

A Cumulant-Based Characterization of the Aggregate Interference Power in Wireless Networks

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Abstract— The importance of characterizing the aggregate interference power generated by a wireless network has increased with the emergence of different types of wireless networks such as ad-hoc networks, sensor networks, and cognitive radios. A cumulant-based characterization of this aggregate interference is an attractive approach. A number of recent papers in literature have dealt with cumulants of the aggregate interference but under specific scenarios. In this paper, we introduce a simple yet comprehensive method to determine the cumulants of the aggregate interference power originating from a wireless network. This method is quite general and applicable for finite and infinite network sizes, and it is flexible to encompass different system and propagation parameters such as large-scale fading, small-scale fading or even composite fading. We also investigate the behavior of these cumulants with respect to changes in the network size and fading distributions.

Index Terms—Aggregate Interference, Cumulants, Fading, Poisson Point Process.

I. INTRODUCTION

The importance of characterizing the aggregate interference generated by a wireless network has some history dating back at least to packet radios [1]. This importance has increased with the emergence of different types of wireless networks over the past two decades, e.g., CDMA [2], ad-hoc and sensor networks [3], spectrum sharing and cognitive radio networks [4]–[6]. Further history and references are provided in [7], [8].

Many papers investigate the characterization of the aggregate interference power by modeling the wireless network as a Poisson field of independent interferers. They reach to closed-form expressions for the characteristic function. However, the inversion of this characteristic function into a closed-form probability density function (PDF) or cumulative distribution function (CDF) is not achievable, except for a few cases. A viable option to overcome this inversion problem is to obtain moments (or cumulants) of the aggregate interference and then to apply moments-based (or cumulants-based) approximations or bounds [2], [4]–[6], [9], [10].

While the moments and cumulants are closely related, cumulants have some properties making them more attractive for characterizing the aggregate interference of a Poisson field of interferers. Among these properties is that the m th cumulant of the sum of independent random variables is equal to the

sum of the individual m th cumulants [11]. Combining this property with the independence property of disjoint Poisson fields provides a powerful tool in characterizing the aggregate interference in a wireless network.

To the best of our knowledge, few papers in literature have dealt with cumulants of the aggregate interference power. These papers focus on specific scenarios. For example, [4] and [5] deal with cumulants for non-fading scenarios; [2] provides an integral form to compute the cumulants for out-of-cell interference in a CDMA networks; and [6] considers an infinite field with an exclusion region. Extending these results and generalizing them for a wide range of scenarios are among the contributions of this paper.

We characterize the distribution of the aggregate interference using cumulants. We provide a very simple yet powerful method to determine the cumulants. The method is flexible enough to be applicable to a wide range of scenarios including, but not limited to, the following: finite fields, infinite fields, different fading distributions (e.g., Rayleigh, Rician, shadowing, generalized- K), and variations in power levels. Moreover, we investigate the behavior of cumulants, and hence the aggregate interference power, with respect to changes in the network size and for various fading distributions.

The rest of the paper is organized as follows. Section II describes the system model and provides some background information on cumulants. Section III presents a simple method to calculate the cumulants of the aggregate interference power. The effects of the network size and fading distributions on cumulants are investigated in Section IV and Section V, respectively. Section VI discusses the cumulants-based approximations of the distribution of the aggregate interference power. Finally, Section VII concludes the paper with some remarks.

II. SYSTEM MODEL AND STATISTICAL PRELIMINARIES

The analysis in this paper is based on modeling a wireless network as a two-dimensional field of interferers deployed over a region of area A with an annular sector shape (ring or disk shapes are special cases). The set of active interfering transmitters (nodes) is assumed to follow a Poisson point process with a homogeneous density λ . Based on this assumption, the number of interferers in a region is a Poisson random

variable with a parameter specified by the multiplication of λ by the area of that region. Moreover, the number of interferers in disjoint regions are independent Poisson random variables [12]. The field of interferers is assumed to have an inner radius of r_o . We call exclusion region the disk $b(O, r_o)$ of radius r_o and centered at the origin. The field has a radial depth of L , making the outer radius of the field $r_o + L$. The field spans over an angle of θ as seen by the victim receiver at the origin, as illustrated in Fig. 1.

