Performance Analysis of Soft-Bit Maximal Ratio Combining in Cooperative Relay Networks

Akram Bin Sediq and Halim Yanikomeroglu

***Abstract*—In digital cooperative relaying, signals from the source-destination and relay-destination links are combined at the destination to achieve spatial diversity. These signals may not necessarily belong to the same modulation scheme due to the varying channel qualities of the two links. Recently, we have proposed the "soft-bit maximum ratio combiner" (SBMRC) as a low complexity diversity combining scheme for signals with different modulation levels. SBMRC exhibits BER performance that is very close to the optimal maximum likelihood detector (MLD), but with much reduced complexity. In this paper, we revisit SBMRC and provide tight lower bound for the BER performance. Since SBMRC has BER performance slightly inferior to MLD, the derived lower bound can also be used as a good approximation for the BER performance of MLD.**

***Index Terms*—Cooperative diversity, diversity combining, diversity analysis, relay networks.**

I. Introduction

NEXT generation wireless networks target to provide high data coverage to meet the demand for emerging applications. The use of digital relays in the wireless network was shown to be a key technology to achieve these requirements [1].

Two key strategies proposed for increasing the data rate in relay networks are cooperative relaying [2], [3], and adaptive modulation and coding (AMC). While the former is used to improve the quality of the links by combining the signals received from the base station (BS) and the relay station(s), the latter is used to optimize the transmission rate according to the channel conditions. In [4], [5], it is shown that the average throughput of the wireless network can be significantly increased by combining the two strategies. When AMC is utilized, the signals reaching the user terminal (UT) from BS and relay station (RS) do not necessarily belong to the same modulation, yet they contain the same information bits. In order to achieve spatial diversity for signals with different modulation levels, selection combining, rather than maximal ratio combining (MRC), has been proposed to be utilized because it has not been known yet how to do MRC for signals with different modulation levels [4], [5]. Beside selection combining, a trivial solution is to force the modulation levels to be the same in all the links, and optimally combine at UT using MRC. However, this solution reduces the benefit of AMC as the performance will be limited by the link which suffers from the most unfavourable channel conditions.

The need for such a technique arises from the nature of relay networks that impose different channel conditions on different links. For example, since the relays are fixed and can be installed at strategic locations, reliable line-of-sight link(s) can be established between BS and RSs, and hence, larger constellations can be used to achieve high data rates. However, because of the mobility of the users, the link(s) from RSs to UT are not necessarily as reliable, so smaller constellations can be used to ensure reliable transmission. In order to achieve spatial diversity at UT, it is imperative to establish an optimal diversity combining scheme for different modulations. Recently, we have proposed soft-bit maximum ratio combiner (SBMRC) in [6] as a low complexity scheme for diversity combining of signals with different modulation levels. SBMRC is in essence identical to log-likelihood ratio (LLR) combining (sometimes also referred to as soft-combining or Chase combining) used in Hybrid Automatic ReQuest (HARQ), although the norm in HARQ is to use the same modulation level in all the retransmissions, as far as we know. In this paper we revisit SBMRC in more details and derive tight lower bound for the BER performance. The main focus of the paper will be on fixed-relay networks, where the links from BS to RSs are assumed to be error-free.

Notation: $CN(0, \sigma^2)$ represents a circular symmetric complex Gaussian random variable with zero mean and variance σ^2 per dimension; for a random variable X, $\bar{X} = E\{X\}$ denotes its mean; $f_{X|y=c}(x)$ is the conditional probability density function of X evaluated at x given that $y = c$; for a complex number C, $\mathcal{R}\{C\}$ and $\mathcal{I}\{C\}$ denote the real and imaginary parts of C, respectively; C^* is the complex conjugate of C; for integer numbers D and E, $rem(D, E)$ denotes the remainder of dividing D by E; $g(x; \mu, \sigma^2)$ denotes the Gaussian probability density function (PDF) with mean μ and variance σ^2, i.e., $g(x; \mu, \sigma^2) = (1/\sigma\sqrt{2\pi})\exp(-(x-\mu)^2/(2\sigma^2))$; $\mathbf{G}^N(x,\boldsymbol{\mu}, \sigma^2)$ denotes the PDF of a Gaussian mixture random variable that consists of N equally-probable Gaussian random variables with the same variance and different means, i.e., $\mathbf{G}^N(x,\boldsymbol{\mu}, \sigma^2) = \frac{1}{N}\sum_{i=1}^{N} g(x, \mu_i,\sigma^2)$, where $\boldsymbol{\mu} = [\mu_1, \mu_2, ..., \mu_N]$.

