Space Diversity for Multi-antenna Multi-relay Channels

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***Abstract*— In this paper we analyze the performance of multiple relay channels when multiple antennas are deployed only at relays. We apply two antenna diversity techniques at relays, namely maximum ratio combining (MRC) on receive and transmit beamforming (TB). We show that with K relays the network can be decomposed into K diversity channels each with a different channel gain, and that the signals can be effectively combined at the destination. We assume that the total number of antennas at all relays is fixed at N. If the total transmit power for all relays are the same as for the source and equally distributed among all the relays, the network capacity will be lower bounded by that of N relay channels each with single antenna, and upper bounded by that of single relay channels with N antennas.**

I. Introduction

It is widely believed that ad hoc networking [1] or multi-hop cellular networks [2] are important new concepts for future generation wireless systems [3], where either mobile or fixed nodes (often referred to as relays) are used to help forward the information to the desired user. One advantage of these structures are that it is possible to unite multiple relays in the network as a “virtual antenna array” to forward the information cooperatively, while appropriate combining at the destination realizes diversity gain. The diversity achieved in this way is often named as *user cooperation diversity* or *cooperative diversity* [4], as it mimics the performance advantages of multiple-input multiple-output (MIMO) systems [5] in exploiting the spatial diversity of the relay channels. The performance limits of space-time codes, which can exploit cooperative diversity, are discussed in [6]–[8] for single-antenna relay networks. For multiple-antenna relay channels where every terminal in the network can be deployed with multiple antennas, studies are mainly concentrated on spatial multiplexing systems [9]–[11].

In this paper we exploit the spatial diversity of the relay channels in a different way from space-time codes based approach. We apply two kinds of antenna combining techniques at the relay, namely maximum ratio combining (MRC) [12] for reception and transmit beamforming (TB) [13] for transmission. Those techniques were often used in point-to-point wireless links to enhance signal-to-noise ratio (SNR) at the output of the receiver by deploying multiple antennas at either transmitter or receiver. In a relay context, we move the multiple antennas to the relays, while the source and the destination are only deployed with a single antenna. Our investigation is based on *digital relaying*, where the relays decode, re-encode and re-transmit the signals. We show that the network with K relays can be decomposed into K diversity channels each with different channel gain, and the signals from all K branch can be effectively combined at the destination. We derive the capacity bounds for this signal combining techniques. Our analysis results can be applied to both ergodic capacity and outage capacity [14] performance.

The rest of this paper is organized as follows. In Section II, the basic system model and assumptions are introduced. Section III introduces the signal combining techniques. The capacity performance analysis are made in section IV. Section V presents and discusses simulation results and finally, conclusions are drawn in Section VI.

II. System Model

We consider a two hop network model with one source, one destination and K relays. We ignore the direct link between source and destination. We also assume that total transmit power of the source and relays are the same and that it is equally distributed among the relays. Each relay processes the received signals independently. We assume that the source and destination are deployed with single antennas, while relay k is deployed with m_k antennas. We assume that the total number of antennas at all relays is fixed to N. This can be expressed as

$$\sum_{k=1}^{K} m_k = N. \tag{1}$$

We restrict our discussion to the case where the channels are slow, frequency-flat fading. The data transmission is over two times slots using two hops. In the first transmission time slot, the source broadcasts the signal to all the relay terminals. The input/output relation for the source to the kth relay is given by

$$\mathbf{r}_k = \sqrt{\eta}\mathbf{h}_k s + \mathbf{n}_k, \tag{2}$$

where $\mathbf{r}_k$ is $m_k \times 1$ receive signal vector. η denotes the transmit power at the source. The s is the transmit signal with covariance 1 and $\mathbf{n}_k$ is the $m_k \times 1$ complex circular additive white Gaussian noise vector at relay k with identity covariance matrix $\mathbf{I}_{m_k}$. The vector $\mathbf{h}_k$ is the $m_k \times 1$ channel transfer matrix from source to the kth relay and can be further expressed as

