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Quantifying the Regularity of Perturbed Triangular Lattices using CoV-Based Metrics for Modeling the Locations of Base Stations in HetNets

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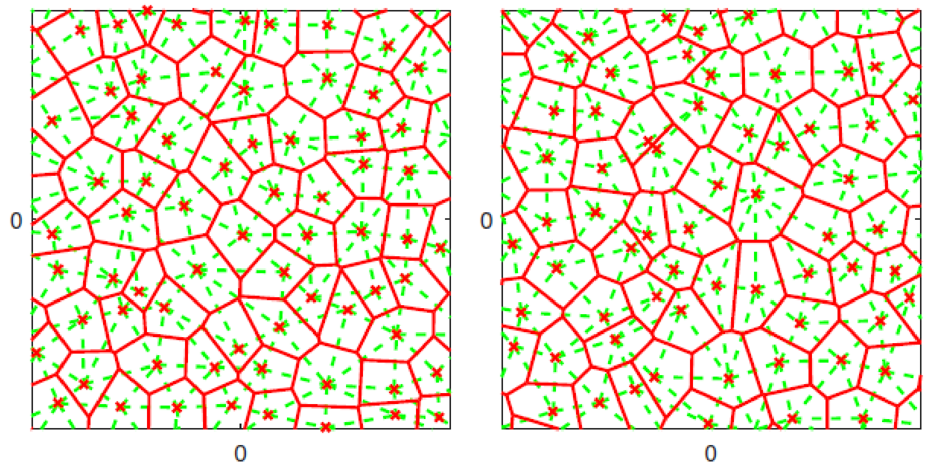
Motivation

Q: Are these two Base station (BS) locations similar in terms of regularity and network performance?



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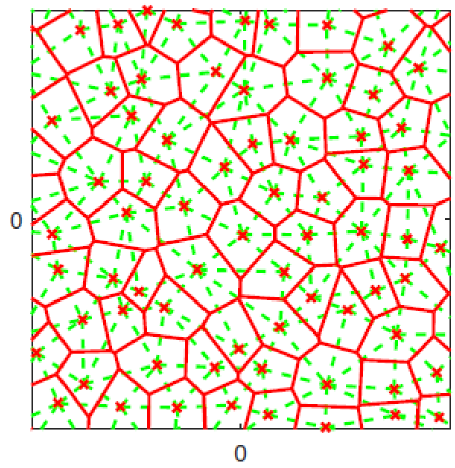

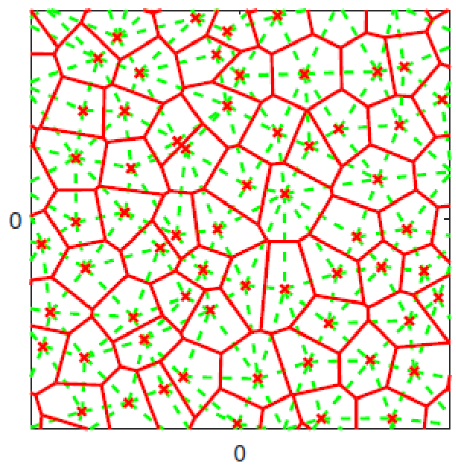

A: Yes, they are alike in terms of regularity and signal-to-interference ratio (SIR)!

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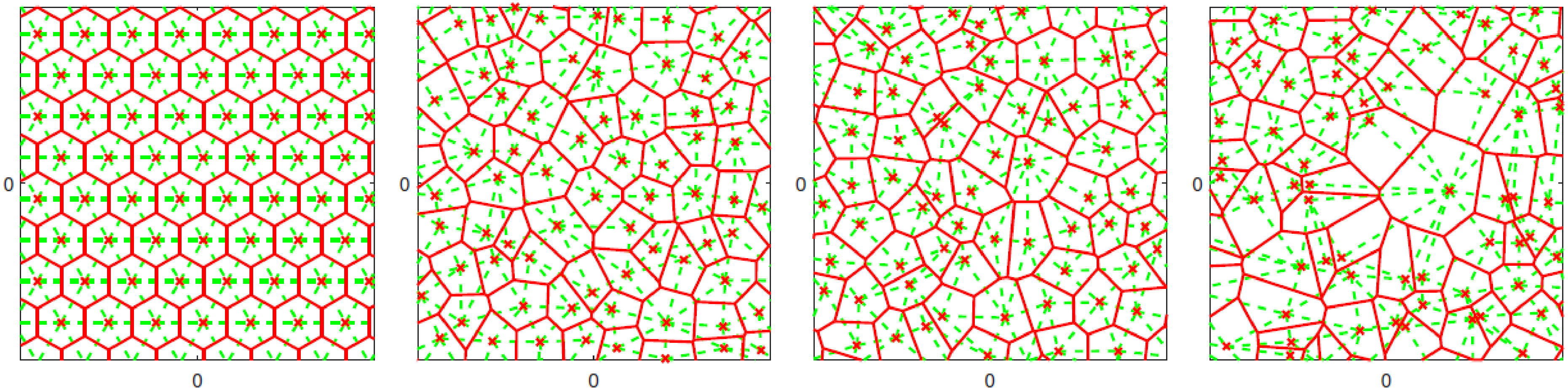

(a) Triangular Lattice (TL) (b) (c) (d) Poisson Point Process (PPP)

A: Yes, they are alike in terms of regularity and signal-to-interference ratio (SIR)!

