

# Threshold Based Distributed Detection That Achieves Full Diversity in Wireless Sensor Networks

(Invited Paper)

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**Abstract**—It is well known that decode and forward relaying protocols, in which the relays decode and re-encode the message, can achieve full diversity when combined with distributed space time coding or relay selection schemes. However, in wireless sensor networks, decoding and re-encoding the message at the relays can be costly due to severe energy limitations. A question arises: is it possible to achieve full diversity when the relays only detect (or demodulate) and forward the message? In this paper a distributed detection scheme is developed that achieves full diversity in wireless sensor network, with the relays only detecting and forwarding the message. It is seen that such a relaying scheme is SNR threshold-based, i.e., the relays will perform detection only if the receive SNR is above a certain threshold. The threshold that achieves full diversity is derived. Furthermore, a single bit feedback scheme is developed to make relaying more spectral efficient, such that the network can achieve the diversity-multiplexing tradeoff of a point-to-point multiple-input single-output (MISO) system in the high SNR regime.

## I. INTRODUCTION

### A. Background

Exploring cooperative diversity in wireless ad hoc or sensor networks has been an extremely active research area over the last five years [2], [3], [7]. Most of the cooperative diversity schemes are based on two relaying protocols, namely decode-and-forward [2] and compress-and-forward [7]. In the decode-and-forward protocol, the relay decodes, re-encodes and forwards the message to the destination. The destination performs joint decoding upon receiving the signal pieces from both the relay and the source. In the compress-and-forward protocol, the relay quantizes the message into different levels and forwards it to the destination. The destination decodes the message by observing the correlation between the signal pieces arriving from both the source and the relay. The compress-and-forward protocol does not require the relay to decode the information, and it outperforms the decode-and-forward protocol in certain scenarios. However, the compress-and-forward protocol is much more complex than the decode-and-forward protocol, since it involves the problem of joint source-channel encoding at the relay, and usually requires the relay to obtain the relay-destination channel state information (CSI) [7].

It is typically the case that, in a sensor network, the battery of each node is an extremely limited resource, while most of the coding schemes can cost significant energy and thus reduce the life time of each sensor node. Simpler relaying protocols that can avoid any form of coding while offering considerable performance gains are highly desired. One such candidate is amplify-and-forward [2], in which the relay simply amplifies and forwards the signals it received from the source. Such a protocol can also be considered as the simplest form of the compress-and-forward protocol, since no form of quantization is assumed at the relay. However, in practice it is difficult to achieve the desired performance offered by such a protocol, primarily due to the randomness of the incoming signal (plus noise) and limited dynamic range of the amplifier at the relay. As a result, signal quantization is still required in order for the amplifier to work well in practice.

### B. Contribution of the Paper

In this paper, we propose a detect-and-forward protocol in which the relay simply performs symbol by symbol detection, and forwards the detected symbol sequence to the destination. Such a scheme is slightly more complex than amplify-and-forward but is easier to implement. It can also be considered as a special form of compress-and-forward, since detection can be considered as the simplest form of quantization. However, it avoids many issues resulting from the standard compress-and-forward protocol, such as joint source-channel coding and the requirement of additional channel information at the relay. We show that such a protocol can achieve the full diversity gain in a multi-node ad hoc or sensor network, provided that the nodes in the network perform detection only if their receive SNRs are above certain thresholds. The thresholds that achieve full diversity are derived.

Specifically, we propose two transmission protocols. In the first protocol, called the MRC-type protocol, each node that acts as a relay detects and transmits the message in a different time slot. The destination performs maximal ratio combining (MRC) to detect the message upon receiving all the signal pieces. We show that the full diversity gain can be obtained

by such a protocol given a proper threshold  $\gamma_{tr}$  at each relay. To further improve the spectral efficiency of transmission, we propose the second protocol, called the ARQ-based protocol. In such a protocol, another threshold  $\gamma_{td}$  is set at the destination. We allow a single bit feedback message (ACK) from the destination to inform the source and the relays whether the receive SNR is above  $\gamma_{td}$  or not. The source will continue transmitting the next message if a positive feedback is received. Otherwise the relays will detect and forward the message to the destination in the following time slots. It is shown that, given proper thresholds, the network offers the same diversity multiplexing tradeoff [1] performance as a multiple-input single-output (MISO) channel.