$$\mathbf{h}_k = \sqrt{\alpha_k}\tilde{\mathbf{h}}_k, \quad (3)$$

where each entry of $\tilde{\mathbf{h}}_k$ are identically independent distributed (i.i.d) complex Gaussian random variables with unit variance. Each factor α_k contains the pathloss and can be written as $\alpha_k = x_k^{-\gamma}$, where x_k is the distance between the source and relay k. The scalar γ denotes the path loss exponent. In the second hop, each relay processes its received signals and re-transmits them to the destination. The signal received at the destination can be written as:

$$y = \sum_{i=1}^{K} \mathbf{g}_k \mathbf{d}_k + n_d, \quad (4)$$

where the vector $\mathbf{g}_k$ is the channel matrix from kth relay to the destination, which might also be written as:

$$\mathbf{g}_k = \sqrt{\beta_k}\tilde{\mathbf{g}}_k, \quad (5)$$

where each entry of $\tilde{\mathbf{g}}_k$ is an i.i.d. complex Gaussian random variables with unit variance. The scalar β_k also contains the pathloss from the kth relay to the destination. The scalar n_d is the complex additive white Gaussian noise at the destination with unit variance. The vector $\mathbf{d}_k$ is the transmit signal vector at relay k, which should meet the total transmit power constraint:

$$\mathrm{E}\left[\|\mathbf{d}_k\|_F^2\right] \leq \frac{\eta m_k}{N}, \quad (6)$$

where $\|\bullet\|_F$ denotes the Frobenius norm. We assume a coherent relay channel configuration context where the kth relay can obtain full knowledge of both backward channel vector $\mathbf{h}_k$ and forward channel vector $\mathbf{g}_k$. For fair comparison, we also assume that for each channel realization, either backward or forward channel coefficients for all N antennas remains the same regardless of the number of relays K. It will not be difficult to see that the conclusions on either Ergodic capacity or outage capacity in this paper also hold if we extend the discuss to a more general case where each antenna is fixed in the network. Fig. 1 gives a description for the system model.

III. Antenna Diversity Techniques in Relay Channels

In this section we apply MRC and TB techniques to the system model described in section II. We assume each relay performs MRC of the received signals, by multiplying the received signal vectors by the vector $\mathbf{h}_k^H / \|\mathbf{h}_k\|_F$. The signals at output of the relay receiver is given by

$$\tilde{r}_k = \sqrt{\eta \sum_{i=1}^{m_k} |h_{i,k}|^2}\, s + \frac{\sum_{i=1}^{m_k} h_{i,k}^* n_{i,k}}{\sqrt{\sum_{i=1}^{m_k} |h_{i,k}|^2}} \quad (7)$$

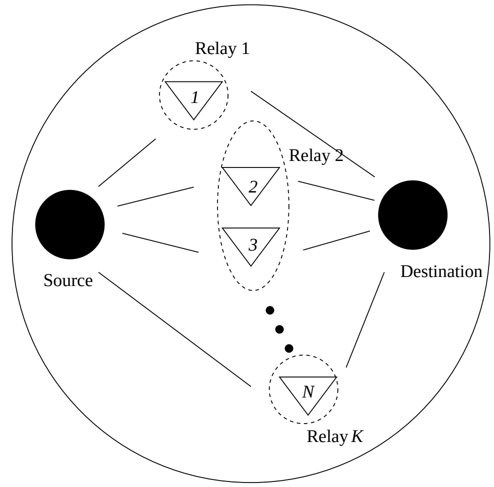


Fig. 1. System model for a two hop network: Source and destination are each deployed with 1 antenna. Totally N antennas are deployed at K relays. For each channel realization, either backward or forward channel coefficients for all N antennas remains the same regardless of the number of relays K.

where $h_{i,k}$ denotes the ith antenna at relay k, and $n_{i,k}$ denotes the noise factor for ith receiver input branch. The SNR at the output of the receiver can be written as:

$$\rho_k^{m_k} = \eta \sum_{i=1}^{m_k} |h_{i,k}|^2. \quad (8)$$

After the relays decode the signals, each relay then performs TB of the transmitted signals. If we denote the transmitted signals as d_k with unit variance, the transmitted signal vector $\mathbf{d}_k$ for relay k can be written as

$$\mathbf{d}_k = \sqrt{\frac{\eta m_k}{N}} \frac{\mathbf{g}_k^H}{\|\mathbf{g}_k\|_F} d_k. \quad (9)$$

