Strategic Densification With UAV-BSs in Cellular Networks

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***Abstract*—Using base stations mounted on an unmanned aerial vehicle (UAV-BSs) is a promising new evolution of wireless networks for the provision of on-demand high data rates. While many studies have explored deploying UAV-BSs in a green field—no existence of terrestrial BSs, this letter focuses on the deployment of UAV-BSs in the presence of a terrestrial network. The purpose of this letter is twofold: 1) to provide supply-side estimation for how many UAV-BSs are needed to support a terrestrial network so as to achieve a particular quality of service and 2) to investigate where these UAV-BSs should hover. We propose a novel stochastic geometry-based network planning approach that focuses on the structure of the network to find strategic placement for multiple UAV-BSs in a large-scale network.**

***Index Terms*—Stochastic geometry, network planning, low altitude platforms, unmanned aerial vehicles, drone placement.**

I. Introduction

CELLULAR network densification is an essential technique to accommodate the rapidly increasing demand for high system and per-user data rate [1]. Unconstrained location, easy relocation, and rapid deployment ability make the UAV-BSs more suitable than T-BSs for temporary densification to capture a sudden rise in traffic [2]. On the contrary, building permanent T-BSs is cost-ineffective: It requires obtaining a permit and high site rental costs [1]. Many of these BSs become underutilized during light traffic periods. Thus, instead of deploying excessive T-BSs to satisfy peak traffic demand, we advocate deploying on-demand UAV-BSs.

Most recent studies focused on finding an efficient placement of UAV-BSs where no T-BSs exist, see [3]–[7]. With the existence of T-BSs, UAVs are employed in [8] as a static aerial relay and in [9] as a mobile aerial relay to assist the communication between a T-BS and a user terminal. In [10], a neural model is used to intelligently allocate multiple UAV-BSs to capacity-strained areas in a cellular network.

Given a terrestrial network suffering from a temporary high demand for data rate, we consider *strategic placement* of UAV-BSs to enhance both the capacity and the coverage probability. Inspired by improving the spatial regularity of the BS locations maximizes the coverage probability and the performance of the network [11]–[14], we propose an aerial-terrestrial network planning approach that relies on the spatial structure of the network to strategically augment (find the best 3D locations) the UAV-BSs to terrestrial network. We choose the horizontal location of the UAV-BSs such that they improve the spatial regularity of the network's BSs (the T-BSs and the projection of the UAV-BS locations on the ground). As for the altitude of the UAV-BSs, each one is adjusted based on the horizontal distance to its nearest BS for even better network throughput compared to random or fixed altitude cases. As such, we fill the coverage gaps and maximize the network's throughput.

Our main contribution is to propose the *strategic placement* of UAV-BSs over an already existing terrestrial network. Notably, although we put forward this letter in the context of UAV-BS placement, it is a general large-scale network planning approach that could be used to design network densification regardless of the cell size or type. In this letter, we do not aim to find the global optimal best-performance placement, but instead, aim to present a proof-of-concept and set in motion a novel research framework that will warrant further investigation. This letter could be envisioned as the dual of the work presented in [13].

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II. Placement and Performance Metrics

A. Placement (Regularity) Metric

To improve the spatial regularity, we need a metric to measure it. The edges of a Delaunay triangulation of a set of points, which are random quantities, can be used as a metric to characterize the spatial organization of that set of points [14], [15]. Specifically, the coefficient of variation (CoV)—the ratio of the standard deviation $\sigma_{\rm D}$ to the mean $\mu_{\rm D}$—of the edges of a Delaunay triangulation is used in [14] and [15] to characterize the spatial regularity/irregularity of BS locations. The metric is defined as $C_{\rm D} = \frac{1}{k_{\rm D}} \cdot \frac{\sigma_{\rm D}}{\mu_{\rm D}}$, where $k_{\rm D} \cong 0.492$ is a normalization factor such that $C_{\rm D} = 1$ for the Poisson point process (PPP). As $C_{\rm D}$ increases, the spatial irregularity increases (i.e., regularity decreases).

B. Performance Metric

1) Median SINR: The signal-to-interference-plus-noise-ratio (SINR) value that 50% of the users in the system can achieve.

2) Normalized Network Capacity: The sum of rates of all users in the system in bits/s/Hz normalized by the sum-rate of a PPP-deployed network (deployed according to a PPP) before augmenting the UAV-BSs.