Motivation

Why do we need a scalar metric for regularity?

- To compare the performance (e.g., SIR) of different spatial patterns and models for BS locations
- 3.4 dB SIR gain between Triangular Lattices (TL) and Poisson point process (PPP) deployments with the same density, for path-loss exponent $\propto = 4$ [Haenggi, 2014]
- Describe BS locations using two scalars: (Density, regularity)
- Tune the internal parameters of point processes

Why do use the PTL models for BS deployment?

- Simple implementation, used in industry
- Span the whole range of regularity
- Tractable [Banani *et al.*, 2015]

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Regularity CoV-Based Metrics

The coefficient of variation (CoVs) of three geometric properties of point processes:

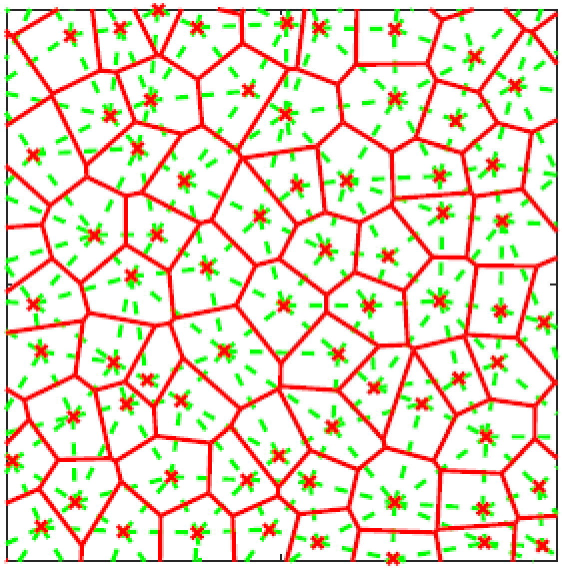
- CoV of the Lengths of Delaunay Triangulation Edges

$$C_{\mathrm{D}} = \frac{1}{k_{\mathrm{D}}} \cdot \frac{\sigma_{\mathrm{D}}}{\mu_{\mathrm{D}}}, \qquad k_{\mathrm{D}} \cong 0.492,$$

- CoV of the Areas of Voronoi Tessellation Cells

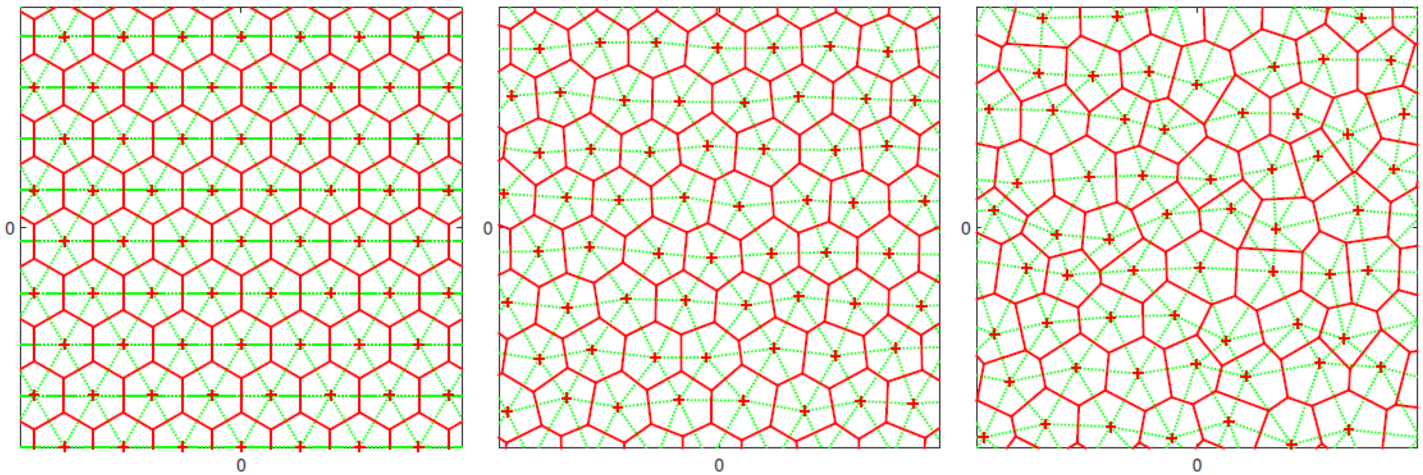
$$C_{\mathrm{V}} = \frac{1}{k_{\mathrm{V}}} \cdot \frac{\sigma_{\mathrm{V}}}{\mu_{\mathrm{V}}}, \qquad k_{\mathrm{V}} \cong 0.529,$$

