Cooperative versus Full-Duplex Communication in Cellular Networks: A Comparison of the Total Degrees of Freedom

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***Abstract*—In this paper, we compare the potential gain that can be obtained from separate-antenna full-duplex transceivers in cellular networks with that obtained from cooperative operation of half-duplex base stations. The gain is characterized in terms of the total degrees of freedom (DoF). In particular, we consider a system composed of two adjacent MIMO base stations. We consider a single time-frequency resource unit that is used by each base station to communicate with one MIMO user. For the full-duplex case, we assume that each node has a configurable transceiver that can allocate some antennas to the uplink and the remainder to the downlink. We provide an upper bound on the total DoF of the full-duplex system and derive the optimal antenna allocation at each node. We compare the derived upper bound for the full-duplex transceivers with the achievable DoF in the case of half-duplex cooperative multipoint transmission. Our results indicate that the achievable DoF in the cooperative case is always greater than or equal to the upper bound on the DoF of the full-duplex system. We further investigate the case of full-duplex cooperative multipoint transmission and show that the maximum DoF gain due to full-duplex operation cannot exceed** 12.5% **of the DoF of the half-duplex cooperative system.**

***Index Terms*—Degrees of freedom, full-duplex separate-antenna transceivers, cooperative communication.**

I. Introduction

The need for 5^{th} Generation wireless networks is driven by the exponential growth in the number of connected devices and mobile data. In order to provide Gigabit-rate data services regardless of the location of the user, future network deployments are expected to be much denser compared to earlier networks. Furthermore, several breakthroughs are needed in the physical and medium access layers, e.g., single-frequency full-duplex processing, cooperative communication, and massive multiple input multiple output (MIMO). However, all these techniques yield interference among the network nodes that should be efficiently managed. The degrees of freedom (DoF) criterion provides a measure of the interference management capability of a network. In most cases, the total DoF is equal to the number of interference-free streams that can be transmitted in the network [1], [2], [3].

Shared-antenna full-duplex wireless systems have recently received considerable attention as they can potentially double the DoF of the system. In these systems, each antenna element can simultaneously transmit and receive in the same frequency band [4]. A single-cell system was considered in [5] where a full-duplex base station (BS) simultaneously communicates with multiple half-duplex mobile stations and an achievable scheme was provided that can double the DoF of the system when the number of mobile stations is sufficiently greater than the number of antennas at the BS.

Full-duplex transmission poses several challenges from the implementation, physical layer, and medium access control layer perspectives [4]. For example, simultaneous transmission and reception from a single node causes the transmitted signals to loop back to the receiver causing self-interference [6]. In addition, full-duplex transmission increases the inter-user interference due to increasing the number of simultaneous transmissions from the network nodes. A full-duplex shared-antenna transceiver was presented in [7] that can provide 40 dB self-interference isolation. However, this is not sufficient for operating full-duplex cellular systems. For a full-duplex BS to achieve link SNR equal to that of its half-duplex counterpart, it must suppress self-interference by more than 106 dB [8].

Separate-antenna full-duplex systems can suppress self-interference more efficiently than shared-antenna systems. Separate-antenna full-duplex transceivers divide the available antennas into two groups; one for transmission and the other for reception. Propagation-domain isolation techniques can then be utilized for self-interference cancellation by using a combination of path loss, cross-polarization, and antenna directionality [9]. A single-cell system with multiple mobile nodes was considered in [10] where the BS is equipped with a full-duplex MIMO transceiver with M_U receive antennas and M_D transmit antennas. In this work, it was shown that the achievable DoF using interference alignment for the full-duplex system is greater than that achieved for a half-duplex system employing $\max\{M_U, M_D\}$ antennas at the BS.

In this paper, we evaluate the maximum gain in DoF that can be obtained by using separate-antenna full-duplex transceivers in a two-cell MIMO wireless network. In particular, we consider a MIMO system composed of two base stations each equipped with M antennas. Each BS communicates with a user equipment (UE) equipped with N antennas. We assume that each node has a separate-antenna full-duplex transceiver

