A Theoretical Characterization of the Multihop Wireless Communications Channel

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Abstract – **This paper provides a theoretical characterization of the multihop wireless communications channel. Four channel models are proposed and developed: the decoded relaying multihop channel, the amplified relaying multihop channel, the decoded relaying multihop diversity channel, and the amplified relaying multihop diversity channel. Two classifications are discussed: decoded relaying versus amplified relaying and multihop versus multihop diversity. When comparing decoded relaying with amplified relaying the primary considerations are noise propagation, error introduction, and delay. When comparing multihop with multihop diversity the primary considerations are performance and complexity. These models are compared, through analysis and simulations, with the singlehop reference channel on the basis of power distribution and probability of error. All models achieve significant gains over the singlehop reference channel, with the amplified relaying models outperforming the decoded relaying models despite noise propagation.**

I. Introduction

This paper is concerned with a proposed wireless system wherein traditional transmission constraints are removed in order to allow direct communication between mobile terminals. This system gives mobile terminals the ability to relay information when they are neither the initial transmitter nor the final receiver. Relaying systems realize a number of benefits over traditional systems in the areas of deployment, connectivity, adaptability and capacity [4,7].

This paper proposes four channel models. The *decoded relaying multihop channel* corresponds to the case where each intermediate terminal digitally decodes and re-encodes the received signal from the immediately preceding terminal before retransmission. The *amplified relaying multihop channel* corresponds to the case where each intermediate terminal simply amplifies the received signal from the immediately preceding terminal before retransmission. The *decoded relaying multihop diversity channel* corresponds to the case where each intermediate terminal combines, digitally decodes and re-encodes the received signals from all preceding terminals before retransmission. The *amplified relaying multihop diversity channel* corresponds to the case where each intermediate terminal simply combines and amplifies the received signals from all preceding terminals before retransmission. The channels with diversity can be viewed as generalizations of the channels without diversity.

Receiver diversity implies the reception of separate, independently faded and shadowed versions of a signal, and is realized here without the requirement for multiple antennas at each terminal. Instead, diversity is generated naturally as each component version is transmitted along a different physical path and experiences different delay characteristics. Diversity combining can then be implemented using traditional techniques such as rake receivers and equalizers depending on the chosen multiple access scheme. Multihop channels without diversity are characterized in more detail in [2]. Multihop channels with diversity are characterized in more detail in [3].

II. System Model

The system model for multihop wireless communications channels is composed of a source terminal, a receiving terminal, and an indeterminate number of intermediate relaying terminals. In Fig. 1 the source terminal is identified as T_1, the destination terminal is identified as T_n and the intermediate terminals are identified as T_2 through T_{n-1} where n is the number of hops along the transmission path.

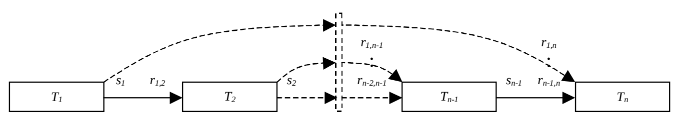

Fig. 1. Generic Multihop Diversity Wireless Communications Channel

Let T_S represent the set of source terminals, T_I represent the set of intermediate terminals, and T_D represent the set of destination terminals. Therefore $T_T = T_S \bigcup T_I$ represents the set of all transmitting terminals and $T_R = T_I \bigcup T_D$ represents the set of all receiving terminals. Let $T_{P(i)}$ represent the set of terminals that transmit a signal received by T_i. The notation used in this paper assumes that $T_{P(i)}$ has cardinality equal to one for channels without diversity and greater than one for channels with diversity. For the communications channel illustrated in Fig. 3, $T_S = \{T_1\}$, $T_I = \{T_2,...T_{n-1}\}$, $T_D = \{T_n\}$, $T_T = \{T_1,...T_{n-1}\}$, $T_R = \{T_2,...T_n\}$ and $T_{P(i)} = \{T_{i-1}\}$ (without diversity) or $T_{P(i)} = \{T_1,...T_{i-1}\}$ (with diversity).