The individual interference power received by a victim receiver at the origin due to the transmission of node i is denoted by I_i . Under the assumption of incoherent addition of interfering signals, the aggregate interference power received by the victim receiver can be expressed as

$$I_A = \sum_{i \in \Lambda} I_i = \sum_{i \in \Lambda} X_i g(r_i), \quad (1)$$

where Λ is a set of interfering nodes, X_i is a positive random variable that can be modeled as the multiplication of deterministic quantities and various random variables reflecting the transmit power, antenna gain, channel attenuation (including multipath and shadow fading) and other factors. In analyzing the aggregate interference of a Poisson field, it is common to assume that X_i s are independent and identically distributed (i.i.d.) random variables [4], [6], [7], [13]. In this paper, we follow the same assumption. The function $g(r_i)$ represents a path-loss model (or more precisely the distance-dependent attenuation), discussed in the following subsection.

A. Path-Loss Models

The most common path-loss model used in literature is

$$g(r_i) = kr_i^{-n}, \quad r_i \geq 0, \quad (2)$$

where k is a constant, r_i is the distance between the interfering node i and the victim receiver, and n is the path-loss exponent. This model is commonly used in contexts similar to the one we consider in this paper due to its mathematical tractability [14]. However, this model suffers from a singularity at $r_i = 0$. Therefore, it is known as a singular (or unbounded) path-loss model [14], [15].

There are non-singular (bounded) path-loss models used in literature as well, e.g., [6], [10], [15], [16]. These non-singular models are similar, with some variations. Following is an example:

$$g(r_i) = \begin{cases} kr_i^{-n}, & r_i \geq r_c \\ kr_c^{-n}, & r_i < r_c \end{cases}, \quad (3)$$

where $r_c > 0$ is the radius at which the slope of the model starts changing.

It is indicated in [15] that the selection of the path-loss model has a significant impact on the characterization of the aggregate interference. Authors in [14] investigated the effect of the unbounded model on the performance analysis of wireless networks and indicated that more realistic performance figures are obtained by using bounded models. We therefore

consider the use of a non-singular model, specifically (3) with $n > 2$. Without loss of generality, we take $k = 1$ assuming its effect is absorbed by X_i .

B. Statistical Preliminaries on Cumulants

The m th cumulant of a random variable X whose characteristic function is $\phi_X(w)$ can be obtained by [11]

$$\kappa_m = \frac{1}{j^m} \left[\frac{d^m \ln(\phi(\omega))}{d\omega^m} \right]_{\omega=0}. \quad (4)$$

Cumulants have some properties that make them more attractive for the problem considered in this paper. Among these properties are the following:

- The m th cumulant of the sum of two independent random variables is equal to the sum of the individual m th cumulants of these independent random variables [11].
- If $Y = \frac{X - \tilde{\mu}}{\sigma}$ where $\tilde{\mu}$ and σ are the mean and the standard deviation of the random variable X respectively, then $\kappa_1(Y) = 0$, and $\kappa_m(Y) = \kappa_m(X)\sigma^{-m} = \kappa_m(X)\kappa_2(X)^{-m/2}$ for $m \geq 2$ [17] where $\kappa_m(Y)$ denotes the m th cumulant of Y .
- Cumulants can be used to obtain some important measures of the distribution, like mean (κ_1), variance (κ_2), skewness ($\kappa_3\kappa_2^{-3/2}$) and kurtosis excess ($\kappa_4\kappa_2^{-2}$) [17].
- The cumulants of a random variable are closely related to its moments. For example, $\kappa_1 = \tilde{\mu}_1$, $\kappa_2 = \mu_2 = \sigma^2$, $\kappa_3 = \mu_3$, and $\kappa_4 = \mu_4 - 3\mu_2^2$ where $\tilde{\mu}_m$ is the m th raw moment, μ_m is m th central moment, and σ^2 is the variance. General expressions relating cumulants to central or raw moments are provided in [17].
- Cumulants of a random variable can be used to approximate the distribution function of that random variable through some approaches like an Edgeworth series expansion [2]. Alternatively, cumulants can be used to determine the moments, and then moments-based approximations can be applied [17].