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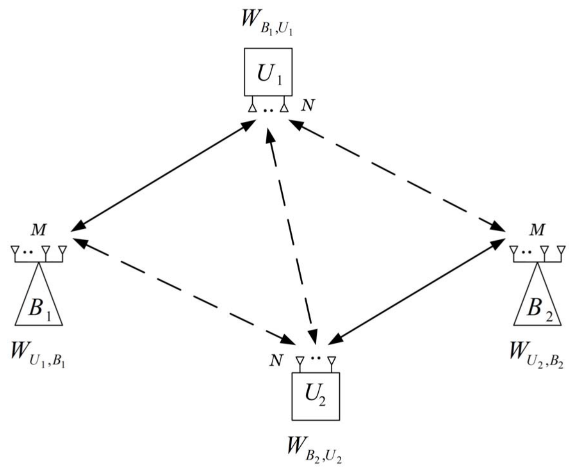

Fig. 1. Two-cell system with MIMO full-duplex transceivers. Solid lines indicate desired signal links while dashed ones indicate interference links.

that can allocate some antennas to uplink and some to the downlink. We consider a single time-frequency resource block that is utilized by each BS to communicate with one of its attached UEs. We investigate the inter-cell and full-duplex interference management capability of the network by deriving an upper bound on the total DoF of the full-duplex system. We solve for the optimal antenna allocation at the BSs and UEs. Although this problem is non-convex, we obtain a closed-form solution for the upper bound on the total DoF and the optimal antenna allocation required to achieve this upper bound.

We compare the full-duplex system with a cooperative half-duplex system with the same total number of antennas at each node. In the cooperative half-duplex system, the two base stations cooperatively transmit to the two users using all the available antennas. We show that the DoF of the half-duplex cooperative system is always greater than or equal to the upper bound on the DoF of the full-duplex noncooperative system. When $M/N = \frac{3}{2}$, cooperative half-duplex communication offers 25% gain in the DoF compared to the maximum possible DoF gain that can result from full-duplex operation. We further investigate the case where the two full-duplex base stations can cooperatively communicate with the two full-duplex UEs and show that the possible relative gain in the DoF of the system due to full-duplex operation cannot exceed 12.5% of the DoF of the half-duplex cooperative system.

The remainder of this paper is organized as follows. In Section II, we present the system model for the full-duplex case. The upper bound on the DoF of the full-duplex system is derived in Section III. The cooperative communication case is studied in Section IV, and the DoF of the two systems are compared. Finally, we present our conclusions in Section V.

II. Full-duplex system model

We consider a two-cell full-duplex system as shown in Fig. 1. We focus on a single time-frequency block that is utilized by each BS to communicate with one UE. We assume that the ith UE, denoted by U_i, communicates only with BS B_i where $i \in \mathcal{I} = \mathcal{I} = \{1,2\}$. We consider the case of full-duplex operation where the uplink and downlink transmissions occur simultaneously in the same frequency band. We assume that the self-interference between the transmitting and receiving antennas of each node is cancelled. This assumption is common in the literature investigating the DoF of full-duplex systems, e.g., [5], [10], [11]. We also assume that there is no interference between the two base stations[1]. Let M and N denote the total number of antennas at each BS and each UE, respectively. We assume that M_T antennas are used for downlink transmission at each BS while M_R antennas are used for uplink reception. Similarly, N_T antennas are used for uplink transmission at each UE while N_R antennas are used for downlink reception. We assume that the channel between any two nodes is block fading and constant during one time slot and that the channel coefficients are independent, identically distributed and drawn from a continuous distribution. Hence, the channel matrices are independent and full-rank almost surely.

Let $\mathcal{N}$ denote the set comprising the *types of nodes* in the system, i.e., $\mathcal{N} = \{B, U\}$ where B indicates that the node is a BS while U indicates that the node is a UE. We use the symbol P_i to refer to the ith node of type P where $P \in \mathcal{N}$ and $i \in \mathcal{I}$. Also, let W_{Q_i,P_i} denote the intended message from node P_i to node Q_i where $P, Q \in \mathcal{N}$, $P \neq Q$ and $i \in \mathcal{I}$. We assume that the messages are mutually independent. The transmitted signal from B_i at time slot n is denoted by $\boldsymbol{x}_{B_i}(n) \in \mathbb{C}^{M_T \times 1}$ while the transmitted signal from U_i at time slot n is denoted by $\boldsymbol{x}_{U_i}(n) \in \mathbb{C}^{N_T \times 1}$ where $i \in \mathcal{I}$. Let $\boldsymbol{H}_{Q_i P_j}(n)$ denote the matrix containing the coefficients of the channel from the transmitter at node P_j to the receiver at node Q_i at time slot n where $P, Q \in \mathcal{N}$ and $i, j \in \mathcal{I}$. The received signal at the jth BS and jth UE are given respectively by

