A Theoretical Characterization of the Multihop Wireless Communications Channel with Diversity

John Boyer, David Falconer, Halim Yanikomeroglu
Department of Systems and Computer Engineering
Carleton University, 1125 Colonel By Drive
Ottawa, Ontario, Canada K1S 5B6

Abstract – **This paper provides a theoretical characterization of the multihop wireless communications channel with diversity, wherein intermediate terminals relay information employing spatial diversity techniques. Two channel models are proposed and developed: one where each intermediate terminal combines, digitally decodes, and re-encodes the received signals from all preceding terminals and the other where each intermediate terminal simply combines and amplifies the received signals from all preceding terminals. These models are compared, through analysis and simulations, with the singlehop reference channel on the basis of probability of outage and probability of error. Both models achieve significant gains over the singlehop reference channel, with the amplified relaying model outperforming the decoded relaying model despite noise propagation.**

I. INTRODUCTION

This paper is concerned with a proposed wireless system wherein traditional transmission constraints are removed in order to allow direct communication between mobile terminals. This system gives mobile terminals the ability to relay information when they are neither the initial transmitter nor the final receiver. Relaying systems realize a number of benefits over traditional systems in the areas of deployment, connectivity, adaptability and capacity [5,9].

This paper proposes two channel models for the case where mobile terminals act as intermediate nodes in wireless communications systems. These are referred to as the *decoded relaying multihop diversity channel* and the *amplified relaying multihop diversity channel*. The decoded relaying multihop diversity channel corresponds to the case where each intermediate terminal combines, digitally decodes, and re-encodes the received signals from all preceding terminals before retransmission. The amplified relaying multihop diversity channel corresponds to the case where each intermediate terminal simply combines and amplifies the received signals from all preceding terminals before retransmission.

This paper focuses on multihop channels with diversity. Receiver diversity implies the reception of separate, independently faded and shadowed versions of a signal, and is realized here without the requirement for multiple antennas at each terminal. Instead, diversity is generated naturally as each component version is transmitted along a different physical path and experiences different delay characteristics. Diversity combining can then be implemented using traditional techniques such as rake receivers and equalizers depending on the multiple access scheme. Further discussion on related implementation issues is provided in [2]. Multihop channels without diversity are characterized in [3].

II. MULTIHOP VERSUS MULTIROUTE DIVERSITY

Before proceeding with the characterization of multihop wireless communications channels employing spatial diversity techniques, it is first necessary to define and distinguish the two classes of techniques: multihop diversity and multiroute diversity. Multiroute diversity results from the concurrent reception of signals that have been transmitted along multiple routes that pass through different intermediate terminals. Multihop diversity results from the concurrent reception of signals that have been transmitted by multiple previous terminals along a single route.

When comparing the potential performance and application of these diversity techniques it is important to note that multiroute diversity is caused by the artificial generation of multiple secondary signals whereas multihop diversity is caused by the natural generation of multiple secondary signals. This distinction between artificial and natural generation is based on the requirement for a redistribution of power within the system. Multiroute diversity requires a redistribution of power among the intermediate terminals whereas multihop diversity requires no such redistribution of power. This analysis assumes a constraint on the total power of the system in order to provide a fair comparison.

A. The Case Supporting Multihop Diversity

The case supporting multihop diversity is straightforward. Since there is no requirement for a redistribution of power within the system, no additional resources must be expended in order to achieve a gain from diversity combining. This is a direct result of the use of omnidirectional antennas. Although a transmission may be intended for a specific terminal, the signal will reach a multitude of other terminals as well. The only cost associated with using multihop diversity results from an increase in terminal receiver complexity.

B. The Case Supporting Multiroute Diversity

Although it is safe to assume that the multihop diversity channel will outperform the multihop channel without diversity, the same is not the case for the multiroute diversity channel. The redistribution of creates two opposing trends in the resulting outage and error performance. The first trend is a decrease due to diversity combining. The second trend is an increase due to lower transmit powers at individual intermediate terminals.