III. CUMULANTS OF I_A

The main contribution of this paper is stated in the following proposition. To value this proposition, it is helpful to imagine that the field of interferers would virtually collapse to a subfield with an effective area A_{eff} , an inner radius r_o , and an outer radius r_{eff} . The average number of interfering nodes within this subfield is N_{eff} .

Proposition 1: The m th cumulant of the distribution of the aggregate interference power received by a victim receiver at the origin from an annulus-shaped Poisson field of i.i.d. interferers is

$$\kappa_m(I_A) = N_{\text{eff}}(m)\tilde{\mu}_m(I_{r_o}), \quad (5)$$

where $N_{\text{eff}}(m)$ is the average number of interfering nodes within a radius of $r_{\text{eff}}(m)$ from the victim receiver, $\tilde{\mu}_m(I_{r_o})$ is the m th raw moment of the distribution of the interference power received by the victim receiver from an interfering node at distance r_o . We then have

$$N_{\text{eff}}(m) = \lambda A_{\text{eff}}(m), \quad (6)$$

where

$$A_{\text{eff}}(m) = \{b(O, r_{\text{eff}}(m)) \cap A\} = \frac{1}{2}\theta [r_{\text{eff}}^2(m) - r_o^2], \quad (7)$$

and

$$r_{\text{eff}}(m) = \hat{r} \sqrt{1 + \frac{2}{mn-2} \left(1 - \left[\frac{\hat{r}}{r_o + L}\right]^{mn-2}\right)}, \quad (8)$$

$$\hat{r} = \max(\min(r_c, r_o + L), r_o).$$

Note that $\tilde{\mu}_m(I_{r_o}) = \tilde{\mu}_m(X)[g(r_o)]^m$ where $\tilde{\mu}_m(X)$ is the m th raw moment of X_i .

Proof: Using Campbell's theory for a Poisson Point Process [7], [12], it can be shown that:

$$\phi_{I_A}(\omega) = \exp\left(-\theta\lambda \int_{r_o}^{r_o+L} \int_0^\infty [1 - e^{j\omega xg(r)}] f_X(x) r dx dr\right), \quad (9)$$

where $\phi_{I_A}(\omega)$ is the characteristic function of I_A , $f_X(x)$ is the PDF of X_i , and $j = \sqrt{-1}$. Denoting the characteristic function of X_i by $\phi_X(\omega)$, (9) can be rewritten as

$$\phi_{I_A}(\omega) = \exp\left(-\theta\lambda \int_{r_o}^{r_o+L} [1 - \phi_X(\omega g(r))] r dr\right). \quad (10)$$

From (4) and (10),

$$\kappa_m(I_A) = \theta\lambda \tilde{\mu}_m(X) \int_{r_o}^{r_o+L} g^m(r) r dr. \quad (11)$$

Then, the proof follows directly from (11) with some mathematical manipulations and arrangements. ■

Equation (5) is simple yet flexible in the sense that it can be applied to a wide range of scenarios such as finite fields, infinite fields (see Section IV), and to different fading distributions (see Section V). Moreover, (5) is applicable for many field's topologies, including when the victim receiver is in the middle of the field or away from it.

IV. EFFECT OF SPATIAL SIZE OF THE FIELD AND INTERFERERS DENSITY ON CUMULANTS

The spatial size of the field is controlled by L , r_o , and θ . The changes in L affect r_{eff} , and hence A_{eff} and κ_m . However, this effect is limited. As L increases, r_{eff} increases but it converges to a finite value regardless of the further increase in L . As m or n increases, this convergence occurs faster. The effect of changes in L is significant only for lower-order cumulants and for L closer to or less than $\max(r_c, r_o)$. As L approaches the value of $\max(r_c, r_o)$, its effect becomes weaker and it will be negligible when $\left(\frac{\max(r_c, r_o)}{r_o + L}\right)^{mn-2} \ll 1$.