$$\mathbf{y}_{B_j}(n) = \sum_{i=1}^{2} \boldsymbol{H}_{B_j,U_i}(n)\boldsymbol{x}_{U_i}(n) + \boldsymbol{z}_{B_j}(n) \quad (1)$$

$$\mathbf{y}_{U_j}(n) = \sum_{i=1}^{2} \boldsymbol{H}_{U_j,B_i}(n)\boldsymbol{x}_{B_i}(n) + \boldsymbol{H}_{U_j,U_k}\boldsymbol{x}_{U_k}(n) + \boldsymbol{z}_{U_j}(n) \quad (2)$$

where $j \in \mathcal{I}$ and $k \neq j$. In (1) and (2), $\boldsymbol{z}_{P_j}(n)$ is the additive noise vector received at node P_j in the nth time slot whose elements are independent circular Gaussian random variables with zero-mean and unit-variance.

Let $\mathbf{y}^n_{P_i}$ denote the sequence containing the received signal vectors at node P_i from time slot 1 up to time slot n where $P \in \mathcal{N}$ and $i \in \mathcal{I}$. The encoder at node P_i is given by

$$\boldsymbol{x}_{P_i}(n) = \mathcal{E}_{P_i}\left(W_{Q_i,P_i}, \mathbf{y}^{n-1}_{P_i}\right). \quad (3)$$

On the other hand, for a transmission block of length L, the decoder at node Q_i can be expressed as

$$\hat{W}_{Q_i,P_i} = \mathcal{D}_{Q_i}\left(W_{P_i,Q_i}, \mathbf{y}^{L}_{Q_i}\right). \quad (4)$$

[1]This assumption is well-motivated by the low elevation angles of the antennas employed at the base stations. The potential presence of such interference can never increase the DoF gain beyond that reported in this work.

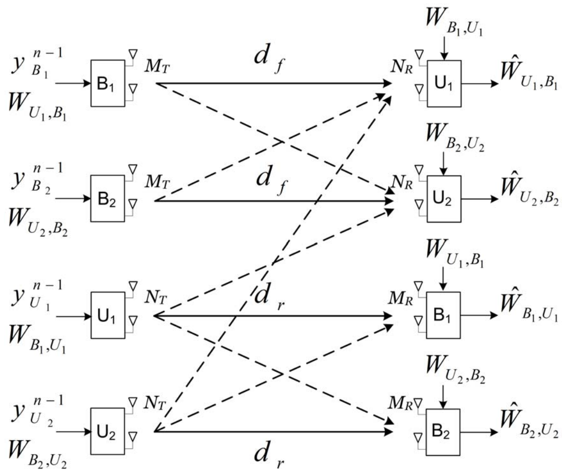

Fig. 2. Equivalent model for the two-cell full-duplex system.

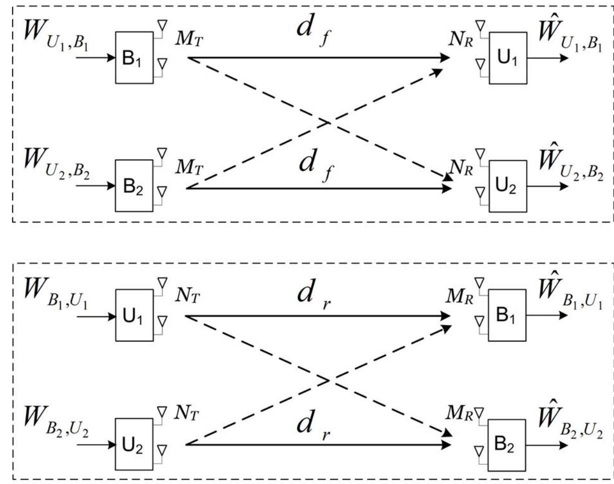

Fig. 3. Resulting system after removing interference links between the UEs.

The total DoF of the system can be defined as

$$D = \max_{M_T, M_R, N_T, N_R} \lim_{\text{SNR}\to\infty} \frac{C(\text{SNR}, M_T, M_R, N_T, N_R)}{\log(\text{SNR})}$$
$$\text{subject to} \quad M_T + M_R = M$$
$$N_T + N_R = N \tag{5}$$

where $C(\text{SNR}, M_T, M_R, N_T, N_R)$ is the total capacity of the network for a given antenna configuration $\{M_T, M_R, N_T, N_R\}$. The DoF represents the total number of interference-free streams that can be transmitted from the sources to the destinations. Due to the symmetry between the two cells, we denote the number of interference-free streams transmitted in the forward link from each BS to its attached UE by d_f and the number of interference-free streams transmitted in the reverse link from each UE to its BS by d_r. As a result, the total DoF of the system is given by $2d_f + 2d_r$.