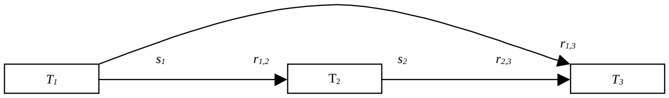

Fig. 1. Example Multihop Diversity Channel

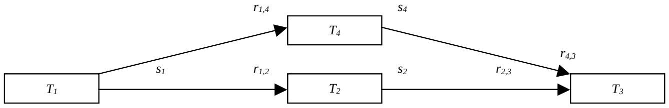

Fig. 2. Example Multiroute Diversity Channel

Consider first the probability of outage due to large-scale lognormal effects. Since the actual average received signal to noise ratio of each route is dependent on the mobility of the respective intermediate terminal relative to the destination terminal, it will change slowly with respect to the period of the signal. The source terminal can therefore determine the best route in terms of actual average received signal to noise ratio and then transmit the signal along that route.

Now consider the probability of error due to small-scale Rayleigh effects. Examine the scenario where the statistics of each of the routes are identical. This is in fact the optimal scenario in support of multiroute diversity because in any other scenario there would be more benefit gained from concentrating all the power along the best route. Analysis in [2] reveals this optimal scenario, and by extension all sub-optimal scenarios, to be generally inferior to the system with a single route for all but very large signal to noise ratios. Coupled with the fact that in realistic implementations received signal to noise ratios will be set to the minimum value able to support a given quality of service class, this questions the potential benefits of using multiroute diversity. From this point forward, all references to diversity within multihop channels will correspond to multihop diversity.

III. SYSTEM MODEL

The system model for multihop wireless communications channels with diversity is composed of a source terminal, a receiving terminal, and an indeterminate number of potential intermediate relaying terminals. In Fig. 3 the source terminal is identified as T_1, the destination terminal is identified as T_n and the intermediate terminals are identified as T_2 through T_{n-1} where *n* is the number of hops along the transmission path.

Let T_S represent the set of source terminals, T_I represent the set of intermediate terminals, and T_D represent the set of destination terminals. Therefore $T_T = T_S \cup T_I$ represents the set of all transmitting terminals and $T_R = T_I \cup T_D$ represents the set of all receiving terminals. Let $T_{P(i)}$ represent the set of terminals that transmit a signal received by T_i. The notation used in this paper assumes that $T_{P(i)}$ has a cardinality greater than or equal to 1 in order to support the characterization of scenarios with diversity. For the communications channel illustrated in Fig. 3, $T_S = \{T_1\}$, $T_I = \{T_2,...T_{n-1}\}$, $T_D = \{T_n\}$, $T_T = \{T_1,...T_{n-1}\}$, $T_R = \{T_2,...T_n\}$ and $T_{P(i)} = \{T_1,...T_{i-1}\}$.

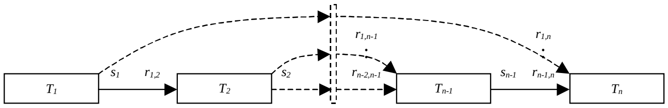

Fig. 3. Generic Multihop Diversity Wireless Communications Channel

Each terminal T_i transmits a signal given by

$$s_i = \sqrt{\varepsilon_i}\, a_t + \beta_i , \tag{1}$$

where ε_i is the transmitted power, a_t is the binary information symbol at time interval t, and β_i is propagated noise. The propagated noise term in (1) is zero for transmitting terminals as well as for intermediate terminals that employ decoded relaying. Each terminal T_i then receives a set of signals given by

$$r_{k,i} = \alpha\sqrt{(L_{k,i}/d_{k,i}^{p})}\,R_{k,i}(\sqrt{\varepsilon_k}\,a_t + \beta_k) + z_{k,i}, k \in P(i) , \tag{2}$$

where α^2 is the free space signal power attenuation factor between the transmitting terminal and an arbitrary reference distance, $d_{k,i}$ is the inter-terminal distance relative to the reference distance, p is the propagation exponent, $L_{k,i}$ is a zero-mean lognormal random variable with variance $\sigma_{L_{k,i}}^2$, $R_{k,i}$ is a complex gaussian (Rayleigh) random variable with mean power $E[R_{P(i),i}^2] = 1$, and $z_{k,i}$ is a zero-mean additive white gaussian noise random variable with variance N_0. Assuming maximal ratio combining, the received signal to noise ratio at T_i is given by