Each terminal T_i transmits a signal given by

$$s_i = \sqrt{\varepsilon_i} a_t + \beta_i , \qquad (1)$$

where ε_i is the transmitted power, a_t is the binary information symbol at time interval t, and β_i is propagated noise. The propagated noise term in (1) is zero for source terminals as well as for intermediate terminals that employ decoded relaying. Each terminal T_i then receives a set of signals given by

$$r_{k,i} = \alpha \sqrt{(L_{k,i} / d_{k,i}^{p})} R_{k,i} (\sqrt{\varepsilon_k} a_t + \beta_k) + z_{k,i}, k \in P(i) , \qquad (2)$$

where α^2 is the free space signal power attenuation factor between the transmitting terminal and an arbitrary reference distance, $d_{k,i}$ is the inter-terminal distance relative to the reference distance, p is the propagation exponent, $L_{k,i}$ is a zero-mean lognormal random variable with variance $\sigma^2_{L_{k,i}}$, $R_{k,i}$ is a complex gaussian (Rayleigh) random variable with mean power $E[R^2_{P(i),i}]=1$, and $z_{k,i}$ is a zero-mean additive white gaussian noise random variable with variance N_0. Assuming maximal ratio combining, the received signal to noise ratio at T_i is given by

$$\gamma_{P(i),i} = \sum_{k\in P(i)} \left(\frac{\alpha^2 \varepsilon_k}{(d^p_{k,i} \big/ L_{k,i}\left|R_{k,i}\right|^2)N_0 + \left|\beta_k\right|^2}\right). \tag{3}$$

where $\left|R_{k,i}\right|^2$ is an exponential random variable with mean $2\sigma^2_{R_{k,i}} = 1$. The calculation of probability of error is dependent on the modulation scheme employed. For the special case of BPSK, the probability of error under fading conditions when $\beta_k = 0, k \in P(i)$ is given in [6] by

$$P_e(\gamma_{P(i),i}) \approx \binom{2K-1}{K} \prod_{k\in P(i)} \left(\frac{1}{2\overline{\gamma_{k,i}}}\right), \overline{\gamma_{k,i}} >> 1, \tag{4}$$

where K is the cardinality of $T_{P(i)}$ and $\overline{\gamma_{k,i}}$ is the expected received signal to noise ratio at T_i for branch k of the diversity combiner.

III. Decoded Relaying

The decoded relaying multihop diversity channel corresponds to the case where each intermediate terminal combines (diversity), digitally decodes and re-encodes the received signal before retransmission. This digital relaying channel does not propagate noise along the multihop channel. The possibility of decoding error is introduced at each intermediate terminal. The channel model is given by (1) through (3) with $\beta_k = 0, k \in P(i)$. The received signal to noise ratio at T_i is given by

$$\gamma_{P(i),i} = \sum_{k\in P(i)} \left(\frac{\alpha^2 \varepsilon_k}{(d^p_{k,i} \big/ L_{k,i}\left|R_{k,i}\right|^2)N_0}\right). \tag{5}$$

The total probability of decoding error for the decoded relaying multihop diversity channel is upper-bounded by

$$P_e \le \sum_{i\in T_R} P_e(\gamma_{P(i),i}). \tag{6}$$

where $P_e(\gamma_{P(i),i})$ is the probability of decoding error at terminal T_i given a received signal to noise ratio of $\gamma_{P(i),i}$.

IV. Amplified Relaying

The amplified relaying multihop diversity channel corresponds to the case where each intermediate terminal simply combines (diversity) and amplifies the received signal before retransmission. This analog relaying channel propagates noise along the multihop channel. The possibility of decoding error is introduced only at the destination terminal. The channel model is composed of a set of individual transmission channels given by (1) through (3). Assuming that each intermediate terminal can track both lognormal shadowing and Rayleigh fading, the amplification factor at each intermediate terminal T_i is given by

$$A_i = \frac{\varepsilon_i}{\sum_{k\in P(i)} (\alpha^2(\varepsilon_k + \left|\beta_k\right|^2) L_{k,i}\left|R_{k,i}\right|^2 \big/ d^p_{k,i} + N_0)}, \tag{7}$$

