Separating the Effect of Independent Interference Sources with Rayleigh Faded Signal Link: Outage Analysis and Applications

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***Abstract*—We show that, for independent interfering sources and a signal link with exponentially distributed received power, the total probability of outage can be decomposed as a simple expression of the outages from the individual interfering sources. We give a mathematical proof of this result, and discuss some immediate implications, showing how it results in important simplifications to statistical outage analysis. We also discuss its application to two active topics of study: spectrum sharing and sum of interference powers (e.g., lognormal) analysis.**

***Index Terms*—Wireless interference, outage probability, Rayleigh fading, spectrum sharing, heterogeneous networks.**

I. Introduction

With the increasing need for spatial spectrum reuse and for co-channel coexistence of heterogeneous wireless networks, the effect of the combined interference from multiple sources is becoming an important topic of study. While this problem has received several decades of theoretical study under various research directions, it remains analytically challenging largely due to the need of finding the distribution of the sum of the random interference powers [1], notably when they are lognormally distributed [2]–[6].

There may be cases in which we wish to study the outage at a receiver due to the sum of independent interfering signal powers, yet the distribution of the constituent interfering powers is unknown. Such a case can be considered in a spectrum sharing scenario where two or more heterogeneous networks share the same spectrum [7]–[11]. Throughout this paper, we only consider spectrum sharing without any spectrum sensing or cognition, which implies that the secondary network necessarily increases the outage probability of primary receivers. The operator of the primary network may be interested in obtaining insights into the additional outage that a receiver would suffer from the deployment of a heterogeneous secondary network, in order to determine the feasibility of spectrum sharing.

In this work, we show that, in the case of independent interfering powers following any distribution, and an independent signal power with exponential distribution, it is possible to separate the outage effect of each interferer. We show this result analytically and exactly, and discuss some of its more immediate consequences for the simplification of outage analysis.

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In Section II, we give the general outage problem as it is often formulated. In Section III, we introduce our main expression for the total outage probability and the mathematical result it is based on, and make some general observations on its consequences to outage analysis. In Section IV, we show how our result can concretely simplify calculations in two important research topics: 1) primary/secondary network sharing scenarios and 2) sum of lognormals modeling. We conclude in Section V.

II. Problem Formulation

Consider a wireless device receiving a useful signal with power S, and suffering from a total received interference of power I. We assume that S is exponentially distributed (due, notably, to Rayleigh fading), while I can be written as

$$I = \sum_{i=1}^{N} I_i, \tag{1}$$

where $\{I_i\}$ is the set of the N independent received interference powers (which may originate from individual transmitters, entire networks or parts thereof, or thermal noise), and the interference powers are assumed to add incoherently (in power) [1]–[3], [6], [12], and are treated as additive white Gaussian noise as far as outage is concerned [2], [12]. We also assume the signal power to be independent of the interference powers.

The outage probability ε on the signal link is obtained from the cumulative distribution function (CDF) of the power ratio S/I:

$$\varepsilon = \mathcal{P}\left(\frac{S}{I} < \beta\right) = \mathcal{P}\left(\frac{S}{\sum_{i=1}^{N} I_i} < \beta\right), \tag{2}$$

where β is the outage threshold in terms of the signal to interference (and noise) power ratio.

III. Analysis

A. Mathematical Result

We introduce a result on random variables (RVs) that will allow us to separate the summation of interference powers under our assumptions. Consider $\{X_i\}$ a set of N independent RVs, and Y an independent exponentially distributed RV. We may then write

$$\mathcal{P}\left(\sum_{i=1}^{N} X_i < Y\right) = \prod_{i=1}^{N} \mathcal{P}\left(X_i < Y\right). \tag{3}$$

The proof is in the Appendix.

B. Separability of the Interference Powers

Applying (3) to the outage expression (2), identifying $X_i = I_i$ and $Y = S/\beta$, and inverting the inequalities gives

$$\varepsilon = 1 - \prod_{i=1}^{N} (1 - \varepsilon_i), \quad \varepsilon_i = \mathcal{P}\left(\frac{S}{I_i} < \beta\right). \tag{4}$$

We have thus expressed the total outage probability ε as a simple algebraic expression of the partial outage probabilities ε_i that would have been caused by each individual interfering source separately (given the same outage threshold β).

The outage probability where *all* the signals are Rayleigh faded has been studied since at least [13]. An expression for the total outage as a function of the Rayleigh faded average powers is given in [14] (see also references therein).