### C. Related Work

Previous work considering the detect-and-forward protocol can be found in [8]–[11]. In [8], the relay adjusts its power continuously in order to offer an effective diversity performance. Similar to amplify-and-forward relaying, this idea will meet difficulties in practical applications. [9]–[11] consider effective combining schemes at the destination for the detect-and-forward protocol. However, all these works require the destination to obtain certain knowledge of the CSI of the source to relay link. This requirement involves much signalling overhead and is difficult to fulfill, especially in sensor networks. We note that compared with [9]–[11], the detection schemes proposed in this paper are *distributed* (among the relays and the destination) rather than centralized (only at the destination).

The rest of the paper is organized as follows. In Section II some preliminaries are introduced. The system model is introduced in Section III. The MRC-type protocol is introduced and analyzed in Section IV and the ARQ-based protocol is introduced and analyzed in Section V. Numerical results are given in Section VI and concluding remarks are given in Section VII.

## II. PRELIMINARIES

### A. Definitions

We first introduce some definitions that will be frequently used in the paper.

*Definition 1 (DMT):* Consider a multiple-input multiple-output (MIMO) system operating at average SNR  $\gamma$  per receive antenna and having rates  $R$  bits per channel use. The multiplexing gain and diversity order are defined as

$$r \triangleq \lim_{\gamma \rightarrow \infty} \frac{R}{\log_2 \gamma} \quad \text{and} \quad d \triangleq - \lim_{\gamma \rightarrow \infty} \frac{\log_2 P_e(R)}{\log_2 \gamma}, \quad (1)$$

where  $P_e(R)$  is the average error probability incurred at transmission rate  $R$ .

*Definition 2:*  $f(x)$  is said to be exponentially equal to  $g(x)$ , denoted by  $f(x) \doteq g(x)$ , if

$$\lim_{x \rightarrow \infty} \frac{\log(f(x))}{\log(g(x))} = 1.$$

The symbol  $\lesssim$  and  $\gtrsim$  are defined accordingly.

Following the above definition, we introduce the following lemma.

*Lemma 1:* Suppose  $X$  is a random variable having the chi-square distribution with  $2L$  degrees of freedom. Then

$$\Pr \left( X < \frac{1}{x} \right) \doteq x^{-L}$$

when being considered as a function of  $x$ .

*Proof:* The proof is straightforward and is omitted. ■

### B. Error Probability for QAM Signals

In this paper we assume QAM constellations for all signals. Note that the analysis can also be applied to other modulation schemes. However, we use QAM because it is diversity-multiplexing tradeoff optimal over Rayleigh fading channels [6]. In the following we introduce the performance of QAM signals. Details regarding these results can be found in Chapter 5 of [6].

Assume we transmit at SNR  $\gamma$ . The error probability for a specific channel realization  $h$  can be approximately written as

$$P_{e|h} \approx Q \left( \sqrt{\alpha \gamma |h|^2} \right)$$

where  $\alpha$  is a factor that is related to the constellation size, which is mainly decided by the rate  $R$ . With a standard QAM constellation, we can approximate  $\alpha$  as

$$\alpha \approx \frac{1}{2^{R+1}} \quad (2)$$

for large rate  $R$ . Assuming  $h$  is a complex Gaussian random variable with zero mean and unit variance, the symbol error probability averaged over all channel realizations can be approximately written as

$$P_e \approx \frac{2^R}{\gamma}$$

for large  $\gamma$ . Now suppose the receiver receives  $L$  branch of signals each undergoing independent and identically distributed (i.i.d.) Rayleigh fading. The error probability after the maximal ratio combining (MRC) can be written approximately as

$$P_e \approx \left( \frac{2^R}{\gamma} \right)^L;$$

that is, the system has a diversity order of  $L$  for any fixed rate  $R$ .

## III. SYSTEM MODEL

We assume a network comprised of one source,  $N$  relays, and one destination. The channel coefficient between the source and the  $i$ th relay, the  $i$ th relay and the destination, and the source and destination are denoted by  $h_{sr_i}$ ,  $h_{r_i d}$  and  $h_{sd}$ , respectively. The channels between each pair of nodes are assumed to be i.i.d. complex Gaussian random variables with zero means and unit variances. An SNR threshold of value  $\gamma_{tr}$  is set at each relay. When the relay receives a message, it measures the value of its received SNR. If the receive SNR is higher than  $\gamma_{tr}$ , the relay will detect the message and forward

it to the destination. Otherwise the relay will remain silent. We call such an operation at the relay *threshold based detection*. In this paper we will propose two different protocols that incorporate threshold based detection at the relay and examine their performance.