The destination receiver simply detects the combined signals from all K relays. If we adjust the transmission data rate so that the signals are correctly decoded at all the relays (i.e. $d_k = s$), the output signal at the destination can be written as:

$$y = s\sum_{k=1}^{K} \sqrt{\frac{\eta m_k}{N} \sum_{i=1}^{m_k} |g_{i,k}|^2} + n_d = s\sum_{k=1}^{K} \tilde{g}_k + n_d \quad (10)$$

It can be seen from (10) that by applying antenna diversity schemes at relays, the networks can be decomposed to K diversity channels each with channel gain $\tilde{g}_k$. The output SNR at the destination receiver can therefore be written as:

$$\rho_d^{m_k} = \left(\sum_{k=1}^{K} \sqrt{\frac{\eta m_k}{N} \sum_{i=1}^{m_k} |g_{i,k}|^2}\right)^2. \quad (11)$$

When all the relays are deployed with a single antenna, there is no traditional maximum ratio combining gain at the relays and the destination. However, the destination still observes a set of equal gain combined [15] amplitude signals from all relays.[1]

[1]Different from [15], the equal gain combining for relay channels is applied at the transmitter instead of the receiver.

Since we assume that the backward and forward channel coefficients for each antennas are kept same for different number of K and m_i. The output SNR at the destination can be rewritten as

$$\rho_d^1 = \frac{\eta}{N}\left(\sum_{k=1}^{K}\sum_{i=1}^{m_i}|g_{i,k}|\right)^2; \tag{12}$$

when all the antenna are deployed in one relay (i.e. $K = 1$ and $m_1 = N$), full diversity gain is achieved among all the N antennas at the relay and also at the destination. The SNR can be rewritten as

$$\rho_d^N = \eta\sum_{k=1}^{K}\sum_{i=1}^{m_i}|g_{i,k}|^2 \tag{13}$$

IV. Capacity Performance

In this section we derive the capacity bounds for the scheme proposed in previous section. The network capacity for digital relaying can be written as

$$C_D^{m_k} = \min\left(C_r^{1,m_1}, C_r^{2,m_2}, \cdots, C_r^{K,m_k}, C_d^{m_k}\right) \tag{14}$$

where $C_r^{k,m_k} = 0.5\log_2\left(1+\rho_k^{m_k}\right)$ denoting the Shannon capacity from source to relay k channel, and $C_d^{m_k} = 0.5\log_2\left(1+\rho_d^{m_k}\right)$ denoting the Shannon capacity from relays to destination channels. The factor 0.5 denotes the half bandwidth compared with non-relay channels.

We firstly analyze channel capacity from the relays to destination link by bounding the $\rho_d^{m_k}$,i.e. the output SNR at the destination.

Lemma 1: For any m_k, $\rho_d^1 \leq \rho_d^{m_k} \leq \rho_d^N$.

Proof: See Appendix. ■

From *Lemma 1*, we can see that

$$C_d^1 \leq C_d^{m_k} \leq C_d^N, \tag{15}$$

where C_d^1 denotes the capacity for relays to destination channels when $K = N$, and C_d^N denotes the capacity for relay to destination channels when $K = 1$. Now also considering the capacity from the source to relays link and extending the analysis to the whole network scenario, we have the following theorem:

Theorem 1: If we denote the network capacity for $K = N$ as C_D^1 and for $K = 1$ as C_D^N, for any m_k, $C_D^1 \leq C_D^{m_k} \leq C_D^N$.