- CoV of the Distances to the Nearest Neighbour



$$C_{\mathrm{N}} = \frac{1}{k_{\mathrm{N}}} \cdot \frac{\sigma_{\mathrm{N}}}{\mu_{\mathrm{N}}}, \qquad k_{\mathrm{N}} = \sqrt{\frac{4-\pi}{\pi}} \cong 0.5227.$$

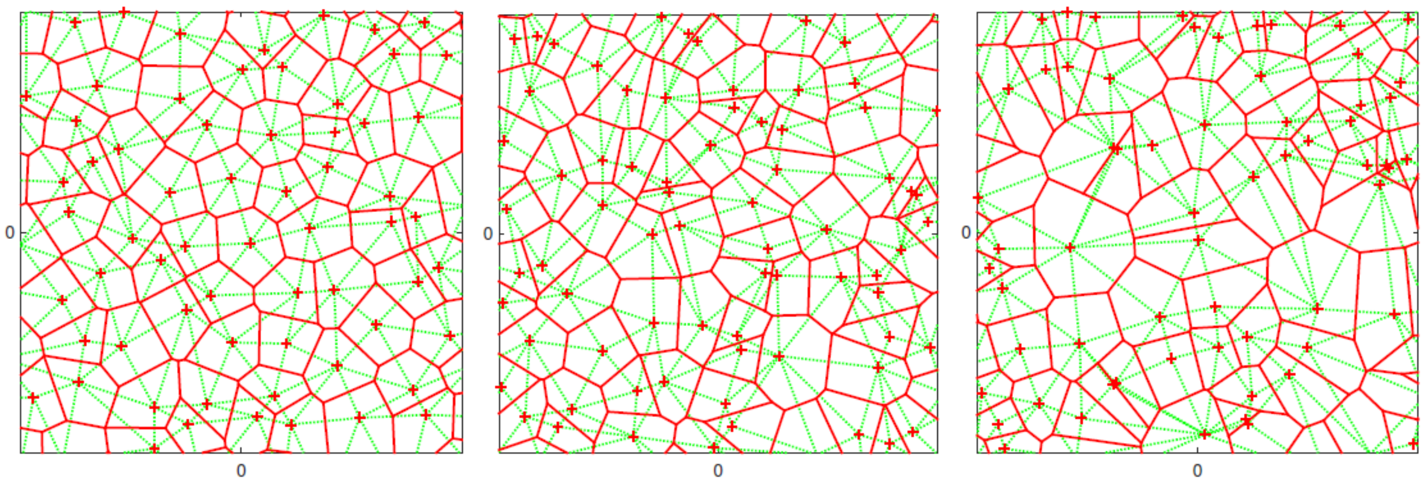
BS locations with different amount of regularity



(a) $C_{\mathrm{D}} = 0$ (TL)

