# QoS-Guaranteed User Association in HetNets via Semidefinite Relaxation

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# Introduction

- The fundamental limitations of existing cellular networks, e.g.,
  - higher data rates,
  - user-coverage in hot-spots and crowded areas,
  - energy consumption.
- To mitigate these limitations, cellular networks have evolved to include low-power base stations (BSs), so-called heterogeneous networks (HetNets).
- HetNet:
  - improving network capacity,
  - eliminating coverage holes in the macro-only system,
  - reducing energy consumption.

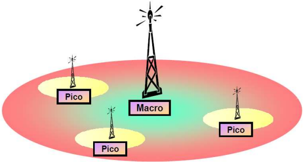


Figure: An example of HetNet

# Introduction (cont'd)

- Disparate transmit powers and BS capabilities of HetNets render user-to-BS association a challenge.
- The problem of user-to-BS association is inherently combinatorial NP-hard and hence difficult to solve.
- Two considerations must be taken into account in selecting of the serving BS of each user:
  - Channel conditions, and
  - Load condition of BSs.
- **Problem statement**: Find the user-to-BS association which ensures that (1) the number of accommodated users is maximized but also that (2) the network resources are efficiently utilized and (3) the users' quality of service (QoS) demands are met.

# Introduction (cont'd)

- For example,

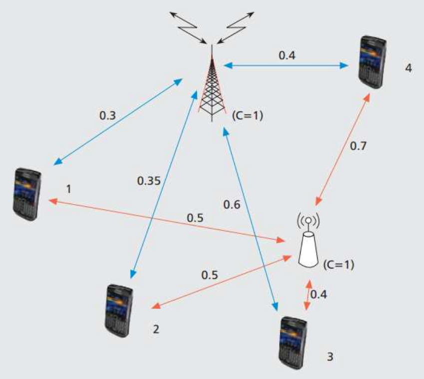


Figure: Load Balancing in HetNet

- Max-SINR: (1, 2, 4) at macro and (3) at pico.
  - (4) cannot be accepted (call blocking).
- Load Balancing: (2, 4) at macro and (1, 3) at pico.

# Related work

- **Cell range expansion** [Guvenc et al., *VTC Fall* 2011].
- Similarities:
  - User association problem in HetNet considered.
- Differences:
  - Solution method re-adjusting cell boundaries by adding a constant bias terms to SINR values.
- Comment:
  - It is a heuristic method. There is no theoretical guidance on the optimal biasing factors in the sense of load balancing or achieving a particular optimization criteria.
  - QoS requirements not considered.

# Related work (cont'd)

- **Lagrange dual decomposition** [Ye et al.,*IEEE Trans. Wireless Commun.* 2013, Shen and Yu, *IEEE J. Sel. Areas Commun.* 2014].
- Similarities:
  - User association in HetNet.
- Differences:
  - Different objective functions presented.
  - Each BS equally shares the total bandwidth among users.
  - Load definition the number of associated users to a BS.
  - Relaxing the binary BS association variables to continuous variables in [0, 1] allows a user to be served by multiple BSs, which may require more overhead to implement.
- Comment:
  - QoS requirements not considered.

# Related work (cont'd)

- **Game theory** [Aryafar et al.,*IEEE Infocom* 2013].
- Similarities:
  - User association in HetNet.
- Differences:
  - Assignment problem thought of as a game among BSs.
  - The Nash equilibrium of the game is found.
- Comment:
  - QoS requirements not considered.
  - Convergence of the algorithms not guaranteed. Even if the algorithms converge, the solution may be far from optimal.

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# Related work (cont'd)

- **Semidefinite Relaxation and Randomization** [Corroy and Mathar, *IEEE Globecom Wkshp.* 2012].
- Similarities:
  - User association in HetNet.
  - Solution approach towards solving the problem.
- Differences:
  - The objective to maximize the sum rate.
  - Each BS equally shares the total bandwidth among users.
  - Load definition the number of associated to a BS.
- Comment:
  - QoS requirements not considered.
  - A simple HetNet with one macro and one pico is considered.

