

Impact of Secondary Users' Field Size on Spectrum Sharing Opportunities

Muhammad Aljuaid and Dr. Halim Yanikomeroglu

Department of Systems and Computer Engineering
Carleton University

IEEE WCNC 2010

Outline

Introduction

- Spectrum Sharing

- Harmful Interference

- Motivation

- System Model

Characterization of the Aggregate Interference

- Mean of the Aggregate Interference

- Variance of the Aggregate Interference

- Upper Bound on the Interference Probability

Spectrum Sharing Opportunities

- Effect of Field Expansion

- Effect of Exclusion Region

Results Beyond WCNC10 Paper

- Cumulants of I_A

- Effect of Field Size on the CCDF of I_A

Spectrum Sharing

- ▶ Radio spectrum: scarce resource but under-utilized.
- ▶ Spectrum sharing: a new spectrum management paradigm.
- ▶ Sharing schemes: overlay & underlay.

Harmful Interference

- ▶ Interference event vs. harmful interference.
- ▶ Different metrics to gauge harmful interference.
- ▶ These metrics $\equiv f(\text{system \& channel parameters})$.

Motivation

- ▶ Field size receives least attention.
- ▶ Usually infinite field size is assumed, e.g., in [Menon05], [Menon06], [Ghassemi08] and [Win09].
- ▶ Impact of field size on spectrum sharing?

Interference Characterization in Large Wireless Networks (1/2)

Literature Overview

- ▶ Many papers investigate interference in large wireless networks using Poisson Point Process, e.g., [Sousa90], [Sousa92], [Ilow98], [Chan01], [Yang03], [Haenggi05], [Menon05], [Menon06], [Weber07], [Hasan07], [Ghasemi08], [Salbaroli09] and [Win09].
- ▶ Using a singular distance-dependent attenuation model leads to having an alpha-stable distribution of the aggregate interference power. This distribution has a closed form expression for the characteristic function but not for the CDF/PDF except for one special case [Sousa90].

Interference Characterization in Large Wireless Networks (2/2)

Literature Overview

- ▶ More realistic performance results are obtained by using non-singular distance-dependent attenuation models [Inaltekin09].
- ▶ If an exclusion region is imposed around the victim receiver or a non-singular distance-dependent attenuation model is used, the distribution of the aggregate interference power has a characteristic function in a closed form expression.
- ▶ However, no closed form expression is known for the distribution function. A numerical inversion of the characteristic function is an option.
- ▶ Alternatively, approximating the distribution of the aggregate interference using a finite set of moments (or cumulants) is a viable option.

Our Approach and Contributions

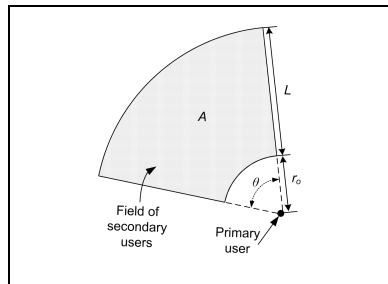
- ▶ We consider a finite field with an exclusion region (an infinite field is a special case of our results).
- ▶ First, we investigate the effect of field size on spectrum sharing by deriving an upper bound on the interference probability using the first two cumulants, i.e., mean and variance.
- ▶ Then, we extend cumulants formulations provided in [Menon06], and approximate the distribution of the aggregate interference based on a finite set of these cumulants.
- ▶ Finally, we repeat the investigation of the effect of the field size on spectrum sharing opportunities utilizing the approximation of the distribution of the aggregate interference.

System Model

- ▶ Field of secondary users sharing a spectrum with a primary user.
- ▶ Aggregate interference power:

$$I_A = \sum_{i \in N} I_i = \sum_{i \in N} r_i^{-n} W_i$$

- ▶ Analysis objective:
Investigate effect of L on I_A and spectrum sharing.