III. Downlink System Model

We consider a network consisting of a set of T-BSs and a set of UAV-BSs. For both sets, unless otherwise stated, all parameters and specifications—including the terrestrial channel model—follow the urban macro (UMa) scenario [16]. For UAV-BSs, we use the air-to-ground channel model in [3].

System Setup: We assume equal resource allocation scheme. Independent of network deployment, the users are spatially distributed according to a PPP with uniform data rate demand.

Each user is connected to a BS (either aerial or terrestrial) that provides the strongest SINR. Neither fading nor shadowing is considered in this scenario. The UAV-BSs and the T-BSs are identical: They transmit at the same fixed power level, have the same carrier frequency, serve with a full buffer, are equipped with a single antenna, and so on. While each T-BS has an omni-directional antenna, a UAV-BS has a directional antenna (due to its high altitude and high line-of-sight opportunity) to reduce interfering with others.

Air-to-Ground Channel Model: We use a generic path loss model proposed in [3]. The model considers line-of-sight (LoS) and non-line-of-sight (NLoS) links as

$$\begin{aligned} \mathrm{PL}_{\mathrm{LoS}} &= 20\log\left(\frac{4\pi f_c d_{ij}}{c}\right) + \eta_{\mathrm{LoS}} + D(\phi_{ij}), \\ \mathrm{PL}_{\mathrm{NLoS}} &= 20\log\left(\frac{4\pi f_c d_{ij}}{c}\right) + \eta_{\mathrm{NLoS}} + D(\phi_{ij}). \end{aligned} \tag{1}$$

The occurrence probability of the LoS link is given by

$$\mathbb{P}_{\mathrm{LoS}}(\theta_{ij}) = \left(1 + \alpha \cdot \exp\left(-\beta[(\theta_{ij} - \alpha])\right)\right)^{-1} \tag{2}$$

and the NLoS link by $\mathbb{P}_{\mathrm{NLoS}}(\theta_{ij}) = 1 - \mathbb{P}_{\mathrm{LoS}}(\theta_{ij})$, where $\theta_{ij} = \sin^{-1}(h_j/d_{ij})$ is the elevation angle (degrees) between a user i and UAV-BS j. Note that h_j is the altitude of the UAV-BS, $d_{ij} = \|u_i - l_j\|$ is the Euclidean distance between a user located at u_i and a UAV-BS located at $l_j \in \mathbb{R}^3$, and f_c is the carrier frequency, where c is the speed of light. Also, η_{LoS} and η_{NLoS} (dB) are environment-dependent losses corresponding to the LoS and NLoS links, respectively. α and β are other environment-dependent parameters. The antenna of the UAV-BSs is perpendicular to the ground, and its *directivity gain* $D(\phi_{ij})$ is specified [16] as $D(\phi_{ij}) = \min[\frac{\phi_{ij}}{15^\circ}, 20\,\mathrm{dB}]$, where $\phi_{ij} = 90 - \theta_{ij}$. The probabilistic path loss can be written as

$$\mathrm{PL} = \mathrm{PL}_{\mathrm{LoS}} \times \mathbb{P}_{\mathrm{LoS}}(\theta_{ij}) + \mathrm{PL}_{\mathrm{NLoS}} \times \mathbb{P}_{\mathrm{NLoS}}(\theta_{ij}). \tag{3}$$

Terrestrial-BS Deployment: The deployment of the T-BSs is modelled using a perturbed triangular lattice (PTL): a repulsive point process that provides variable spatial regularity. The uniform PTL is a result of the independent uniform random displacement on a disk for each point (BS location) of a triangular lattice from its original location [15].

IV. Strategic Placement Method

The best horizontal placement of the UAV-BSs must decrease the spatial irregularity of the network, which can be achieved by spreading out to the greatest possible extent the horizontal location of the new BSs from each other and from the existing BSs. One approach to achieving the highest spatial regularity, i.e., the farthest distribution of the points, is to maximize the sum of the distance between the nodes [17] as we formulate in Eq. (4). As for the altitude, it must be chosen such that it provides the highest signal strength inside the targeted area without interfering with the surrounding BSs.