# Problem formulation

- $P_i$: the transmit power of BS $i$,
- $g_{ij}$: the average channel gain,
- The average SINR between BS $i$ and the user $j$:

$$SINR_{ij} = \frac{P_i g_{ij}}{\sum\limits_{k \in B,\, k \neq i} P_k g_{kj} + \sigma_N},$$

- The bandwidth efficiency to a user $j$ from BS $i$: $\eta_{ij} = \log_2(1 + SINR_{ij})$ [bps/Hz],

# Problem formulation (cont'd)

- $t_i$: total available resources of BS $i$ and $t_i = t_M$ for macro BSs and $t_i = t_P$ for pico BSs
- $Q_j$: demanded data rate of user $j$
- $W$: bandwidth of an RB
- The amount of resource allocated: $b_{ij} = \lceil Q_j / (W \eta_{ij}) \rceil$ and $\hat{b}_{ij} = b_{ij} / t_i$ (**given input**)
- $x_{ij} \in \{0, 1\}$ : assignment indicator variable (**optimization variable**)
- The load of BS $i$: $\ell_i = \sum_{j \in U_i} \hat{b}_{ij} x_{ij}$

# Problem formulation (cont'd)

Find the optimal user-to-BS association that ensures maximizing the number of accommodated users and simultaneously minimizing the number of expended resources:

$$\max_{x_{ij}} \quad \rho \sum_{i \in B} \sum_{j \in U} x_{ij} - (1-\rho) \sum_{i \in B} \sum_{j \in U} b_{ij} x_{ij},$$

- Total resource limit for the $i$-th BS: $\sum_{j \in U_i} b_{ij} x_{ij} \leq t_i, \quad i \in B,$
- User-to-BS association: $\sum_{i \in B_j} x_{ij} \leq 1, \quad j \in U,$
- Binary association variable: $x_{ij} \in \{0, 1\}, \quad i \in B, \ j \in U_i,$
- $\rho \in [0, 1]$ parametrizes a family of objectives,
- The optimal choice of the value of $\rho \in \left( \frac{\sum_{i \in B} t_i}{1 + \sum_{i \in B} t_i}, 1 \right)$.

# Semidefinite relaxation

- $\Psi = \begin{bmatrix} \phi & \beta \\ \beta^T & 1 \end{bmatrix}$, where $\phi = \beta\beta^{\mathbf{T}}$ and $\beta = 2\mathbf{x} - \mathbf{1}$.

$$\max_{\boldsymbol{\Psi}} \quad \frac{\rho}{2}\mathrm{Tr}(\mathbf{A}_1\boldsymbol{\Psi}) - \frac{1-\rho}{2}\mathrm{Tr}(\mathbf{A}_b\boldsymbol{\Psi}), (\text{a linear function in } \Psi) \tag{1a}$$

$$\text{subject to} \quad \frac{1}{2}\mathrm{Tr}(\mathbf{A}_{d_i}\boldsymbol{\Psi}) \le t_i, \quad i \in B, \ (\text{a linear inequality in } \Psi) \tag{1b}$$

$$\frac{1}{2}\mathrm{Tr}(\mathbf{A}_{e_j}\boldsymbol{\Psi}) \le 1, \quad j \in U, \ (\text{a linear inequality in } \Psi) \tag{1c}$$

$$\mathrm{diag}(\boldsymbol{\Psi}) = 1, \ (\text{a linear inequality in } \Psi) \tag{1d}$$

$$\boldsymbol{\Psi} \succeq \mathbf{0}, \ (\text{positive semidefinite constraint}) \tag{1e}$$

$$\mathrm{rank}(\boldsymbol{\Psi}) = 1. \ (\text{non-linear constraint}) \tag{1f}$$

- Semidefinite programming is an extension of linear programming to the space of symmetric matrices.
- Non-convex rank-1 constraint is removed based on the premise of solving strategy.