(b) $C_{\mathrm{D}} \approx 0.2$

(c) $C_{\mathrm{D}} \approx 0.4$

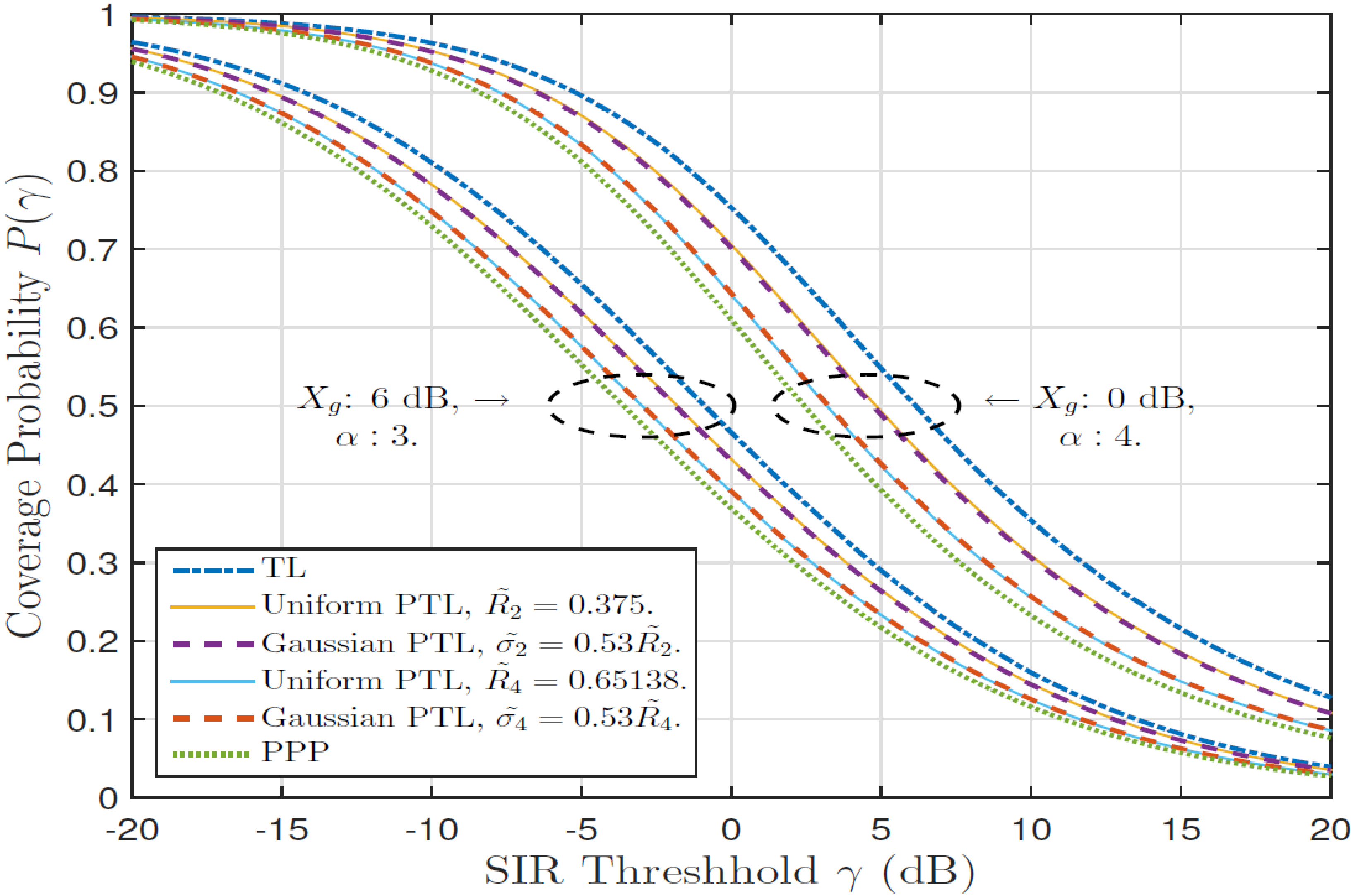


(d) $C_{\mathrm{D}} \approx 0.6$

(e) $C_{\mathrm{D}} \approx 0.8$

(f) $C_{\mathrm{D}} \approx 1$ (PPP)

Effect of regularity on SIR



Perturbed Triangular Lattices (PTLs)

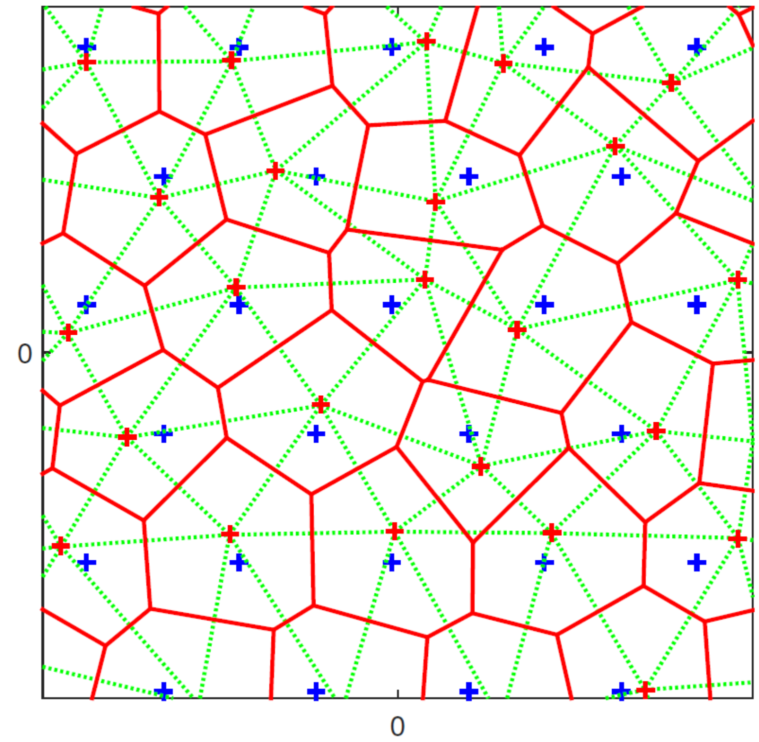
Why do use the PTL models for BS deployment?

- Simple implementation, used in industry
- Span the whole range of regularity
- Tractable (Banani, Adve, and Eckford, 2015)