Interference Probability

A harmful interference metric [Ghasemi08] and [Win09]

- ▶ Non-harmful interference:

$$P(I_A \geq I_{th}) \leq \beta$$

⇒ spectrum sharing allowed

- ▶ Harmful interference:

$$P(I_A \geq I_{th}) > \beta$$

⇒ spectrum sharing NOT allowed

Mean of the Aggregate Interference

Formulation

- ▶ Mean of I_A (i.e., $\sum_{i \in N} r_i^{-n} W_i$):

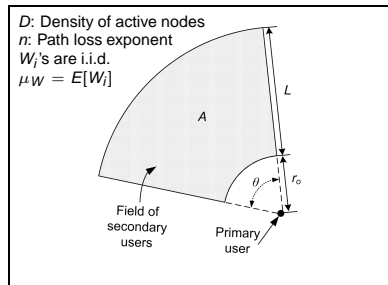
$$\mu_A = \frac{1}{n-2} D \theta r_o^{2-n} \mu_W \times \left[1 - \left(\frac{r_o}{r_o + L} \right)^{n-2} \right]$$

- ▶ For $L \ll r_o$:

$$\mu_A \simeq D \theta r_o^{1-n} L \mu_W$$

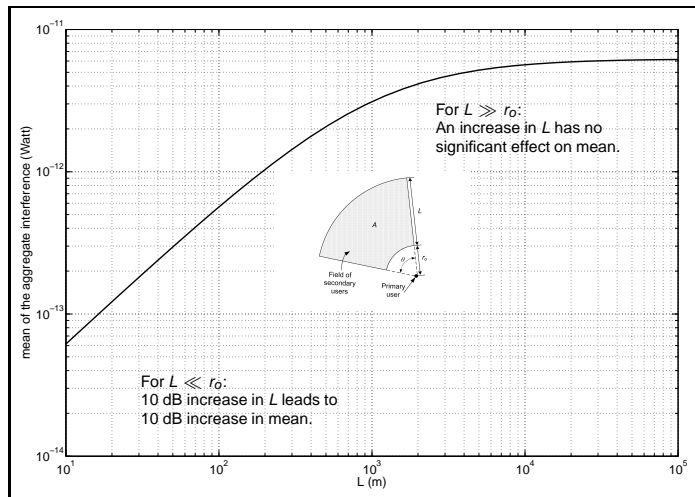
- ▶ For $L \gg r_o$:

$$\mu_A \simeq \frac{1}{n-2} D \theta r_o^{2-n} \mu_W$$



Mean of the Aggregate Interference

Effect of Field Size



Variance of the Aggregate Interference

Formulation

- ▶ Variance of I_A (i.e., $\sum_{i \in N} r_i^{-n} W_i$):

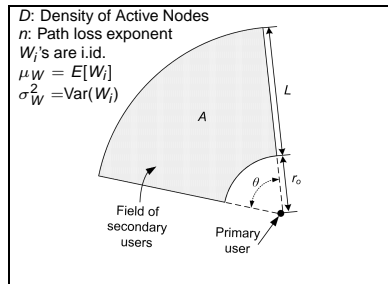
$$\sigma_A^2 = \frac{1}{2n-2} D\theta r_o^{2-2n} \mu_W^2 \left(1 + \frac{\sigma_W^2}{\mu_W^2}\right) \times \left[1 - \left(\frac{r_o}{r_o + L}\right)^{2n-2}\right]$$

- ▶ For $L \ll r_o$:

$$\sigma_A^2 \simeq D\theta r_o^{1-2n} L \mu_W^2 \left(1 + \frac{\sigma_W^2}{\mu_W^2}\right)$$