C. Some Useful Consequences

Some interesting observations immediately result from (4):

1) The difficulty of finding the CDF of the sum I of N independent RVs I_i is removed. One only has to find the CDFs of the ratios of the exponentially distributed RV S and the individual RVs I_i. Furthermore, there may be fewer than N such ratios to compute if some of the interference powers I_i are statistically identical (in particular, if all the interference powers have the same statistics, there is then only one ratio to evaluate).
2) The total outage probability ε can be obtained directly from the partial outage probabilities ε_i without the need to know the models of the underlying interferences, which is useful when the partial outage probabilities are obtained from simulation, field measurements, or even usage statistics of a working network.
3) If the interference powers are statistically dependent (due, e.g., to correlated shadowing [2], [4], [6]), our result can still be useful if the interferers can be grouped in such a way that the interferences are independent across the groups. Then, in order to calculate the outage probability, the CDFs of the sum interferences (or the partial outage probabilities) need to be found only within those groups, but not globally. For example, assume a set of 5 interference sources $\{I_1, \ldots, I_5\}$, statistically dependent among themselves, and another set of 5 interferers $\{I_6, \ldots, I_{10}\}$, also dependent. We assume that the two sets are independent among themselves. If we can find (through some other method) the outage probability ε_{1-5} caused only by the first set of interferers (with total power $I_{1-5} = \sum_{i=1}^{5} I_i$) and similarly ε_{6-10} caused only by the second set (with total power $I_{6-10} = \sum_{i=6}^{10} I_i$), then the probability of outage caused by *all* the interferers is $\varepsilon = 1 - (1 - \varepsilon_{1-5})(1 - \varepsilon_{6-10})$ (always assuming a Rayleigh faded signal link).

Our result has important implications in simplifying the analysis of outage caused by multiple interference sources. In the next section, we show how our result can concretely be applied to two research directions that have already received much attention.

IV. Applications to Current Research

Our result in (4) has immediate applications in simplifying various outage calculations, e.g., when a secondary network shares the spectrum without sensing the primary network's activity, thereby increasing the outage probability. The result also simplifies outage probability calculations when the interference is modeled as the sum of independent lognormal RVs.

A. Spectrum Sharing between Primary and Secondary Network

A direct application of our result with $N = 2$ can be seen in the spectrum sharing scenario, where we want to find the additional outage at a primary network receiver due to the deployment of a secondary network, while avoiding the potentially complex tasks of characterizing the interference from either network.

Consider a typical primary receiver, experiencing an outage probability ε_1 due to co-channel interfering primary transmitters (in the absence of the secondary network). We call ε_{T} the maximum outage probability allowed at a primary receiver. It then follows from (4) that the secondary network must be designed in such a way that the outage probability ε_2 caused by its interference alone (in the absence of the primary network's co-channel interferers) satisfies

$$\varepsilon_2 \leq \frac{\varepsilon_{\mathrm{T}} - \varepsilon_1}{1 - \varepsilon_1}. \tag{5}$$

This can be achieved by varying the size or density of the secondary transmitters, their transmit power and protocol, and similar parameters [1], [8]–[11].

B. Outage Analysis Using the Sum of Lognormal Random Variables

The study of the distribution of the sum of several lognormal RVs has received much research attention for several decades [2], [3], and still attracts significant interest [4]–[6]. The research is primarily (but not exclusively) in the field of wireless communications, where it is motivated by the model in which each interference source suffers (possibly correlated) lognormal shadowing. In this case, each I_i is modeled as a lognormal RV, and the challenge is to find the sum distribution of I. However, no closed-form solution exists [4], [5] even for the simplest cases, and in fact there exist many approximating methods that trade accuracy off against simplicity.

It is important to see the context of this research: the goal of finding the sum distribution of the interference is not necessarily an end in itself. Its main use is as an intermediate step in finding the distribution of the signal-to-interference-(and possibly noise)-power ratio, and hence the outage probability [2], [3], [12]. Our result (4) shows that, given independent interference powers and an exponentially distributed received signal power, the unsolved problem of a sum of lognormal RVs disappears, and essentially reduces to the problem of the outage from a single lognormal interferer:

$$\varepsilon_i = \mathcal{P}\left(\frac{S}{I_i} < \beta\right) = \mathcal{P}\left(\sqrt{S} \cdot I_i^{-1/2} < \sqrt{\beta}\right). \tag{6}$$

Now, $\sqrt{S}$ follows a Rayleigh distribution, while ${I_i}^{-1/2}$ is an independent lognormal RV, hence the problem reduces the computation of $M \leq N$ different probabilities from the Suzuki distribution[1], given M statistically distinct lognormal RVs.

The result can also be extended to the case where the interference powers are not lognormal: notably they may include small-scale fading (lognormal-times-fading power, as in [12]), and path loss based on random positions [6]. It remains the case that the most difficult probability calculation, i.e., the summation of random powers, need not be performed.

V. Conclusion

We have shown that, under the assumption of independent received interference powers, and an exponentially distributed received signal power (e.g., due to Rayleigh fading), the outage probability due to all the interfering sources can easily be decomposed into the partial outage probabilities that would be caused by the interferers individually. It is therefore not necessary to know the distribution of the total interference power to find the outage probability of the system, nor in fact even that of the individual interference powers, as long as the corresponding partial outage probabilities are known.