A. Horizontal Placement

Given a set $\mathcal{A} = \{a_1, a_2, \ldots, a_{\mathrm{N_t}}\} \subset \mathbb{R}^2$ of the locations of $\mathrm{N_t}$ terrestrial BSs and a set $\mathcal{C} = \{c_1, c_2, \ldots, c_{\mathrm{N_d}}\} \subset \mathbb{R}^2$ representing $\mathrm{N_d}$ predefined potential horizontal locations of UAV-BSs, we aim to select M locations from the set $\mathcal{C}$ for UAV-BSs to hover over such that the spatial regularity of all the nodes is maximized. The percentage of the augmented UAV-BSs to the T-BSs is $\zeta = (M/\mathrm{N_t}) \times 100\%$.

Algorithm 1: Horizontal Placement Algorithm

Input : T-BS locations set $\mathcal{A}$, candidate locations set $\mathcal{C}$, and the desired number of the UAV-BSs M.
Output : A set $\mathcal{S} = \{s_1, s_2, \ldots, s_M\}$ of UAV-BS horizontal locations that maximizes the network regularity.

$\mathcal{S} \Leftarrow \{\ \}$; $\quad \mathcal{B} \Leftarrow \{\mathcal{A}\}$
while $|\mathcal{S}| < M$ **do**
  Find $c_d \in \mathcal{C}$, such that $\mathbf{dist}(c_d, \mathcal{B}) = \max\{\mathbf{dist}(c_i, \mathcal{B}){:}c_i \in \mathcal{C}\ \forall\ i\}$
  $\mathcal{S} \Leftarrow \mathcal{S} \cup \{c_d\}$; $\quad \mathcal{B} \Leftarrow \mathcal{B} \cup \{c_d\}$; $\quad \mathcal{C} \Leftarrow \mathcal{C} \setminus \{c_d\}$
end
return $\mathcal{S}$

Let the set $\mathcal{N}$ be the concatenation of the sets $\mathcal{A}$ and $\mathcal{C}$. Formally, $\mathcal{N} = \mathcal{A}\|\mathcal{C} = \{a_1, a_2, \ldots, a_{\mathrm{N_t}}, c_1, c_2, \ldots, c_{\mathrm{N_d}}\} \subset \mathbb{R}^2$. Members of $\mathcal{N}$ are indexed by j, and for simplicity we refer to them only by their indexes. The members of $\mathcal{N}$ with index $j \leq \mathrm{N_t}$ belong to $\mathcal{A}$ and those with index $j > \mathrm{N_t}$ belong to $\mathcal{C}$.

A binary quadratic formulation for maximizing the sum of the distance between the nodes is as follows:

$$\underset{z_j}{\text{maximize}} \quad \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{N}} w_{ij} z_i z_j \tag{4a}$$

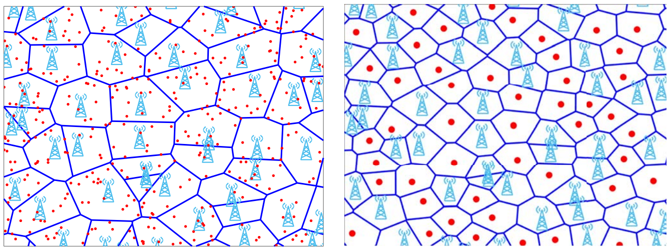
$$\text{subject to} \quad \sum_{j\in\mathcal{N}} z_j = p \tag{4b}$$

$$z_j \in \{0, 1\}, \qquad j \in \mathcal{N} \tag{4c}$$

$$z_j = 1, \qquad \forall j \leq \mathrm{N_t}, \quad j \in \mathcal{N}. \tag{4d}$$

The Boolean variable z_j indicates whether a node location is selected. The distance between the node i and j is defined by w_{ij}, and of course $w_{ii} = 0$. The total number of the selected nodes $p = \mathrm{N_t} + M$. The constraint (4d) forces all terrestrial BSs to be selected.

