\gamma_i = 2(2^{R_i}-1)/\eta_{i,d}$.

Having noted that all the relay protocols use twice the bandwidth used by direct transmission, we define the following protocol to enable a fair comparison between relaying and direct transmission [16]: In Time-Division Direct Transmission (TDDT), users are divided into two sets of equal size. Assuming M is even, each set has $M/2$ users. In the first time slot, the first set of streams are transmitted from their assigned antennas and the second set follows in the second slot. The system outage of this protocol is given by:

$$\begin{aligned} P_o^{TDDT} &= 1-\Big(\left(1-P_o^{u_1\to d}(M/2,N,\mathbf{W_{d1}})\right) \\ &\qquad \times\left(1-P_o^{u_2\to d}(M/2,N,\mathbf{W_{d2}})\right)\Big) \end{aligned} \tag{4}$$

where $\mathbf{W_{d1}}$ and $\mathbf{W_{d2}}$ are the $M/2\times M/2$ weight matrices for the two groups.

B. Relaying Protocols

Let $\mathbf{Y_{d1}}$ and $\mathbf{Y_{d2}}$ be the received signals at the destination in the first and second time slots, respectively. These received signals can be represented as

$$\mathbf{Y_{d1}} = \mathbf{A}\mathbf{W_d}\mathbf{X} + \mathbf{N_{d1}} \tag{5}$$

$$\mathbf{Y_{d2}} = \mathbf{G}\mathbf{W_r}\hat{\mathbf{X}} + \mathbf{N_{d2}}, \tag{6}$$

where $\mathbf{W_r}$ is a diagonal matrix whose entries depend on the average SNR at the destination due to the relay transmission. We assume that the relay allocates the power uniformly for all active antennas. Hence, $\mathbf{W_r}$ is given by $\mathbf{W_r} = \sqrt{\eta_r}\,\mathbf{I}_M$. Similarly, we represent the user-relay channel as:

$$\mathbf{Y_r} = \mathbf{B}\mathbf{W_{ur}}\mathbf{X} + \mathbf{N_r}, \tag{7}$$

where $\mathbf{W_{ur}}$ depends on the average received SNRs.

If the user-relay channel is in outage, i.e. at least one of the users is in outage in this channel ($\hat{\mathbf{X}} \neq \mathbf{X}$), we assume that data of some users will be decoded incorrectly at the destination, causing a system outage.

[1]This second slot can be used by the users to transmit their next data blocks. Here, however, for simplicity, we assume that if relay cannot decode, the second time slot is not used by any of the terminals.

[2]In this representation decoding order is decreasing user index, user M is decoded first.

We denote the outage probabilities of user-relay channel and relay-destination channel as $P_o^{u\to r}(M,K,\mathbf{W_{ur}})$ and $P_o^{r\to d}((M,N,\mathbf{W_r}))$, respectively. These can be computed as in (3). $P_o^{u,r\to d}(M,N,\mathbf{W_r},\mathbf{W_d})$ denotes the outage probability after combining at the destination, given that the relay decoded all the streams correctly. Then, the outage probability of the three protocols are given by:

$$\begin{aligned} P_o^{CDR} &= P_o^{u\to r} + (1-P_o^{u\to r})\,P_o^{r\to d}, && (8)\\ P_o^{NDR} &= P_o^{u\to r} + (1-P_o^{u\to r})P_o^{u,r\to d}, && (9)\\ P_o^{SDR} &= P_o^{u\to r}P_o^{u\to d} + (1-P_o^{u\to r})P_o^{u,r\to d}, && (10)\end{aligned}$$

where all the arguments are dropped to simplify notation.

V. Combining Methods for Diversity Relaying Protocols

In this section we investigate the outage at the destination, given that the relay decodes all the users correctly. We propose two methods for detecting $\mathbf{X}$ based on $\mathbf{Y_{d1}}$ and $\mathbf{Y_{d2}}$: *Joint ZF-DF (JZF-DF)* and *Parallel ZF-DF (PZF-DF)*.