$$\gamma_{P(i),i} = \sum_{k \in P(i)} \left(\frac{\alpha^2 \varepsilon_k}{(d_{k,i}^{p} \big/ L_{k,i}\left|R_{k,i}\right|^2)N_0 + \left|\beta_k\right|^2}\right) . \tag{3}$$

where $\left|R_{k,i}\right|^2$ is an exponential random variable with mean $2\sigma_{R_{k,i}}^2 = 1$. The probability of outage due to lognormal shadowing when $\beta_k = 0$ is given in [7] by

$$\Pr[\gamma_{P(i),i} < \gamma] \approx Q\left(\frac{10\log(\overline{\gamma_{P(i),i}} \big/ \gamma)}{\sigma_{L_{Z(i)}}}\right), \tag{4}$$

where γ is an arbitrary signal to noise ratio that must be met in order to maintain communication and $\sigma_{L_{Z(i)}}^2$ is the variance of the lognormal approximation of (3) determined using Fenton's method [1,8]. The calculation of probability of error is dependent on the modulation scheme employed. For the special case of BPSK, the probability of error under fading conditions when $\beta_k = 0, k \in P(i)$ is given in [6] by

$$P_e(\gamma_{P(i),i}) \approx \binom{2K-1}{K} \prod_{k \in P(i)} \left(\frac{1}{2\overline{\gamma_{k,i}}}\right), \overline{\gamma_{k,i}} >> 1 , \tag{5}$$

where K is the cardinality of $T_{P(i)}$ and $\overline{\gamma_{k,i}}$ is the expected received signal to noise ratio at T_i for branch k of the diversity combiner.

IV. Decoded Relaying

The decoded relaying multihop diversity channel corresponds to the case where each intermediate terminal combines, digitally decodes and re-encodes the received signals from all preceding terminals before retransmission. This digital relaying channel does not propagate noise along the multihop channel. The possibility of decoding error is introduced at each intermediate terminal.

The channel model is given by (1) through (3) with $\beta_k = 0, k \in P(i)$. The received signal to noise ratio at T_i is given by

$$\gamma_{P(i),i} = \sum_{k\in P(i)} \left(\frac{\alpha^2 \varepsilon_k}{(d_{k,i}^{p} \big/ L_{k,i}\left|R_{k,i}\right|^2) N_0}\right). \tag{6}$$

The total probability of outage for the decoded relaying multihop diversity channel is given by

$$P_o = 1 - \prod_{i\in T_R} (1 - \Pr[\gamma_{P(i),i} < \gamma]), \tag{7}$$

where $\Pr[\gamma_{P(i),i} < \gamma]$ is the probability of outage at terminal T_i given a received signal to noise ratio of $\gamma_{P(i),i}$. This value can be upper-bounded by

$$P_o \le \sum_{i\in T_R} \Pr[\gamma_{P(i),i} < \gamma]. \tag{8}$$

The total probability of decoding error for the decoded relaying multihop diversity channel is given by

$$P_e \approx 1 - \prod_{i\in T_R} (1 - P_e(\gamma_{P(i),i})), \tag{9}$$

where $P_e(\gamma_{P(i),i})$ is the probability of decoding error at terminal T_i given a received signal to noise ratio of $\gamma_{P(i),i}$. This value can be upper-bounded by

$$P_e \le \sum_{i\in T_R} P_e(\gamma_{P(i),i}). \tag{10}$$

V. Amplified Relaying

The amplified relaying multihop diversity channel corresponds to the case where each intermediate terminal simply combines and amplifies the received signals from all preceding terminals before retransmission. This analog relaying channel propagates noise along the multihop channel. The possibility of decoding error is introduced only at the destination terminal.