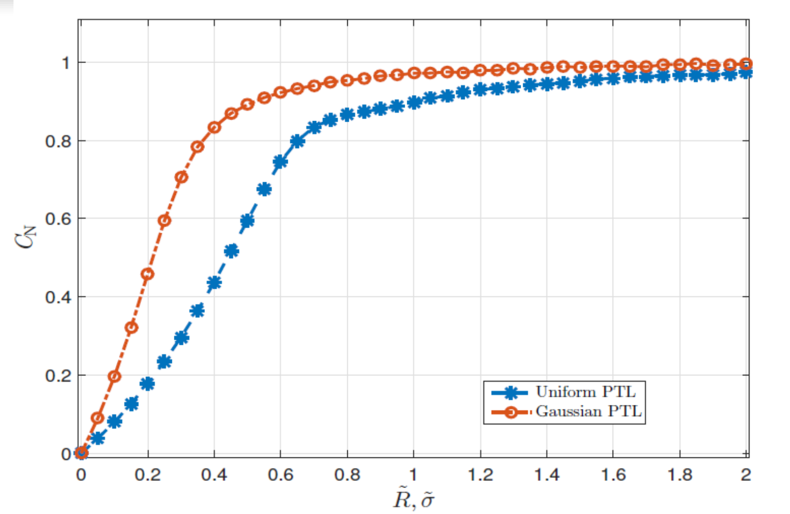
Perturbed Triangular Lattices (PTLs)

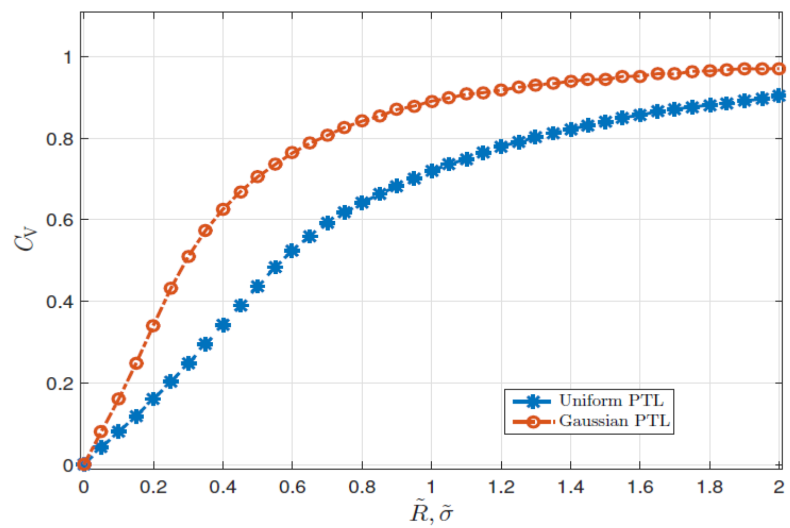
How to generate the PTL?

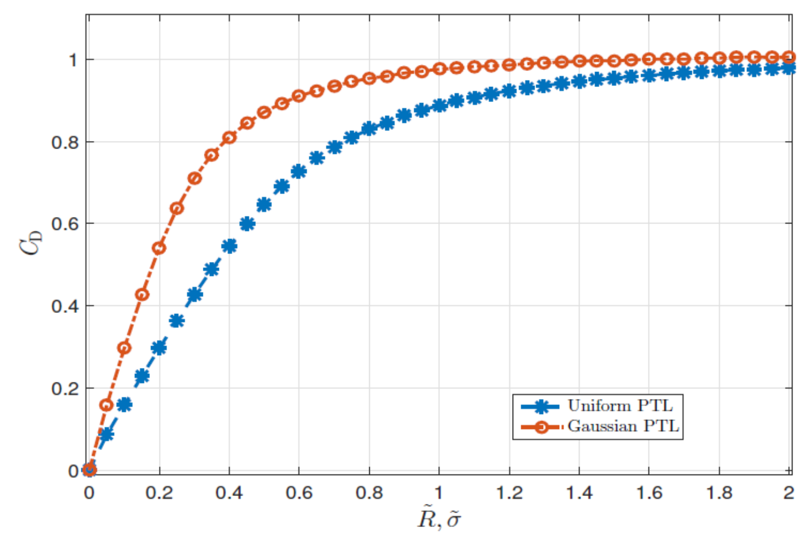
- Start with triangular lattice (in blue)
- Independent perturbation (e.g, uniform on disc , or Gaussian)
- Tunable amount of regularity