#### IV. THE MRC-TYPE PROTOCOL

In the first scheme, the source and the  $N$  relays transmit each message in different time slots. If a relay decides not to transmit the message, it will remain silent and no transmission occurs in that time slot. Therefore, it takes  $N + 1$  time slots to finish transmitting each message. When the destination receives all the signal pieces after the  $N + 1$  time slots, it performs MRC and detects the message. We call this protocol the MRC-type protocol. In the following we propose a threshold  $\gamma_{tr}^*$  that achieves the full diversity order  $(N + 1)$  of the network. The results are summarized in the following theorem.

*Theorem 1:* The following threshold  $\gamma_{tr}^*$  is sufficient for achieving the full diversity  $(N + 1)$  of the network when the MRC-type protocol is performed:

$$\gamma_{tr}^* = \frac{2}{\alpha} M \log c \gamma$$

for any  $M \geq N$ , where  $c$  is a positive constant and  $\alpha$  can be approximated by (2) for large  $R$ .

*Proof:* To facilitate the analysis of the diversity-multiplexing tradeoff which is to appear later, in the following proof we directly replace  $\alpha$  with its approximation  $\frac{1}{2^{R+1}}$ . Also for notational simplicity, we assume  $c = 1$ .

The total system error probability can be written as

$$P_e = P_d + P_r \quad (3)$$

where  $P_d$  is the error probability when no relay forwards the message and  $P_r$  is the error probability when there is *at least* one relay forwarding the message.  $P_d$  can be written as

$$P_d \approx \left( \Pr \left( |h_{sr_i}|^2 \gamma < \gamma_{tr} \right) \right)^N \frac{2^R}{\gamma} \quad (4)$$

$$\doteq \left( \frac{2^{R+2} M \log \gamma}{\gamma} \right)^N \frac{2^R}{\gamma}, \quad (5)$$

where *Lemma 1* is used in going from (4) to (5). When  $\gamma \rightarrow +\infty$ , the log term becomes insignificant and  $P_d$  has a diversity order of  $N + 1$ .

Now we investigate  $P_r$ , which can be further written as

$$P_r = \sum_{n=1}^N P_r(n) P_e(n) \quad (6)$$

where  $P_r(n)$  is the probability with which a group of  $n$  relays decide to detect and forward the message, and  $P_e(n)$  is the error probability conditioned on this event.  $P_r(n)$  can

be further written as

$$P_r(n) = \binom{N}{n} \Pr \left( \gamma |h_{sr_i}|^2 > \gamma_{tr} \right)^n \Pr \left( \gamma |h_{sr_i}|^2 < \gamma_{tr} \right)^{N-n} \quad (7)$$

$$\leq \binom{N}{n} \Pr \left( \gamma |h_{sr_i}|^2 < \gamma_{tr} \right)^{N-n} \quad (8)$$

$$\doteq \left( \frac{2^{R+2} M \log \gamma}{\gamma} \right)^{N-n} \quad (9)$$

Thus  $P_r(n)$  has diversity order of  $N - n$ . Furthermore,  $P_e(n)$  can be bounded as

$$P_e(n) \leq \sum_{\tilde{n}=1}^n P_r(\tilde{n}) + P_{coop}(n), \quad (10)$$

where  $P_r(\tilde{n})$  is the probability with which  $\tilde{n}$  of the  $n$  relays detect the signal *incorrectly*, and  $P_{coop}(n)$  denotes the error probability at the destination when all the  $n$  relays detect the signal correctly. From the results in Section II.B, we know that

$$P_{coop}(n) \approx \left( \frac{2^R}{\gamma} \right)^{n+1}. \quad (11)$$

$P_r(\tilde{n})$  can be further bounded as

$$P_r(\tilde{n}) \leq \binom{n}{\tilde{n}} P_e \left( |h_{sr_i}|^2 \gamma > \gamma_{tr} \right)^{\tilde{n}},$$

where  $P_e \left( |h_{sr_i}|^2 \gamma > \gamma_{tr} \right)$  denotes the error probability averaged over those  $h_{sr_i}$  that satisfy  $|h_{sr_i}|^2 \gamma > \gamma_{tr}$ . Following the proof of *Lemma 3* in our previous work [5],  $P_e \left( |h_{sr_i}|^2 \gamma > \gamma_{tr} \right)$  can be up bounded by

$$P_e \left( |h_{sr_i}|^2 \gamma > \gamma_{tr} \right) < \frac{1}{\gamma} Q \left( \sqrt{\frac{1}{2^{R+1}} \gamma_{tr}} \right).$$