II. System Model

We consider a multihop network of L transmitting nodes, (comprising of $L-1$ RSs and a BS), and a receiving UT, all having a single antenna. This layout is shown in Fig. 1. The RSs are used to assist a UT which suffers from poor channel

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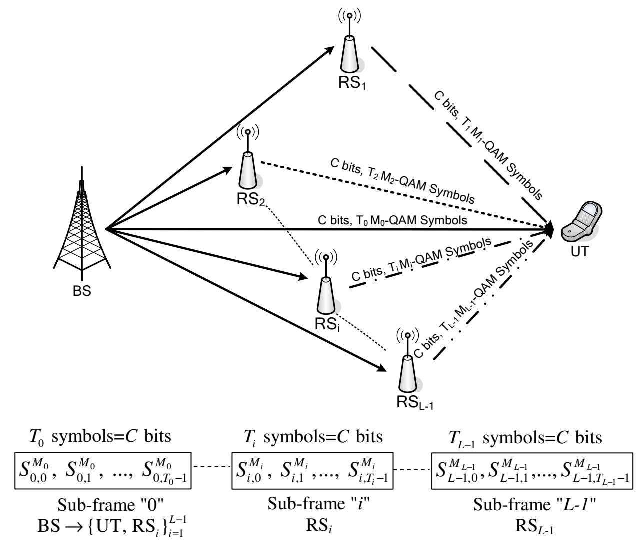

Fig. 1. System model and the corresponding frame structure.

conditions. The RSs decode the signals they receive from BS and forward them to the UT.

The transmitting nodes transmit on L orthogonal channels, i.e., they do not interfere with each other. For simplicity, we consider time-division multiple access (TDMA) to insure orthogonal transmission from all the nodes. The transmitting node i, (BS or RS_i) where $i \in \{0, 1, ..., L-1\}$, uses square M_i Quadrature Amplitude Modulation (M_i-QAM) with Gray coding. Each M_i-QAM symbol carries K_i bits, where $K_i = \log_2(M_i)$ and M_i is the ith modulation level. Without loss of generality, the M_i-QAM constellation has an average energy per bit equal to unity.

The frame is divided into L sub-frames, i.e., one sub-frame for each transmitting node. All sub-frames contain the same sequence of bits, denoted by $\{s_0, s_1, ..., s_{C-1}\}$. The ith sub-frame consists of T_i M_i-QAM symbols, each denoted by $S_{i,j}^{M_i}$, where $j \in \{0, 1, ..., T_i - 1\}$. The symbol $S_{i,j}^{M_i}$ contains the bit sequence $\{s_{jK_i+0}, s_{jK_i+1}, ..., s_{(j+1)K_i-1}\}$. Note that different nodes will be assigned different number of symbols, depending on their modulation schemes, i.e., $T_i = C/K_i$. Since T_i is an integer, C must be a common multiple of $\{K_0, K_1, ..., K_{L-1}\}$. Without loss of generality, the Least Common Multiple (LCM) will be used.

In the zeroth sub-frame, BS broadcasts T_0 M_0-QAM symbols to all RSs and the UT. Since the RSs can be installed at strategic locations, the BS to RSs links can be made very reliable as it is possible in most of the cases to have line-of-sight transmission in these links. Thus, the RSs can decode the signals with negligible errors [5]. In the ith sub-frame, RS_i forwards T_i symbols to UT using M_i-QAM modulation. The frame structure is depicted in Fig. 1.

In the ith sub-frame and the jth symbol, the received signal at UT is $r_{i,j}^{M_i}$ and given by $r_{i,j}^{M_i} = \alpha_i \, S_{i,j}^{M_i} + n_{i,j}$. The complex additive white Gaussian noise (AWGN) is represented by $n_{i,j}$, and it is modeled as $CN(0, N_0/2)$. The channel coefficient between the transmitting node i and UT is denoted by α_i and it captures both path-loss distant dependant attenuation and small scale fading due to multipath propagation. The multipath fading is assumed to be slow, i.e., the channel does not change for the duration of a whole sub-frame. It is assumed that α_i's are known at the receiver and are modeled as independent $CN(0, \sigma_i^2)$, with $\sigma_i^2 = E\{|\alpha_i^2|\}$. If full channel state information (CSI) is available at BS, deciding whether to communicate through relays or directly, and optimizing the modulation levels for all the transmitting nodes, can improve the end-to-end throughput drastically. Such optimization is studied extensively (for instance, [4], [5]) and will not be repeated here. We emphasize that relays are used opportunistically, i.e., whenever the end-to-end throughput using the relays is larger than that of the direct link. In this work, we focus only on the cases when the relays are used. The instantaneous signal-to-noise ratio (SNR) per bit of the link from node i to UT is $\gamma_i = |\alpha_i|^2 \textbf{SNR}$, and the average SNR is denoted as $\bar{\gamma}_i = \sigma_i^2 \textbf{SNR}$, where **SNR** is a reference SNR equal to $\textbf{SNR} = E_b/N_0$.

