Low-Complexity Detection of *M*-ary PSK Faster-than-Nyquist Signaling

Ebrahim Bedeer*, Halim Yanikomeroglu**, Mohamed Hossam Ahmed***

*Ulster University, Belfast, UK
**Carleton University, Ottawa, ON, Canada
***Memorial University, St. John's, NL, Canada

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Agenda

- Introduction
- FTN Signaling System Model
- Our FTN Signaling Contributions
  - Quasi-Optimal Detection (High SE)
  - Symbol-by-Symbol Detection (Low SE)
  - *M*-ary QAM Detection
  - *M*-ary PSK Detection
- Conclusions

Introduction

- Orthogonality is an advantage and a constraint.
- Nyquist limit is more of a guideline than a rule.
- Nyquist limit simplifies receive design by avoiding ISI.
- Faster-than-Nyquist (FTN signaling) intentionally introduce ISI to improve SE.
- Detection: Increased complexity

Introduction

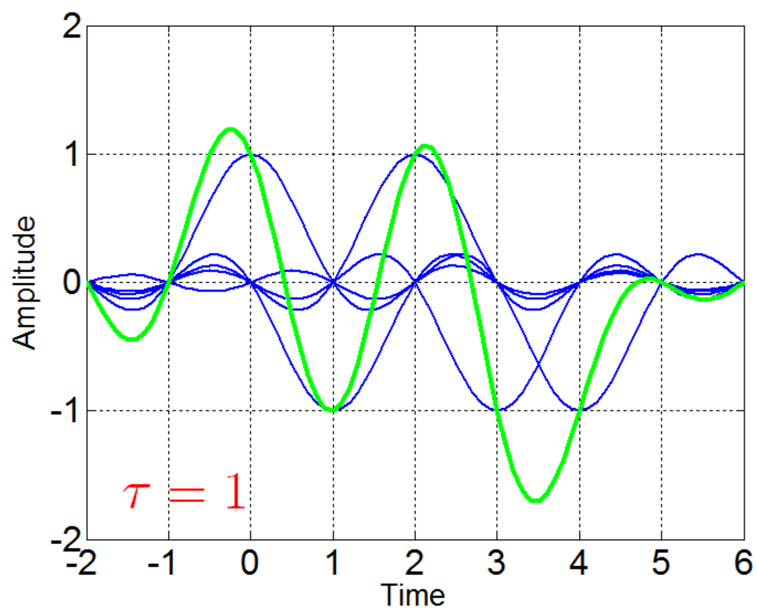
- FTN signaling *concept* exists at least since 1968 [Saltzberg-68].
- FTN signaling *term* coined by Mazo in 1975 [Mazo-75].
- Mazo Limit: FTN does not affect minimum distance of uncoded sinc binary transmission up to a certain range.
- Mazo Limit: 1/0.802 → 25% faster than Nyquist
  → 25% in spectral efficiency.
- Much faster Mazo limit: Possible, but with some SNR penalty.

Saltzberg B. Intersymbol interference error bounds with application to ideal bandlimited signaling. IEEE Transactions on Information Theory. July 1968; 14(4):563-8.

Mazo JE. Faster-than-Nyquist signaling. The Bell System Technical Journal. Oct. 1975; 54(8):1451-62.

FTN Signaling Basic Idea

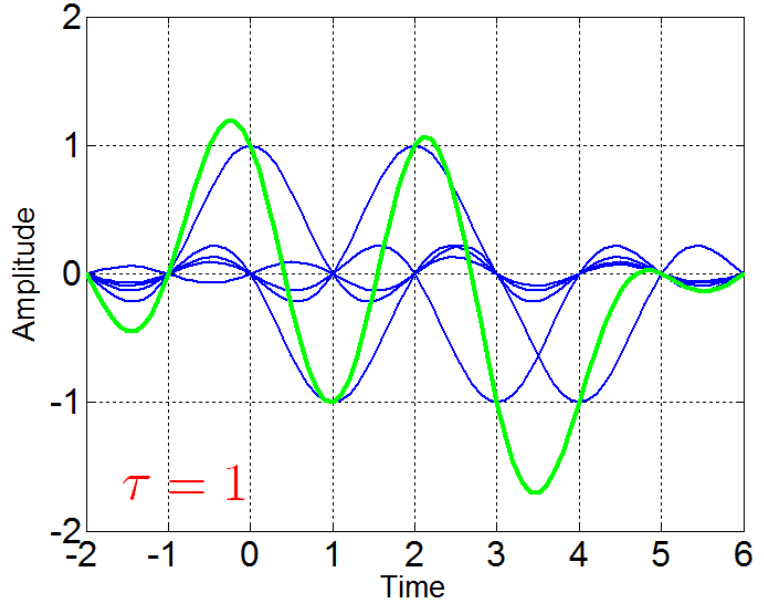
$$s(t) = \sqrt{E_s} \sum_n a_n \, g(t - n\,T)$$

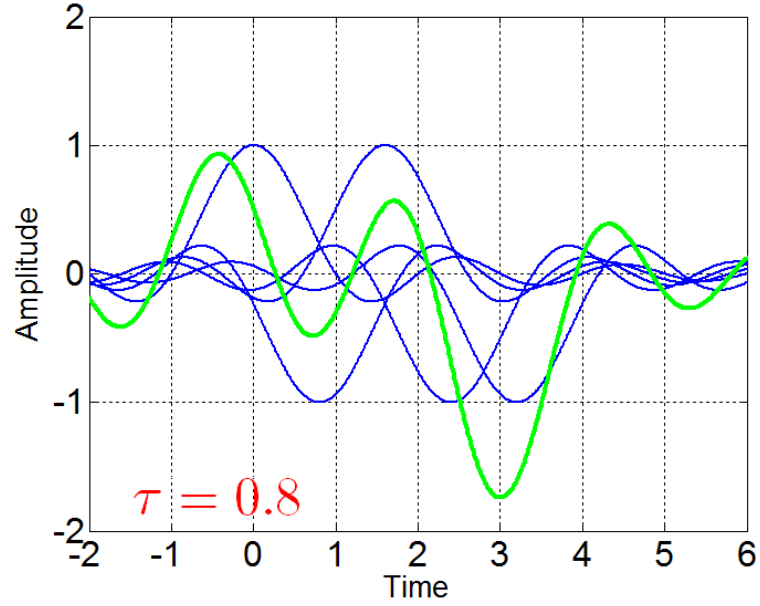


$$2 \quad \text{symbols/s/Hz}$$

FTN Signaling Basic Idea

$$s(t) = \sqrt{E_s} \sum_n a_n\, g(t - n\, T) \qquad\qquad s(t) = \sqrt{E_s\, \tau} \sum_n a_n\, g(t - n\, \tau\, T)$$