Our result makes important simplifications in the calculation of outage probability, which is applicable in a variety of scenarios, and notably in the case of spectrum sharing, as well as in the case of sum of lognormal RVs interference modeling. It naturally extends to include noise powers as well. It has the advantage of being simple and exact, and can be used in practical scenarios with possibly complex and intractable interfering sources in order to get insights into the effects of those sources.

Appendix

Proof of (3): We can write the left hand side of (3) as

$$\mathbb{E}\left(\mathcal{P}\left(\sum_{i=1}^{N} X_i < Y \middle| X_1, X_2, \ldots, X_N\right)\right). \tag{7}$$

Let μ be the mean of the exponentially distributed RV Y. Then we can write the above as

$$\mathbb{E}\left(\exp\left(-\mu \sum_{i=1}^{N} X_i\right)\right) = \mathbb{E}\left(\prod_{i=1}^{N} \exp\left(-\mu X_i\right)\right). \tag{8}$$

Now, since $X_1, X_2, \ldots, X_N$ are independent RVs, we can write the above as

$$\prod_{i=1}^{N} \mathbb{E}\left(\exp\left(-\mu X_i\right)\right), \tag{9}$$

which is equivalent to

$$\prod_{i=1}^{N} \mathcal{P}\left(X_i < Y\right). \tag{10}$$

■

[1]This is a well-established numerical calculation: e.g., the SuzukiDistribution$[\mu, \nu]$ function in Wolfram *Mathematica*.

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References

[1] M. Win, P. Pinto, and L. Shepp, "A mathematical theory of network interference and its applications," *Proc. IEEE*, vol. 97, pp. 205–230, Feb. 2009.

[2] A. A. Abu-Dayya and N. C. Beaulieu, "Outage probabilities in the presence of correlated lognormal interferers," *IEEE Trans. Veh. Technol.*, vol. 43, pp. 164–173, Feb. 1994.

[3] N. C. Beaulieu and Q. Xie, "An optimal lognormal approximation to lognormal sum distributions," *IEEE Trans. Veh. Technol.*, vol. 53, pp. 479–489, Mar. 2004.

[4] M. Di Renzo, L. Imbriglio, F. Graziosi, and F. Santucci, "Smolyak's algorithm: a simple and accurate framework for the analysis of correlated log–normal power–sums," *IEEE Commun. Lett.*, vol. 13, no. 9, pp. 673–675, Sep. 2009.

[5] C. Tellambura and D. Senaratne, "Accurate computation of the MGF of the lognormal distribution and its application to sum of lognormals," *IEEE Trans. Commun.*, vol. 58, no. 5, pp. 1568–1577, May 2010.

[6] S. S. Szyszkowicz, F. Alaca, H. Yanikomeroglu, and J. S. Thompson, "Aggregate interference distribution from large wireless networks with correlated shadowing: an analytical–numerical–simulation approach," *IEEE Trans. Veh. Technol.*, vol. 60, no. 6, pp. 2752–2764, July 2011.

[7] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Process. Mag.*, vol. 24, no. 3, pp. 79–89, May 2007.

[8] K. Huang, V. K. N. Lau, and Y. Chen, "Spectrum sharing between cellular and mobile ad hoc networks: transmission-capacity trade-off," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1256–1267, Sep. 2009.

[9] Z. Chen, C. Wang, X. Hong, J. Thompson, S. A. Vorobyov, X. Ge, H. Xiao, and F. Zhao, "Aggregate interference modeling in cognitive radio networks with power and contention control," submitted to *IEEE Trans. Commun.*, Aug. 2010. Available: http://arxiv.org/abs/1008.1043.

[10] A. S. Kahlon, S. S. Szyszkowicz, S. Periyalwar, and H. Yanikomeroglu, "Identification of spectrum sharing opportunities for a finite field secondary network through an exact outage expression under Rayleigh fading," in *Proc. 2011 IEEE International Symposium on Personal Indoor and Mobile Radio Communications*, pp. 1–5.

[11] A. S. Kahlon, S. Periyalwar, H. Yanikomeroglu, and S. S. Szyszkowicz, "Outage in a cellular network overlaid with an ad hoc network: the uplink case," in *Proc. 2011 IEEE International Symposium on Personal Indoor and Mobile Radio Communications*, pp. 1–5.

[12] C. Fischione and M. D'Angelo, "An approximation of the outage probability in Rayleigh-lognormal fading scenarios," UC Berkeley, Tech. Rep., Mar. 2008.

[13] K. W. Sowerby and A. G. Williamson, "Outage probability calculations for a mobile radio system having multiple Rayleigh interferers," *Electron. Lett.*, vol. 23, no. 11, pp. 600–601, May 1987.

[14] S. Kandukuri and S. Boyd, "Optimal power control in interference-limited fading wireless channels with outage-probability specifications," *IEEE Trans. Wireless Commun.*, vol. 1, no. 1, pp. 46–55, Jan. 2002.