Proof: Considering the SNR $\rho_k^{m_k}$ for the source to the kth relay link, if we denote it as ρ_n^1 for $K = N$ and ρ_1^N for $K = 1$, it can be shown that

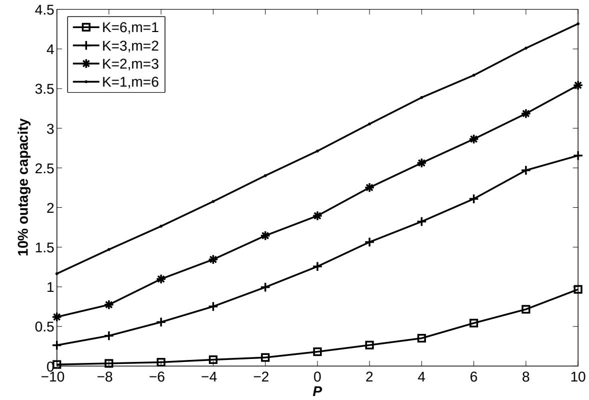
$$\min\left(\rho_n^1\right) \leq \min\left(\rho_k^{m_k}\right) \leq \rho_1^N. \tag{16}$$

Therefore, we have the following:

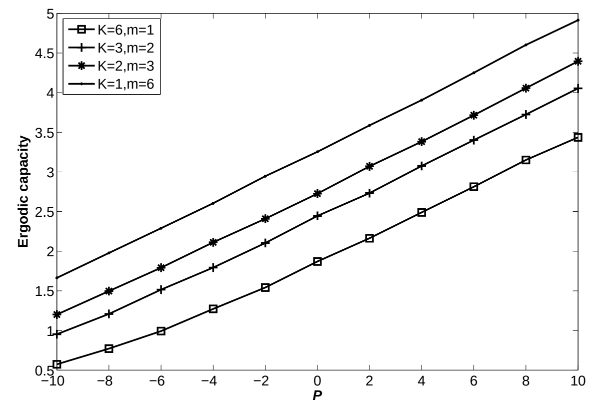
$$\min\left(C_r^{1,1}, \cdots, C_r^{N,1}\right) \leq \min\left(C_r^{1,m_1}, \cdots, C_r^{K,m_k}\right) \leq C_r^{1,N}. \tag{17}$$

Combining (17) and (15), we thus complete the proof. ■

From the above analysis we have shown that for the signal combining techniques discussed in the paper, the network capacity will be lower bounded by that of N relay channels each with single antenna, and upper bounded by that of a single relay channel with N antennas. This means that even there are more relays, the increased "equal gain combining" gain at the destination can not compensate for the loss of maximum ratio combining gain at the relay and the destination when numbers of antennas at each relay are reduced.



(a) 10% outage capacity



(b) Ergodic capacity

Fig. 2. Capacity of single MIMO relay channels for different number of relays K, while each relay is deployed with m antennas. (a) 10% outage capacity. (b) Ergodic capacity.

V. Simulation Results

We calculate the both ergodic capacity and 10% outage capacity (in bits per channel use) for 1000 channel realizations regarding different values of η, denoted as *P* in the figures. In this simulation example we assume that the distance between source and destination is normalized. The relays are uniformly and randomly located in the middle region between the source and relays. Therefore x_k is set to 0.5. We assume the total number of antennas at relays (N) is 6 and we also assume that all K relays have the same number of antennas m. Fig. 2 shows the capacity performance. We can see that for different (K, m), the capacity is always upper bounded by $(1, 6)$ and lower bounded by $(6, 1)$. These results verify the analysis made in this paper. Furthermore, we can see through the simulation that larger m and small K might give larger benefit, since larger m allows more freedom of cooperation among the antennas at each relay. Therefore when m reaches N (K reduces to 1), full cooperation are made among all the antennas to give rise to the best performance.

VI. Conclusions

In this paper we analyze the performance of multiple relay channels when multiple antennas are deployed only at relays. We apply antenna diversity techniques at relays which are known as maximum ratio combining and transmit beamforming. We show that with K relays the network can be decomposed into K diversity channels each with a different channel gain, while the signals can be effectively combined at the destination. If we assume that the total number of antennas at all relays are fixed to N and total transmit power at all relays are normalized, the network capacity will be lower bounded by that of N relay channels each with single antenna, and upper bounded by that of single relay channels with N antennas.