After receiving all the sub-frames, UT can utilize the signals received from L independent branches, and achieve spatial diversity.

We remark that the most general case of $L-1$ relays is considered for mathematical completeness. However, given that each relay requires an orthogonal channel, it will be difficult in practice to have more than two relays due to the limited radio resources.

III. The Optimal Detector: The Maximum Likelihood Detector (MLD)

For additive white Gaussian noise, the MLD reduces to a minimum distance classifier [7]. Consequently, the MLD decides on the sequence $\{\hat{s}_0, ..., \hat{s}_{C-1}\}$ that satisfies the following criterion:

$$[\hat{s}_0, ..., \hat{s}_{C-1}] = \arg \min_{s_0, ..., s_{C-1}} \sum_{i=0}^{L-1} \sum_{j=0}^{T_i - 1} \left| r_{i,j}^{M_i} - \alpha_i S_{i,j}^{M_i}(s_{jK_i}, s_{jK_i+1}, ..., s_{jK_i+(K_i-1)}) \right|^2 \quad (1)$$

where $S_{i,j}^{M_i}(s_{jK_i}, s_{jK_i+1}, ..., s_{jK_i+(K_i-1)})$ represents the bit to symbol mapping for square M-QAM with Gray-coding. The followings are examples of 4-QAM, 16-QAM, and 64-QAM Gray coded symbols [6]

$$\begin{aligned} &S^4(s_0, s_1) = s_0 d_4 - j s_1 d_4, \\ &S^{16}(s_0, s_1, s_2, s_3) = s_0(-s_1 + 2) d_{16} - j s_2(-s_3 + 2) d_{16}, \\ &S^{64}(s_0, s_1, s_2, s_3, s_4, s_5) = s_0(-s_1(-s_2 + 2) + 4) d_{64} \\ &\qquad\qquad - j s_3(-s_4(-s_5 + 2) + 4) d_{64}, \end{aligned} \quad (2)$$

where d_M is a constant used to fix the energy per bit to unity and it is given by

$$d_M = \sqrt{\frac{3 \log_2 M}{2(M-1)}}. \quad (3)$$

For example, $d_4 = 1, d_{16} = 0.6325$, and $d_{64} = 0.378$.

Although MLD achieves the optimum performance (in minimizing the sequence error rate), it has exorbitant computational complexity. In general, the MLD requires 2^C computations which means that its complexity grows exponentially with C. For this reason, we are motivated to investigate practical schemes with much reduced complexity and with performance comparable to that of the MLD.

IV. Soft-Bit Maximal Ratio Combiner (SBMRC)

In the MLD, the complexity arises from the fact that different modulation schemes carry different number of bits per symbol. As a result, bit-by-bit (or symbol-by-symbol) decoding is not possible. In order to avoid such high complexity, the received M_i-QAM soft symbol, $r_{i,j}$, is mapped into K_i soft-bits. Then, decoding can be preformed on the soft-bits, which results in bit-by-bit detection, rather than detecting a sequence of bits jointly. To extract soft-bits from a soft-symbol, the LLR can be used. The LLR is a well known technique and it is being used in many channel coding schemes (see for example [8] and [9]). It is also used in [10] for diversity combining of signals that belong to the same modulation level, but with different bit to symbol mapping. Combining was preformed by adding the soft-bits. In this work, we use the same LLR concept and explain how to use this concept to get close to optimum performance in combining signals with different modulations levels.

For the Gray coded M_i-QAM schemes described by (2), the LLR can be well approximated by the following recursive expression [8], [9], [11]

$$\tilde{s}_{i,jK_i+k} = \begin{cases} d_{M_i}\mathcal{R}\{\alpha_i^* r_{i,j}^{M_i}\}, & k=0, \\ 2^{\frac{K_i}{2}-k} d_{M_i}^2 |\alpha_i|^2 - |\tilde{s}_{i,jK_i+k-1}|, & 0<k\leq \frac{K_i}{2}-1, \\ -d_{M_i}\mathcal{I}\{\alpha_i^* r_{i,j}^{M_i}\}, & k=\frac{K_i}{2}, \\ 2^{K_i-k} d_{M_i}^2 |\alpha_i|^2 - |\tilde{s}_{i,jK_i+k-1}|, & \frac{K_i}{2}<k\leq K_i-1, \end{cases} \tag{4}$$

where $\tilde{s}_{i,jK_i+k}$ is the (jK_i+k)th soft-bit generated from the received soft symbol $r_{i,j}^{M_i}$. In other words, k denotes the position of the soft-bit in the soft-symbol. As it will be shown later, the approximation given by (4) results in negligible degradation in the BER performance, as compared to that of the MLD.