A. Joint ZF-DF (JZF-DF)

Assuming correct decoding of all M streams at the relay, the equivalent system is given by:

$$\mathbf{Y_e} = \mathbf{H_e}\mathbf{X} + \mathbf{N_e} \tag{11}$$

where

$$\mathbf{H_e} = \begin{bmatrix}\mathbf{A}\mathbf{W_d}\\ \mathbf{G}\mathbf{W_r}\end{bmatrix} \tag{12}$$

and $\mathbf{Y_e} = [\mathbf{Y_{d1}}^T\ \mathbf{Y_{d2}}^T]^T$, $\mathbf{N_e} = [\mathbf{N_{d1}}^T\ \mathbf{N_{d2}}^T]^T$. Then, the destination decodes the equivalent $M\times 2N$ system based on (11).

B. Parallel ZF-DF (PZF-DF)

This detection method is based on parallel zero forcing and per stream combining. Let i be the index of the stream to be detected. First, the outputs $\mathbf{Y_{d1}}$ and $\mathbf{Y_{d2}}$ are filtered independently to nullify the interference of the streams yet to be detected. Next, the filtered signals are combined using maximal ratio combining, which is the optimal combining, since ZF filtering suppresses all inter-stream interference. After the stream is detected and decoded based on the combined output, the contribution of stream i is subtracted both from $\mathbf{Y_{d1}}$ and $\mathbf{Y_{d2}}$.

This procedure can be mathematically represented as follows. After the *QR* decomposition, we have

$$\mathbf{A} = \mathbf{Q_A}\mathbf{R_A} \text{ and } \quad \mathbf{G} = \mathbf{Q_G}\mathbf{R_G},$$

where $\mathbf{Q_A}, \mathbf{Q_G} \in \mathbb{C}^{M,M}$ are unitary matrices satisfying $\mathbf{Q_A^H}\mathbf{Q_A} = \mathbf{Q_G^H}\mathbf{Q_G} = \mathbf{I}_M$ and $\mathbf{R_A}, \mathbf{R_G} \in \mathbb{C}^{M,N}$ are upper triangular matrices whose diagonal elements are positive. Then, we can write:

$$\begin{aligned} \tilde{\mathbf{Y}}_\mathbf{1} &= \mathbf{R_A}\mathbf{W_d}\mathbf{X} + \tilde{\mathbf{N}}_\mathbf{1} && (13)\\ \tilde{\mathbf{Y}}_\mathbf{2} &= \mathbf{R_G}\mathbf{W_r}\mathbf{X} + \tilde{\mathbf{N}}_\mathbf{2}, && (14)\end{aligned}$$

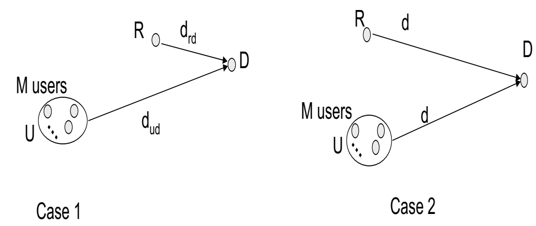

Fig. 2. Illustration of *case 1* and *case 2* assuming that the average channel gain is a function of the distance between the transmit antenna and the receive antenna. In *case 1*, $|U_iD| = d$ for all $i = 1, 2, \ldots, M$. In *case 2*, in addition to this condition, $|RD| = d$. However, in both cases, the users can have arbitrary distance to the relay.

where $\tilde{\mathbf{Y}}_\mathbf{1} = \mathbf{Q_A^H}\mathbf{Y_{d1}}$, $\tilde{\mathbf{Y}}_\mathbf{2} = \mathbf{Q_G^H}\mathbf{Y_{d2}}$ and $\tilde{\mathbf{N}}_\mathbf{1}, \tilde{\mathbf{N}}_\mathbf{2}$ are statistically equivalent to $\mathbf{N_{d1}}$ and $\mathbf{N_{d2}}$. In this notation, decoding order is in terms of decreasing user index. ZF-DF decoding corresponds to decoding the last stream first and cancelling the effect of this codeword from all upper streams. To decode user j, destination combines j-th row of $\tilde{\mathbf{Y}}_\mathbf{1}$ and $\tilde{\mathbf{Y}}_\mathbf{2}$ using maximal ratio combining:

$$\begin{aligned} \mathbf{Y_c}(j,:) &= \mathbf{R_A}(j,j)\mathbf{W_d}(j,j)\mathbf{Y_{d1}}(j,:)\\ &+ \mathbf{R_G}(j,j)\mathbf{W_r}(j,j)\mathbf{Y_{d2}}(j,:). && (15)\end{aligned}$$