Appendix
Proof of *Lemma 1*

We firstly prove that $\rho_d^1 \le \rho_d^{m_k}$. We write the following

$$\sqrt{\rho_d^{m_k}} - \sqrt{\rho_d^1} = \sum_{k=1}^{K} \left(\underbrace{\sqrt{\frac{\eta m_k}{N} \sum_{i=1}^{m_k} |g_{i,k}|^2}}_{A_k} - \underbrace{\sqrt{\frac{\eta}{N}} \sum_{i=1}^{m_k} |g_{i,k}|}_{B_k} \right). \tag{18}$$

To compare A_k with B_k, we write

$$A_k^2 - B_k^2 = \frac{\eta}{N} \left(m_k \sum_{i=1}^{m_k} |g_{i,k}|^2 - \left(\sum_{i=1}^{m_k} |g_{i,k}| \right)^2 \right) \tag{19}$$

$$= \frac{\eta}{N} \left((m_k - 1) \sum_{i=1}^{m_k} |g_{i,k}|^2 \right) - \frac{\eta}{N} \left(\sum_{i,j=1;i\neq j}^{m_k} |g_{i,k}| \, |g_{j,k}| \right). \tag{20}$$

Note that

$$(m_k - 1) \sum_{i=1}^{m_k} |g_{i,k}|^2 = \sum_{i=1}^{m_k} \sum_{j=1,j\neq i}^{m_k} |g_{j,k}|^2 \tag{21}$$

$$= 0.5 \sum_{i,j=1;i\neq j}^{m_k} \left(|g_{i,k}|^2 + |g_{j,k}|^2 \right). \tag{22}$$

So (20) can be further written as:

$$A_k^2 - B_k^2 = \frac{\eta}{2N} \sum_{i,j=1;i\neq j}^{m_k} \left(|g_{i,k}|^2 - 2 |g_{i,k}| \, |g_{j,k}| + |g_{j,k}|^2 \right)$$
$$= \frac{\eta}{2N} \sum_{i,j=1;i\neq j}^{m_k} \left(|g_{i,k}| - |g_{j,k}| \right)^2 \ge 0.$$

So $A_k \ge B_k$ and therefore $\rho_d^1 \le \rho_d^{m_k}$.

Next we prove that $\rho_d^N \ge \rho_d^{m_i}$. For simplicity, we denote

$$a_k = \sum_{i=1}^{m_k} |g_{i,k}|^2 \tag{23}$$

in equation (11) and (13). Then $\rho_d^N - \rho_d^{m_i}$ can be written as

$$\rho_d^N - \rho_d^{m_i} = \frac{\eta}{N} \left(\sum_{k=1}^{K} (N - m_k) \, a_k - \sum_{i,j=1;i\neq j}^{K} \sqrt{m_i m_j a_i a_j} \right). \tag{24}$$

Note the constraint by (1) in section II, we have the following:

$$(N - m_k) = \sum_{i=1,i\neq k}^{K} m_i. \tag{25}$$

Putting (25) into (24), we have the following:

$$\rho_d^N - \rho_d^{m_i} = \frac{\eta}{N} \left(\sum_{k=1}^{K} \sum_{i=1,i\neq k}^{K} m_i a_k - \sum_{i,j=1;i\neq j}^{K} \sqrt{m_i a_i m_j a_j} \right). \tag{26}$$

Note the following:

$$\sum_{k=1}^{K} \sum_{i=1,i\neq k}^{K} m_i a_k = \sum_{i,j=1;i\neq j}^{K} (m_i a_j)$$
$$= 0.5 \left(\sum_{i,j=1;i\neq j}^{K} (m_i a_j) + \sum_{i,j=1;i\neq j}^{K} (m_j a_i) \right).$$

We can further write (26) as follows:

$$\rho_d^N - \rho_d^{m_i} = \frac{\eta}{2N} \sum_{i,j=1;i\neq j}^{K} (m_i a_j) - \frac{\eta}{N} \sum_{i,j=1;i\neq j}^{K} \sqrt{m_i a_j m_j a_i} + \frac{\eta}{2N} \sum_{i,j=1;i\neq j}^{K} (m_j a_i)$$
$$= \frac{\eta}{2N} \sum_{i,j=1;i\neq j}^{K} \left(\sqrt{m_i a_j} - \sqrt{m_j a_i} \right)^2.$$

Therefore $\rho_d^N \ge \rho_d^{m_i}$ and $\rho_d^1 \le \rho_d^{m_k} \le \rho_d^N$.

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