and the received signal to noise ratio at the destination terminal can be expressed recursively as

$$\gamma_{P(D),D} \approx \sum_{\substack{k\in P(D)\\ k\neq S}} (\gamma^{-1}_{P(k),k} + \psi^{-1}_{k,D})^{-1} + \psi_{S,D}, \tag{8}$$

where $\psi_{k,D}$ is the received signal to noise ratio $\gamma_{k,D}$ at terminal T_D for branch k of the diversity combiner with $\beta_k = 0$.

The total probability of decoding error for the amplified relaying multihop diversity channel is given by

$$P_e = P_e(\gamma_{P(D),D}) \approx \binom{2K-1}{K} \prod_{k\in P(D)} \left(\frac{1}{2\overline{\gamma_{k,D}}}\right), \overline{\gamma_{k,D}} >> 1, \tag{9}$$

where $P_e(\gamma_{P(D),D})$ is the probability of decoding error at the destination terminal given a received signal to noise ratio of $\gamma_{P(D),D}$, K is the cardinality of $T_{P(D)}$, and $\overline{\gamma_{k,D}}$ is the expected received signal to noise ratio at the destination terminal for branch k of the diversity combiner.

V. Simulation Results

In order to visualize the discussion, the results presented thus far are applied in two simulations and compared against the reference channel on the basis of probability of error. A BPSK modulation scheme is used for simplicity of exposition. The example multihop channel is composed of $n+1$ terminals: source T_1, intermediate T_2 through T_n and destination T_{n+1}. The coordinates of the channel are normalized with respect to the distance between the source and destination terminals such that $d_{1,n+1} = 1$. The propagation exponent is $p = 4$. The lognormal shadowing components are independent with zero-mean and variance $\sigma^2_{L_{P(i),i}} = 12$ dB. The Rayleigh fading components are independent with mean power $E[R^2_{P(i),i}] = 1$. The threshold signal to noise ratio for outage calculations is $\gamma = 6$ dB. For the purpose of simplifying the comparison, and without loss of generality, the free space signal power attenuation factor is $\alpha^2 = 1$. Optimal power distribution is assumed for each of the channel models with the total power constrained to the reference power ε_0.

For the first simulation the intermediate terminals are fixed so that they divide the direct path between the source and destination terminals into *n* equal length segments. This serves to validate the theory presented thus far as well as illustrate the power gain that can be realized under an optimal placement of the intermediate terminals with respect to the source and destination terminals. For the second simulation the single intermediate terminal is placed at a set of locations uniformly distributed across a unit square. The source and destination terminals are located at (0,0) and (1,0) respectively. The intermediate terminal ranges from 0 to 1 along the x-axis and -½ to ½ along the y-axis. This serves to illustrate the robustness of the channel models with respect to distance from the optimal placement of the intermediate terminal.

Figs. 2-5 show the simulated error performance under optimal intermediate terminal placement. The theoretical characterizations (6) and (9) are represented by dashed lines and indicate good agreement with the simulated results. Figs. 6-9 show the variation of the error performance with respect to the position of the intermediate terminal. A horizontal plane indicates the error performance of the singlehop reference channel. The graphs indicate that the performance gain with respect to the reference channel is fairly sensitive to the relative position of the intermediate terminal. Further discussion of these results is presented in [1], [2] and [3]. Discussion related to the problem of selecting intermediate terminals is presented in [8].

VI. Discussion

The results presented in this paper provide a firm foundation for the characterization of multihop channels with diversity and attest to the importance of good decisions when selecting intermediate terminals. Although there are significant advantages to be gained from employing multihop channels, this paper also highlights a set of important related issues for consideration. These include the delay characteristics of the channel, spatial diversity combining, relaying in the same channel, power control, and node complexity.