The channel model is composed of a set of individual transmission channels given by (1) through (3). Assuming that each intermediate terminal can track both lognormal shadowing and Rayleigh fading, the amplification factor at each intermediate terminal T_i is given by

$$A_i = \frac{\varepsilon_i}{\sum\limits_{k\in P(i)} (\alpha^2(\varepsilon_k + \left|\beta_k\right|^2) L_{k,i} \left|R_{k,i}\right|^2 \big/ d_{k,i}^{p} + N_0)}. \tag{11}$$

Although this amplification factor is exact, the resulting mathematical characterization of the channel model is extremely complex. In order to simplify the amplification factor, an approximation can be made where the noise terms are removed from the denominator of (11). This approximation yields an upper bound on the amplification factor, which is tight provided that the signal to noise ratio at the terminal under consideration is significantly greater than 1. The amplification factor at each terminal is then given by

$$A_i \le \frac{\varepsilon_i}{\sum\limits_{k\in P(i)} (\alpha^2 \varepsilon_k L_{k,i} \left|R_{k,i}\right|^2 \big/ d_{k,i}^{p})}, \tag{12}$$

and received signal to noise ratio at the destination terminal can be expressed recursively as

$$\gamma_{P(D),D} \approx \sum_{\substack{k\in P(D)\\ k\neq S}} (\gamma_{P(k),k}^{-1} + \psi_{k,D}^{-1})^{-1} + \psi_{S,D}, \tag{13}$$

where $\psi_{k,D}$ is the received signal to noise ratio $\gamma_{k,D}$ at terminal T_D for branch k of the diversity combiner with $\beta_k = 0$.

The probability of outage for the amplified relaying multihop diversity channel at the destination terminal is given by

$$P_o = \Pr[\gamma_{P(D),D} < \gamma] \approx Q\left(\frac{10\log(\overline{\gamma_{P(D),D}} \big/ \gamma)}{\sigma_{L_{Z(D)}}}\right), \tag{14}$$

where $\Pr[\gamma_{P(D),D} < \gamma]$ is the probability of outage at the destination terminal given a received signal to noise ratio of $\gamma_{P(D),D}$ and $\sigma_{L_{Z(D)}}^2$ is the variance of the lognormal approximation of (13) determined using Fenton's method. The application of this method is discussed in detail in [3].

The total probability of decoding error for the amplified relaying multihop diversity channel is given by

$$P_e = P_e(\gamma_{P(D),D}) \approx \binom{2K-1}{K} \prod_{k\in P(D)} \left(\frac{1}{2\overline{\gamma_{k,D}}}\right), \overline{\gamma_{k,D}} >> 1, \tag{15}$$

where $P_e(\gamma_{P(D),D})$ is the probability of decoding error at the destination terminal given a received signal to noise ratio of $\gamma_{P(D),D}$, K is the cardinality of $T_{P(D)}$, and $\overline{\gamma_{k,D}}$ is the expected received signal to noise ratio at the destination terminal for branch k of the diversity combiner.

VI. Simulation Results

In order to visualize the discussion, the results presented thus far are applied in two simulations and compared against the reference channel on the basis of probability of outage and probability of error. A BPSK modulation scheme is used for simplicity of exposition. The example multihop channel is composed of three terminals: source T_1, intermediate T_2 and destination T_3. The coordinates of the channel are normalized with respect to the distance between the source and destination terminals such that $d_{1,3} = 1$. The propagation exponent is $p = 4$. The lognormal shadowing components are independent with zero-mean and variance $\sigma^2_{L_{P(i),i}} = 12$ dB. The Rayleigh fading components are independent with mean power $E[R^2_{P(i),i}] = 1$. The threshold signal to noise ratio for outage calculations is $\gamma = 6$ dB. For the purpose of simplifying the comparison, and without loss of generality, the free space signal power attenuation factor is $\alpha^2 = 1$. Optimal power distribution is assumed for each of the channel models with the total power constrained to the reference power ε_0.

For the first simulation the intermediate terminals are fixed so that they divide the direct path between the source and destination terminals into n equal length segments. This serves to validate the theory presented thus far as well as illustrate the power gain that can be realized under an optimal placement of the intermediate terminals with respect to the source and destination terminals. For the second simulation the single intermediate terminal is placed at a set of locations uniformly distributed across a unit square. The source and destination terminals are located at (0,0) and (1,0) respectively. The intermediate terminal ranges from 0 to 1 along the x-axis and -½ to ½ along the y-axis. This serves to illustrate the robustness of the channel models with respect to distance from the optimal placement of the intermediate terminal.