The SBMRC adds the soft-bits in a way similar to MRC, hence the name SBMRC. In other words, the SBMRC decides on the bit $\hat{s}_l$, for $l \in \{0,1,...,C-1\}$, according to the following criterion

$$\begin{cases} \hat{s}_l = 1, & \text{if } \bar{s}_l > 0, \\ \hat{s}_l = -1, & \text{otherwise}, \end{cases} \tag{5}$$

where $\bar{s}_l$ is a sum of the soft-bits received from different links and is given by

$$\bar{s}_l = \sum_{i=0}^{L-1} \tilde{s}_{i,l}. \tag{6}$$

The block diagram of this scheme is shown in Fig. 2.

Since calculating the soft-bits costs one simple computation per bit, the SBMRC requires LC computations. This means that the SBMRC's complexity grows linearly with C as opposed to the MLD which has complexity growing exponentially with C.

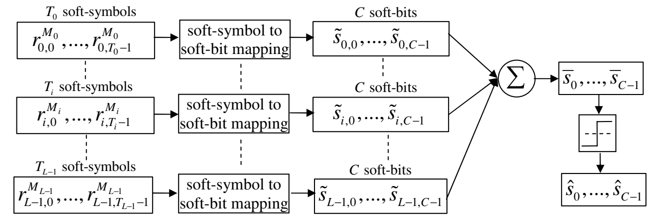

Fig. 2. The SBMRC block diagram.

The structure of SBMRC does not only reduce detection complexity, but also makes it mathematically tractable to find a closed-form approximation of the BER, unlike the MLD.

V. BER performance analysis of SBMRC

To find the average BER, we start in Section V-A by finding $f_{\bar{s}_l|s_l=\pm1}(x)$. Then, in Section V-B we develop tight bounds on the instantaneous BER. Finally, in Section V-C we develop tight bounds on the average BER by averaging over the PDFs of the SNRs.

A. Obtaining $f_{\bar{s}_l|s_l=\pm1}(x)$

To find $f_{\bar{s}_l|s_l=\pm1}(x)$, we need first to find the conditional PDF of the soft-bits $f_{\tilde{s}_{i,k_i}|s_{k_i}=\pm1}(x)$. From (4), it is easy to see that the marginal conditional PDFs of $\tilde{s}_{i,jK_i+k}$ are the same as those of $\tilde{s}_{i,k}$, for $j=0,1,...,T_i-1$. Furthermore, from the similarity of the expression of the bits in the real and imaginary parts, it can be easily shown that the conditional PDF expression of the soft-bit $\tilde{s}_{i,k}$ is the same as that of $\tilde{s}_{i,k-K_i/2}$, for $k \geq K_i/2$. Thus, it suffices to provide the conditional PDFs expression of $\tilde{s}_{i,k_i}$ for $k_i \in \{0,1,...,K_i/2-1\}$, where $k_i = rem(l, K_i/2)$.

The conditional PDFs of the soft-bits are given by (7) at the bottom of the this page [6]. In (7), $\eta_{i,k_i} = 2^{\frac{K_i}{2}-k_i} d_{M_i}^2 |\alpha_i|^2$ and $n \in \{0,1,...,K_i/2-1\}$. The vector $\boldsymbol{\lambda}_n^{\pm}$ contains all the possible values of $\mathcal{R}\{S^{M_i}\}$ given that $s_n = \pm1$.

For high SNR, where $|\alpha_i|^2/N_0 >> 1$, the truncated part of the PDF (caused by the absolute value operation in (4) where $\tilde{s}_{i,k_i} > \eta_{i,k_i}$) becomes negligible. Applying this approximation and evaluating the recursive expression in (7) yields the following approximation

$$f_{\tilde{s}_{i,k_i}|s_{k_i}=\pm1}(x) \approx \mathbf{G}^{N_{i,k_i}}(x; \pm\boldsymbol{\mu}_{i,k_i}, d_{M_i}^2|\alpha_i|^2 N_0/2). \tag{8}$$

In (8), the mth element of the vector $\boldsymbol{\mu}_{i,k_i}$ is given by

$$\boldsymbol{\mu}_{i,k_i}(m) = (2m+1)|\alpha_i|^2 d_{M_i}^2, \tag{9}$$

where $m \in \{0,...,N_{i,k_i}-1\}$ and $N_{i,k_i} = \sqrt{M_i}/2^{k_i+1}$.