Note that  $Q(x) < e^{-x^2/2}$ ,  $x > 0$ . We thus further obtain

$$P_e \left( |h_{sr_i}|^2 \gamma > \gamma_{tr} \right) < \frac{1}{\gamma} e^{-\frac{1}{2^{R+1}} \gamma_{tr}}.$$

Since  $\gamma_{tr} = 2^{R+2} M \log \gamma$ , it follows that

$$P_e \left( |h_{sr_i}|^2 \gamma > \gamma_{tr} \right) \leq \frac{1}{\gamma^{M+1}}. \quad (12)$$

On substituting (11) and (12) into (10), we obtain

$$P_e(n) \leq \frac{1}{\gamma^{M+1}} + \left( \frac{2^R}{\gamma} \right)^{n+1}. \quad (13)$$

Further substituting (9) and (13) into (6), we obtain

$$P_r \leq \frac{1}{\gamma^{M+1}} + \left( \frac{2^R}{\gamma} \right)^{N+1} (\log \gamma)^N. \quad (14)$$

When  $\gamma$  approaches infinity, the term  $\log \gamma$  can be ignored. Thus the full diversity gain  $N + 1$  can be achieved provided that  $M \geq N$ , and the proof is complete. ■

*Remark 1:* From the proof of *Theorem 1* we can see that, compared with a perfectly coded scheme (e.g., the relaying

protocol in [3]), the threshold based detection scheme suffers from an error probability increase proportional to  $(\log \gamma)^N$ . We conjecture that such loss may represent the coding gain when perfect encoding and decoding is performed *at the relay*. We also note that the optimal  $M$  is equal to  $N$  in the sense that any higher  $M$  will not help to increase the system diversity gain but will introduce an additional increase in error probability in proportion to at most  $M^N$  (see (9) and (4)).

To motivate the second protocol, we first analyze the diversity multiplexing tradeoff for the MRC-type protocol. Since the transmission of each message is divided into  $N + 1$  time slots, the effective rate becomes  $\bar{R} = \frac{1}{N+1}R$ . For high SNR, we assume  $\bar{R} = r \log \gamma$ . Thus  $R = (N + 1)r \log \gamma$ . On substituting this into (14), we can bound the error probability as

$$P_e \leq \gamma^{-(N+1)(1-(N+1)r)}.$$

Thus the diversity multiplexing tradeoff offered by such a scheme is

$$d = (N + 1)(1 - (N + 1)r).$$

While offering a diversity gain of  $N + 1$ , the MRC-type protocol suffers from spectral inefficiency. Specifically, its highest multiplexing gain is only  $1/(N + 1)$  times of that of direct transmission due to its compulsory  $N + 1$  transmission time slots for each message. Thus a more spectrally efficient protocol is required. In the following section we will introduce one such protocol called the ARQ-based protocol.

## V. THE ARQ-BASED PROTOCOL

In the ARQ-based protocol, an SNR threshold  $\gamma_{td}$  is set at the destination as well. After the destination receives the message in the first time slot, it compares its receive SNR  $\gamma |h_{sd}|^2$  with  $\gamma_{td}$ . If the receive SNR is higher than  $\gamma_{td}$ , the destination will detect the message. Meanwhile, it sends an ACK back to the relay and the source. Upon reception of the ACK, the relays will remain silent and the source will continue transmitting the next message. If the receive SNR is lower than  $\gamma_{td}$ , the destination will send an one bit negative acknowledge (NACK) to the relay and the destination. Upon the reception of the NACK, each relay will perform threshold ( $\gamma_{tr}$ ) based detection in a separate time slot. The destination, upon reception of all the signal pieces from the relays, performs maximal ratio combining to combine all of them and performs detection. The source will wait for all the  $N$  relays to process their messages before transmitting the next message. In such a protocol, it takes either one time slot or  $(N + 1)$  time slots to transmit each message, depending on the channel quality of the source to destination link. Note that when the NACK is sent, the signal piece from direct transmission is no longer used in the MRC process at the destination. The operation at the destination can be considered as a selection between the signal from the direct link and those from the relay links. This system has a transmission rate  $R$  when the destination sends an ACK, and  $\frac{1}{N+1}R$  when the destination sends a NACK.