# Randomization Method

**Approach:**

- **Phase-1**: The semidefinite relaxation generates a positive semidefinite covariance matrix together with an upper bound on the objective.
- **Phase-2**: Using Randomization, we exploit output of Phase-1 to compute good approximate solutions with provable approximation accuracies.

**Steps:**

- For $j = 1, \ \ldots, \ J$
  - Generate a random vector sample: $\delta^j \sim \mathcal{N}(\mathbf{z}^*, \mathbf{Z}^* - \mathbf{z}^* {\mathbf{z}^*}^T)$.
  - Find the candidate solution: $\tilde{\beta} = \mathrm{sgn}(\delta^j)$.
  - Find the candidate binary solution: $\tilde{\mathbf{x}}^j = 0.5(\tilde{\beta} + \mathbf{1})$.
  - Determine the feasibility of the candidate solution:
- Select the best among the feasible solutions, which has the highest objective function value and assign it to $\mathbf{x}^*$.

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**Algorithm 1:** Proposed algorithm via SDR

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**Input**: **b** and $t_i$, $i = 1, \ldots, B$.

**Output**: $\mathbf{x}^*$

**Relax the original non-convex problem:** Drop the rank-1 constraint and convert the non-convex problem into a convex formulation.

**Solve the semidefinite programming problem:** Find the optimization variables of the relaxed problem, $\mathbf{z}^*$, $\mathbf{Z}^*$ and $\mathbf{R}^*$.

**for** $j = 1 : J$ **do**

    **Generate a random vector sample:** Obtain a random vector drawn from the Gaussian distribution, $\delta^j \sim \mathcal{N}(\mathbf{z}^*, \mathbf{Z}^* - \mathbf{z}^*\mathbf{z}^{*T})$.

    **Find the candidate solution:** Quantize the entries of the realization of $\delta^j$, $\tilde{\beta} = \text{sgn}(\delta^j)$.

    **Find the candidate binary solution:** Using simple mathematical manipulation, obtain the candidate solution, $\tilde{\mathbf{x}}^j = 0.5(\tilde{\beta} + \mathbf{1})$.

    **Determine the feasibility of the candidate solution:** Check the constraints:

    **if** *They are satisfied* **then**

        Record $\tilde{\mathbf{x}}^j$.

**Find the best solution:** Select the best among the feasible solutions, which has the highest objective function value and assign it to $\mathbf{x}^*$.

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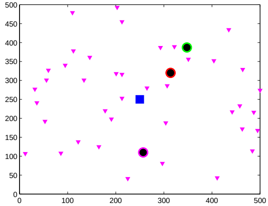
# Simulations

## Simulation models and parameters

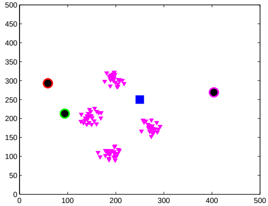
| Parameter | Assumption or Value |
|---|---|
| Transmit power of macro BS | 40 W |
| Transmit power of pico BSs | 1 W |
| Noise power at all receiver | -114 dBm |
| Shadowing standard deviation | 8 dB |
| Path loss between BSs and users | L(d)=34+40log(d) |
| Number of RBs of macro BS | 50 |
| Number of RBs of pico BSs | 25 |
| Number of Gaussian samples | 100 |
| Optimization Solver | CVX-SDPT3 solver |
| User spatial distribution | uniform and hotspot |