Equation (8) shows that the conditional PDF of the soft-bits is well approximated by a Gaussian mixture and the number

$$f_{\tilde{s}_{i,k_i}|s_n=\pm1}(x) = \begin{cases} \mathbf{G}^{\sqrt{M_i}/2}(x; \boldsymbol{\lambda}_n^{\pm} d_{M_i}|\alpha_i|^2, d_{M_i}^2|\alpha_i|^2 N_0/2), & k_i = 0, \\ f_{\tilde{s}_{i,k_i-1}|s_{i,n}=\pm1}(x-\eta_{i,k_i}) + f_{\tilde{s}_{i,k_i-1}|s_{i,n}=\pm1}(-x+\eta_{i,k_i}), & 0<k_i\leq\frac{K_i}{2}-1, x\leq\eta_{i,k_i}, \\ 0, & 0<k_i\leq\frac{K_i}{2}-1, x>\eta_{i,k_i}. \end{cases} \tag{7}$$

of components of the mixture $N_{i,k}$ depends on the position of the soft-bit k_i and the modulation level M_i. The high accuracy of (8) is demonstrated in [12, pp. 36-40]. Using the approximation given by (8), it can be shown that $f_{\bar{s}_l|s_l=\pm 1}(x)$ can be expressed as follows

$$f_{\bar{s}_l|s_l=\pm 1}(x) \approx \mathbf{G}^{N_{\bar{s}_l}}(x, \pm \boldsymbol{\mu}_{\bar{s}_l}, \sigma^2_{\bar{s}_l}), \tag{10}$$

where $N_{\bar{s}_l} = \prod_{i=0}^{L-1} N_{i,k_i}$, $\sigma^2_{\bar{s}_l} = \sum_{i=0}^{L-1} d^2_{M_i} |\alpha_i|^2 N_0/2$, and $\boldsymbol{\mu}_{\bar{s}_l}$ is a $1 \times N_{\bar{s}_l}$ vector that consists of the $N_{\bar{s}_l}$ summands in the expression $\sum_{j_0=0}^{N_0-1} \ldots \sum_{j_{L-1}=0}^{N_{L-1}} \pi_{j_0,j_1,\ldots,j_{L-1}}$ where $\pi_{j_0,j_1,\ldots,j_{L-1}} = \sum_{i=0}^{L-1} \mu_{i,j_i}$. The derivation of (10) is omitted due to space limitation and can be found in [12, pp. 42-43].

B. Finding the instantaneous BER

Assuming an equiprobable source (i.e., $p(s_l = 1) = p(s_l = -1) = \frac{1}{2}$), the instantaneous BER can be written as

$$BER_{inst} = \frac{1}{C} \sum_{l=0}^{C-1} p(\bar{s}_l < 0 | s_l = 1). \tag{11}$$

Using (10), $p(\bar{s}_l < 0|s_l = 1)$ can be expressed as

$$\begin{aligned} p(\bar{s}_l < 0|s_l = 1) &= \int_{-\infty}^{0} \mathbf{G}^{N_{\bar{s}_l}}(x, \boldsymbol{\mu}_{\bar{s}_l}, \sigma^2_{\bar{s}_l}) \\ &= \int_{-\infty}^{0} \frac{1}{N_{\bar{s}_l}} \sum_{i=0}^{N_{\bar{s}_l}-1} g(x, \mu_{\bar{s}_l,i}, \sigma^2_{\bar{s}_l}) = \frac{1}{N_{\bar{s}_l}} \sum_{i=0}^{N_{\bar{s}_l}-1} Q(\frac{\mu_{\bar{s}_l,i}}{\sigma_{\bar{s}_l}}) \end{aligned} . \tag{12}$$

Because the Q function is a positive monotonic decreasing function, the previous expression can be bounded as follow

$$\frac{1}{N_{\bar{s}_l}} Q(\frac{\min(\boldsymbol{\mu}_{\bar{s}_l})}{\sigma_{\bar{s}_l}}) < p(\bar{s}_l < 0|s_l = 1) < Q(\frac{\min(\boldsymbol{\mu}_{\bar{s}_l})}{\sigma_{\bar{s}_l}}). \tag{13}$$

Since the Q function dies out exponentially, the lower bound becomes very tight for high γ_i. Note that in the previous bound we only need to evaluate $\min(\boldsymbol{\mu}_{\bar{s}_l})$, rather than evaluating all the elements of $\boldsymbol{\mu}_{\bar{s}_l}$.