2 symbols/s/Hz $\qquad\qquad$ $\dfrac{2}{\tau}$ symbols/s/Hz

Extension of Mazo Limit

- Other pulse shapes (root-raised cosine, Gaussian, …)
- Non-binary transmission
- Frequency domain

Our FTN Publications

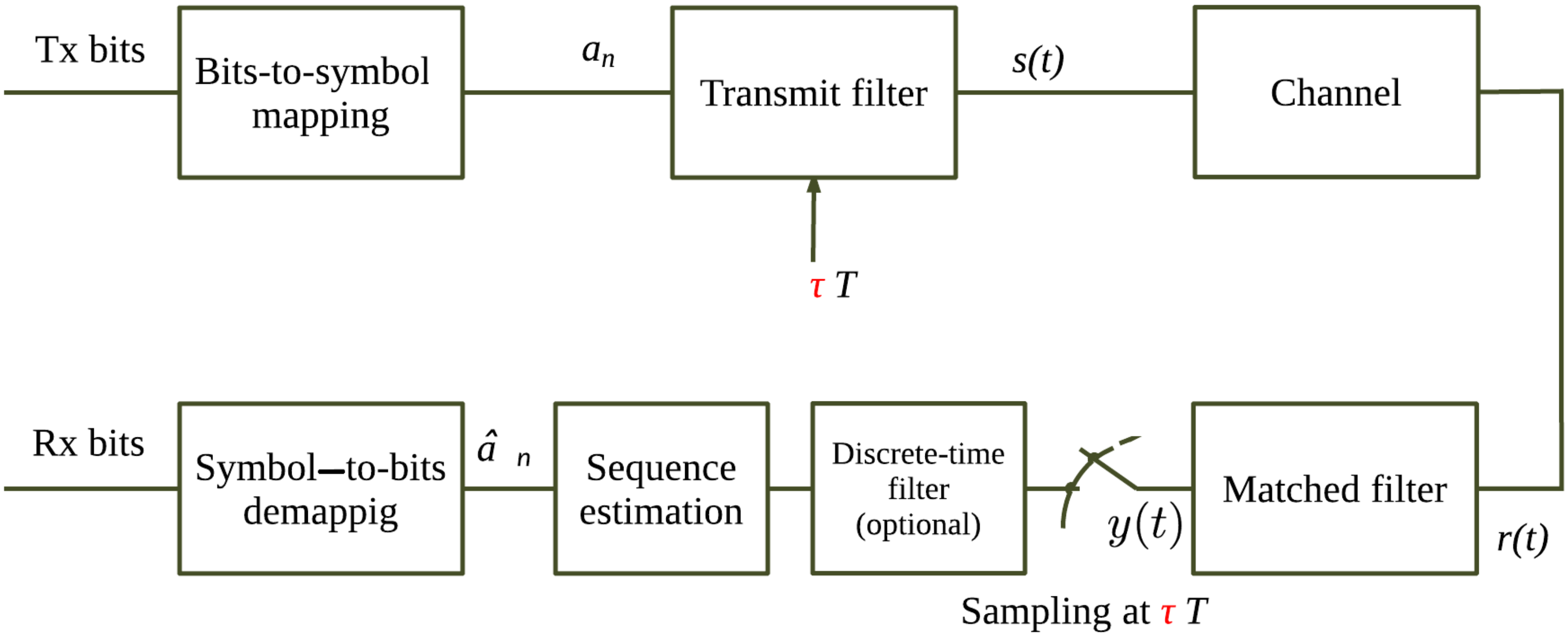
Ebrahim Bedeer, Halim Yanikomeroglu, and Mohamed H. Ahmed, "Quasi-optimal sequence estimation of binary faster-than-Nyquist signaling", *IEEE ICC 2017*, Paris, France.

Ebrahim Bedeer, Mohamed H. Ahmed, and Halim Yanikomeroglu, "A very low complexity successive symbol-by-symbol sequence estimator for binary faster-than-Nyquist signaling", *IEEE Access*, March 2017.

Ebrahim Bedeer, Mohamed H. Ahmed, and Halim Yanikomeroglu, "Low-complexity detection of high-order QAM faster-than-Nyquist signaling", *IEEE Access*, July 2017.

Ebrahim Bedeer, Halim Yanikomeroglu, and Mohamed H. Ahmed, "Low-Complexity Detection of *M*-ary PSK Faster-than-Nyquist Signaling", *IEEE WCNC 2019 Workshops*, Marrakech, Morocco.

FTN Block Diagram



$$y_k = \underbrace{\sqrt{\tau\, E_s}\, a_k\, g(0)}_{\text{desired symbol}} + \underbrace{\sqrt{\tau\, E_s}\, \sum_{n=1,\, n\neq k}^{N} a_n\, g((k-n)\tau T)}_{\text{ISI}} + q(k\tau T)$$

Our FTN Signaling Contributions

Quasi-Optimal Detection (High SE)

Ebrahim Bedeer, Halim Yanikomeroglu, and Mohamed H. Ahmed, "Quasi-optimal sequence estimation of binary faster-than-Nyquist signaling", *IEEE ICC 2017*, Paris, France.