Then, stream j is decoded based on $\mathbf{Y_c}(j,:)$ and its effect on $\tilde{\mathbf{Y}}_\mathbf{1}(1:j-1,:)$ and $\tilde{\mathbf{Y}}_\mathbf{2}(1:j-1,:)$ are cancelled. By continuing this process for all the streams, output SNR for user j, which is decoded as the i-th stream ($j = M-i+1$) is given by:

$$\rho_\mathbf{i} = \frac{1}{2}\left(\eta_{j,d}z^{(1)}(2(N-M+i)) + \eta_r z^{(2)}(2(N-M+i))\right) \tag{16}$$

where $z^{(1)}(m)$ and $z^{(2)}(m)$ are i.i.d. with $\chi^2(m)$ distribution.

C. Outage Performances of JZF-DF and PZF-DF

Here, we analyze the performances of JZF-DF and PZF-DF under the assumption that the relay decoded all the users correctly. We introduce two special topologies for which the performance comparison of JZF-DF and PZF-DF is easier. In *case 1*, it is assumed that all the users have the same average SNR to the destination. Hence, $\mathbf{W_r} = \sqrt{\eta_r}\,\mathbf{I}_M$ and $\mathbf{W_d} = \sqrt{\eta_d}\,\mathbf{I}_M$. In *case 2*, which is a special case of *case 1*, the relay signal and the direct channel have the same average SNR, $\mathbf{W_r} = \mathbf{W_d} = \sqrt{\eta}\,\mathbf{I}_M$. Fig. 2 illustrates these special topologies.

1) JZF-DF: We note that, unlike individual channel matrices $\mathbf{A}$ and $\mathbf{G}$, for general diagonal $\mathbf{W_d}$ and $\mathbf{W_r}$, $\mathbf{H_e}$ is not a normal data matrix[3]. Thus, many useful results on normal data matrices do not apply to this problem.

Hence, for JZF-DF, only two special cases introduced above will be considered. Clearly, in *case 2*, JZF-DF is equivalent to direct transmission with $2N$ receive antennas instead of N.

[3]A random matrix is a normal data matrix if all of its row vectors are i.i.d. complex normal random vectors with arbitrary covariance matrix. Each column vector, however, must have identity covariance matrix [22].

Hence, output SNR for the ith stream is chi-square distributed with $2(2N-M+i)$ degrees of freedom and diversity order is increased by N for all streams compared to the direct transmission.

In *case 1*, we can represent the equivalent channel as a Rayleigh channel with receive correlation [23] [24]:

$$\mathbf{H_e} \;=\; \mathbf{H_r}\mathbf{H_w}, \tag{17}$$

where

$$\mathbf{H_r} = \begin{bmatrix} \sqrt{\eta_d}\,\mathbf{I}_N & \mathbf{0} \\ \mathbf{0} & \sqrt{\eta_r}\,\mathbf{I}_N \end{bmatrix} \tag{18}$$

and $\mathbf{H_w} \in \mathbb{C}^{2N\times M}$ is a Rayleigh channel, whose elements are i.i.d. and distributed as $\mathbf{H_w}(i,j) \sim \mathcal{CN}(0,1)$. Exact CDF of output SNR for ZF receiver in a correlated Rayleigh MIMO channel is given in [25][4].

2) PZF-DF: In *case 2*, (16) simplifies to

$$\rho_i \;=\; \frac{1}{2}\eta\, z\left(4(N-M+i)\right) \tag{19}$$

where $z(m) \sim \chi^2(m)$. Hence, it is clear that, PZF-DF combining at the destination doubles the diversity order at each stage and the diversity order for the i-th user is $2(N-M+i)$, which is smaller than or equal to the one achieved by JZF-DF $(2N-M+i)$, for all $i=1,2,\ldots,M$ for any $M \geq 2$.