It is not difficult to find that

$$\min(\boldsymbol{\mu}_{\bar{s}_l}) = \sum_{i=0}^{L-1} |\alpha_i|^2 d^2_{M_i}. \tag{14}$$

Using the previous expression, the argument of the Q function in (13) can be written as

$$\begin{aligned} \min(\boldsymbol{\mu}_{\bar{s}_l})/\sigma_{\bar{s}_l} &= \frac{\sum_{i=0}^{L-1} |\alpha_i|^2 d^2_{M_i}}{\sqrt{\sum_{i=0}^{L-1} |\alpha_i|^2 d^2_{M_i} N_0/2}} = \sqrt{\frac{\sum_{i=0}^{L-1} |\alpha_i|^2 d^2_{M_i}}{N_0/2}} \\ &= \sqrt{2 \sum_{i=0}^{L-1} d^2_{M_i} \gamma_i} = \sqrt{2\gamma_{out}}, \end{aligned} \tag{15}$$

where

$$\gamma_{out} = \sum_{i=0}^{L-1} d^2_{M_i} \gamma_i. \tag{16}$$

This suggest that the output SNR of the SBMRC is a weighted sum of the individual SNRs, and these weights depend on the modulation levels of the signals to be combined. This again shows the similarities between conventional MRC and SBMRC. It is important to note that the argument of the Q function is independent of the index l.

By substituting (15) and (13) in (12), we get

$$\tau Q(\sqrt{2\gamma_{out}}) < BER_{inst} < Q(\sqrt{2\gamma_{out}}), \tag{17}$$

where τ is a constant that is related to the modulation levels of the signals to be combined and it is given by

$$\tau = \frac{1}{C} \sum_{l=0}^{C-1} \frac{1}{N_{\bar{s}_l}}. \tag{18}$$

For example, $\tau = 0.75$ for combining QPSK and 16-QAM, $\tau = 0.5833$ for combining QPSK and 64-QAM and $\tau = 0.5$ for combining 16-QAM and 64-QAM or for combining QPSK, 16-QAM and 64-QAM.

From the bounds on the BER_{inst} given by (17), we can make the following observations:

1) Recall that BER_{inst} for MRC, where the signals to be combined have the same modulation level M, is proportional to $Q(\sqrt{2d^2_M \sum_{i=0}^{L-1} \gamma_i})$ [7], which means that both the SBMRC and MRC achieve the same post-processing SNR in this case.
2) Let $\gamma_{(i)}$ denotes the ith smallest γ_i, i.e., $\gamma_{(0)} \leq \gamma_{(1)} \leq \ldots \leq \gamma_{(L-1)}$. If $\{\gamma_{(i)}\}_{i=0}^{L-1}$ are known at the BS, then, to maximize the post-processing SNR at UT, the modulation levels should be assigned such that $d_{M_{(0)}} \leq d_{M_{(1)}} \leq \ldots \leq d_{M_{(L-1)}}$, which means that $M_{(0)} \geq M_{(1)} \geq \ldots \geq M_{(L-1)}$. In other words, lower modulation levels should be used in the links that experience better channel conditions, and vice versa. Note that this rule is reversed in conventional AMC without diversity combining.

C. Finding the average BER

To get bounds on the average BER, the expression in (17) is averaged over the PDF of γ_{out}. Because of the similarity between the output SNR expression for SBMRC and for classical MRC, the same procedure for finding the average BER (when MRC is employed) can be used. The details of this procedure are given in [7], and only the final result is shown here. The average BER can be bounded as

$$\begin{aligned} BER &> \tfrac{1}{2}\tau \sum_{i=0}^{L-1} \pi_i \left[1 - \sqrt{\frac{d^2_{M_i}\bar{\gamma}_i}{1+d^2_{M_i}\bar{\gamma}_i}}\right], \\ BER &< \tfrac{1}{2} \sum_{i=0}^{L-1} \pi_i \left[1 - \sqrt{\frac{d^2_{M_i}\bar{\gamma}_i}{1+d^2_{M_i}\bar{\gamma}_i}}\right], \end{aligned} \tag{19}$$

where $\pi_i = \prod_{j=0, j\neq i}^{L-1} \frac{d^2_{M_i}\bar{\gamma}_i}{d^2_{M_i}\bar{\gamma}_i - d^2_{M_j}\bar{\gamma}_j}$.

These bounds can be approximated for $\gamma_i \gg 1$ as

$$\begin{aligned} BER &> \tau \begin{pmatrix} 2L-1 \\ L \end{pmatrix} (\prod_{i=0}^{L-1} \frac{1}{4d^2_{M_i}\sigma^2_i}) \mathbf{SNR}^{-L}, \\ BER &< \begin{pmatrix} 2L-1 \\ L \end{pmatrix} (\prod_{i=0}^{L-1} \frac{1}{4d^2_{M_i}\sigma^2_i}) \mathbf{SNR}^{-L}. \end{aligned} \tag{20}$$

TABLE I
LOSS IN SNR (dB) AT BER=10^{-3} FOR SC AND SBMRC IN COMPARISON TO THE OPTIMUM MLD.