This protocol is similar to the incremental relaying protocol proposed in [2] in the sense that they both use one bit feedback from the relays. One can think of this protocol as a special form of ARQ [4] with the additional feature of cooperation from the relays. However, unlike the coded system, specific thresholds are required at the receivers in order to make the protocol efficient. In the following we will derive thresholds  $\gamma_{tr}^*$  and  $\gamma_{td}^*$  such that the network will achieve the same diversity-multiplexing tradeoff as that of an  $(N + 1) \times 1$  point-to-point MISO system. Specifically, we state the results in the following theorem.

*Theorem 2:* The following thresholds  $\gamma_{tr}^*$  and  $\gamma_{td}^*$  are sufficient for achieving the diversity multiplexing tradeoff of  $d = (N + 1)(1 - r)$  in a  $N$  relay network when the ARQ-based protocol is performed:

$$\gamma_{td}^* = \frac{2}{\alpha} M \log c_1 \gamma, \quad \text{and} \quad \gamma_{tr}^* = \frac{2}{\alpha} (M - 1) \log c_2 \gamma$$

for any  $M \geq N$ , where  $c_1$  and  $c_2$  are constants and  $\alpha$  can be approximated by (2) for large  $R$ .

*Proof:* For notational simplicity we assume  $c_1 = c_2 = 1$ . The error probability as a function of  $R$  and  $\gamma$  can be expressed as

$$P_e = \Pr(\gamma |h_{sd}|^2 > \gamma_{td}^*) P_e(\gamma |h_{sd}|^2 > \gamma_{td}^*) + \Pr(\gamma |h_{sd}|^2 < \gamma_{td}^*) P_{rd} \quad (15)$$

where  $P_e(\gamma |h_{sd}|^2 > \gamma_{td}^*)$  is the error probability for direct transmission given  $\gamma |h_{sd}|^2 > \gamma_{td}^*$ , and  $P_{rd}$  is the error probability when the destination chooses to receive the signals from the relays. Following the proof of *Theorem 1*, it is not difficult to see that

$$P_e \leq \frac{1}{\gamma^{M+1}} + \frac{2^{R+2} M \log \gamma}{\gamma} \times \sum_{n=1}^N \left( \frac{2^{R+2} (M - 1) \log \gamma}{\gamma} \right)^{N-n} \left( \frac{1}{\gamma^M} + \left( \frac{2^R}{\gamma} \right)^n \right) \doteq \frac{1 + 2^R \log \gamma}{\gamma^{M+1}} + \left( \frac{2^R}{\gamma} \right)^{N+1} (\log \gamma)^N \quad (16)$$

$$\doteq \frac{2^R}{\gamma^{M+1}} + \left( \frac{2^R}{\gamma} \right)^{N+1}, \quad (17)$$

where we ignore the effect of  $\log \gamma$  from (16) to (17).

Since the system rate is changing due to the adaptive nature of the protocol. We need to obtain the relationship between the rate  $R$  and the system expected spectral efficiency, denoted by  $\bar{R}$ , in order to obtain the diversity multiplexing tradeoff. Here we use an approach similar to that in [2] when analyzing the performance of incremental relaying. Specifically,  $\bar{R}$  can be

written as

$$\begin{aligned} \bar{R} &= R \Pr\left(\gamma |h_{sd}|^2 > \gamma_{td}^*\right) \\ &\quad + \frac{1}{N+1} R \left(1 - \Pr\left(\gamma |h_{sd}|^2 > \gamma_{td}^*\right)\right) \end{aligned} \quad (18)$$

$$= R \times \exp\left(-\frac{\gamma_{td}^*}{\gamma}\right) + \frac{1}{N+1} R \left(1 - \exp\left(-\frac{\gamma_{td}^*}{\gamma}\right)\right) \quad (19)$$

$$= \frac{N}{N+1} R \times \exp\left(-\frac{2^{R+2} M \log \gamma}{\gamma}\right) + \frac{1}{N+1} R. \quad (20)$$