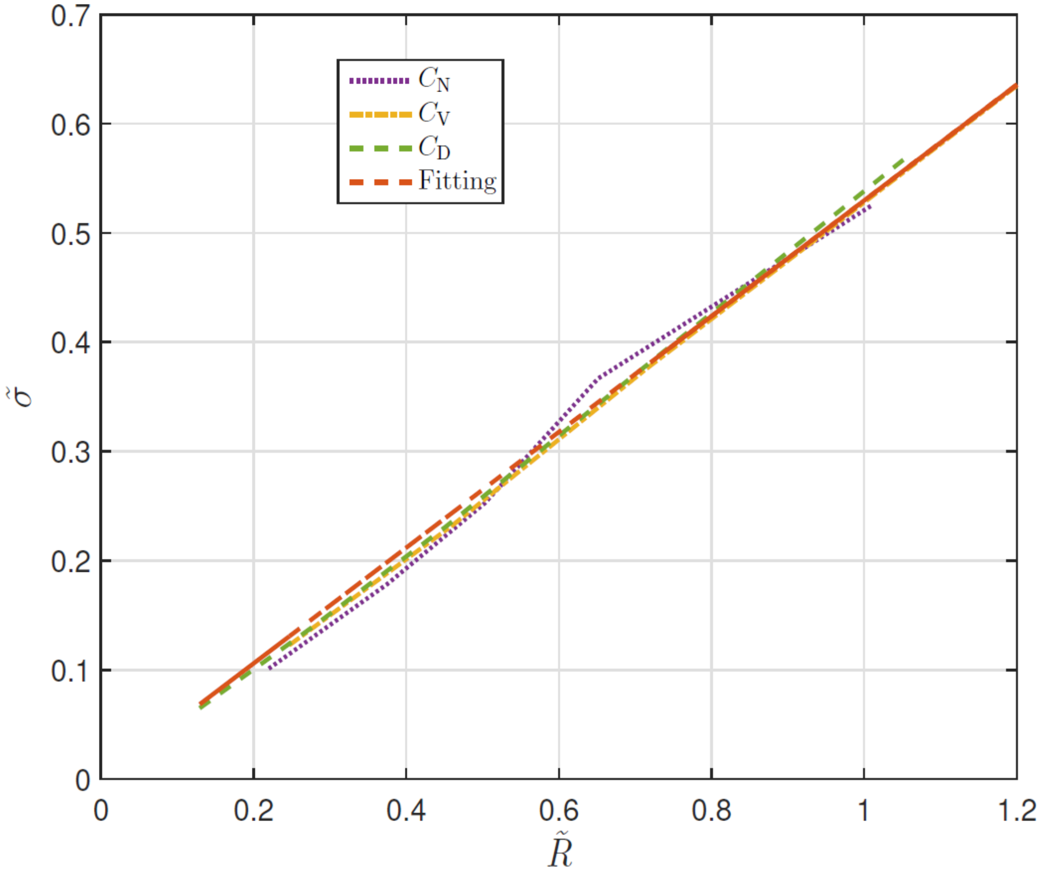
Regularity of PTLs





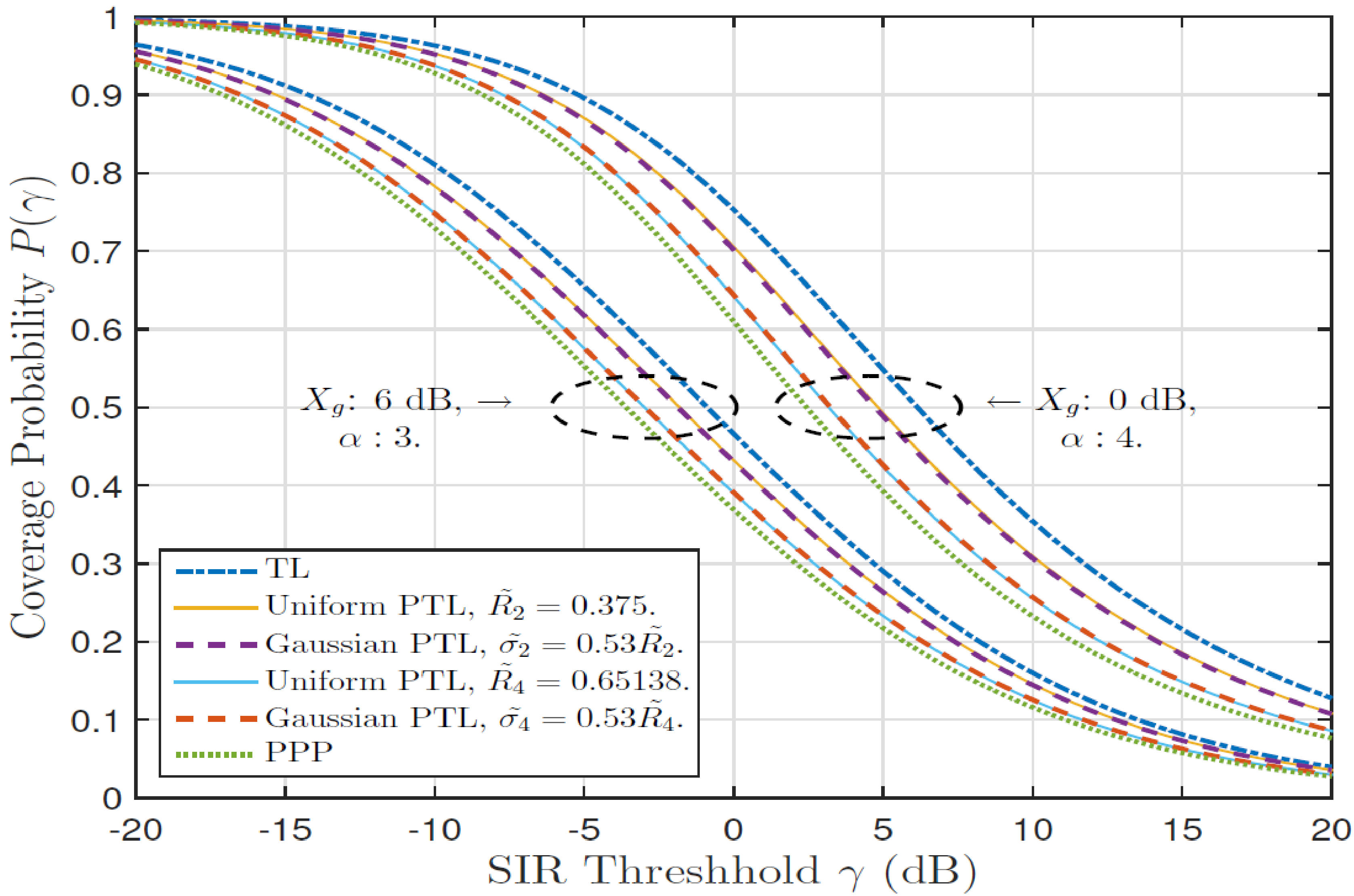


Matching Gaussian and Uniform PTL



$$\tilde{\sigma} \approx 0.53\tilde{R}.$$

Matching Gaussian and Uniform PTL