Scenario \ Scheme	SC	SBMRC
M_0=4, M_1=16	2.30	0.02
M_0=4, M_1=64	4.10	0.09
M_0=16, M_1=64	2.73	0.08
M_0=4, M_1=4, M_2=16	3.49	0.07
M_0=4, M_1=4, M_2=64	6.48	0.27
M_0=4, M_1=16, M_2=16	3.36	0.09
M_0=16, M_1=16, M_2=64	3.93	0.13
M_0=16, M_1=64, M_2=64	3.63	0.09

The upper bound in (20) gives an explicit proof that the SBMRC achieves a diversity order of L, i.e., full diversity (as expected). As stated earlier, the lower bound is very tight for high SNR and can be used as a very good approximation of BER.

VI. Analytical and Simulation Results

To compare the performance of conventional selection combining (SC), SBMRC, and MLD, we consider relay networks with L=2 (single relay) and L=3 (two relays). For the sake of presentation, we assume the average channel conditions to be the same in all the links, i.e., $\bar{\gamma}_i = \bar{\gamma}$ for $i \in \{0, 1, ..., L-1\}$. Table I shows the loss in SNR of all schemes compared to the optimum MLD scheme for different scenarios. The SC suffers from high performance loss that ranges from 2.3 to 6.48 dB depending on the scenario. On the other hand, SBMRC has very close performance to the MLD scheme (degradation is less than 0.3 dB). Note that the loss in SNR is measured at a BER of 10^{-3} which is a reasonable value for uncoded schemes. Nevertheless, it is observed that the loss in SNR for SBMRC vanishes at very low BER. For these reasons, SBMRC is the most attractive scheme from the practical point of view; it is very simple to implement, with negligible performance degradation compared to the MLD scheme.

Fig. 3 and Fig. 4 show the simulation results for the BER performance of SBMRC for combining signals with different modulation levels, for one and two relays, respectively. The lower bound given by (19) is evaluated for each scenario and plotted in the same figure. It is clear that the lower bound is very tight for all scenarios and it can be used as a very good approximation for evaluating the BER for SBMRC. This validates the accuracy of approximating the conditional PDF by a Gaussian mixture.

It is also clear that all curves in Fig. 3 decay two orders of magnitude per decade (diversity order of 2) and all curves in Fig. 4 decay three orders of magnitude per decade (diversity order of 3). Consequently, SBMRC achieves full diversity, as proven by (20).

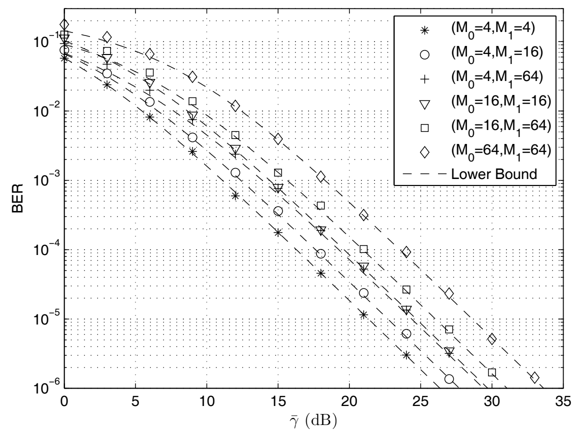

Fig. 3. BER performance of diversity combining using SBMRC, for $L = 2$.

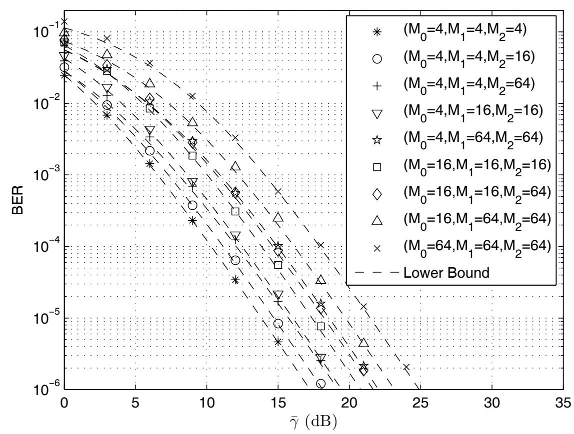

Fig. 4. BER performance of diversity combining using SBMRC, for $L = 3$.