From (16), we observe that output SNR is a weighted sum of two chi-square random variables with even degrees of freedom. Theorem 2.4 of [26] gives the exact CDF of the weighted sum of an arbitrary number of chi-square random variables in terms a finite sum of chi-square CDFs. Applying this theorem to our case, we obtain:

$$Pr\{\rho_i > x\} \;=\; 1 - \sum_{j=1}^{2}\sum_{s=1}^{g_i} \alpha_{js} Pr\left\{\chi^2(2s) > \frac{x}{\lambda_j}\right\} \tag{20}$$

where

$$\alpha_{1s} \;=\; f(g_i,s)\left(\frac{\lambda_2}{\lambda_1}\right)^{g_i-s}\left(\frac{\lambda_1-\lambda_2}{\lambda_1}\right)^{-2g_i+s} \tag{21}$$

$$\alpha_{2s} \;=\; f(g_i,s)\left(\frac{\lambda_1}{\lambda_2}\right)^{g_i-s}\left(\frac{\lambda_2-\lambda_1}{\lambda_2}\right)^{-2g_i+s}, \tag{22}$$

$$f(g_i,s) = (-1)^{g_i-s}\frac{(2g_i-s-1)!}{(g_i-s)!(g_i-1)!}, \tag{23}$$

$g_i = N-M+i$, $\lambda_1 = \eta_d/2$ and $\lambda_2 = \eta_r/2$. Note that (20) gives the exact outage probability for the i-th user (given cooperation) for general $\mathbf{W_r}$ and $\mathbf{W_d}$.

VI. Results

We compare system outage of all protocols and combining methods. All terminals transmit with the same power per antenna in each protocol. A topology as in *case 1* is considered. η_{rd} is 9.03 dB worse than the η_{ur} and η_{ud} is 14.31 dB worse than η_{ur}. The x-axis shows the SNR of the user-destination

[4]The derivation in [25] assumes that $\mathbf{R_{RX}} = \mathbf{H_r}\mathbf{H_r}^H$ has distinct eigenvalues and its final results do not apply to our problem. Hence, for the numerical results we present in this paper, we derived the CDF of output SNRs for a system with channel matrix given in (17) and (18).

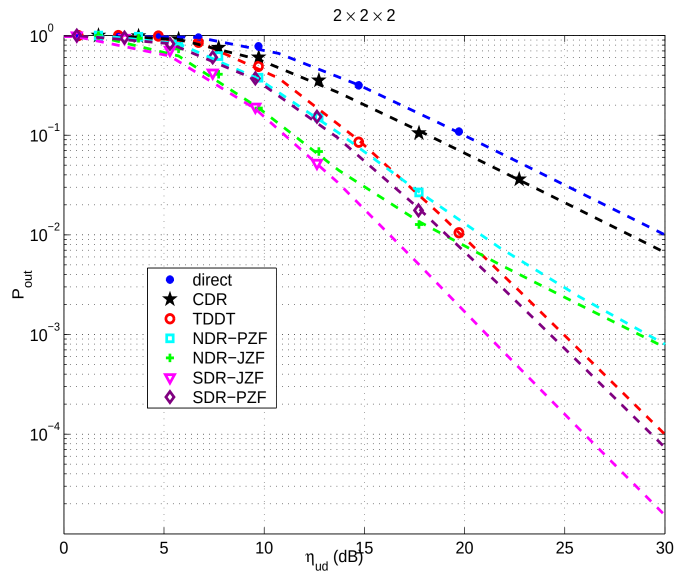

Fig. 3. System outage probability for a $2\times2\times2$ system as a function of average direct link SNR. Markers show simulation results while dashed lines show analytical results.