As an example, Fig. 2 shows that the convergence value for κ_1 is almost reached when $L > 100\text{m}$ for a field with no exclusion region ($r_o = 0$). However, κ_2 is almost at the convergence value when $L > 2\text{m}$. The higher-order cumulants converge faster (at lower values of L). Therefore, doubling L or even expanding it to infinity has a negligible (or practically no) effect on κ_m provided that κ_m is almost at the convergence value.

If there is an exclusion region, e.g., $r_o=10\text{m}$, then the convergence value will be almost reached when $L > 1\text{Km}$ for the first cumulant and $L > 20\text{m}$ for the second cumulant. A smaller L will be required to closely approach the convergence level for higher cumulants, as seen in Fig. 3.

Interestingly, the value of r_{eff} converges to $\max(r_c, r_o) \sqrt{1 + \frac{2}{mn-2}}$ as $L \rightarrow \infty$. From this, we may conclude that κ_m is controlled mainly by the region which is close to the victim receiver. The dominant region for κ_1 (the average) is wider than that for κ_2 (the variance). It shrinks as m increases. On the other hand, this dominant region expands as r_o (the exclusion region) increases.

The exclusion region has an effect on A_{eff} of an order of r_o^2 (provided that $r_o > r_c$). Moreover, r_o affects the value of $\tilde{\mu}_m(I_{r_o})$ by an order of r_o^{-mn} . Therefore, the net effect of r_o on κ_m is in the order of r_o^{2-mn} , which suggests that increasing the value of r_o is an effective way to lower the aggregate interference. However, increasing r_o may contradict the performance objectives of the wireless network [5], [18], [19]. Therefore, an optimal tradeoff is to be found.

Regarding the effect of the active node density (λ), it has a linear effect on all cumulants. Therefore, it is one of the important parameters that could be used to control the level of interference at the victim receiver.

Similarly, θ has a linear effect on the cumulants. However, it is limited to the range $[0, 2\pi]$. It may reflect the effectiveness of using a directional antenna at the victim receiver.

The effect of the increase in the network size (spatial size and node density) on the average of the aggregate interference is considered in [3] and [20]. Moreover, the effect on the variance is discussed in [21]. However, to the best of our knowledge, this paper is the first to address the effect of the spatial size on all cumulants of the aggregate interference.

V. EFFECT OF THE DISTRIBUTION OF X ON CUMULANTS

The random variable X encompasses many system and channel parameters. It might be modeled as the multiplication of some deterministic and random variables reflecting the effect of different parameters, such as fluctuations in power level and antenna gains, multipath fading, and shadow fading¹.

$$X_i = \prod_l X_{i,l}, \quad (12)$$

where $X_{i,l}$ is a deterministic or a random variable. (A similar representation is used in [7].)

From (5) and $\tilde{\mu}_m(I_{r_o}) = \tilde{\mu}_m(X)[g(r_o)]^m$, it is clear that the distribution of X_i has a major influence on κ_m , and hence the distribution of the aggregate interference power. A similar observation on the influence of the fading distribution, equivalently of the distribution of X_i , on the aggregate interference appears in [15].

Following are some examples of κ_m under different distributions of X_i . While X_i can be more general than just fading, examples given here consider fading distributions only.

¹Equation (12) is general enough to account for the adjacent-channel interference. However, it is not discussed here due to space limitation.

A. Multipath Fading

A general model that could be used to reflect the multipath fading on the interference power is the Gamma distribution (under the assumption of Nakagami fading for the interference signal). The PDF of the Gamma distribution is

$$f_X(x) = \left(\frac{\nu}{\Omega}\right)^\nu \frac{x^{\nu-1}}{\Gamma(\nu)} e^{-\frac{\nu}{\Omega}x}, \quad x > 0, \nu \geq \frac{1}{2}, \quad (13)$$

where ν is the shape parameter, $\Gamma(\cdot)$ is the Gamma function, and Ω is the average power, i.e., $E[X]$, which is commonly assumed to be equal to unity [22]. The m th raw moment of the Gamma distribution with $\Omega = 1$ can be expressed as

$$\tilde{\mu}_m(X) = \nu^{-m} \frac{\Gamma(\nu + m)}{\Gamma(\nu)}. \quad (14)$$

Therefore,

$$\kappa_m(I_A) = N_{\text{eff}}(m) [g(r_o)]^m \nu^{-m} \frac{\Gamma(\nu + m)}{\Gamma(\nu)}. \quad (15)$$

The exponential distribution (corresponding to Rayleigh fading for the interference signal) is a special case of the Gamma distribution, when $\nu = 1$.