Fig. 2 shows the equivalent system model after separating the transmit and receive sections of each transceiver. The resulting system in Fig. 2 is a partly-connected asymmetric 4-user MIMO interference channel with causal output feedback and knowledge of the message of the transmitter of each full-duplex node at its co-located receiver. The K-user $L_T \times L_R$ fully-connected MIMO interference channel was studied in [12], [13] and $L_T L_R/(L_T + L_R)$ DoF per user was shown to be achievable. The achieved DoF coincides with the upper bound on the DoF when $K \geq (L_T + L_R)/\gcd(L_T, L_R)$ where $\gcd(M_T, M_R)$ denotes the greatest common divisor of L_T and L_R. The DoF of the 3-user $L_T \times L_R$ MIMO interference channel was characterized in [14]. The results of [14] were shown to be useful also for characterizing the DoF of the 4-user SISO interference channel in [15]. However, the resulting system in Fig. 2 differs from the classical 4-user interference channel in two aspects. The first one is the absence of some interference links, i.e., partial connectivity, and the second is the presence of causal output feedback and message feedback. The problem is further complicated due to the flexibility of choosing the antenna configuration in (5) Hence, the results of [12]–[15] cannot be directly applied to the system in Fig. 2.

III. Bounding the DoF of the full-duplex system

In this section, we derive an upper bound on the total DoF of the full-duplex system shown in Fig. 2. First, let us assume that the antenna allocation at each node is given, i.e., each BS uses M_T antennas for transmission and M_R antennas for reception while each UE uses N_T antennas for transmission and N_R antennas for reception, where $M_T + M_R = M$ and $N_T + N_R = N$. Let us remove the interference links from each transmitting UE on the other receiving UE. This operation cannot decrease the DoF of the system. The resulting system is composed of two isolated two-user MIMO interference channels. We can also remove the output feedback $\mathbf{y}_{P_i}^{n-1}$ at transmitter P_i where $P \in \mathcal{N}$ and $i \in \mathcal{I}$, and knowledge of message W_{P_i,Q_i} at receiver Q_i, where $P, Q \in \mathcal{N}$, $P \neq Q$, and $i \in \mathcal{I}$, due to the independence between the transmitted messages and the absence of any interaction between the two interference channels. The resulting system is shown in Fig. 3. Applying the results on the DoF of the two-user MIMO interference channel in [1] on the two isolated channels in Fig. 3 yields

$$2d_f \leq \min\{2M_T, 2N_R, \max\{M_T, N_R\}\} \tag{6}$$
$$2d_r \leq \min\{2N_T, 2M_R, \max\{N_T, M_R\}\}. \tag{7}$$

Starting from the system model in Fig. 2, we can obtain a third upper bound on the DoF of the system as follows. We first eliminate all the messages that are not associated with node U_2 transmitter or node U_1 receiver, i.e., only the messages W_{B_2,U_2} and W_{U_1,B_1} remain. Next, we transform the 4-user partially-connected interference channel in Fig. 2 into a 2-user Z-interference channel as follows. We start by grouping the transmitting nodes B_1, B_2, and U_1, into a terminal T_1 (with $2M_T + N_T$ antennas) that transmits the message W_{U_1,B_1} and has knowledge of $\mathbf{y}_{B_1}^{n-1}, \mathbf{y}_{B_2}^{n-1}$, and $\mathbf{y}_{U_1}^{n-1}$. Furthermore, we group the receiving nodes U_2, B_1 and B_2 into the terminal T_2 with $2M_R + N_R$ antennas and prior knowledge of both messages W_{U_1,B_1} and W_{B_2,U_2}. Note that all operations described so far cannot reduce the total DoF of the system and therefore do not contradict our upper bound argument [11], [16]. Since T_2 has prior knowledge of W_{U_1,B_1} and $\mathbf{y}_{B_1}^{n-1}$, it can reconstruct $\boldsymbol{x}_{B_1}(n)$ and subtract its contribution from its received signal. Hence, T_2 can eliminate the interference signal resulting from the transmission of T_1. The resulting system is shown in Fig. 4 which is a two-user Z-channel with output feedback and prior knowledge of all messages at terminal T_2. Using the results on the DoF of the MIMO Z-channel in [16] and noting that the additional information about W_{U_1,B_1} and W_{B_2,U_2} at T_2 will not increase the DoF,[2] we obtain the following upper bound