Modified Sphere Decoding (MSD)

- Noise covariance matrix can be exploited to develop MSD.
- Estimated data symbols can be found using MSD as

$$\left\lceil z_N - \frac{d}{R_{N,N}} \right\rceil \leq a_N \leq \left\lfloor z_N + \frac{d}{R_{N,N}} \right\rfloor .$$

$$\begin{aligned} a_{N-1} &\geq \left\lceil z_{N-1} - \frac{\hat{d} - R_{N-1,N}(z_N - a_N)}{R_{N-1,N-1}} \right\rceil , \\ a_{N-1} &\leq \left\lfloor z_{N-1} + \frac{\hat{d} + R_{N-1,N}(z_N - a_N)}{R_{N-1,N-1}} \right\rfloor , \end{aligned}$$

Simulation Results

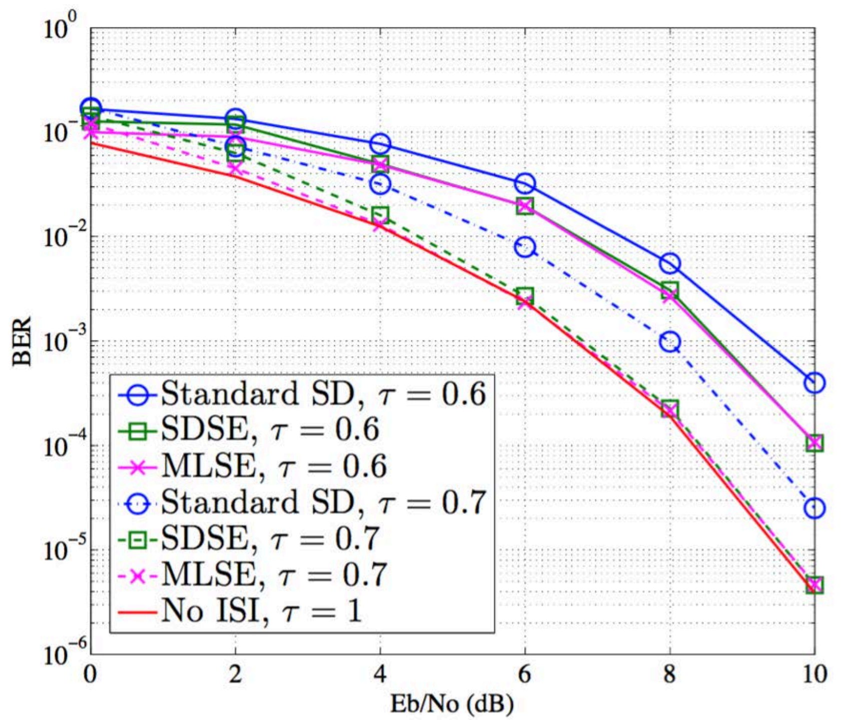


Fig. 2: BER performance of binary FTN detection versus $\frac{E_b}{N_o}$ using the standard SD-based and proposed SDSEs at $\beta = 0.3$ and $\tau = 0.6$ and 0.7.

Simulation Results

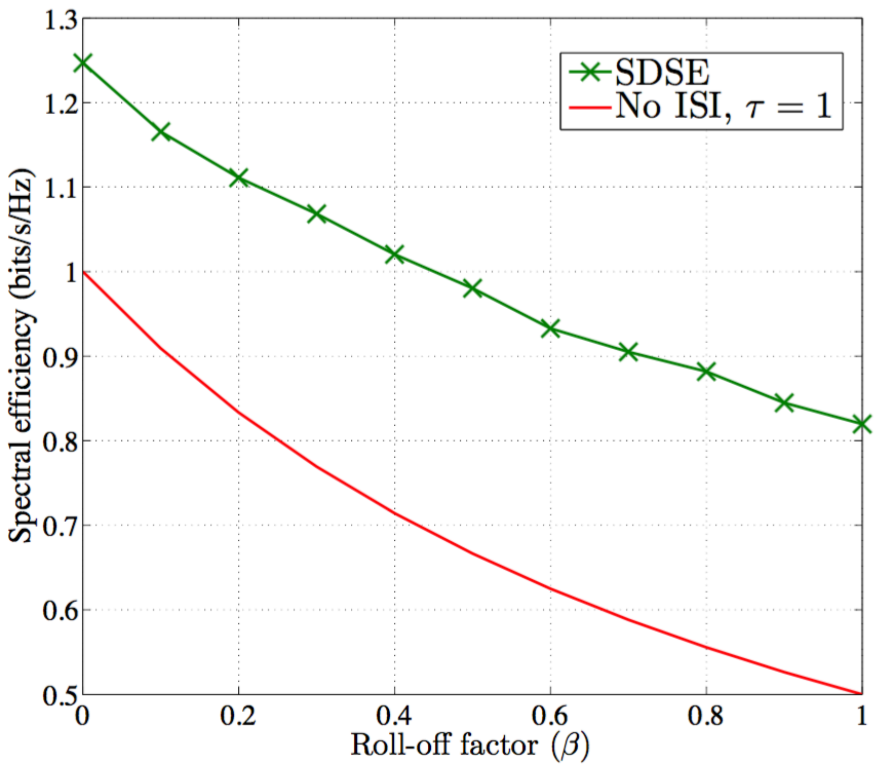
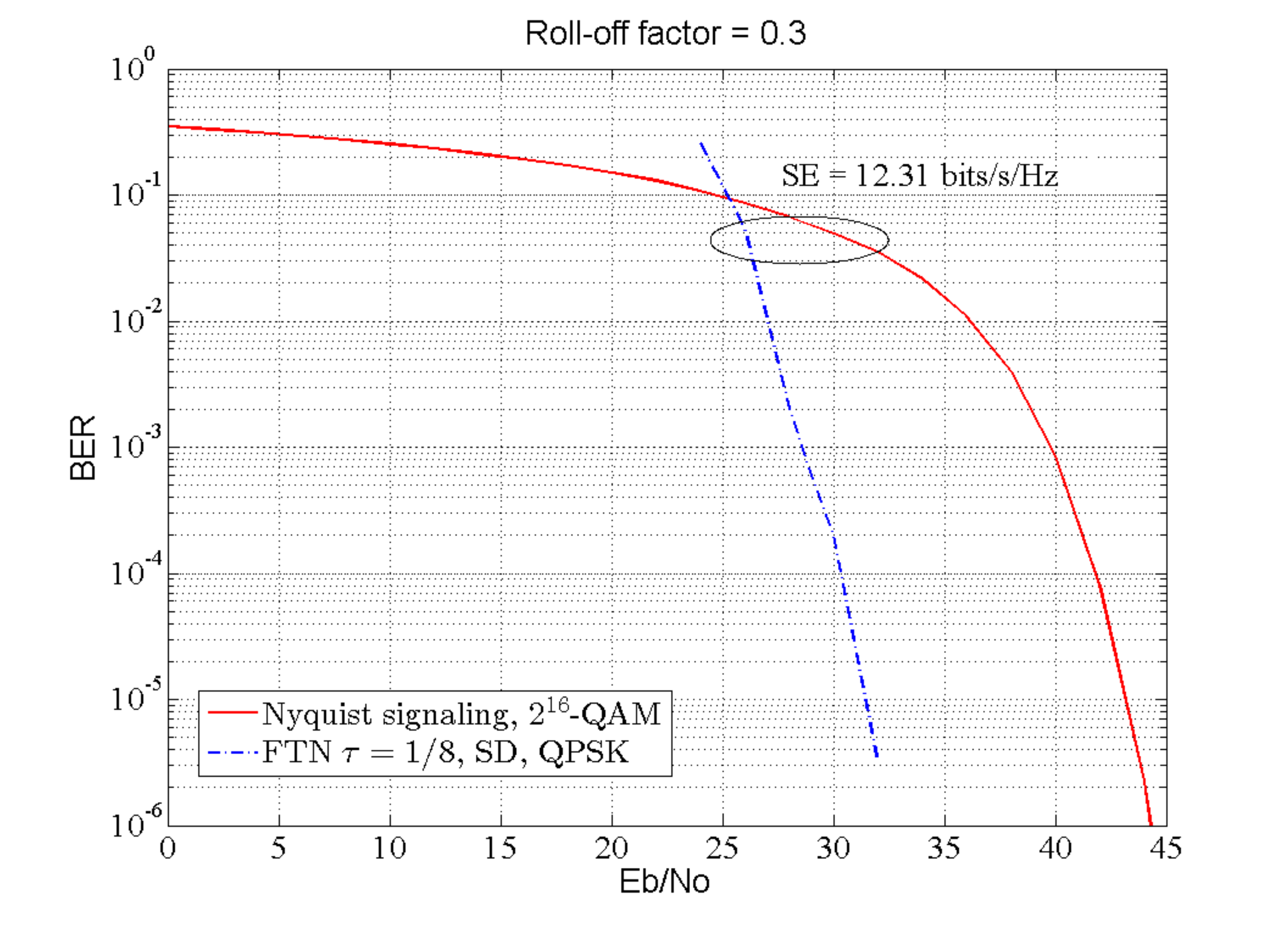