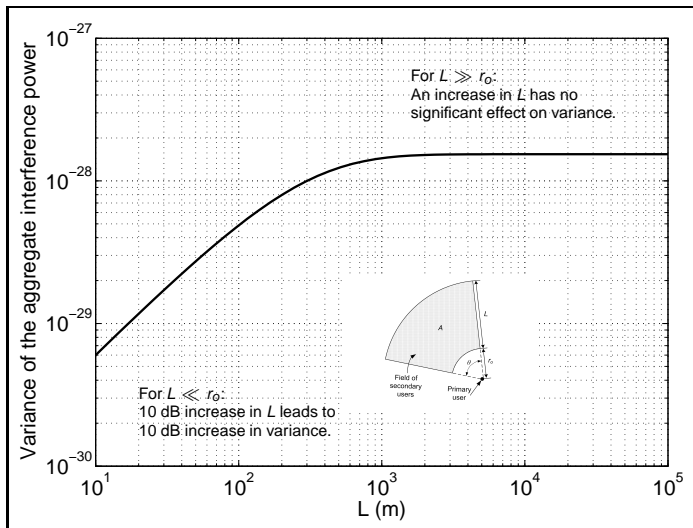
- ▶ For $L \gg r_o$:

$$\sigma_A^2 \simeq \frac{1}{2n-2} D\theta r_o^{2-2n} \mu_W^2 \left(1 + \frac{\sigma_W^2}{\mu_W^2}\right)$$



Variance of the Aggregate Interference

Effect of Field Size



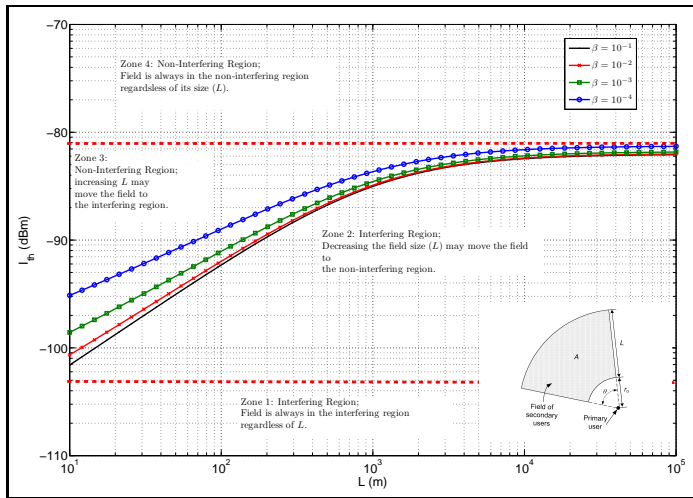
Upper Bound on the Interference Probability

Formulation

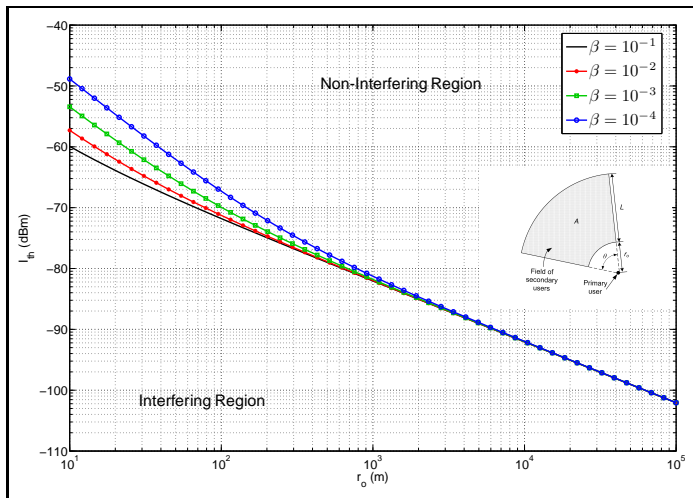
- ▶ Based on Chebyshev inequality, interference probability is bounded by:

$$P(I_A \geq I_{th}) \leq \frac{\sigma_A^2}{(I_{th} - \mu_A)^2}$$

Effect of Field Expansion



Effect of Exclusion Region



Cumulants of I_A

- ▶ Cumulants of I_A (i.e., $\sum_{i \in N} r_i^{-n} W_i$):