$$d_f + d_r \leq \max\{N_T, N_R\}. \tag{8}$$

The upper bounds on d_f and d_r presented in (6)–(8) depend on the number of antennas allocated for transmission and reception at each node. In order to obtain the optimum antenna allocation, we maximize the total DoF of the system for a given number of antennas at each BS and each UE subject to the constraints on d_f and d_r in (6)–(8), i.e.,

$$\begin{aligned} \max_{d_f,d_r,M_T,N_T} \quad & 2d_f + 2d_r \\ \text{subject to} \quad & d_f \leq \min\left\{M_T, N-N_T, \frac{1}{2}\max\{M_T, N-N_T\}\right\} \\ & d_r \leq \min\left\{N_T, M-M_T, \frac{1}{2}\max\{N_T, M-M_T\}\right\} \\ & d_f + d_r \leq \max\{N_T, N-N_T\} \\ & 0 \leq N_T \leq N \\ & 0 \leq M_T \leq M. \end{aligned} \tag{9}$$

The above problem is non-convex due to the $\max\{\cdot\}$ function in the R.H.S. of the first three constraints. Note that we did not include any integer constraints on N_T or M_T in (9). Dropping the integer constraints does not contradict our upper bound argument, however, it might yield non-achievable antenna configurations even with symbol extension as some channel matrices might drop rank. Nevertheless, the solution of (9) yields a valid upper bound on the DoF of the system.

Theorem 1. *The total DoF of the full-duplex two-cell system shown in Fig. 2 with N antennas at each UE and M antennas at each BS is upper bounded by*

$$D_{FD} \leq \begin{cases} \min\left\{2M, \frac{2}{3}M + \frac{2}{3}N, \frac{4}{3}N\right\} & 0 \leq M < \frac{7}{6}N \\ \min\left\{\frac{4}{5}M + \frac{2}{5}N, 2N\right\} & M \geq \frac{7}{6}N \end{cases} \tag{10}$$

Proof. Theorem 1 is obtained by solving the optimization problem in (9) as shown in Appendix A. □

[2] The converse proof of the DoF of the Z-channel in [16] relies on the ability of receiver U_1 to decode its intended messages and reducing the noise at receiver U_1 to enable it to decode the message intended for T_2. Since receiver U_1 in Fig. 4 does not have any prior information about the transmitted messages, the DoF of the system cannot be improved beyond those in [16].

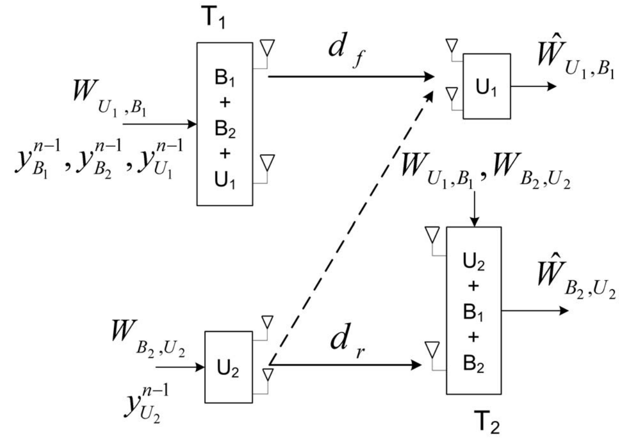

Fig. 4. Resulting system after message reduction and cooperation.

IV. Cooperative Multipoint Transmission System

In this section, we consider the cooperative multipoint transmission scenario, i.e., instead of attaching each UE to a single BS, the two base stations can communicate simultaneously with the two UEs. When all nodes are half-duplex, the system reduces to a two-user MIMO X channel with M antennas at each transmitter and N antennas at each receiver whose total DoF is given by

$$D_{\text{HD}}^{\text{X}} = \min\left\{2M, 2N, \frac{4}{3}\max\{M, N\}\right\} \tag{11}$$

which is achievable through interference alignment [16].