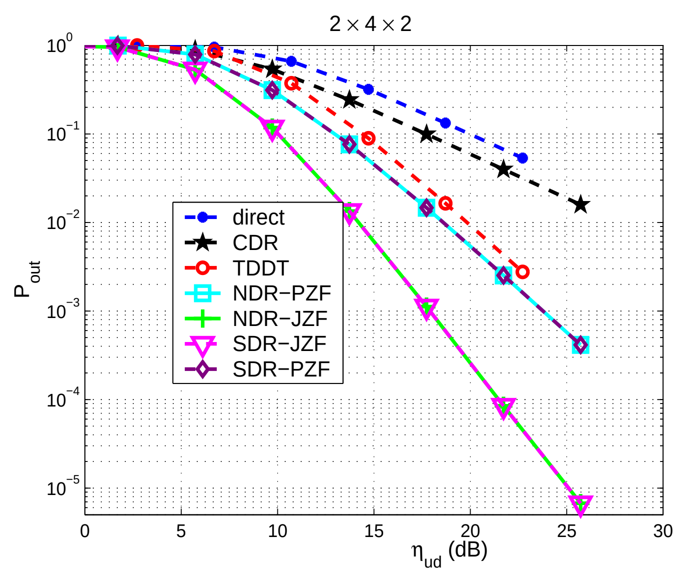

Fig. 4. System outage probability for a $2\times4\times2$ system as a function of average direct link SNR.

channel for direct transmission. For other protocols we plot the outage for the same total energy as the direct transmission. We considered path-loss and Rayleigh fading only. Target SNR γ_i is taken as 10 dB for all users.

In Fig. 3, we plot the system outage probability for all protocols. Here, we plot both analytical and Monte Carlo simulated system outage curves. Analytical curves are obtained from (3), (4), (8)-(10) and the derivations in Section V-C. In simulations, output SNRs and outage rates are calculated based on a large number of randomly generated channel matrices. Having validated the analytical expressions in Fig. 3, in Fig. 4 we plot (analytical) system outage probability for a $2\times4\times2$ system.

As expected, conventional relaying provides only a constant SNR gain over direct transmission. In Fig. 3, we observe that NDR is limited by the source-relay channel, regardless of the combining method used at the destination and it is outperformed by TDDT at high average SNRs. For this system, we can conclude that the relaying protocol (NDR vs. SDR) is the dominant factor that determines outage performance. In Fig. 4, however, we see that NDR and SDR have almost identical outage performance and the outage probability is mostly determined by the combining scheme.

VII. Discussion and Conclusions

This paper discusses the potential benefits of using a multi-antenna relay in the spatial multiplexing of independent single antenna users communicating with a common multi-antenna destination. Digital relaying protocols of fixed vs. selective and conventional vs. diversity kind were considered. For diversity relaying, which has not been tackled so far in the context of multiple users/streams, we proposed two ZF-DF type combiners/decoders (JZF-DF and PZF-DF) to be used at the destination. We derived outage expressions, in closed-form for some special cases, and evaluated the performance for selective and fixed relaying protocols.

Our study indicates that under Rayleigh fading and path loss, diversity relaying has significant advantages over conventional relaying, as in single antenna relaying. Selective diversity relaying is crucial when the user-relay channel has lower diversity than the cooperative channel from users & relay to the destination, which is expected in most practical configurations. When user-relay channel is sufficiently reliable, which, for example, happens if the relay has a large number of antennas, diversity protocols are the most advantageous compared to conventional relaying. In this case, the performances of NDR and SDR for the same combining method are very close. Then, the outage probability is determined by the combining method used by the destination rather than the protocol used by the relay and JZF-DF has a considerably better performance than PZF-DF.

Once the relay decodes the user signals, it can encode them jointly using a space-time code. However, we do not expect this strategy to improve the overall system outage unless user-relay channel is very reliable and the second hop (or the cooperative hop) is the dominant cause of outage.

References

[1] E. Telatar, “Capacity of multi-antenna gaussian channels,” *European Transactions on Telecommunications*, vol. 10, pp. 585–595, Nov. 1999.

[2] G. J. Foschini and M. J. Gans, “On limits of wireless communications in a fading environment when using multiple antennas,” *Wireless Personal Communications*, vol. 6, March 1998.

[3] P. Wolniansky, G. Foschini, G. Golden, and R. Valenzuela, “V-BLAST: an architecture for realizing very high data rates overthe rich-scattering wireless channel,” in *Proc. of URSI International Symposium on Signals, Systems, and Electronics*, 1998.

[4] V. Tarokh, H. Jafarkhani, and A. Calderbank, “Space-time block coding for wireless communications: performance results,” *IEEE Journal on Selected Areas in Communications*, vol. 17, 1999.