The problem is difficult to solve optimally on a large scale. (It is an NP-hard as similar problems are known to be NP-hard, see [17].) Therefore, we propose a greedy-type heuristic algorithm to deal with it. The proposed algorithm consists of two main sets: the candidates set $\mathcal{C}$ and the solution set $\mathcal{B}$. The candidates set represents the possible horizontal locations of UAV-BSs, while the solution set contains the T-BS locations as well as the already chosen UAV-BS horizontal location. The solution set of this algorithm is initialized by the locations of the T-BSs $\mathcal{A}$. Then, at each iteration, a new UAV-BS horizontal location that is the furthest apart from all points in this solution set is added until the required number of the UAV-BSs M is achieved. Algorithm 1 illustrates the pseudo-code of the UAV-BSs' horizontal placement algorithm. Its complexity at most is $O(N^3)$, where $N = |\mathcal{N}| = \mathrm{N_t} + \mathrm{N_d}$. A notation $\mathbf{dist}(a_i, a_j) = \|a_i - a_j\|$ denotes the Euclidean distance between the points a_i and a_j, and the notation $\mathbf{dist}(a_i, \mathcal{B})$ denotes the distance between the point a_i and its nearest neighbour point in the set $\mathcal{B}$. Namely, $\mathbf{dist}(a_i, \mathcal{B}) = \min\{\mathbf{dist}(a_i, a_j){:}\ a_j \in \mathcal{B}, a_j \neq a_i,\ \forall\ j\}$. Also, $|\mathcal{B}|$ indicates the number of points in the set $\mathcal{B}$. Fig. 1 depicts the horizontal placement using Algorithm 1. In practical implementations, a cloud-based entity [2] could be used to implement as well as to collect, store, and manage the information required to execute this Algorithm.



(a) (b)

Fig. 1. (a) Arbitrary T-BSs deployment (blue towers) with random candidate location (red dots). (b) T-BSs with the projection of the UAV-BSs on the ground (red disks), as selected from the random candidates using Algorithm 1.

B. Vertical Placement

We adjust the altitude of the UAV-BSs ($\mathcal{H} = \{h_1, h_2, \ldots, h_M\}$) to provide maximum coverage. We approximate the coverage region for each UAV-BS to be a disk. The radius of the coverage disk R_j is half the horizontal distance to the nearest neighbour BS, either another UAV-BS or a T-BS. From [4], we observe that the optimal altitude h_j depends only on the radius of the coverage region R_j and the optimal elevation angle θ_{opt}. The altitude can be found using [4]

$$h_j = k \times R_j \tan(\theta_{\text{opt}}), \tag{5}$$

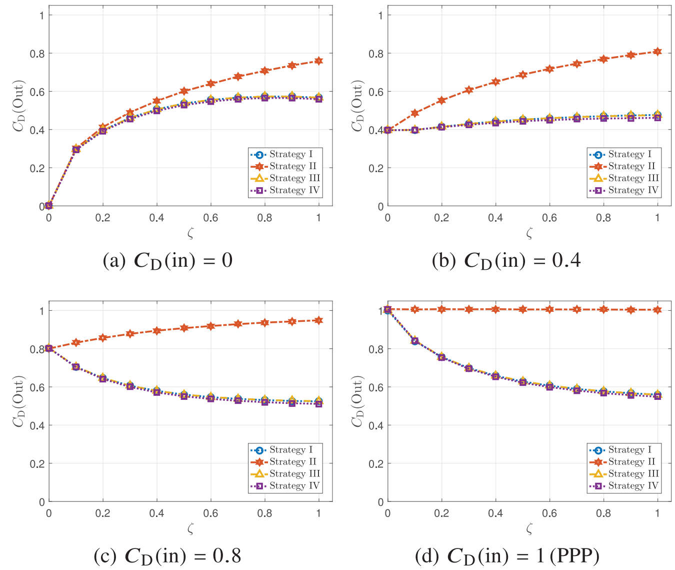
where $0 < k \leq 1$ is a scaling factor used to reduce the altitude of the UAV-BS so as to back off the interference. In this letter, using a line search method, we select $k = 0.6$ for all UAV-BSs.

We refer to the strategic placement above as *Strategy* I; it is a placement based merely on the long-time average of the user locations. To enrich the discussion, we also introduce three slightly different placement strategies. 1) *Strategy* II: random horizontal placement with fixed altitude. 2) *Strategy* III: strategic horizontal placement, as in Algorithm 1, with random altitude. 3) *Strategy* IV: strategic horizontal placement, as in Algorithm 1, but with further readjustment flexibility that allows the horizontal location of the UAV-BS to be located exactly in the centre of the user locations within its coverage region, defined by its Voronoi cell. The altitude is then found according to Eq. (5).