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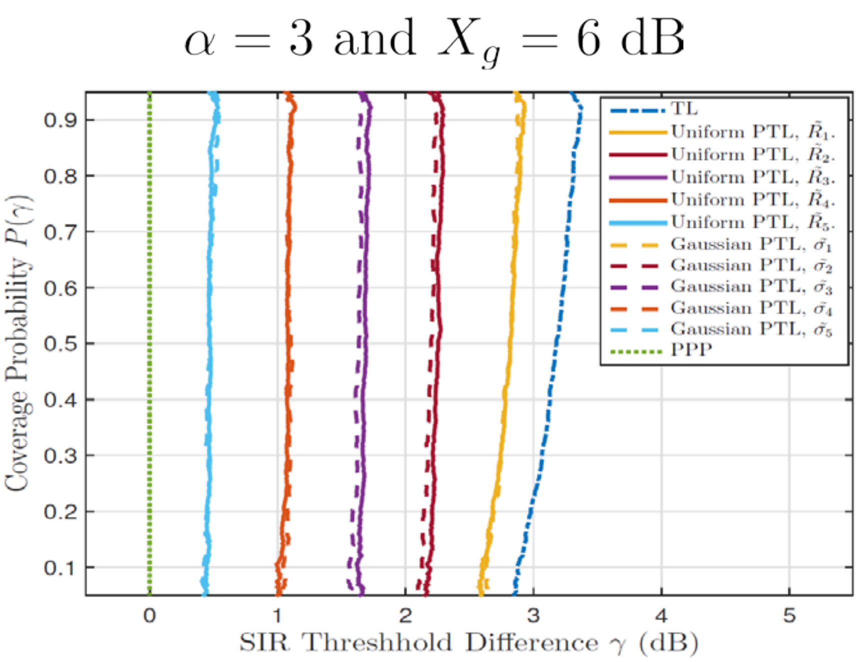
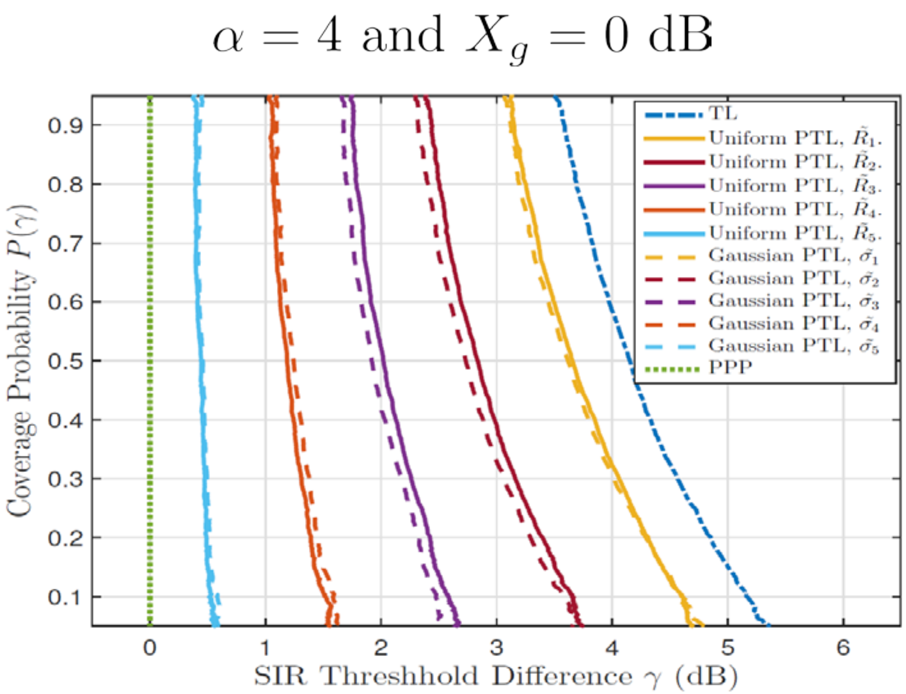
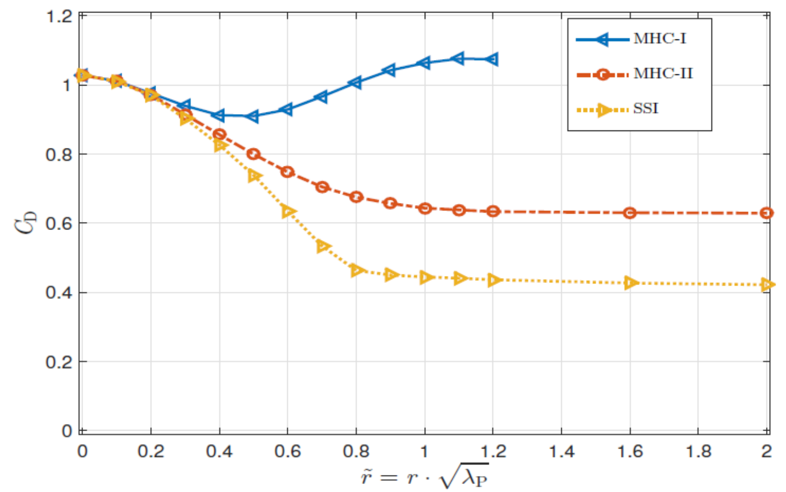
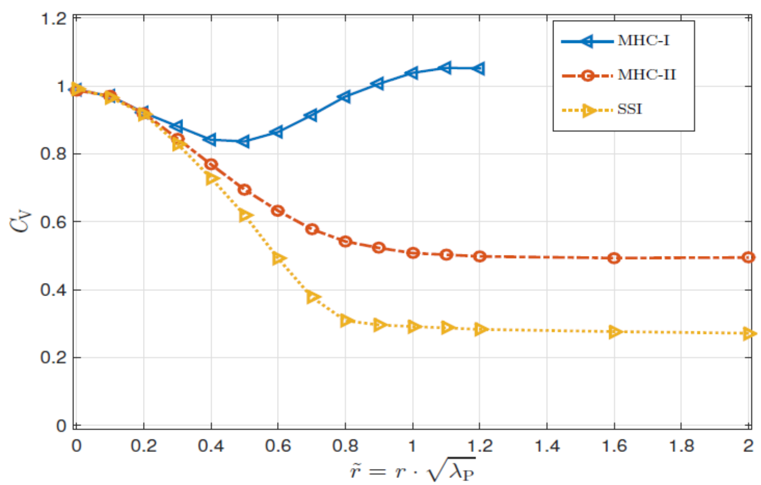