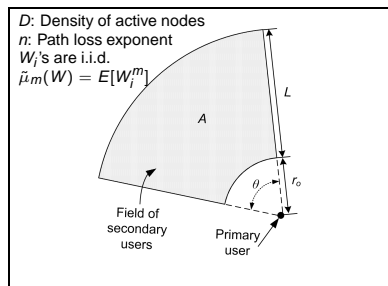
$$\kappa_m(I_A) = \frac{1}{nm-2} D\theta \tilde{\mu}_m(W) r_o^{2-mn} \\ \times \left[1 - \left(\frac{r_o}{r_o + L} \right)^{mn-2} \right]$$

- ▶ For $L \ll r_o$:

$$\kappa_m(I_A) \simeq D\theta r_o^{1-mn} L \tilde{\mu}_m(W)$$

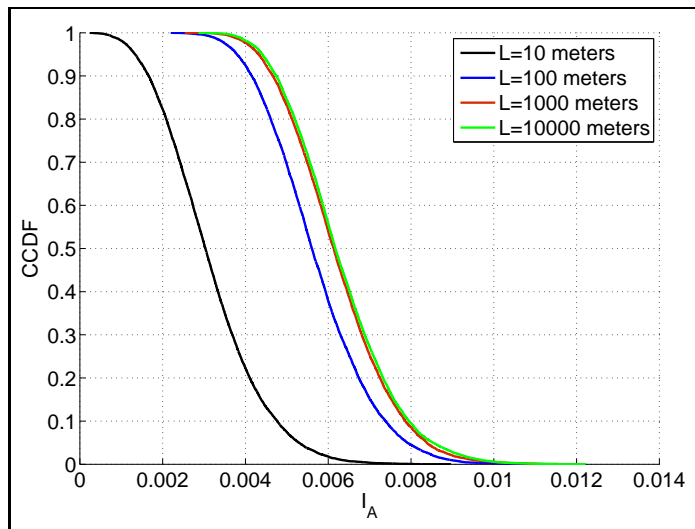
- ▶ For $L \gg r_o$:

$$\kappa_m(I_A) \simeq \frac{1}{nm-2} D\theta \tilde{\mu}_m(W) r_o^{2-mn}$$

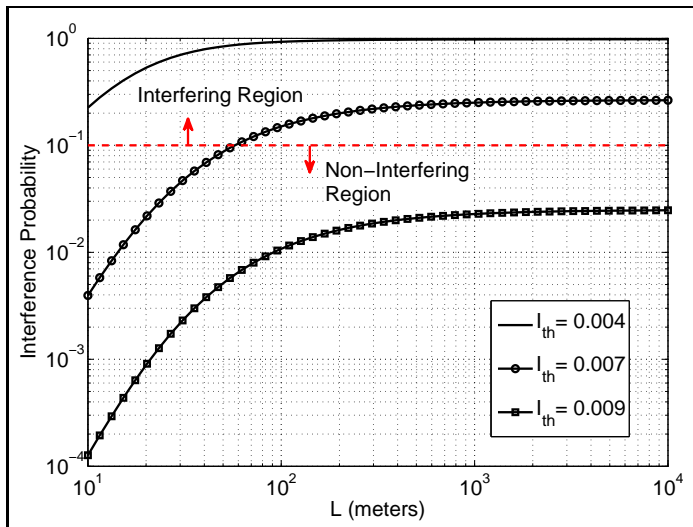


Effect of Field Size on the CCDF of I_A

Simulation



Effect of Field Size on Spectrum Sharing (2)



Summary

- ▶ Asymptotic results for infinite fields:
 - ▶ Applicable for finite but relatively large fields.
 - ▶ Too conservative otherwise.
- ▶ Spectrum sharing vs. field size:
 - ▶ In some cases, small reduction in size may create spectrum sharing opportunities.
 - ▶ In some other cases, huge increase in size may not eliminate spectrum sharing opportunities.
- ▶ In certain cases, concurrent and continuous spectrum sharing is possible without the need for cognitive radio functionalities.

Thank you