V. RESULTS

We carry out the simulation over 1000 Monte Carlo runs. We execute the experiment for T-BSs deployed with four different spatial regularity values—similar to [13, Fig. 1]—as generated using the PTL model. We consider a network of around 100 T-BSs deployed with an average inter-site distance of 500 m [16], which is established on an approximately 4.6 km × 4.6 km geographical region. An average of 2000 random users are uniformly distributed over the whole region, but only users within the central area of 2.8 km × 2.8 km are considered in the simulation to eliminate the boundary effects. For the same reason, the set of the candidate horizontal locations of the UAV-BSs—a large set of discrete random locations—is generated inside the *convex haul* of T-BS locations. Network parameters are set according to the UMa scenario [16]. Since we consider an urban environment, the parameters of the air-to-ground channel model (α, β, η_{LoS}, η_{NLoS}) are given by (9.6117, 0.1581, 1 dB, 20 dB) [3], [4], respectively. Hence, the optimal elevation angle $\theta_{\text{opt}} = 42.44°$ [6] for this environment, as it is found by numerically solving [3, eq. (13)].

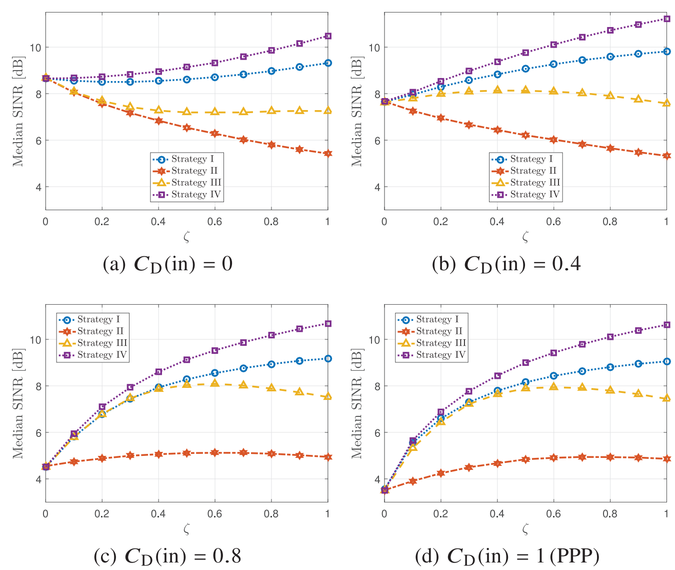
In Figs. 2–4, each sub-figure represents a specific spatial deployment of T-BSs with certain regularity values



(a) $C_D(\text{in}) = 0$ (b) $C_D(\text{in}) = 0.4$

(c) $C_D(\text{in}) = 0.8$ (d) $C_D(\text{in}) = 1$ (PPP)

Fig. 2. The $C_D(\text{out})$ of the network as a function of the augmentation percentage ζ. Each sub-figure represents a specific regularity value $C_D(\text{in})$ of the original network deployment (similar to [13, Fig. 1]).

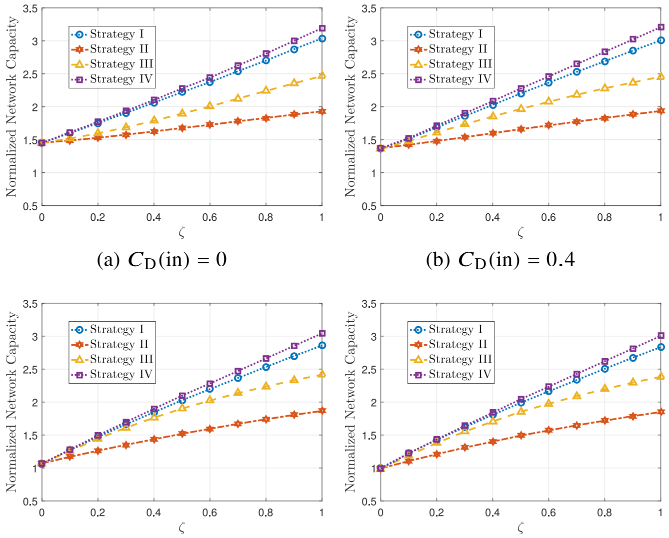


(a) $C_D(\text{in}) = 0$ (b) $C_D(\text{in}) = 0.4$

(c) $C_D(\text{in}) = 0.8$ (d) $C_D(\text{in}) = 1$ (PPP)

Fig. 3. The median SINR as a function of the ratio of augmentation of the UAV-BSs ζ for different network deployment regularity $C_D(\text{in})$.