Next, we consider the case in which full-duplex separate-antenna transceivers are employed at each node of the cooperative system. In this case, the encoder and decoder of node P_i are given respectively by

$$\begin{aligned} \boldsymbol{x}_{P_i}(n) &= \mathcal{E}_{P_i}\left(W_{Q_i,P_i}, W_{Q_j,P_i}, \mathbf{y}_{P_i}^{n-1}\right) & (12) \\ \left[\hat{W}_{P_i,Q_j}, \hat{W}_{P_i,Q_i}\right] &= \mathcal{D}_{P_i}\left(W_{Q_i,P_i}, W_{Q_j,P_i}, \mathbf{y}_{P_i}^{N}\right) & (13) \end{aligned}$$

where $P, Q \in \mathcal{N}$, $i, j \in \mathcal{I}$, $P \neq Q$ and $i \neq j$. For example, BS B_1 simultaneously encodes its downlink messages for U_1 and U_2 into the transmitted signal vector $\boldsymbol{x}_{B_1}(n)$ and utilizes its received signal in addition to its transmitted messages in decoding the messages of U_1 and U_2 transmitted in the uplink. We can apply the same analysis as that in Section III to obtain upper bounds on the DoF. By removing the interference links from each transmitting UE on the other receiving UE, we obtain two independent MIMO X-channels that yield the following upper bounds

$$\begin{aligned} 2d_f^{\text{X}} &\leq \min\left\{2M_T, 2N_R, \frac{4}{3}\max\{M_T, N_R\}\right\} & (14) \\ 2d_r^{\text{X}} &\leq \min\left\{2N_T, 2M_R, \frac{4}{3}\max\{N_T, M_R\}\right\} & (15) \end{aligned}$$

where d_f^{X} and d_r^{X} denote the total number of streams transmitted from each BS and each UE respectively. In addition, the same upper bound in (8) can be derived, i.e.,

$$d_f^{\text{X}} + d_r^{\text{X}} \leq \max\{N_T, N_R\}. \tag{16}$$

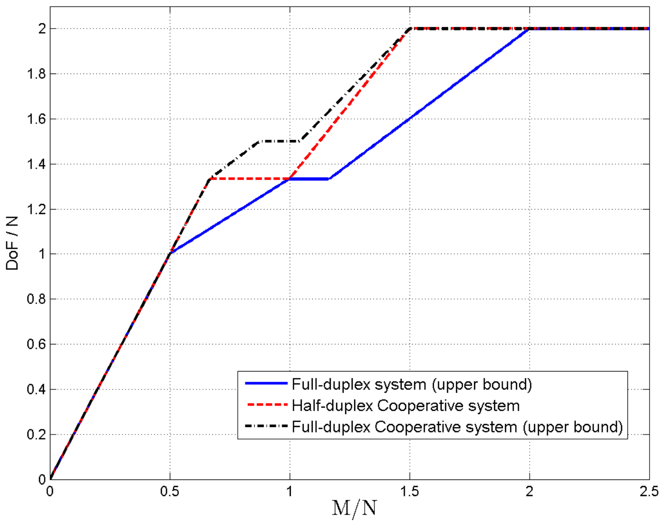

Fig. 5. Total DoF for different values of M/N.

An optimization problem similar to (9) can be formulated to find the antenna configuration that maximizes the total DoF of the full-duplex system subject to the constraints in (14)–(16).

Theorem 2. *The total DoF of the full-duplex cooperative system with N antennas at each UE and M antennas at each BS is upper bounded by*

$$D_{FD}^{X} \leq \begin{cases} \min\left\{2M, \frac{4}{5}(M+N), \frac{3}{2}N\right\} & 0 \leq M < \frac{25}{24}N \\ \min\left\{2N, \frac{12}{11}M + \frac{4}{11}N\right\} & M \geq \frac{25}{24}N \end{cases} \tag{17}$$

Proof. Theorem 2 can be obtained using the same technique in Appendix A. The details have been omitted to avoid repetition. □

Fig. 5 shows the upper bound on the total DoF of the full-duplex system obtained from Theorem 1 and the total DoF of the half-duplex cooperative system in (11). We can see that the achievable DoF of the cooperative system is always greater than or equal to the upper bound on the DoF of the full-duplex system. When $M/N = \frac{3}{2}$, cooperative communication yields at least 25% gain in DoF compared to full-duplex communication. Fig. 5 also shows the upper bound on the DoF of the system when full-duplex capability is added to all nodes in the cooperative system which is given in Theorem 2. We can see from this figure that adding the full-duplex capability to the transceivers in the cooperative case does not yield significant additional gain in the DoF where the maximum DoF gain cannot exceed 12.5% of the DoF of the half-duplex cooperative system. In addition, when $M/N \leq 2/3$ or $M/N \geq 3/2$, half-duplex cooperative operation is optimal. As expected, full-duplex separate-antenna transceivers increase the interference as more nodes are allowed to transmit simultaneously which diminishes the gain that can arise due to the flexibility of choosing the antenna configuration. On the other hand, cooperative communication adds more flexibility to the network without incurring any more interference on the system.