Figs. 4-7 show the simulated outage and error performance of the decoded relaying multihop diversity channel and the amplified relaying multihop diversity channel respectively. The theoretical characterizations (8), (10), (19), and (20) are represented by dashed lines and indicate good agreement with the simulated results. Figs. 8-9 show the variation of the error performance of the decoded relaying multihop diversity channel and amplified relaying multihop diversity channel with respect to the position of the intermediate terminal. A horizontal plane indicates the error performance of the singlehop reference channel. The graphs represents the theoretical characterization presented in (10) and (20) and indicate that the performance gain with respect to the reference channel is fairly sensitive to the relative position of the intermediate terminal.

VII. CONCLUSION

Not surprisingly, the multihop channels significantly outperform the singlehop reference channel. What is somewhat unexpected, however, is that the amplified relaying channel experiences performance gains that are greater than the decoded relaying channel. An explanation for this result lies in the structure of the decoded relaying receiver implied by (7) through (10). Whereas the amplified relaying channel responds gracefully to severe signal degradation on any individual link between two terminals, the decoded relaying channel produces an error at the destination receiver if any individual link between two terminals produces an error. The relevant question therefore becomes whether the decoded relaying receiver can be modified to increase system performance.

These results attest to the importance of good decisions when selecting intermediate terminals. Although the performance gain is significant when the intermediate terminal is positioned close to the midpoint between the source and destination terminals, the gain becomes negligible and in some cases negative as the distance with respect to that midpoint position increases. That generality aside, it is evident that the amplified relaying multihop diversity channel is less sensitive to intermediate terminal position. Further discussion related to the problem of selecting intermediate terminals is presented in [10]. Discussion of important issues relevant to potential implementations is included in [2] and [4].

The results presented in this paper provide a firm foundation for the characterization of multihop channels with diversity. The mathematical characterizations outlined are very tractable and enable the quick comparison of the proposed channels with the singlehop reference channel. The results suggest that there are significant advantages to be gained from employing multihop channels, and indicate a number of interesting areas for further development. In the future, these results will be applied to generalized architectures for cellular and ad-hoc systems in order to provide a comparison in terms of coverage, relay usage distribution, interference distribution, and system capacity.

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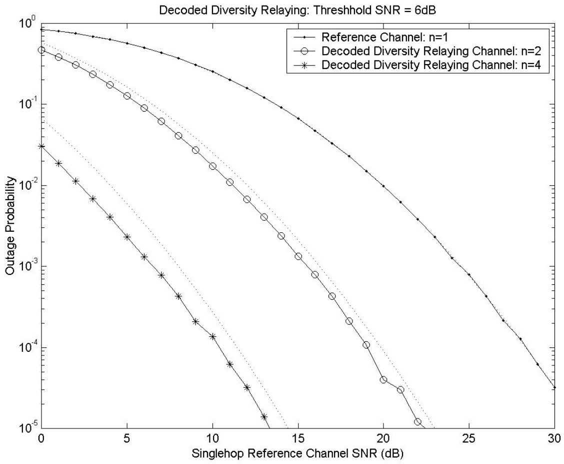