Fig. 4: Spectral efficiency comparison of binary FTN signaling versus β using the proposed SDSE and Nyquist signaling at BER = 10^{-4}.

Simulation Results

Spectral Efficiency
SE= log2(M) x [1/(1+β)] x (1/τ) bits/s/Hz



Our FTN Signaling Contributions

Symbol-by-Symbol Detection (Low SE)

Ebrahim Bedeer, Mohamed H. Ahmed, and Halim Yanikomeroglu, "A very low complexity successive symbol-by-symbol sequence estimator for binary faster-than-Nyquist signaling", *IEEE Access*, March 2017.

Successive Symbol-by-Symbol Sequence Estimation (SSSSE)

- Received sample

$$y_k = \underbrace{G_{1,L}\, a_{k-L+1} + \ldots + G_{1,2}\, a_{k-1}}_{\text{ISI from previous } L-1 \text{ symbols}} + \underbrace{G_{1,1}\, a_k}_{\text{Current symbol to be estimated}} + \underbrace{G_{1,2}\, a_{k+1} + \ldots + G_{1,L}\, a_{k+L-1}}_{\text{ISI from upcoming } L-1 \text{ symbols}} .$$

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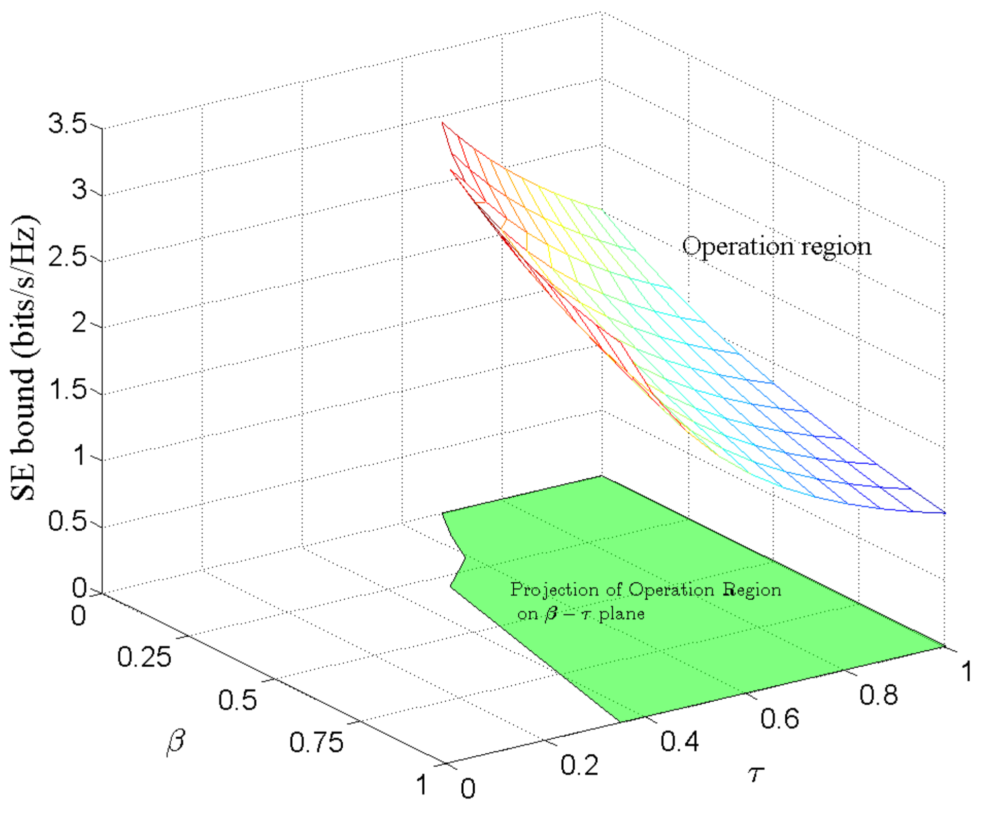
- Perfect estimation condition for QPSK FTN signaling