V. Conclusion

We have investigated the maximum gain in DoF that can be obtained by using full-duplex separate-antenna transceivers in a MIMO wireless network composed of two base stations communicating with two UEs. We have derived upper bounds on the total DoF of the system. We have also considered the case of cooperative BS operation where each BS can communicate with the two UEs simultaneously. Our results indicate that the DoF gain with separate-antenna full-duplex transceivers is always less than or equal to the gain due to cooperative communication. Furthermore, the maximum DoF gain due to adding the full-duplex capability to the transceivers in the cooperative case cannot exceed 12.5% of the DoF of the half-duplex cooperative system.

Appendix A
Proof of Theorem 1

We will start the proof with the following lemma that shows that the upper bound on the DoF of the network in Fig. 2 is symmetric, i.e., the reciprocal network obtained by switching the roles of the transmitters and the receivers has the same upper bound on the total DoF.

Lemma 1. *Let $\left\{d_f^\star, d_r^\star, M_T^\star, N_T^\star\right\}$ be the optimum solution of* (9)*, then $d_f = d_r^\star$, $d_r = d_f^\star$, $M_T = M - M_T^\star$, $N_T = N - N_T^\star$ is also an optimum solution of* (9).

Proof. Lemma 1 can be simply proved by noting that substituting with $d_f = d_r^\star$, $d_r = d_f^\star$, $M_T = M - M_T^\star$, $N_T = N - N_T^\star$ yields the same value of the objective function while satisfying all the constraints of problem (9). □

As a result, we can assume that the optimum antenna allocation satisfies $N_T \leq N_R$, i.e., $N_T \leq \frac{N}{2}$ without any loss of optimality. In order to solve the non-convex problem, we divide the feasible set into four convex subsets and maximize the objective function over each subset. The optimum solution of (9) is the one that yields the maximum value of the objective function among the four subproblems. For example, the first subproblem is obtained by assuming that $M_T \leq N - N_T$ and $N_T \leq M - M_T$. Adding these assumptions to the problem in (9) together with the constraint $N_T \leq \frac{N}{2}$ yields the following convex problem written in matrix form

$$\min_{\boldsymbol{v}} \boldsymbol{c}^T \boldsymbol{v} \qquad \text{subject to } \boldsymbol{A}\boldsymbol{v} \preceq \boldsymbol{b} \tag{18}$$

where $\boldsymbol{v} = [d_f, d_r, N_T, M_T]^T$, $\boldsymbol{c} = [-2, -2, 0, 0]^T$, $(\cdot)^T$ denotes the transpose operator, and

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 1 & -1 & 0 & 1 & 1 & -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & \frac{1}{2} & 1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}^T \tag{19}$$

$$\boldsymbol{b} = \begin{bmatrix} 0 & \frac{N}{2} & N & 0 & \frac{M}{2} & M & N & 0 & \frac{N}{2} & 0 & M \end{bmatrix}^T. \tag{20}$$

Similarly, we can write the remaining three subproblems such that the feasible set of (9) is covered by the four subproblems.

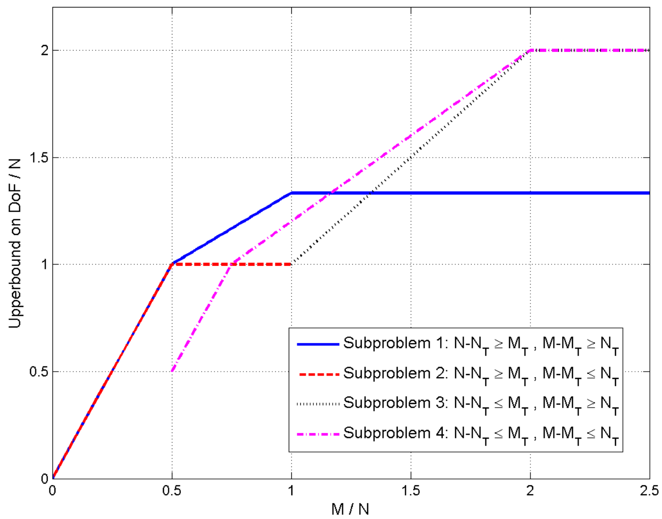

Fig. 6. Upper bounds on the total DoF obtained by solving the four subproblems of Problem (9).