Fig. 4. Probability of Outage for Decoded Relaying Multihop Diversity Channel

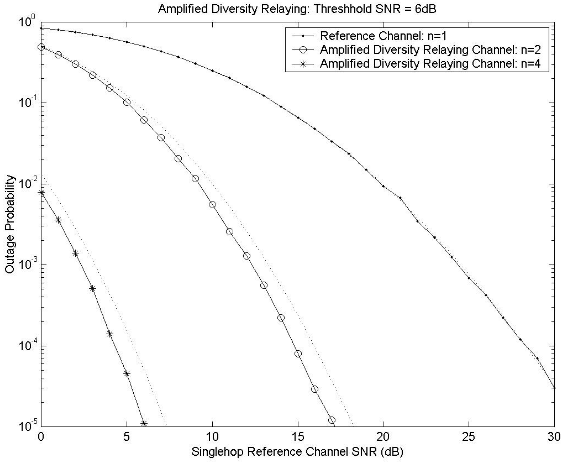

Fig. 5. Probability of Outage for Amplified Relaying Multihop Diversity Channel

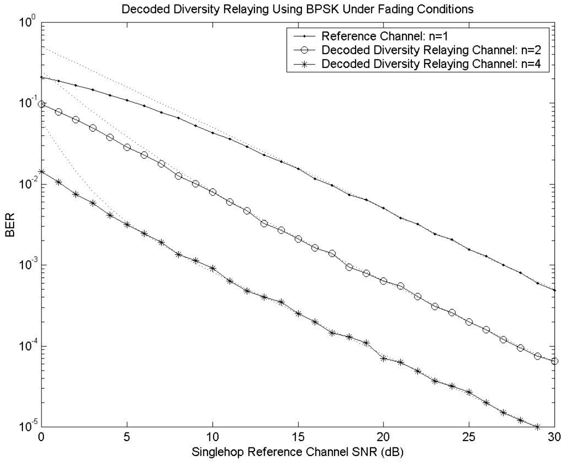

Fig. 6. Probability of Error for Decoded Relaying Multihop Diversity Channel

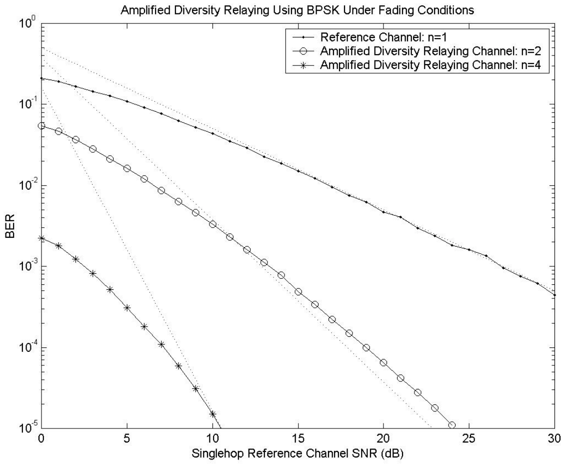

Fig. 7. Probability of Error for Amplified Relaying Multihop Diversity Channel

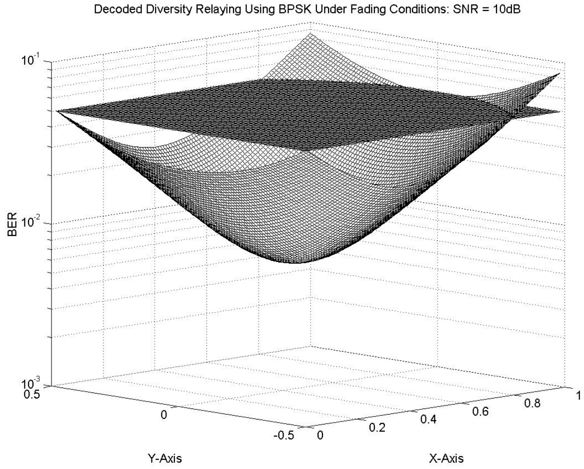

Fig. 8. Probability of Error for Decoded Relaying Multihop Diversity Channel

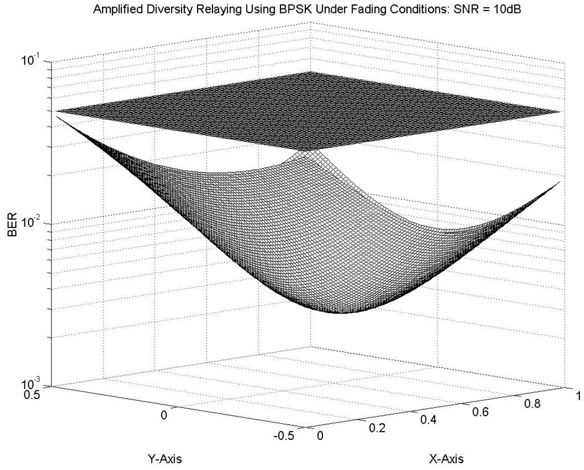

Fig. 9. Probability of Error for Amplified Relaying Multihop Diversity Channel