Any value of  $\bar{R}$  can result from several values of  $R$ . Note that a smaller  $R$  will result in a lower system error probability (e.g., see (17)). Therefore, for each  $\bar{R}$  we always choose the smallest value of the corresponding  $R$ , which we denote by  $h^{-1}(\bar{R})$ . Note that  $\bar{R} \leq R$  and  $\bar{R} \rightarrow R$  as  $\gamma \rightarrow \infty$ , and  $\bar{R}/R$  is monotonically decreasing in  $R$  for each  $\gamma$ . Assume  $P_e \rightarrow g(R)$  as  $\gamma \rightarrow \infty$ , in which  $g(R)$  is a function of  $R$ . According to *Claim 3* in [2], it follows that the system error probability  $P_e$  as a function of  $\bar{R}$ , now written as  $P_e(h^{-1}(\bar{R}))$ , has the following property.

$$P_e(h^{-1}(\bar{R})) \doteq g(R) \leq \frac{2^{\bar{R}}}{\gamma^{M+1}} + \left(\frac{2^{\bar{R}}}{\gamma}\right)^{N+1}. \quad (21)$$

Let  $\bar{R} = r \log \gamma$ . We further obtain

$$P_e \leq \gamma^{-(M+1-r)} + \gamma^{-(N+1)(1-r)}. \quad (22)$$

Clearly,  $P_e$  will always be dominated by the second term if  $M \geq N$ . The proof is thus complete. ■

*Remark 2:* It is interesting to observe that the minimal  $\gamma_{tr}^*$  is less than the minimal  $\gamma_{td}^*$ . Specially, if only one relay exists, there is no threshold requirement at the relay, i.e.,  $\gamma_{tr}^* = 0$ . This is because the event of using the relays is conditioned on the event that direct transmission fails, which happens with a fairly small probability (with diversity order of one). Thus the threshold requirements at the relays are less stringent. In fact, setting  $\gamma_{tr}^*$  to be the same as  $\gamma_{td}^*$  will result in an increase in the error probability due to a loss of power gain from the relays, since for those relays whose receive SNRs are sufficiently high, only part of them will transmit. One can also observe from (22) that even for some  $M < N$ , the system can still achieve the diversity multiplexing tradeoff of  $(N+1)(1-r)$  for sufficiently large  $r$ . This is mainly due to the fact that as the  $r$  increases, the requirement for system error probability becomes lower while the need for spectral efficiency becomes higher. Thus a smaller  $M$  will not affect the overall system performance. The diversity multiplexing tradeoff curves for different schemes are compared in Fig. 1.

## VI. NUMERICAL RESULTS

For simplicity, we use 4-QAM constellation. Thus the data rate  $R$  for direct transmission is 2 bits per channel use. The value of the  $\alpha$  inside the Q function for such constellation size is known to be 1. Note that the approximation (2) is not used here since it is only valid for large  $R$ .

Fig. 2 shows the performance of different schemes in a single relay network. For the MRC-type protocol, we set  $\gamma_{tr} =$

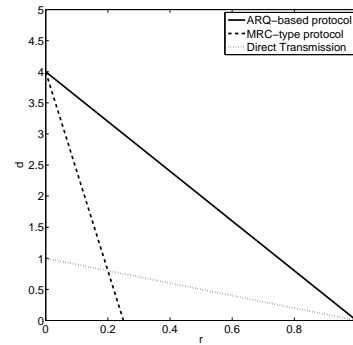


Fig. 1. Diversity multiplexing tradeoffs for different schemes for  $N = 3$ .

$2 \log \gamma$ . For the ARQ-based protocol, we set  $\gamma_{td} = 2 \log \gamma$  and  $\gamma_{tr} = 0$ . We also plot the system performance on assuming a perfect source-relay link. In this scenario the network mimics a MISO channel with two transmit antennas. Clearly both the MRC-type protocol and the ARQ-based protocol offer the same diversity gain as the MISO channel. One can also observe that the MRC-type protocol offers a higher power gain than the ARQ-based protocol. This is because the ARQ-based protocol performs selection combining at the destination, which offers less power gain than MRC.