$$|G_{1,1}\, \Re\{a_k\}| > |G_{1,2}\, \Re\{a_{k+1}\} + \ldots + G_{1,L}\, \Re\{a_{k+L-1}\}|,$$

$$|G_{1,1}\, \Im\{a_k\}| > |G_{1,2}\, \Im\{a_{k+1}\} + \ldots + G_{1,L}\, \Im\{a_{k+L-1}\}|,$$

Operating region of SSSSE



Successive Symbol-by-Symbol Sequence Estimation (SSSSE)

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$$|G_{1,1}\, \Re\{a_k\}| > |G_{1,2}\, \Re\{a_{k+1}\} + \ldots + G_{1,L}\, \Re\{a_{k+L-1}\}|,$$

$$|G_{1,1}\, \Im\{a_k\}| > |G_{1,2}\, \Im\{a_{k+1}\} + \ldots + G_{1,L}\, \Im\{a_{k+L-1}\}|,$$

- Estimated symbol

$$\hat{a}_k = \text{quantize}\left\{y_k - (G_{1,L}\, \hat{a}_{k-L+1} + \ldots + G_{1,2}\, \hat{a}_{k-1})\right\},$$

Successive Symbol-by-Symbol with go-back-*K* Sequence Estimation (SSSgb*K*SE)

- Received sample

$$y_k = \underbrace{G_{1,L}\, a_{k-L+1} + \ldots + \underbrace{G_{1,K+1}\, a_{k-K} + \ldots + G_{1,2}\, a_{k-1}}_{\text{Previous } K \text{ symbols to be re-estimated}}}_{\text{ISI from previous } L-1 \text{ symbols}} + \underbrace{G_{1,1}\, a_k}_{\text{Current symbol to be re-estimated}}$$

$$+ \underbrace{G_{1,2}\, a_{k+1} + \ldots + G_{1,L}\, a_{k+L-1}}_{\text{ISI from upcoming } L-1 \text{ symbols}}.$$

- Estimated symbol

$$\hat{a}_k = \text{quantize}\Big\{ y_k - \big(\underbrace{G_{1,L}\, \hat{a}_{k-L+1} + \ldots + \underbrace{G_{1,K+1}\, \hat{\hat{a}}_{k-K} + \ldots + G_{1,2}\, \hat{\hat{a}}_{k-1}}_{\text{ISI from previous } K \text{ data symbols with improved estimation accuracy}} \big)}_{\text{ISI from the previous } L-1 \text{ data symbols}} \Big\}.$$

Simulation Results

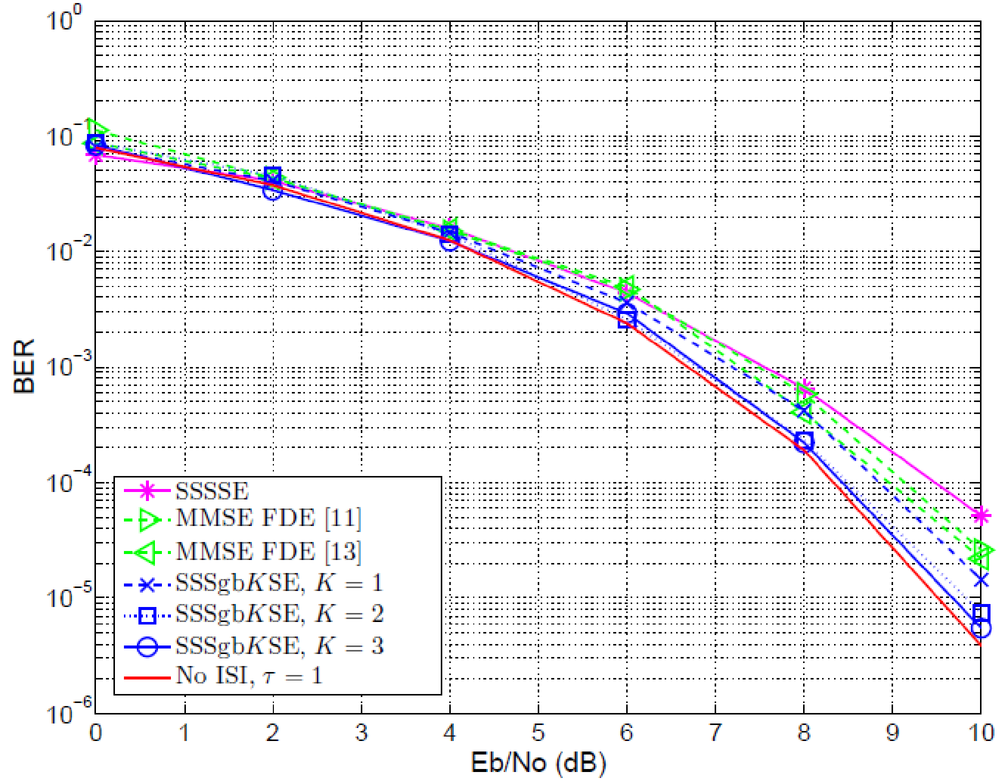


Fig. 4: BER performance of QPSK FTN sequence estimation as a function of $\frac{E_b}{N_o}$ using the proposed SSSSE, proposed SSSgb$K$SE, and FDEs in [11], [13] at $\beta = 0.3$ and SE of 1.71 bits/sec/Hz.