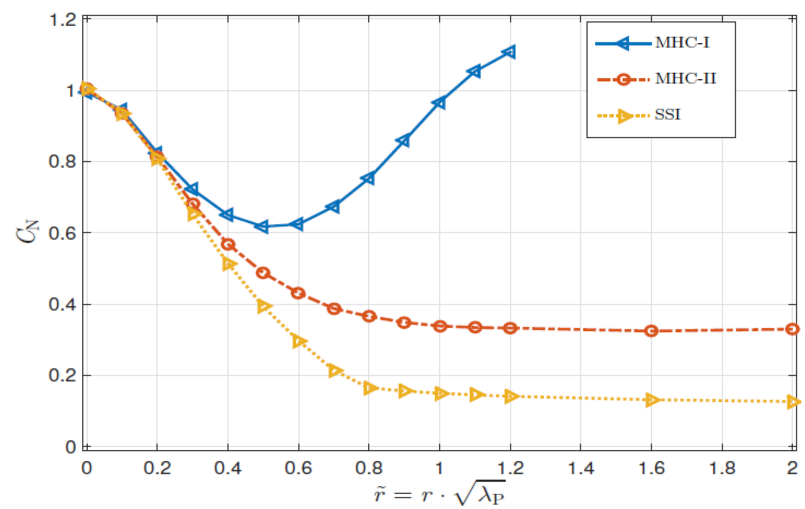


Matching and interchanging the two PTL models, within about 0.1 dB error in SIR.

Hard-core Models







F. Lagum, S. Szyszkowicz, and H. Yanikomeroglu, “CoV-Based Metrics for Quantifying the Regularity of Hard-Core Point Processes for Modeling Base Station Locations,” IEEE Wireless Commun. Lett., vol. 5, no. 3, pp. 276–279, June 2016.

Conclusion

- We proposed a novel approach for mapping between two spatial models; Specifically, uniform PTL and Gaussian PTL using CoV-based metrics as an intermediate step.

- We found a simple relation to match internal parameters two PTL.

- We advocates modeling the placement of different types of BSs in HetNets using one of the PTL models, because of their simple and efficient implementation, their full regularity range (from the TL to the PPP).

Extensions to this work

- Fitting real BS location data to RPP models using CoV-based metrics.
- Fitting different types of RPP models to each other.
- Ultimately, we would like to describe the spatial structure of any wireless network using only two scalars: the density of the BSs and a regularity metric value.