B. Shadow Fading

Shadow fading is usually modeled by a lognormal random variable whose mean and standard deviation in the dB domain are zero and σ_{dB} , respectively. Let $X_i = 10^{S_i/10}$, representing the shadowing effect in the linear domain, where $S_i \sim \mathcal{N}(0, \sigma_{\text{dB}}^2)$. After a proper normalization of X_i , it can be shown that

$$\tilde{\mu}_m(X) = e^{\frac{1}{2} \left(m \frac{\ln 10}{10} \sigma_{\text{dB}}\right)^2}, \quad (16)$$

and

$$\kappa_m(I_A) = N_{\text{eff}}(m) [g(r_o)]^m e^{\frac{1}{2} \left(m \frac{\ln 10}{10} \sigma_{\text{dB}}\right)^2}. \quad (17)$$

C. Composite Fading

If the wireless channel suffers from multipath and shadow fading simultaneously, X_i can be modeled as the product of two independent random variables [7]. Assuming these random variables are Gamma (for multipath) and lognormal (for shadowing), the cumulants of the normalized aggregate interference can be expressed as

$$\kappa_m(I_A) = N_{\text{eff}}(m) [g(r_o)]^m e^{\frac{1}{2} \left(m \frac{\ln 10}{10} \sigma_{\text{dB}}\right)^2} \nu^{-m} \frac{\Gamma(\nu + m)}{\Gamma(\nu)}. \quad (18)$$

VI. CUMULANTS-BASED APPROXIMATIONS OF THE DISTRIBUTION OF I_A

The distribution of I_A can be approximated using a finite set of its cumulants. For example, an Edgeworth expansion is used in [2], [4], and [6] to approximate the PDF of the aggregate interference. The Edgeworth expansion is a simple approach used to expand the logarithm of the characteristic function using its Taylor series expansion. Based on this expansion and knowing the cumulants of a random variable, the PDF of this random variable can be approximated [2]. A detailed

discussion of the Edgeworth expansion for approximating a PDF is provided in [17].

It is important to highlight that while the Edgeworth series expansion is an asymptotic expansion for the PDF, the finite Edgeworth series should be applied with some caution: it is applicable for moderately-skewed distributions only. Conditions under which the Edgeworth finite approximation can be used are discussed in [23].

If the distribution of the aggregate interference has a positive skewness, I_A can be approximated by a lognormal random variable. An enhanced version of the lognormal approximation, called shifted lognormal approximation, is proposed in [24], and used in [6] to approximate the distribution of the aggregate interference power. The shifted lognormal approximation is a three-parameter approximation. These parameters are obtained from the first three cumulants. Figure 4 shows an example of approximating the PDF of I_A by an Edgeworth series expansion and a shifted lognormal PDF. A Gaussian PDF is included in the figure as a reference.

As an alternative, cumulants can be used to calculate the moments. Then, moments-based approximations or bounds can be applied to approximate the distribution of I_A [17].

VII. CONCLUSIONS

In this work, we characterized the aggregate interference power generated by a wireless network through the approach of cumulants. We introduced a simple yet comprehensive method to calculate the cumulants. Our method is applicable for finite and infinite networks, and is flexible to encompass different system and propagation parameters including large-scale fading, small-scale fading, and composite fading. We also discussed the behavior of these cumulants with respect to changes in the network size and fading distributions. Moreover, we highlighted some cumulants-based approximations of the distribution of the aggregate interference power.