[11] S. Sugiura, "Frequency-domain equalization of faster-than-Nyquist signaling," *IEEE Wireless Commun. Lett.*, vol. 2, no. 5, pp. 555–558, Oct. 2013.

[13] T. Ishihara and S. Sugiura, "Frequency-domain equalization aided iterative detection of faster-than-Nyquist signaling with noise whitening," in *Proc. IEEE International Conference on Communications (ICC)*, May 2016, pp. 1–6.

Simulation Results

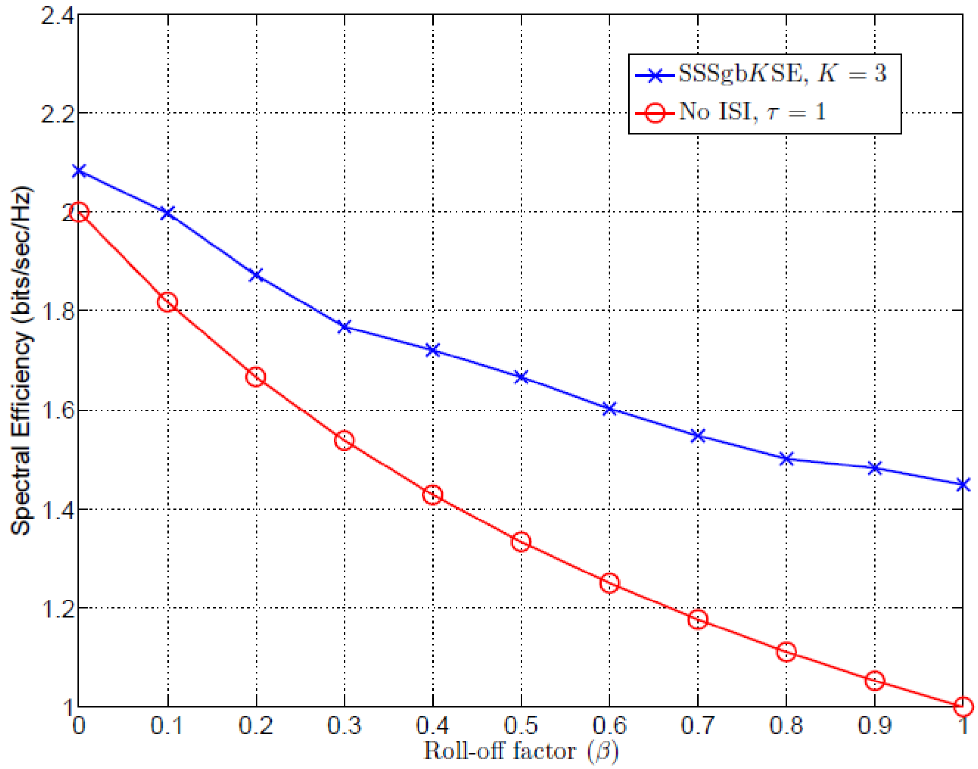


Fig. 6: Spectral efficiency of QPSK Nyquist and FTN signaling as a function of β using the proposed SSSgb$K$SE at BER = 10^{-4}.

Our FTN Signaling Contributions

M-ary PSK Detection

Ebrahim Bedeer, Halim Yanikomeroglu, and Mohamed H. Ahmed, "Low-Complexity Detection of *M*-ary PSK Faster-than-Nyquist Signaling", *IEEE WCNC 2019 Workshops*, Marrakech, Morocco.

FTN Detection Problem

- Received sample

$$y_k = \underbrace{\sqrt{\tau\, E_s}\, a_k\, g(0)}_{\text{desired symbol}} + \underbrace{\sqrt{\tau\, E_s} \sum_{n=1,\, n\neq k}^{N} a_n\, g((k-n)\tau T) + q(k\tau T)}_{\text{ISI}}$$

- Received sampled signal in vector format

$$\mathbf{y}_{\mathrm{c}} = \sqrt{\tau\, E_s}\, \mathbf{a} \star \mathbf{g} + \mathbf{q}_{\mathrm{c}}$$

- Received sampled signal after (optional) whitening filter

$$\mathbf{y}_{\mathrm{w}} = \sqrt{\tau\, E_s}\, \mathbf{a} \star \mathbf{v} + \mathbf{q}_{\mathrm{w}}$$

FTN Detection Problem

- Received sampled signal

$$\mathbf{y}_{\mathrm{w}} = \mathbf{V}\mathbf{a} + \mathbf{q}_{\mathrm{w}}$$

- Maximum likelihood detection problem

$$\min_{\boldsymbol{a} \in \mathcal{D}} \quad \|\boldsymbol{y}_{\mathrm{w}} - \boldsymbol{V}\boldsymbol{a}\|_2^2$$

$$\text{s. t.} \quad \mathcal{D} = \left\{ a = e^{j\frac{(2m-1)\pi}{M}} \middle| m = 1, \ldots, M \right\},$$

NP-hard

- Can be solved in polynomial time complexity using ideas from semidefinite relaxation and Gaussian randomization

Proposed FTN Detection Scheme

$$\begin{aligned} \min_{\boldsymbol{a} \in \mathcal{D},\, \boldsymbol{A} \in \mathbb{C}^{N}} \quad & \operatorname{tr}\{\boldsymbol{V}^{\mathrm{H}} \boldsymbol{V} \boldsymbol{A}\} - 2\Re\{\boldsymbol{a}^{\mathrm{H}} \boldsymbol{V}^{\mathrm{H}} \boldsymbol{y}_{\mathrm{w}}\} \\ \text{s. t.} \quad & |a_n| = 1, \qquad n = 1, \ldots, N, \\ & \begin{bmatrix} \boldsymbol{A} & \boldsymbol{a} \\ \boldsymbol{a}^{\mathrm{H}} & 1 \end{bmatrix} \succeq 0. \end{aligned}$$