# Simulations (cont'd)

## Uniform (homogeneous) distribution

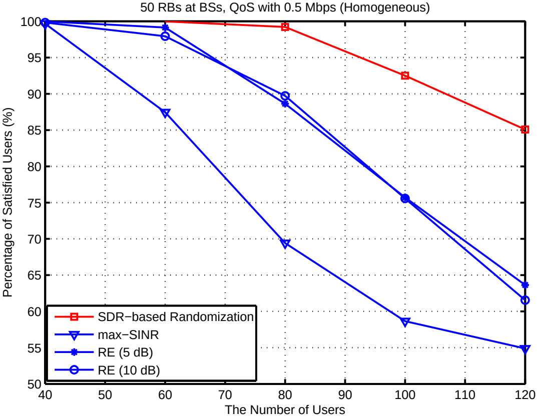



## Hotspot (heterogeneous) distribution



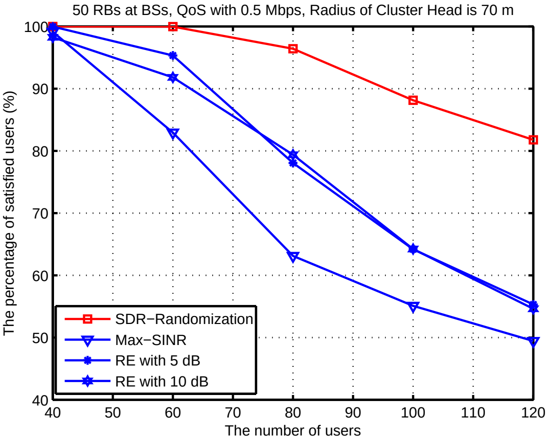
# Simulations (cont'd)

## Uniform (homogeneous) distribution



# Simulations (cont'd)

## Hotspot (heterogeneous) distribution



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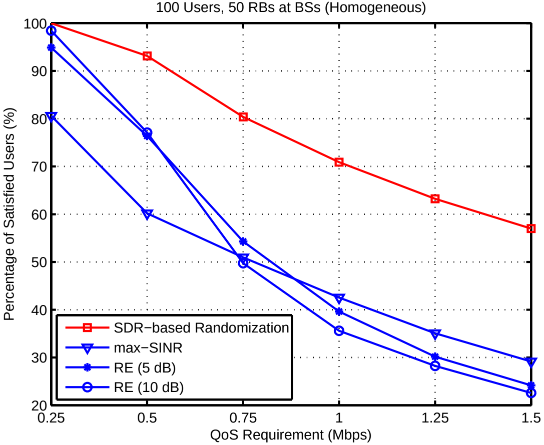
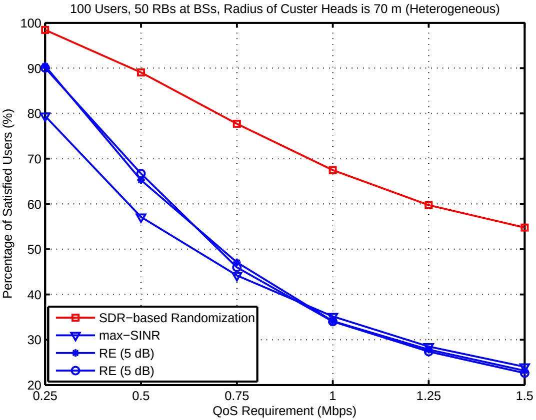
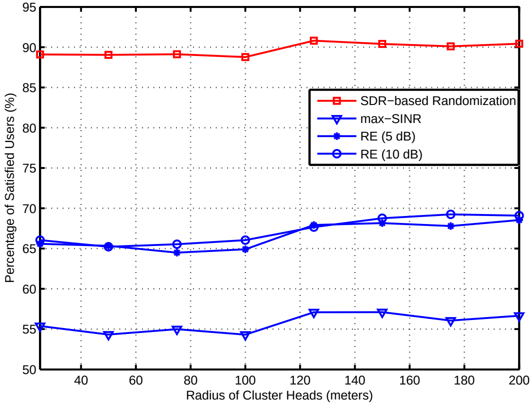


# Simulations (cont'd)

## Hotspot (heterogeneous) distribution



# Simulations (cont'd)



# Conclusion

- Since the aim of service providers is to serve as many users as possible, the proposed technique will increase the number of satisfied users.
- The proposed technique based on semidefinite relaxation and Gaussian randomization.
  - Polynomial complexity of $\mathcal{O}\left((|B||U|)^{4.5}\log(1/\epsilon) + (|B||U|)^2 J\right)$, where $|\cdot|$ represents the cardinality, $J$ is the number of random samples,
  - Provable approximation accuracy.