Decoded relaying incurs significantly more delay than amplified relaying. Although both suffer from additional propagation delay in comparison to the reference channel due to the indirect nature of the transmission path, the decoded relaying channel also incorporates an additional processing delay at each terminal. Unlike the propagation delay, this additional processing delay has a non-trivial impact on the delay characteristics of the channel, and may in fact make the decoded relaying channel unsuitable for delay sensitive transmissions.

This will also affect the application of spatial diversity combining techniques to the decoded relaying channel. Although the destination terminal will receive the transmitted signal from multiple intermediate terminals concurrently, the signal received along each path will be separated in time by at least the duration of one frame. This will render infeasible the use of multipath technologies like conventional rake receivers. In order to apply spatial diversity combining techniques, complex receiver structures will have to be developed that have the ability to buffer the received signals and calculate cross-correlation metrics on the different signals across a number of frames.

Current literature [4] places the restriction that relaying must be performed in a separate channel for concern that feedback from the transmitter may obscure the received signal. However, this restriction may be relaxed as methods for removing this feedback are introduced, including digital subtraction of the transmitted signal and intelligent placement of a separate receive antenna. It is therefore interesting to compare the channels with and without this restriction. With this restriction, both relaying channels are forced to use more system resources per communications link. Without this restriction, the amplified relaying channel requires no additional resources and the presented results can be compared directly to the reference channel. The decoded relaying channel will encounter the problems outlined in the discussion on spatial diversity combining.

Power control in both the decoded relaying and amplified relaying channels is very different from power control in the reference channel. Of specific interest is the problem of how to propagate power control information to individual terminals along a transmission route. Since the terminals communicate independently and are not always directly connected to a base station, any power control algorithm used must by distributed. A relevant note is that this problem corresponds very closely to the problem of propagating routing information to individual terminals along a transmission route. It should therefore be possible to leverage the routing algorithms proposed for the network layer of multihop wireless communications systems.

Given that both relaying channels are significantly more complex than the reference channel, it is not surprising that there is a corresponding increase in terminal complexity. For both relaying channels, this increased complexity includes more complex power control and routing algorithms, the capability of handling multiple communications signals from different sources concurrently, and more complex antenna structures if the same channel is used for relaying. In addition, the decoded relaying channel also includes increased complexity at the receiver in order to apply spatial diversity combining techniques.

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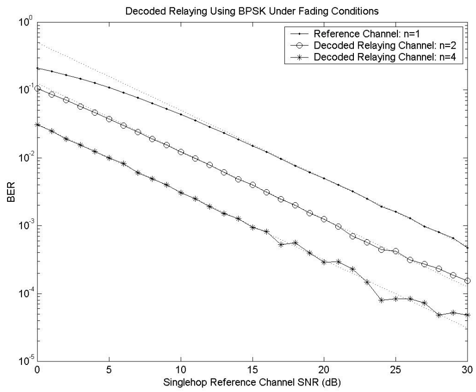