Fig. 3 further shows the performance of different schemes in a two relay network. For the MRC-type protocol, we set  $\gamma_{tr} = 4 \log \gamma$ . For the ARQ-based protocol, we set  $\gamma_{td} = 4 \log \gamma$  and  $\gamma_{tr} = 2 \log \gamma$ . The same observations can be made as those for Fig. 2. One might question the performance advantage of the relaying protocols for lower SNR after observing both figures. When the SNR is low, the power gain becomes a more important factor and one might need to shift the curves by changing the values of the constants  $c$ ,  $c_1$  and  $c_2$ . Searching the values of these constants that optimize the error probability performance is an interesting problem, which is beyond the scope of this paper.

In order to show the advantages of the ARQ-based protocol over the MRC-type protocol, we plot the system effective transmission rate  $\bar{R}$  for the ARQ-based protocol given the constellation size of 4-QAM (i.e., 2 bits per transmission for direct transmission). From Fig. 4 it is clear that  $\bar{R}$  approaches 2 bits per channel use as the SNR increases. Compared with MRC-type protocol, which has effective rate of 1 bit per channel use, the transmission rate is doubled.

It is also of interest to see whether the optimal thresholds that minimize the system end-to-end error probability follow those forms proposed in *Theorem 1* and *Theorem 2*. Fig. 5 offers an answer through numerical results. It shows the optimal thresholds for the MRC-type protocol for different values of  $N$  that minimize the system error probability expressed in (3). Clearly for higher SNR values the curves become linear in the semi-log plot. Thus the optimal threshold increases logarithmically as a function of  $\gamma$ . Also it increases with increasing number of relays. As a result, we conjecture that

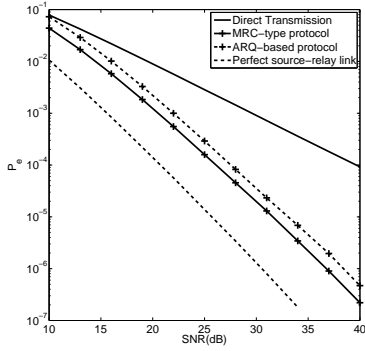


Fig. 2. Error probability for different relaying schemes when  $N = 1$ .

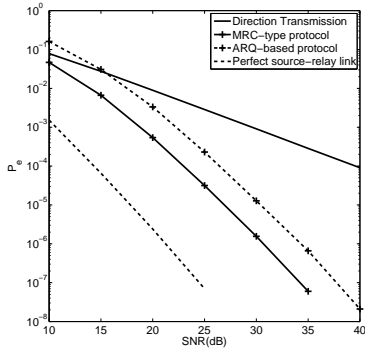


Fig. 3. Error probability for different relaying schemes when  $N = 2$ .

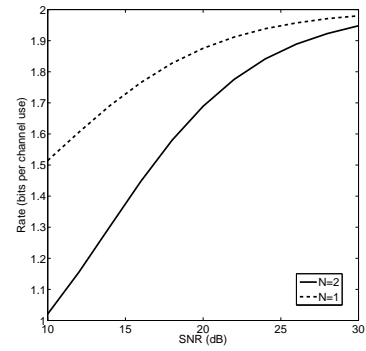


Fig. 4. Transmission rate for the ARQ-based protocol.

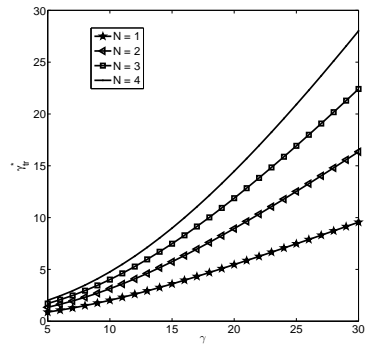


Fig. 5. Optimal thresholds for the MRC-type protocol.

the optimal threshold follows the form proposed in this paper for high SNR with careful choices of  $c$ ,  $c_1$  and  $c_2$ . Simulation of the ARQ-based protocol is more involved and is omitted.

## VII. CONCLUSIONS

In this paper, we have proposed two different relaying protocols for threshold based distributed detection in sensor networks, in which each node performs symbol by symbol detection depending on certain SNR thresholds. Both protocols require no coding and offer the full diversity gain of the network, while being capable of saving significant energy cost caused by coding. More specifically, the proposed ARQ-based protocol is shown to offer the network the same diversity-multiplexing tradeoff as a point-to-point MISO channel. For both protocols, the SNR thresholds at different nodes that achieve full diversity have been derived and are shown to increase logarithmically with the link SNRs and linearly with the number of relays in the network.

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