In Fig. 5, we compare the BER performance of SBMRC with SC , in single-relay and two-relay networks with $\{M_0 = 4, M_1 = 64\}$, and $\{M_0 = 4, M_1 = 16, M_2 = 64\}$, respectively. We observe that both schemes achieve the same diversity order. However, SBMRC achieves a higher SNR gain (or coding gain) represented by a horizontal shift to the left of the BER curve compared to SC. SBMRC has an SNR gain of about 3.8 dB and 5.2 dB, over SC, in the first and second scenarios, respectively.

VII. Conclusions

In the recent literature, diversity combining of signals with different modulation levels in cooperative relay networks has been addressed by means of selection combining, which is far from optimal. SBMRC is a low complexity scheme for diversity combining of signals with different modulation levels. SBMRC exhibits BER performance that is very close to that of the optimal MLD, with much reduced complexity. In addition to presenting simulation results of the BER performance of SBMRC, a very tight lower bound is derived for the BER expression of SBMRC. Since SBMRC has BER performance slightly inferior to that of the MLD, the derived lower bound can also be used as a very good approximation for the BER of MLD.

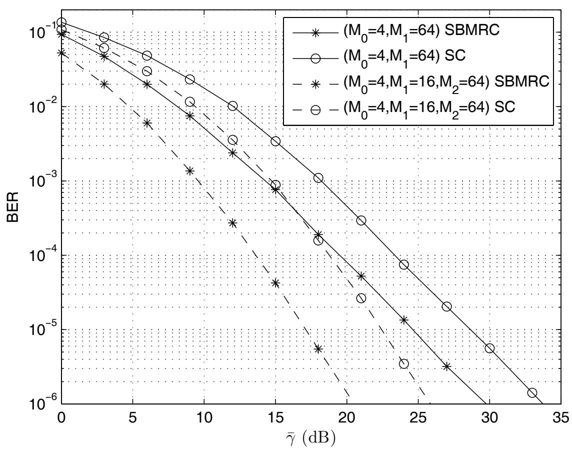

Fig. 5. BER performance of SC and SBMRC.

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REFERENCES

[1] R. Pabst, B. H. Walke, D. C. Schultz, P. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. D. Falconer, and G. P. Fettweis, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Commun. Mag.*, vol. 42, no. 9, pp. 80–89, Sept. 2004.

[2] J. N. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[3] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. Commun.*, vol. 52, no. 10, pp. 1820–1830, Oct. 2004.

[4] S. Hares, H. Yanikomeroglu, and B. Hashem, "Diversity- and AMC (adaptive modulation and coding)-aware routing in TDMA multihop networks," in *Proc. IEEE GLOBECOM*, San Francisco, CA, USA, Dec. 2003.

[5] B. Can, H. Yanikomeroglu, F. Onat, E. Carvalho, and H. Yomo, "Efficient cooperative diversity schemes and radio resource allocation for IEEE 802.16j," in *Proc. IEEE WCNC*, Las Vegas, USA, Apr. 2008.

[6] A. Bin Sediq and H. Yanikomeroglu, "Diversity combining of signals with different modulation levels in cooperative relay networks," in *Proc. IEEE VTC2008-Fall*, Calgary, Alberta, Canada, Sept. 2008.

[7] J. G. Proakis, *Digital Communications*. McGraw-Hill, 2000.

[8] S. L. Goff, A. Glavieux, and C. Berrou, "Turbo-codes and high spectral efficiency modulation," in *Proc. ICC'94*, New Orleans, LA, USA, May 1994.

[9] V. Aue and R. Nuessgen, "Method for generating soft-bit information from gray coded signals." U.S. Patent 20040096007, May 2004.

[10] C. Wengerter, A. G. E. von Elbwart, E. Seidel, G. Velev, and M. Schmitt, "Advanced hybrid ARQ technique employing a signal constellation rearrangement," in *Proc. IEEE VTC2002-Fall*, Vancouver, BC, Canada, Sept. 2002.

[11] K. S. Kim, K. Hyun, C. W. Yu, Y. O. Park, D. Yoon, and S. K. Park, "General log-likelihood ratio expression and its implementation algorithm for Gray-Coded QAM signals," *ETRI J.*, vol. 28, no. 3, pp. 291–300, June 2006.

[12] A. Bin Sediq, "Diversity combining of signals with different modulation levels and constellation rearrangement in cooperative relay networks," master's thesis, Carleton University, Department of Systems and Computer Engineering, 2008, [Online]. Available: www.sce.carleton.ca/faculty/yanikomeroglu/Pub/AkramBinSediq-MAScThesis-15Sep09.pdf.