[5] N. Laneman, D. Tse, and G. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Transactions on Information Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[6] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity–part I: System description,” *IEEE Transactions on Communications*, vol. 51, pp. 1927–1938, Nov. 2003.

[7] H. Bolukbasi, H. Yanikomeroglu, D. Falconer, and S. Periyalwar, “Feasibility of providing high data coverage in cellular fixed relay networks.” *World Wireless Research Forum (WWRF)*, meeting no. 12, 3-4 November 2004, Toronto, Canada.

[8] A. Florea and H. Yanikomeroglu, “On the optimal number of hops in infrastructure-based fixed relay networks,” in *Proc. of IEEE Globecom, St. Louis, MO, USA*, Nov. 2005.

[9] R. Pabst, B. H. Walke, D. C. Schultz, P. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. D. Falconer, and G. P. Fettweis, “Relay-based deployment concepts for wireless and mobile broadband cellular radio,” *IEEE Communications Magazine*, vol. 42, no. 9, 2004.

[10] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, “Multihop diversity in wireless relaying channels,” *IEEE Transactions on Communications*, vol. 52, pp. 1820–1830, Oct. 2004.

[11] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, “On the maximum diversity order of wireless relaying networks.” submitted to *IEEE Transactions on Communications*, Nov. 2005.

[12] A. Adinoyi and H. Yanikomeroglu, “Multi-antenna aspects of parallel fixed wireless relays,” in *Proc. of IEEE Wireless Communications and Networking Conference (WCNC)*, 2006.

[13] Z. Lin, E. Erkip, and A. Stefanov, “Cooperative regions for coded cooperative systems,” in *Proc. of Globecom Communication Theory Symposium*, Dec. 2004.

[14] H. Bölcskei, R. U. Nabar, O. Oyman, and A. J. Paulraj, “Capacity scaling laws in MIMO relay networks.” *IEEE Transactions on Wireless Communications*, to appear in 2006.

[15] Y. Fan and J. Thompson, “On the outage capacity of MIMO multihop networks,” in *Proc. of IEEE Globecom*, Nov. 2005.

[16] F. A. Onat, H. Yanikomeroglu, and S. Periyalwar, “Adaptive multi-stream relaying,” in *Proc. of IEEE Canadian Conference on Electrical and Computer Engineering (CCECE)*, 2006.

[17] G. Foschini, D. Chizhik, M. Gans, C. Papadias, and R. Valenzuela, “Analysis and performance of some basic space-time architectures,” *IEEE Journal on Selected Areas in Communications*, vol. 21, 2003.

[18] J. H. Winters, J. Salz, and R. D. Gitlin, “The impact of antenna diversity on the capacity of wireless communication systems,” *IEEE Transactions on Communications*, vol. 42, 1994.

[19] T. Guess, H. Zhang, and T. Kotchiev, “The outage capacity of BLAST for MIMO channels,” in *Proc. of IEEE International Conference on Communications (ICC'03)*, 2003.

[20] R. Horn and C. Johnson, *Matrix Analysis*. Cambridge University Press, 1990.

[21] R. J. Muirhead, *Aspects of Multivariate Statistical Theory*, pp. 100,101. John Wiley and Sons Inc., 1982.

[22] K. V. Mardia, J. T. Kent, and J. M. Bibby, *Multivariate Analysis*. Academic Press, 1979.

[23] M. Kiessling and J. Speidel, “Asymptotically tight bound on SER of MIMO zero-forcing receivers with correlated fading,” in *Proc. of International Symposium on Information Theory (ISIT'04)*, 2004.

[24] H. Shin and J. H. Lee, “Capacity of multiple-antenna fading channels: Spatial fading correlation, double scattering, and keyhole,” *IEEE Transactions on Information Theory*, vol. 49, 2003.

[25] M. Kiessling, “MIMO zero-forcing receivers Part I: Multivariate statistical analysis.” submitted for publication, available at http://www.inue.uni-stuttgart.de/publications/pub_2005/kiessling_ZF_performance.pdf.

[26] G. E. P. Box, “Some theorems on quadratic forms applied in the study of analysis of variance problems, I. Effect of inequality of variance in the one-way classification,” *The Annals of Mathematical Statistics*, vol. 25, 1954.