1) **Input:** The inputs to our SDR-based algorit received samples $\boldsymbol{y}_{\mathrm{c}}$, the whitened matched filter impulse response, pulse shape $p(t)$ (ISI matrix $\boldsymbol{G}$), and the Gaussian randomization number of iterations L.
2) Solve the relaxed convex optimization problem and find the optimal solutions ($\boldsymbol{A}^*$ and $\boldsymbol{a}^*$).
3) Generate random variable $\boldsymbol{\xi}_\ell$ as $\boldsymbol{\xi}_\ell \sim \mathcal{N}(\boldsymbol{a}^*, \boldsymbol{A}^* - \boldsymbol{a}^* \boldsymbol{a}^{*\mathrm{H}})$, $\ell = 1, \ldots, L$.
4) Find ℓ_{op}

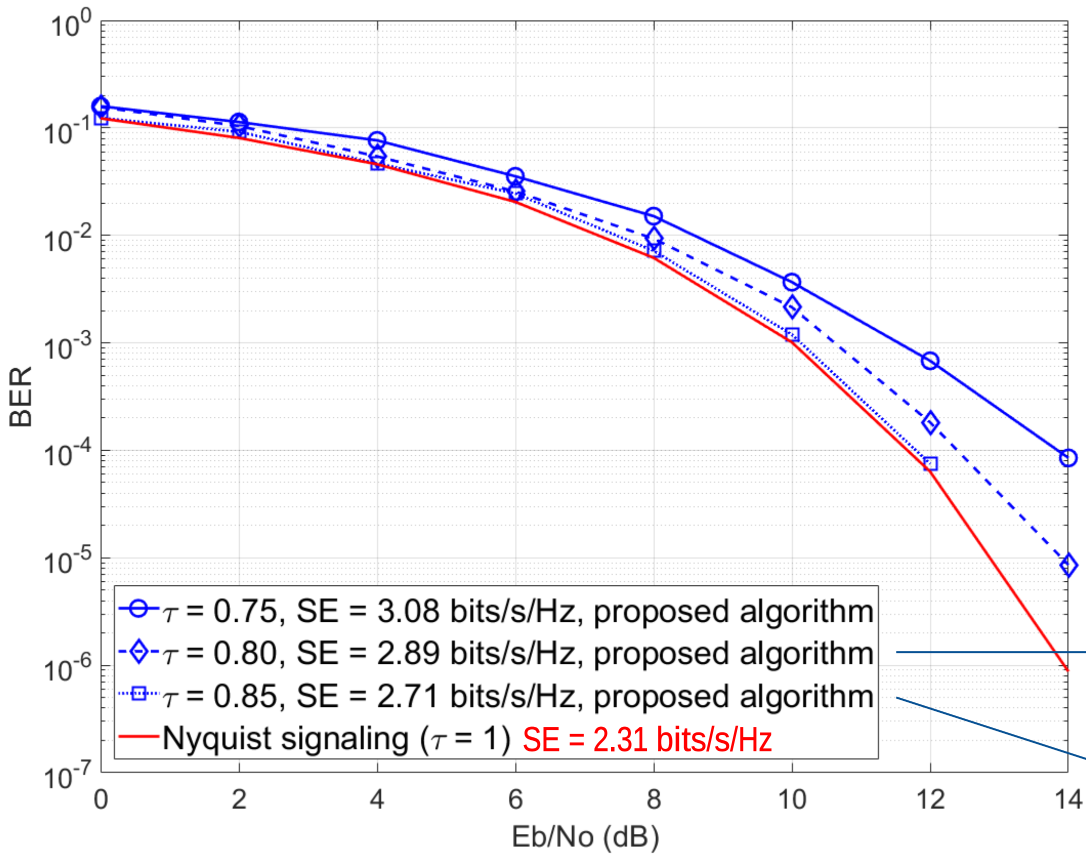
$$\ell_{\mathrm{op}} = \arg \min_{\ell = 1, \ldots, L} \operatorname{tr}\{\boldsymbol{V}^{\mathrm{H}} \boldsymbol{V} \breve{\boldsymbol{a}}_\ell \breve{\boldsymbol{a}}_\ell^{\mathrm{H}}\} - 2\Re\{\breve{\boldsymbol{a}}_\ell^{\mathrm{H}} \boldsymbol{V}^{\mathrm{H}} \boldsymbol{y}_{\mathrm{w}}\},$$

5) Calculate candidate suboptimal solutions to the FTN detection problem in (8) as $\hat{\boldsymbol{a}} = \breve{\boldsymbol{a}}_{\ell_{\mathrm{op}}}$.
6) **Output:** The estimated data symbol vector $\hat{\boldsymbol{a}}$.