($C_D(\text{in})$)—similar to [13, Fig. 1]—and depicts four placement strategies. The figures show that the performance of the network depends on its spatial structure. In Fig. 2, we see that the strategic horizontal placement of the UAV-BSs improves the spatial regularity of the network ($C_D(\text{out})$). As a particular case, adding more points to an entirely regular (i.e., hexagonal layout) or highly regular spatial pattern has to result in a deterioration of the overall regularity, which is the case in Figs. 2a and 2b. However, this may be the best achievable regularity when more points are added to a very regular spatial pattern. (Indeed, the best placement algorithm either much improves the regularity or provides the least degradation in regularity.) The spatial regularity of the network is similar for both *Strategy* I and *Strategy* IV, i.e., the average horizontal location of the UAV-BSs over all realizations is the same for both strategies.



(a) $C_{\mathrm{D}}(\mathrm{in}) = 0$ (b) $C_{\mathrm{D}}(\mathrm{in}) = 0.4$

(c) $C_{\mathrm{D}}(\mathrm{in}) = 0.8$ (d) $C_{\mathrm{D}}(\mathrm{in}) = 1$ (PPP)

Fig. 4. Normalized network capacity as a function of the UAV-BS augmentation ratio ζ for different network deployment regularity $C_{\mathrm{D}}(\mathrm{in})$.

Figs. 3 and 4 show the median SINR and the normalized sum-rate as a function of the UAV-BSs augmentation ratio.[1] They indicate that *Strategy* IV provides best median SINR and normalized sum-rate followed by *Strategy* I. For example, for a PPP-deployed terrestrial network (Figs. 3d and 4d), the median SINR and the normalized sum-rate for *Strategy* IV and *Strategy* I are (10.63 dB, 2.99) and (9.05 dB, 2.48), respectively. Remarkably, they both are much better than *Strategy* II (4.8 dB, 1.85). In fact, *Strategy* IV simply provides further tuning to the statistical approach of *Strategy* I. The disadvantage of *Strategy* IV is that it requires instantaneous UAV-BS movement to track the mobile users. The frequent repositioning of the UAVs may be energy inefficient and would be computationally more expensive. (In this strategy, the additional complexity includes the complexity of Voronoi tessellation $O(p \log p)$, where p is the number of the BSs and the complexity of point-in-polygon algorithm $O(n \log n)$, where n is the number of users.)

For a terrestrial network deployed according to a hexagonal layout, we observe that adding more UAV-BSs results in a better sum-rate even for *Strategy* II, although the median SINR deteriorates—look at Fig. 3a and Fig. 4a together. Noteworthy, adding more nodes to an entirely regular deployment deteriorates the regularity and the SINR but provides more capacity.

Given the required increase in the capacity, we can estimate how many UAV-BSs are needed using Fig. 4. For example, the sum-rate of a PPP-deployed network (Fig. 4d) can be doubled by adding UAV-BSs at the augmentation ratio $\zeta = 0.48$.

VI. Conclusion

We provided a framework for improving network capacity by strategically placing UAV-BSs to assist a cellular network. Based on the maximization of the spatial regularity of the network's structure, we proposed a novel large-scale network planning approach that helps to decide where UAV-BSs hover to maintain connectivity, fill the coverage holes and to boost network capacity. We showed that the best place for UAV-BSs is where their projection on the ground improves the spatial regularity of the network. We observed that even the random augmentation of the UAV-BSs enhances network throughput (yet it could be associated with deterioration of the spatial regularity and the SINR); the strategic augmentation maximizes this throughput. Allowing additional horizontal location adjustment based on the exact location of the users provides further gain. However, in practice, we need to trade off the energy consumption and computation time with the throughput reward. Future works could extend this framework to the non-uniform distribution of the users.

[1] UAV-BSs augmentation ratio could be greater than 1 ($\zeta > 1$), i.e., more UAV-BSs than T-BSs. Hence, we expect the performance continues to improve until it reaches either a peak—and then declines—or a plateau; the exact behaviour critically depends on the terrestrial and aerial channel models.

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