Fig. 2. Error for Decoded Relaying Multihop Channel

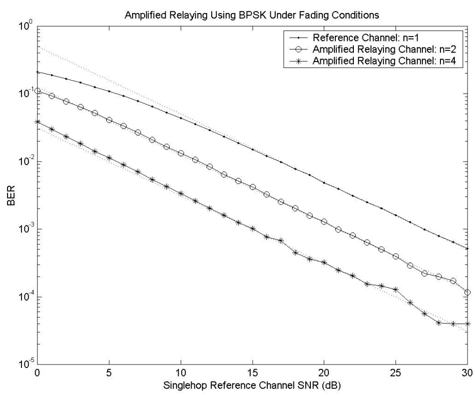

Fig. 3. Error for Amplified Relaying Multihop Channel

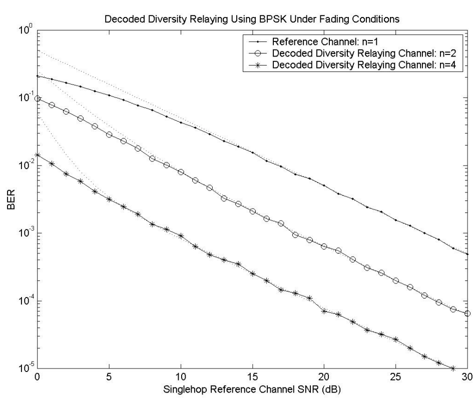

Fig. 4. Error for Decoded Relaying Multihop Diversity Channel

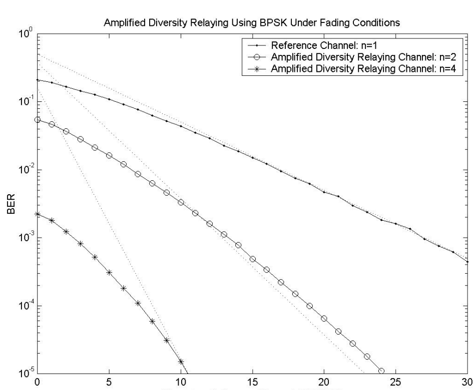

Fig. 5. Error for Amplified Relaying Multihop Diversity Channel

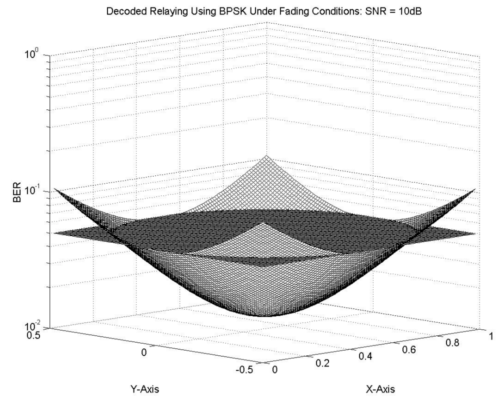

Fig. 6. Error Robustness of Decoded Relaying Multihop Channel

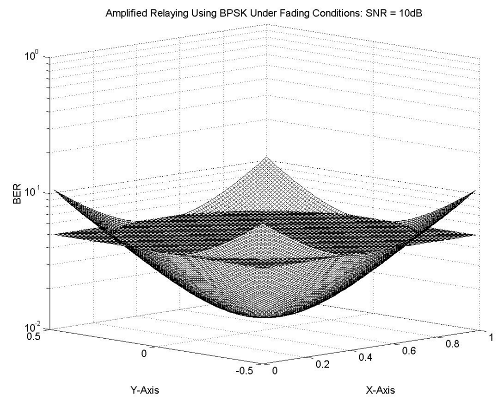

Fig. 7. Error Robustness of Amplified Relaying Multihop Channel

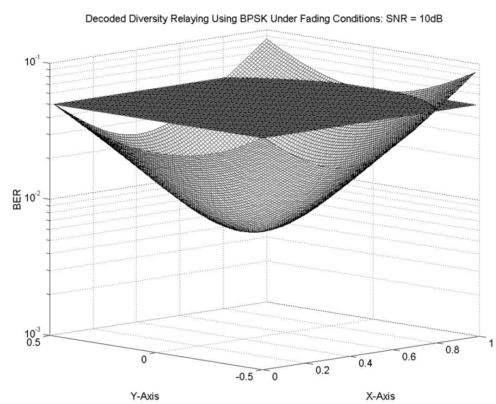

Fig. 8. Error Robustness of Decoded Relaying Multihop Diversity Channel

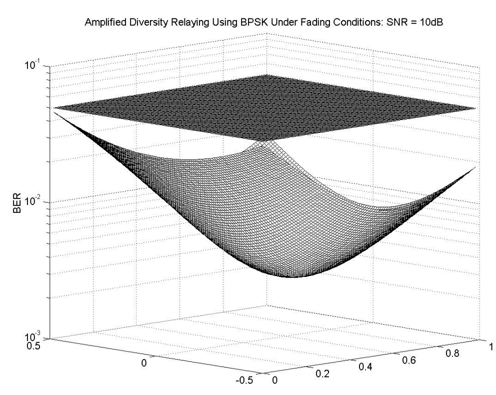

Fig. 9. Error Robustness of Amplified Relaying Multihop Diversity Channel