Simulation Results

Spectral Efficiency
SE= log2(M) x [1/(1+β)] x (1/τ) bits/s/Hz

8-PSK
Roll-off factor: β = 0.3



Mazo limit: τ = 0.802

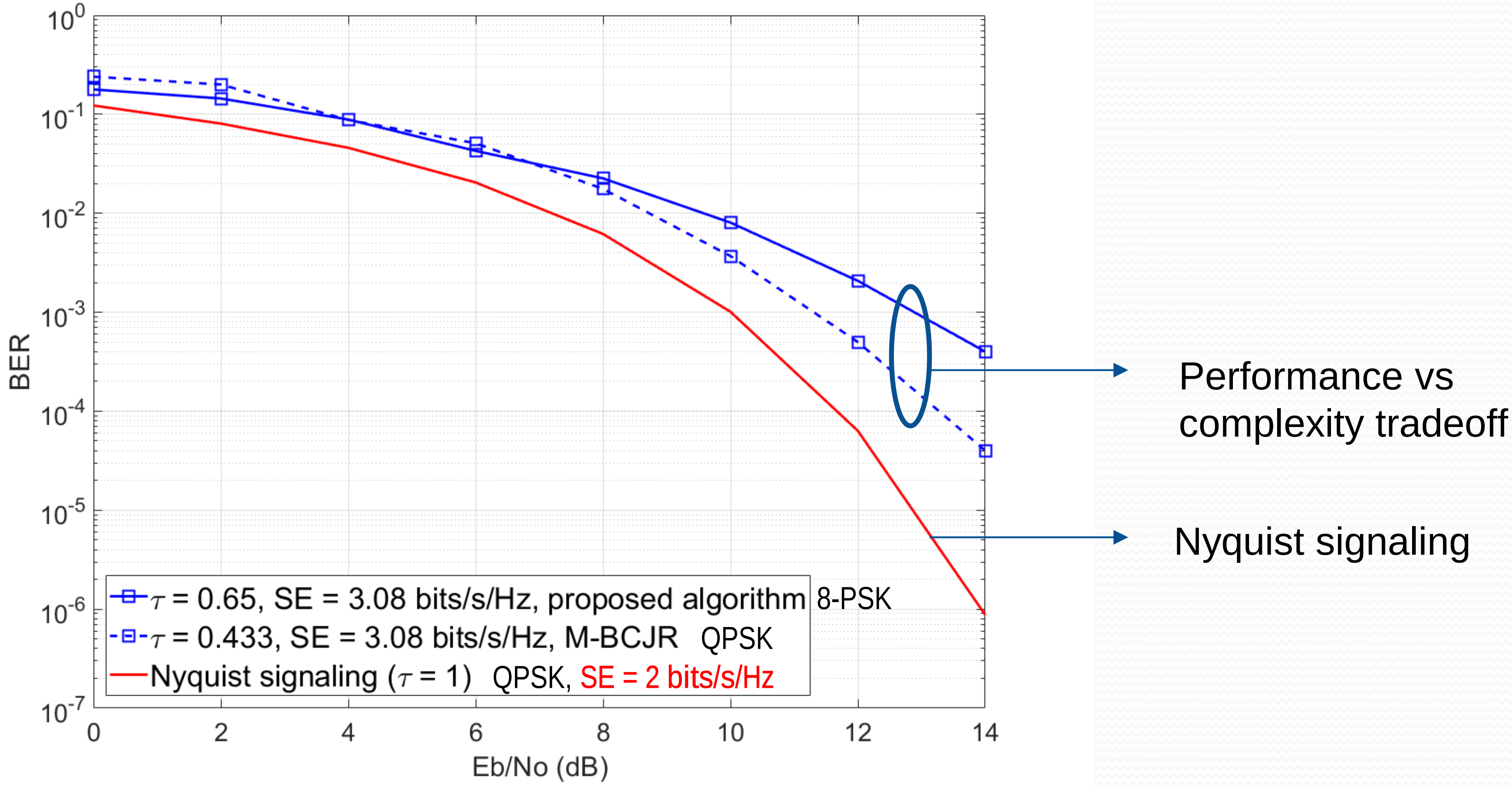
17% increase in SE
→ excellent

Simulation Results

Spectral Efficiency

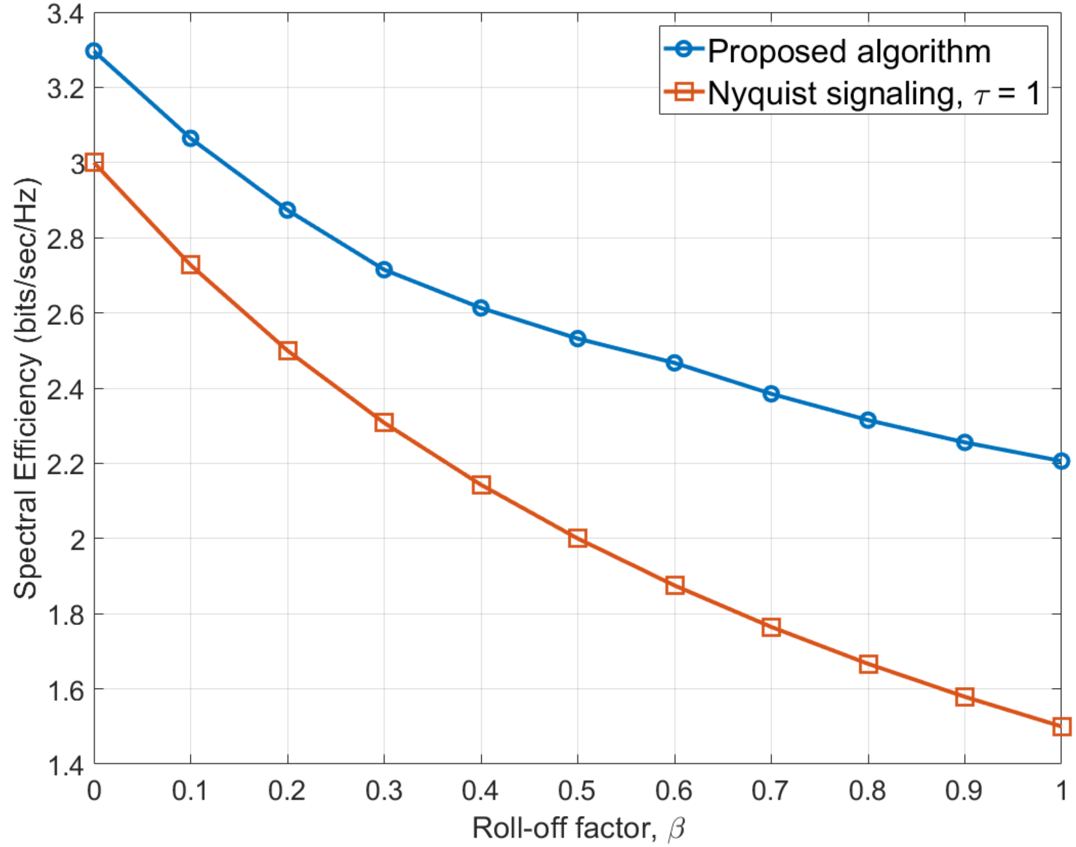
SE= log2(M) x [1/(1+β)] x (1/τ) bits/s/Hz

Roll-off factor: β = 0.5



J. B. Anderson and A. Prlja, "Turbo equalization and an M-BCJR algorithm for strongly narrowband intersymbol interference," ISIT 2010.

Simulation Results



Conclusions

- FTN signaling is promising to increase the SE.
- Tradeoff between performance and complexity.
- Gain of FTN increases at higher values of SE.
- Channel coding?
- AI / machine learning?