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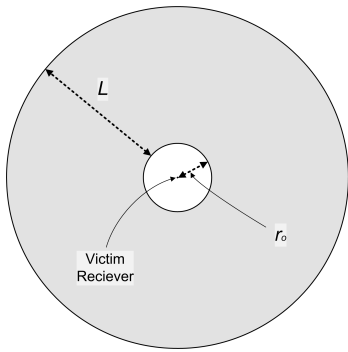


Fig. 1. Interferers field layout ($\theta = 2\pi$).

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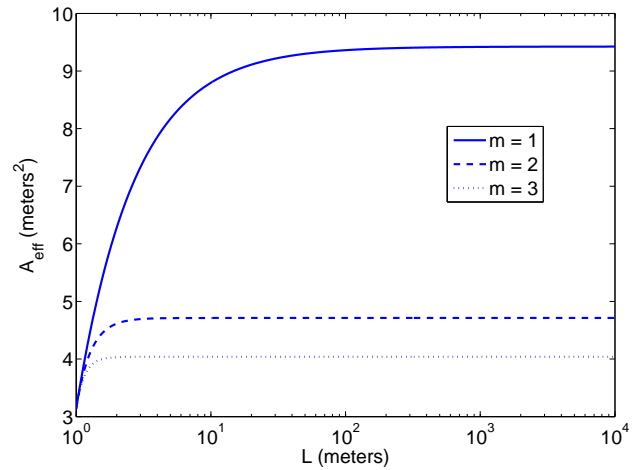


Fig. 2. The effect of L on A_{eff} , and hence on κ_m for the case of no exclusion region ($r_o = 0$) around the victim receiver ($r_c = 1$ meter, $\theta = 2\pi$ and $n = 3$).

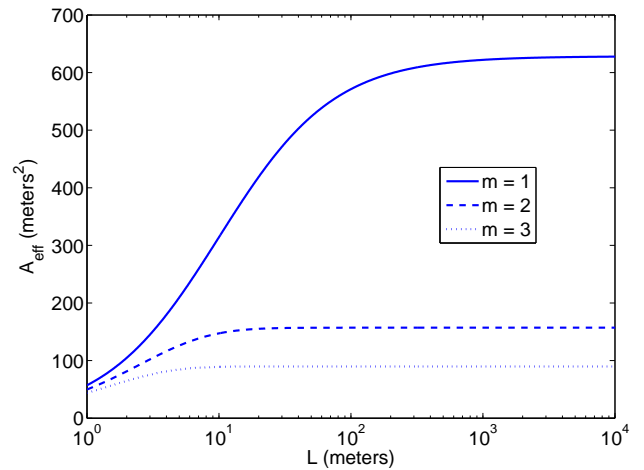


Fig. 3. The effect of L on A_{eff} , and hence on κ_m for the case of an exclusion region of $r_o = 10$ meters ($r_c = 1$ meter, $\theta = 2\pi$ and $n = 3$).

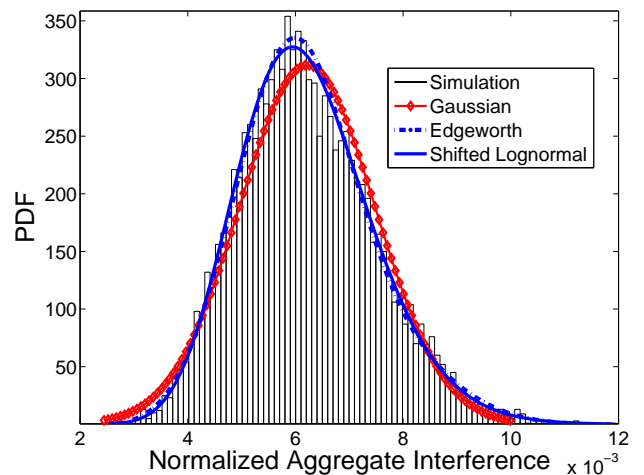


Fig. 4. The Monte Carlo simulation and different approximations of the PDF of I_A ($r_o = 10$ meters, $L = 1000$ meters, $r_c = 1$ meter, $\theta = 2\pi$ and $n = 3$).