The Lagrange dual of problem (18) is given by

$$\begin{aligned} \max_{\boldsymbol{\lambda}} \quad & -\boldsymbol{b}^T\boldsymbol{\lambda} \\ \text{subject to} \quad & \boldsymbol{A}^T\boldsymbol{\lambda} + \boldsymbol{c} = \boldsymbol{0}, \qquad \boldsymbol{\lambda} \succeq 0 \end{aligned} \tag{21}$$

where $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_{11}]^T$. If we can find primal and dual solutions such that strong duality holds, i.e., $\boldsymbol{c}^T\boldsymbol{v}^\star + \boldsymbol{b}^T\boldsymbol{\lambda}^\star = 0$, then $\boldsymbol{v}^\star$ is primal optimal and $\boldsymbol{\lambda}^\star$ is dual optimal. First, let us consider the case where $\frac{N}{2} \leq M \leq N$. In this case, it can be shown that the optimal primal variable is given by

$$\boldsymbol{v}^\star = \left[\frac{2N}{3} - \frac{M}{3}, -\frac{N}{3} + \frac{2M}{3}, -\frac{N}{3} + \frac{2M}{3}, \frac{2N}{3} - \frac{M}{3}\right]^T \tag{22}$$

while all the optimal dual variables are equal to zero except for $\lambda_1^\star = \frac{2}{3}$, $\lambda_2^\star = \frac{4}{3}$, $\lambda_4^\star = \frac{2}{3}$, and $\lambda_5^\star = \frac{4}{3}$. In this case the optimum value of the dual problem in (21) is given by $-\frac{2}{3}(N+M)$ which is equal to that of the primal problem. This proves the optimality of $\boldsymbol{v}^\star$ in (22) for $\frac{N}{2} \leq M \leq N$. Similarly, we can show that $\boldsymbol{v}^\star = [M, 0, M, 0]$ is optimal for $0 \leq M \leq \frac{N}{2}$ and $\boldsymbol{v}^\star = [\frac{N}{3}, \frac{N}{3}, \frac{N}{3}, \frac{N}{3}]^T$ is optimal for $M > N$. Therefore, the optimum value of subproblem 1 is given by

$$2d_f^{(1)} + 2d_r^{(1)} = \min\left\{2M, \frac{2}{3}(M+N), \frac{4N}{3}\right\}. \tag{23}$$

Similarly, we can formulate and solve the remaining three subproblems. Fig. 6 shows the upper bounds on the DoF of the system obtained by solving the four subproblems. We can see from this figure that the antenna allocation of subproblem 1 yields the best upper bound for $0 \leq M \leq \frac{7N}{6}$ while subproblem 4 yields the best upper bound for $M > \frac{7N}{6}$. The optimal value of subproblem 4 is

$$2d_f^{(4)} + 2d_r^{(4)} = \min\left\{2M - \frac{N}{2}, \frac{4M}{5} + \frac{2N}{5}, 2N\right\} \quad \text{for } M \geq \frac{N}{2} \tag{24}$$

where the problem is infeasible for $M < \frac{N}{2}$. Combining (23) and (24) yields (10). The optimum antenna allocation that yields this upper bound is given by

$$N_T^\star = \min\left\{\left(\frac{2M}{3} - \frac{N}{3}\right)^+, \frac{N}{3}, \left(-\frac{2M}{5} + \frac{4N}{5}\right)^+\right\} \tag{25}$$

$$M_T^\star = \begin{cases} M & 0 \leq M < \frac{N}{2} \\ \max\left\{-\frac{M}{3} + \frac{2N}{3}, \frac{N}{3}\right\} & \frac{N}{2} \leq M < \frac{7N}{6} \\ \min\left\{\frac{6M}{5} - \frac{2N}{5}, M\right\} & M \geq \frac{7N}{6} \end{cases} \tag{26}$$

where the operator $(x)^+$ refers to $\max\{x, 0\}$. □

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